

RESEARCH PAPER

# Multi-Period Sustainability Performance in the Presence of Discrete and Bounded Measures

Monireh Jahani Sayyad Noveiri<sup>1</sup> & Sohrab Kordrostami<sup>\*2</sup>

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## ABSTRACT

*Sustainability performance assessment is a significant aspect of making sustainable decisions for organizations. Measuring sustainability performance of firms in a time span, covered in several periods, leads to more rational decision-making and planning by managers. Furthermore, in many application fields, there are discrete and bounded measures. However, there has been no systematic effort to analyze sustainability performance of Decision-Making Units (DMUs) in multiple periods of time when discrete and bounded factors are presented. Therefore, approaches based on Data Envelopment Analysis (DEA) are proposed in this paper to tackle this problem. To illustrate this issue in more detail, the performance of systems is measured for all dimensions, including economic, social, and environmental ones and for each period. Moreover, the overall multi-period sustainability performance and sustainability performance of each period are estimated using the suggested one-stage methods. Then, the sustainability performance is investigated for conditions in which internal relationships among economic, social, and environmental indicators are presented. Moreover, sustainability performance changes and performance changes of dimensions are addressed. An example and a case study are provided to explain our proposed approach. Results show that the introduced ideas are practical and effective.*

**KEYWORDS:** Sustainability Performance, Data Envelopment Analysis, Multi-Period, Discrete, Bounded.

## 1. Introduction

Sustainable development is an important aspect in today's societies. Forwarding sustainable growth needs to follow economic, social, and environmental goals, simultaneously. Measuring the sustainability of systems can facilitate determining the sustainability level and the efficiency in each sustainability dimension. Obviously, more beneficial information can be obtained from sustainability performance analysis in a time span containing multiple periods of time.

In the literature, there are approaches to measuring sustainability performance. Readers can refer to [1] for more information. One of the prevailing approaches for this purpose is

the non-parametric Data Envelopment Analysis (DEA) approach, originally proposed by Charnes et al. [2]. Zhou et al. [3] provided an organized investigation on DEA utilizations in sustainability. Galán-Martín et al. [4] proposed an enhanced DEA method that applied the concept of 'order of efficiency' to assess sustainability. In their approach, sustainability has been analyzed in a specific period of time. Zhao et al. [5] developed a method based upon DEA to measure the performance of systems that each of them designed as an economic and environmental subunit and a social subunit. Hassanzadeh et al. [6] introduced input-oriented and output-oriented inverse semi-oriented models and measured the sustainability of countries in the presence of negative data. Tajbakhsh and Shamsi [7] designed a non-parametric index to assess the sustainability performance of countries in one period when undesirable and integer materials are presented. Amirteimoori et al. [8] assessed the sustainability

\* Corresponding author: Sohrab Kordrostami  
kordrostami@liau.ac.ir

1. Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran.
2. Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran.

performance in the existence of undesirable outputs over time.

In the DEA context, there are methods for measuring the efficiency of Decision-Making Units (DMUs) in several periods of time. Park and Park [9] evaluated the aggregative efficiency of multi-period systems. Kao and Liu [10] utilized and adopted a relational network model to calculate the overall and period efficiencies of multi-period processes. Afterwards, Kao and Hwang [11] computed the multi-period efficiency of two-stage systems. Esmaeilzadeh and Kazemi Matin [12] extended multi-period DEA models considering more complex internal relations for the sub-processes of each entity. Kordrostami and Jahani Sayyad Noveiri [13] proposed a DEA-based method to analyze the efficiency of the multi-period production systems when imprecise data are presented. Jablonsky [14] presented efficiency and super-efficiency multi-period DEA models. Kordrostami et al. [15] investigated multi-period efficiency scores and efficiency changes where there were undesirable outputs. Furthermore, the multi-period efficiency of firms with negative factors is measured in the study by Kordrostami and Jahani Sayyad Noveiri [16]. According to the DEA literature, there is no investigation to analyze the multi-period sustainability performance of systems with discrete and bounded data. Nevertheless, there are occasions in the real world that the sustainability of systems should be measured in multiple periods while discrete and bounded measures are appeared. For instance, the number of employees and the number of products are integer variables. In addition, the satisfaction degree and the rate of on-time delivery can be taken into account as bounded factors.

Through a survey of the literature, one can found that there are studies within DEA that have addressed the efficiency of DMUs with integer and/or bounded factors.

Lozano and Villa [17] suggested DEA patterns to appraise the performance of DMUs where integer inputs and outputs appeared. Subsequently, Kuosmanen and Kazemi Matin

[18] described principles to evaluate the efficiency and finding targets under constant returns to scale (CRS) case when integer factors were detected. Then, Kazemi Matin and Kuosmanen [19] developed integer-valued DEA models under alternative returns to scale assumptions including variable, non-increasing and non-decreasing. Chen et al. [20] measured efficiency and super-efficiency in the presence of integer undesirable factors. Further to that, Chen et al. [21] designed models based on DEA to analyze the relative efficiency of DMUs in the presence of bounded, discrete data and Likert scales. Then, after that, Chen et al. [22] evaluated efficiency and super-efficiency of NBA players utilizing bounded integer DEA models. A mixed-integer linear programming problem has been developed by Kazemi Matin and Emrouznejad [23] to estimate the performance of DMUs with integer input-output variables and bounded outputs.

As literature reviews and Table 1 show, no study has estimated multi-period sustainability performance in the presence of discrete and bounded measures. In this paper, approaches on the basis of DEA are proposed to analyze the sustainability performance of systems in a time span where discrete and bounded measures appeared. To illustrate, the relative efficiency is evaluated for each sustainability dimension and each period while bounded and discrete measures are available. The overall multi-period sustainability is also calculated. Indeed, the approaches proposed by Jablonsky [14] and Chen et al. [21] are extended to assess multi-period sustainability performance in the presence of discrete and bounded factors. Next, the presented method is generalized for situations where internal relationships among dimensions are observed. After introducing two models, input-oriented and output-oriented, the efficiency changes between years are addressed. These variations between the whole sustainability performance and each dimension of the sustainability in economic, environmental, and social terms are explored.

Tab 1. Comparative review of some studies explored in Section 1

| Source                            | Three dimensions of sustainability | Bounded | Integer-valued | Multi-period |
|-----------------------------------|------------------------------------|---------|----------------|--------------|
| Tajbakhsh and Shamsi [7]          | *                                  |         | *              |              |
| Galán-Martín et al. [4]           | *                                  |         |                |              |
| Lozano and Villa [17]             |                                    |         | *              |              |
| Kuosmanen and Kazemi Matin [18]   |                                    |         | *              |              |
| Kazemi Matin and Kuosmanen [19]   |                                    |         | *              |              |
| Chen et al. [20]                  |                                    |         | *              |              |
| Chen et al. [21]                  |                                    | *       | *              |              |
| Chen et al. [22]                  |                                    | *       | *              |              |
| Kazemi Matin and Emrouznejad [23] |                                    | *       | *              |              |
| Jablonsky [14]                    |                                    |         |                | *            |
| Park and Park [9]                 |                                    |         |                | *            |
| Kao and Liu [10]                  |                                    |         |                | *            |
| Amirteimoori et al. [8]           | *                                  |         |                | *            |
| Our approach                      | *                                  | *       | *              | *            |

The paper is organized as follows: the approach of Chen et al. [21] to address bounded and discrete data and, also, Jablonsky's technique [14] to study the relative efficiency of systems in several years are reviewed in Section 2. Then, approaches are proposed to assess the multi-period sustainability performance with bounded and discrete measures in Section 3. Afterwards, an example and a case study of gas companies are provided in Section 4 to exemplify the suggested method herein. Finally, conclusions are revealed in Section 5.

## 2. Preliminaries

This section begins with the explanation of the approach proposed by Chen et al. [10] to deal with discrete and bounded items. Then, Jablonsky's method [8] to analyze the relative efficiency of DMUs in several periods of time is investigated.

### 2-1. Approach of Chen et al. [21] for handling discrete and bounded factors

By considering  $x_{io}$  and  $y_{ro}$  as the  $i^{\text{th}}$  input and the  $r^{\text{th}}$  output of the unit under consideration,  $DMU_o$ , Chen et

al. [21] provided the following model to measure the relative efficiency of  $DMU_o$  and determine the projection points when discrete and bounded data are presented:

$$\begin{aligned}
 ef^* = \text{Min} \quad ef &= \frac{1}{m} \sum_{i=1}^m \alpha_i \\
 \text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij} &\leq \tilde{x}_{io}, i = 1, 2, \dots, m, \\
 \tilde{x}_{io} &\leq \alpha_i x_{io}, i = 1, 2, \dots, m, \\
 L_{i_{Bnd}} &\leq \tilde{x}_{i_{Bnd}o} \leq U_{i_{Bnd}}, i_{Bnd} \in I_{Bnd}, \\
 \tilde{x}_{i_{Int}o} &\text{ integer}, i_{Int} \in I_{Int}, \\
 \sum_{j=1}^n \lambda_j y_{rj} &\geq y_{ro}, r = 1, 2, \dots, s, \\
 \lambda_j &\geq 0, j = 1, \dots, n.
 \end{aligned} \tag{1}$$

in which  $\lambda_j (j = 1, \dots, n)$  and  $\alpha_i (i = 1, \dots, m)$  are intensity and efficiency variables, respectively.

For  $DMU_j (j = 1, \dots, n)$ , inputs  $x_{ij} (i = 1, \dots, m)$  may be integer, non-integer, bounded and unbounded variables. Therefore, inputs are split into integer and continuous factors as

indicated by  $x_{i_{Int}j}$  and  $x_{i_{Cont}j}$  such that  $I_{Int} \cup I_{Cont} = \{1, \dots, m\}$ ; bounded and unbounded

items are symbolized by  $x_{i_{Bnd}j}$  and  $x_{i_{Unb}j}$  while  $I_{Bnd} \cup I_{Unb} = \{1, \dots, m\}$ .  $I_{Int}$  and  $I_{Bnd}$  may have common components. Moreover,

$y_{rj} (r = 1, \dots, s)$  show outputs.  $\tilde{x}_{i_{Int}o}, i_{Int} \in I_{Int}$  are integer variables.  $\tilde{x}_{i_{Bnd}o}, i_{Bnd} \in I_{Bnd}$  also indicate bounded variables. It is clear that  $L_{i_{Bnd}}$  and

$U_{iBnd}$  show lower and upper bounds of input  $i$ , respectively. The unit under evaluation is called efficient if and only if  $ef^* = 1$ .

Notice that Model (1) is a non-radial and input-oriented model. For investigating some other forms, readers can refer to [21, 22].

## 2-2. Multi-period efficiency

Let us consider  $n$  DMUs; their performance should be evaluated in  $T$  periods of time, i.e.,  $t = 1, \dots, T$ . Each observed  $DMU_j$  ( $j = 1, \dots, n$ ) is

represented by a set of inputs  $x_{ij}^{(t)}$  ( $i = 1, \dots, m$ )

and outputs  $y_{rj}^{(t)}$  ( $r = 1, \dots, s$ ) for period  $t$ . Jablonsky [14] introduced the following model to analyze the efficiency of the unit under evaluation  $P$  in several periods of time:

$$e_p^* = \text{Max } e_p = \sum_{t=1}^T \theta_p^t / T$$

$$\text{s.t. } \sum_{j=1}^n \lambda_j^t x_{ij}^t \leq x_{ip}^t, i = 1, 2, \dots, m; t = 1, \dots, T, \quad (2)$$

$$\sum_{j=1}^n \lambda_j^t y_{rj}^t \geq \theta_p^t y_{rp}^t, r = 1, 2, \dots, s; t = 1, \dots,$$

$$\lambda_j^t \geq 0, j = 1, \dots, n; t = 1, \dots, T.$$

in which  $\lambda_j^t$  shows intensity variables for the unit  $j$  and the period  $t$ . The optimal objective function value of Model (2) is more than or equal to 1, that is,  $e_p^* \geq 1$ .  $e_p^* = 1$  implies that  $DMU_p$  is the whole efficient. In another way, it is inefficient at least in one period when  $e_p^* > 1$ . To illustrate, the multi-period efficiency is defined as the average of the efficiency of periods. It is noted that Model (2) is an output-oriented and radial form. Readers can refer to [14] for further details in this regard.

In the next section, the extension of the approach of Chen et al. [21] and, also,

In summary, all notations and terms used in this section are described as follows:

$d \in D$ : sustainability dimensions  $D = \{\text{economic, environmental, social}\}$ ,

$t = 1, \dots, T$ : periods,

$\lambda_j^t$ : intensity variables for economic dimension in period  $t$ ,

$\beta_j^t$ : intensity variables for environmental dimension in period  $t$ ,

Jablonsky's method [14] is provided to measure multi-period sustainability performance in the presence of discrete and bounded factors.

## 3. The Proposed Approach

In what follows, approaches based on DEA are provided to measure the sustainability performance of entities comprising three dimensions of economic, social, and environmental where integer and bounded values are presented. In doing so, we take  $n$

DMUs,  $DMU_j$  ( $j = 1, \dots, n$ ), that use  $m^d$  inputs  $x_{ij}^{(t)d}$  ( $i = 1, \dots, m^d$ ), produce  $s^d$  desirable outputs  $y_{rj}^{(t)d}$  ( $r = 1, \dots, s^d$ ), and emit  $B^d$

undesirable outputs  $z_{bj}^{(t)d}$  ( $b = 1, \dots, B^d$ ) while they should be evaluated in  $T$  periods of time ( $t = 1, \dots, T$ ). It is assumed that undesirable outputs are weakly disposable by the following [24]. Weak disposability of outputs means that  $\theta y$  ( $0 \leq \theta \leq 1$ ) can be generated proportionally if the output vector  $y$  can be generated. Firstly, to analyze the multi-period sustainability performance with discrete and bounded factors, Model (2) is introduced in which

$d \in D = \{\text{economic, environmental, social}\}$ . It is assumed that inputs may be integer, non-integer, bounded or unbounded in this stage. Therefore, inputs for dimension  $d$  and period  $t$  have been split into integer and continuous

factors that are shown by  $x_{intj}^{(t)d}$  and  $x_{contj}^{(t)d}$  that  $I_{int}^{(t)d} \cup I_{cont}^{(t)d} = \{1, \dots, m^d\}$ . Moreover, bounded and

unbounded inputs have been denoted by  $x_{ibndj}^{(t)d}$  and  $x_{iunbj}^{(t)d}$  while  $I_{bnd}^{(t)d} \cup I_{und}^{(t)d} = \{1, \dots, m^d\}$ .  $\tilde{x}_{imO}^{(t)d}$ ,  $i_{int}^{(t)d} \in I_{int}^{(t)d}$ , and  $\tilde{x}_{ibndO}^{(t)d}, i_{bnd}^{(t)d} \in I_{bnd}^{(t)d}$  indicate integer and bounded variables for period  $t$  and dimension  $d$ .

- $\gamma_j^t$ : intensity variables for social dimension in period  $t$ ,  
 $\Omega^*$ : the overall multi-period sustainability variable in the input orientation,  
 $\theta_o^t$ : the performance of  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\alpha_o^t$ : sustainability performance of  $DMU_o$  for period  $t$ ,  
 $L_{iBnd}^{(t)(d)}$ : lower bound of input  $i$  for dimension  $d$  and period  $t$ ,  
 $U_{iBnd}^{(t)(d)}$ : upper bound of input  $i$  for dimension  $d$  and period  $t$ ,  
 $L_{rBnd}^{(t)(d)}$ : lower bound of desirable output  $r$  for dimension  $d$  and period  $t$ ,  
 $U_{rBnd}^{(t)(d)}$ : upper bound of desirable output  $r$  for dimension  $d$  and period  $t$ ,  
 $L_{bBnd}^{(t)(d)}$ : lower bound of undesirable output  $b$  for dimension  $d$  and period  $t$ ,  
 $U_{bBnd}^{(t)(d)}$ : upper bound of undesirable output  $b$  for dimension  $d$  and period  $t$ ,  
 $x_{ij}^{(t)(d)}$ :  $i$  th input of  $DMU_j$  for dimension  $d$  and period  $t$ ,  
 $x_{io}^{(t)(d)}$ :  $i$  th input of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{x}_{io}^{(t)(d)}$ :  $i$  th input variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{x}_{int o}^{(t)(d)}$ :  $i$  th integer input variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{x}_{iBnd o}^{(t)(d)}$ :  $i$  th bounded input variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $y_{rj}^{(t)(d)}$ :  $r$  th desirable output of  $DMU_j$  for dimension  $d$  and period  $t$ ,  
 $y_{ro}^{(t)(d)}$ :  $r$  th desirable output of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{y}_{ro}^{(t)(d)}$ :  $r$  th desirable output variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{y}_{rint o}^{(t)(d)}$ :  $r$  th integer desirable output variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{y}_{rBnd o}^{(t)(d)}$ :  $r$  th bounded desirable output variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $z_{bj}^{(t)(d)}$ :  $b$  th undesirable output of  $DMU_j$  for dimension  $d$  and period  $t$ ,  
 $z_{bo}^{(t)(d)}$ :  $b$  th undesirable output of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{z}_{bo}^{(t)(d)}$ :  $b$  th integer undesirable output variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{z}_{bint o}^{(t)(d)}$ :  $b$  th integer undesirable output variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $\tilde{z}_{bBnd o}^{(t)(d)}$ :  $b$  th bounded undesirable output variable of the unit under evaluation  $DMU_o$  for dimension  $d$  and period  $t$ ,  
 $w_d^t$ : the predefined preference by the decision-maker for dimension  $d$  and period  $t$ ,  
 $\Phi^*$ : the variable for obtaining the overall multi-period sustainability index in the output orientation.

Therefore, the following input-oriented model, which is under the constant returns to scale assumption, is proposed:

$$\Omega^* = \min_{\theta^t, \lambda_j^t, \beta_j^t, \gamma_j^t} \Omega = \sum_{t=1}^T \alpha^t / T \quad (3)$$

s.t. (economic)

$$\begin{aligned} \sum_{j=1}^n \lambda_j^t x_{ij}^{(t)ec} &\leq \tilde{x}_{io}^{(t)ec}, \quad i=1, \dots, m^{ec}, t=1, \dots, T, \\ \tilde{x}_{io}^{(t)ec} &\leq \theta_o^{t, economic} x_{io}^{(t)ec}, \quad i=1, \dots, m^{ec}, t=1, \dots, T, \\ L_{i_{Bnd}}^{(t)ec} &\leq \tilde{x}_{i_{Bnd}o}^{(t)ec} \leq U_{i_{Bnd}}^{(t)ec}, \quad i_{i_{Bnd}}^{(t)ec} \in I_{i_{Bnd}}^{(t)ec}, \\ \tilde{x}_{i_{Int}o}^{(t)ec} &\text{ integer}, \quad i_{Int}^{(t)ec} \in I_{Int}^{(t)ec}, \\ \sum_{j=1}^n \lambda_j^t y_{rj}^{(t)ec} &\geq y_{ro}^{(t)ec}, \quad r=1, \dots, s^{ec}, t=1, \dots, T, \\ \sum_{j=1}^n \lambda_j^t z_{bj}^{(t)ec} &= \theta_o^{t, economic} z_{bo}^{(t)ec}, \quad b=1, \dots, B^{ec}, t=1, \dots, T, \end{aligned}$$

(environmental)

$$\begin{aligned} \sum_{j=1}^n \beta_j^t x_{ij}^{(t)en} &\leq \tilde{x}_{io}^{(t)en}, \quad i=1, \dots, m^{en}, t=1, \dots, T, \\ \tilde{x}_{io}^{(t)en} &\leq \theta_o^{t, environmental} x_{io}^{(t)en}, \quad i=1, \dots, m^{en}, t=1, \dots, T, \\ L_{i_{Bnd}}^{(t)en} &\leq \tilde{x}_{i_{Bnd}o}^{(t)en} \leq U_{i_{Bnd}}^{(t)en}, \quad i_{i_{Bnd}}^{(t)en} \in I_{i_{Bnd}}^{(t)en}, \\ \tilde{x}_{i_{Int}o}^{(t)en} &\text{ integer}, \quad i_{Int}^{(t)en} \in I_{Int}^{(t)en}, \\ \sum_{j=1}^n \beta_j^t y_{rj}^{(t)en} &\geq y_{ro}^{(t)en}, \quad r=1, \dots, s^{en}, t=1, \dots, T, \\ \sum_{j=1}^n \beta_j^t z_{bj}^{(t)en} &= \theta_o^{t, environmental} z_{bo}^{(t)en}, \quad b=1, \dots, B^{en}, t=1, \dots, T, \end{aligned}$$

(social)

$$\begin{aligned} \sum_{j=1}^n \gamma_j^t x_{ij}^{(t)so} &\leq \tilde{x}_{io}^{(t)so}, \quad i=1, \dots, m^{so}, t=1, \dots, T, \\ \tilde{x}_{io}^{(t)so} &\leq \theta_o^{t, social} x_{io}^{(t)so}, \quad i=1, \dots, m^{so}, t=1, \dots, T, \\ L_{i_{Bnd}}^{(t)so} &\leq \tilde{x}_{i_{Bnd}o}^{(t)so} \leq U_{i_{Bnd}}^{(t)so}, \quad i_{i_{Bnd}}^{(t)so} \in I_{i_{Bnd}}^{(t)so}, \\ \tilde{x}_{i_{Int}o}^{(t)so} &\text{ integer}, \quad i_{Int}^{(t)so} \in I_{Int}^{(t)so}, \\ \sum_{j=1}^n \gamma_j^t y_{rj}^{(t)so} &\geq y_{ro}^{(t)so}, \quad r=1, \dots, s^{so}, t=1, \dots, T, \\ \sum_{j=1}^n \gamma_j^t z_{bj}^{(t)so} &= \theta_o^{t, social} z_{bo}^{(t)so}, \quad b=1, \dots, B^{so}, t=1, \dots, T, \\ \theta_o^{t,d} &\leq 1, \end{aligned}$$

$$\alpha_o^t = \frac{\sum_{d \in D} w_d^t \theta_o^{t,d}}{\sum_{d \in D} w_d^t}$$

$$\lambda_j^t, \beta_j^t, \gamma_j^t \geq 0.$$

**Definition 1.** A production system  $DMU_o$  is called overall multi-period sustainable if and only if  $\Omega^* = 1$ ; in other words, it is efficient in each period and each dimension. Thus, the system is said to be overall multi-period

unsustainable if  $\Omega^* < 1$ , i.e., it is inefficient at least in one period or one dimension.

Values  $\alpha_o^t$  that denote the sustainability performance for each period are less than or equal to one. In other words,  $\alpha_o^t \leq 1$ . For each

period<sup>t</sup>, the index  $\alpha_o^t$  is obtained as the weighted average of the efficiency scores of dimensions. As defined, we have also the optimal  $\theta_o^{*t,d} \leq 1$  which indicates that the efficiency for each dimension <sup>d</sup> and each period <sup>t</sup> is less than or equal to 1.

According to [8], some definitions are described related to sustainability as follows:

**Definition 2.** If  $\theta_o^{t,economic} = \theta_o^{t,environmental} = 1$  for  $t = 1, \dots, T$ , the unit under examination is called economic-environmental sustainable, and vice versa.

**Definition 3.** If  $\theta_o^{t,economic} = \theta_o^{t,social} = 1$  for  $t = 1, \dots, T$ , the unit under assessment is said to be economic-social sustainable, and vice versa.

**Definition 4.**  $DMU_o$  is called social-environmental sustainable if and only if  $\theta_o^{t,social} = \theta_o^{t,environmental} = 1$  for  $t = 1, \dots, T$ .

**Theorem 1.** Model (3) is always feasible.

**Proof.** See

$$\lambda_o^t = 1, \lambda_j^t = 0, j \neq o, \theta_o^{t,economic} = 1, \beta_o^t = 1, \beta_j^t = 0, j \neq o, \theta_o^{t,environmental} = 1,$$

$$\gamma_o^t = 1, \gamma_j^t = 0, j \neq o, \theta_o^{t,social} = 1, \alpha_o^t = 1,$$

$$\Omega^* = 1, x_{ij}^{(t)d} = \tilde{x}_{ij}^{(t)d}, i = 1, \dots, m^d, d \in D, t = 1, \dots, T.$$

It is clear that it is a feasible solution for Model (3). Therefore, this model is always feasible.  $\square$

As mentioned earlier, Model (3) is an input-oriented multi-period form.

In this stage, an extended output-oriented form is provided to analyze the sustainability performance of systems with input and output measures that may be bounded and/or integer. Thus, we have:

$$\begin{aligned} \Phi^* = & \underset{\theta^{t,e}, \lambda_j^t, \beta_j^t, \gamma_j^t}{Max} \quad \Phi = \sum_{t=1}^T \alpha^t / T \\ s.t. \quad & (economic) \\ & \sum_{j=1}^n \lambda_j^t x_{ij}^{(t)ec} \leq \tilde{x}_{io}^{(t)ec}, \quad i = 1, \dots, m^{ec}, t = 1, \dots, T, \\ & \tilde{x}_{io}^{(t)ec} \leq x_{io}^{(t)ec}, i = 1, \dots, m^{ec}, t = 1, \dots, T, \\ & L_{i_{Bnd}^o}^{(t)ec} \leq \tilde{x}_{i_{Bnd}^o}^{(t)ec} \leq U_{i_{Bnd}^o}^{(t)ec}, i_{i_{Bnd}^o}^{(t)ec} \in I_{i_{Bnd}^o}^{(t)ec}, \\ & \tilde{x}_{i_{Int}^o}^{(t)ec} \text{ integer}, i_{i_{Int}^o}^{(t)ec} \in I_{i_{Int}^o}^{(t)ec}, \\ & \sum_{j=1}^n \lambda_j^t y_{rj}^{(t)ec} \geq \tilde{y}_{ro}^{(t)ec}, \quad r = 1, \dots, s^{ec}, t = 1, \dots, T, \\ & \tilde{y}_{ro}^{(t)ec} \geq \theta_o^{t,economic} y_{ro}^{(t)ec}, \\ & L_{r_{Bnd}^o}^{(t)ec} \leq \tilde{y}_{r_{Bnd}^o}^{(t)ec} \leq U_{r_{Bnd}^o}^{(t)ec}, r_{r_{Bnd}^o}^{(t)ec} \in O_{r_{Bnd}^o}^{(t)ec}, \\ & \tilde{y}_{r_{Int}^o}^{(t)ec} \text{ integer}, r_{r_{Int}^o}^{(t)ec} \in O_{r_{Int}^o}^{(t)ec}, \\ & \sum_{j=1}^n \lambda_j^t z_{bj}^{(t)ec} = \tilde{z}_{bo}^{(t)ec}, \quad b = 1, \dots, B^{ec}, t = 1, \dots, T, \\ & z_{bo}^{(t)ec} = \tilde{z}_{bo}^{(t)ec}, b = 1, \dots, B^{ec}, t = 1, \dots, T, \\ & L_{b_{Bnd}^o}^{(t)ec} \leq \tilde{z}_{b_{Bnd}^o}^{(t)ec} \leq U_{b_{Bnd}^o}^{(t)ec}, b_{b_{Bnd}^o}^{(t)ec} \in B_{b_{Bnd}^o}^{(t)ec}, \\ & \tilde{z}_{b_{Int}^o}^{(t)ec} \text{ integer}, b_{b_{Int}^o}^{(t)ec} \in B_{b_{Int}^o}^{(t)ec}, \end{aligned} \tag{4}$$

(environmental)

$$\sum_{j=1}^n \beta_j^t x_{ij}^{(t)en} \leq \tilde{x}_{io}^{(t)en}, \quad i = 1, \dots, m^{en}, t = 1, \dots, T,$$

$$\tilde{x}_{io}^{(t)en} \leq \theta_o^{t \text{ environmental}} x_{io}^{(t)en}, \quad i = 1, \dots, m^{en}, t = 1, \dots, T,$$

$$L_{i_{Bnd}}^{(t)en} \leq \tilde{x}_{i_{Bnd}o}^{(t)en} \leq U_{i_{Bnd}}^{(t)en}, \quad i_{i_{Bnd}}^{(t)en} \in I_{i_{Bnd}}^{(t)en},$$

$$\tilde{x}_{i_{Bnd}o}^{(t)en} \text{ integer}, i_{i_{Bnd}}^{(t)en} \in I_{i_{Bnd}}^{(t)en},$$

$$\sum_{j=1}^n \beta_j^t y_{rj}^{(t)en} \geq \tilde{y}_{ro}^{(t)en}, \quad r = 1, \dots, s^{en}, t = 1, \dots, T,$$

$$\tilde{y}_{ro}^{(t)en} \geq \theta_o^{t \text{ environmental}} y_{ro}^{(t)en}, \quad r = 1, \dots, s^{en}, t = 1, \dots, T,$$

$$L_{r_{Bnd}}^{(t)en} \leq \tilde{y}_{r_{Bnd}o}^{(t)en} \leq U_{r_{Bnd}}^{(t)en}, \quad r_{r_{Bnd}}^{(t)en} \in O_{r_{Bnd}}^{(t)en},$$

$$\tilde{y}_{r_{Bnd}o}^{(t)en} \text{ integer}, r_{r_{Bnd}}^{(t)en} \in O_{r_{Bnd}}^{(t)en},$$

$$\sum_{j=1}^n \beta_j^t z_{bj}^{(t)en} = \tilde{z}_{bo}^{(t)en}, \quad b = 1, \dots, B^{en}, t = 1, \dots, T,$$

$$\tilde{z}_{bo}^{(t)en} = \tilde{z}_{bo}^{(t)en}, \quad b = 1, \dots, B^{en}, t = 1, \dots, T,$$

$$L_{b_{Bnd}}^{(t)en} \leq \tilde{z}_{b_{Bnd}o}^{(t)en} \leq U_{b_{Bnd}}^{(t)en}, \quad b_{b_{Bnd}}^{(t)en} \in B_{b_{Bnd}}^{(t)en},$$

$$\tilde{z}_{b_{Bnd}o}^{(t)en} \text{ integer}, b_{b_{Bnd}}^{(t)en} \in B_{b_{Bnd}}^{(t)en},$$

(social)

$$\sum_{j=1}^n \gamma_j^t x_{ij}^{(t)so} \leq \tilde{x}_{io}^{(t)so}, \quad i = 1, \dots, m^{so}, t = 1, \dots, T,$$

$$\tilde{x}_{io}^{(t)so} \leq x_{io}^{(t)so}, \quad i = 1, \dots, m^{so}, t = 1, \dots, T,$$

$$L_{i_{Bnd}}^{(t)so} \leq \tilde{x}_{i_{Bnd}o}^{(t)so} \leq U_{i_{Bnd}}^{(t)so}, \quad i_{i_{Bnd}}^{(t)so} \in I_{i_{Bnd}}^{(t)so},$$

$$\tilde{x}_{i_{Bnd}o}^{(t)so} \text{ integer}, i_{i_{Bnd}}^{(t)so} \in I_{i_{Bnd}}^{(t)so},$$

$$\sum_{j=1}^n \gamma_j^t y_{rj}^{(t)so} \geq \tilde{y}_{ro}^{(t)so}, \quad r = 1, \dots, s^{so}, t = 1, \dots, T,$$

$$\tilde{y}_{ro}^{(t)so} \geq \theta_o^{t \text{ social}} y_{ro}^{(t)so}, \quad r = 1, \dots, s^{so}, t = 1, \dots, T,$$

$$L_{r_{Bnd}}^{(t)so} \leq \tilde{y}_{r_{Bnd}o}^{(t)so} \leq U_{r_{Bnd}}^{(t)so}, \quad r_{r_{Bnd}}^{(t)so} \in O_{r_{Bnd}}^{(t)so},$$

$$\tilde{y}_{r_{Bnd}o}^{(t)so} \text{ integer}, r_{r_{Bnd}}^{(t)so} \in O_{r_{Bnd}}^{(t)so},$$

$$\sum_{j=1}^n \gamma_j^t z_{bj}^{(t)so} = \tilde{z}_{bo}^{(t)so}, \quad b = 1, \dots, B^{so}, t = 1, \dots, T,$$

$$\tilde{z}_{bo}^{(t)so} = \tilde{z}_{bo}^{(t)so}, \quad b = 1, \dots, B^{so}, t = 1, \dots, T,$$

$$L_{b_{Bnd}}^{(t)so} \leq \tilde{z}_{b_{Bnd}o}^{(t)so} \leq U_{b_{Bnd}}^{(t)so}, \quad b_{b_{Bnd}}^{(t)so} \in B_{b_{Bnd}}^{(t)so},$$

$$\tilde{z}_{b_{Bnd}o}^{(t)so} \text{ integer}, b_{b_{Bnd}}^{(t)so} \in B_{b_{Bnd}}^{(t)so},$$

$$\theta_o^{t \text{ d}} \geq 1,$$

$$\alpha_o^t = \frac{\sum_{d \in D} w_d^t \theta_o^{td}}{\sum_{d \in D} w_d^t}$$

$$\lambda_j^t, \beta_j^t, \gamma_j^t \geq 0.$$

**Definition 5.** The overall multi-period sustainability in Model (4) is defined as  $1/\Phi^*$ . The unit under consideration is said to be overall multi-period sustainable if and only if

$1/\Phi^* = 1$ ; otherwise, it is unsustainable for at least one period and one dimension.

Furthermore, the sustainability performance

for each period  $t$  can be determined by  $1/\alpha_o^{t*}$ .

The entity is called sustainable in period  $t$  if

and only if  $1/\alpha_o^{t*} = 1$ . In another respect, it is



unsustainable in period  $t$  if and only if  $1/\alpha_o^{t*} < 1$ . Similarly, the efficiency of the unit under evaluation for each period and each dimension is defined as  $1/\theta_o^{*td}$ , and it is efficient if and only if  $1/\theta_o^{*td} = 1$ .

Moreover,  $(\tilde{x}_{lo}^{*(t)d}, \tilde{y}_{ro}^{*(t)d}, \tilde{z}_{bo}^{*(t)d}), d \in D$  shows

the target point of  $DMU_o$ .

Likewise, economic-environmental, economic-social, and social-environmental sustainability can be defined in this case. It can also be shown that Model (4) is always feasible.

Due to the possibility of internal relationships among economic, social, and environmental items, we investigate the following cases for economic and social issues:

- $d_{lj}^{(t)d}$  is treated as a desirable integer item <sup>$l$</sup>  for  $DMU_j$  ( $j = 1, \dots, n$ ) in both economic and social respects.
- o If it is the input for both perspectives, the next constraints are included in Model (4):

Economic member

$$\sum_{j=1}^n \lambda_j^t d_{lj}^{(t)ec} \leq \tilde{d}_{lo}^{(t)ec}, t = 1, \dots, T,$$

$$\tilde{d}_{lj}^{(t)ec} \leq d_{lo}^{(t)ec}, t = 1, \dots, T,$$

$$\tilde{d}_{l_{int}o}^{(t)ec} \text{ integer}, l_{int}^{(t)ec} \in I_{int}^{(t)ec},$$

Social member

$$\sum_{j=1}^n \gamma_j^t d_{lj}^{(t)so} \leq \tilde{d}_{lo}^{(t)so}, t = 1, \dots, T,$$

$$\tilde{d}_{lj}^{(t)so} \leq d_{lo}^{(t)so}, t = 1, \dots, T,$$

$$\tilde{d}_{l_{int}o}^{(t)so} \text{ integer}, l_{int}^{(t)so} \in I_{int}^{(t)so},$$

- o If it is the desirable output for both perspectives, the following expressions are included in Model (4):

Economic member

$$\sum_{j=1}^n \lambda_j^t d_{lj}^{(t)ec} \geq \tilde{d}_{lo}^{(t)ec}, t = 1, \dots, T,$$

$$\tilde{d}_{lj}^{(t)ec} \geq \theta_o^{economic} d_{lo}^{(t)ec},$$

$$\tilde{d}_{l_{int}o}^{(t)ec} \text{ integer}, l_{int}^{(t)ec} \in O_{int}^{(t)ec},$$

Social member

$$\sum_{j=1}^n \gamma_j^t d_{lj}^{(t)so} \geq \tilde{d}_{lo}^{(t)so}, t = 1, \dots, T,$$

$$\tilde{d}_{lj}^{(t)so} \geq \theta_o^{social} d_{lo}^{(t)so}, t = 1, \dots, T,$$

$$\tilde{d}_{l_{int}o}^{(t)so} \text{ integer}, l_{int}^{(t)so} \in O_{int}^{(t)so},$$

- It is considered as an undesirable integer output item <sup>$l$</sup>  in both respects.

Economic member

$$\sum_{j=1}^n \lambda_j^t d_{lj}^{(t)ec} = \tilde{d}_{lo}^{(t)ec}, t = 1, \dots, T,$$

$$d_{lo}^{(t)ec} = \tilde{d}_{lo}^{(t)ec}, t = 1, \dots, T,$$

$$\tilde{d}_{l_{int}o}^{(t)ec} \text{ integer}, l_{int}^{(t)ec} \in B_{int}^{(t)ec},$$

Social member

$$\sum_{j=1}^n \gamma_j^t d_{lj}^{(t)so} = \tilde{d}_{lo}^{(t)so}, t = 1, \dots, T,$$

$$d_{lo}^{(t)so} = \tilde{d}_{lo}^{(t)so}, t = 1, \dots, T,$$

$$\tilde{d}_{l_{int}o}^{(t)so} \text{ integer}, l_{int}^{(t)so} \in B_{int}^{(t)so},$$

- The item <sup>$l$</sup>  is an integer desirable output item from one perspective and an integer undesirable output from another.
- o We suppose that the item <sup>$l$</sup>  is the integer desirable output of the economic aspect and the integer undesirable output of the social aspect. Thus, we have:

Economic member

$$\sum_{j=1}^n \lambda_j^t d_{lj}^{(t)ec} \geq \tilde{d}_{lj}^{(t)ec}, t = 1, \dots, T,$$

$$\tilde{d}_{lj}^{(t)ec} \geq \theta_o^{economic} d_{lj}^{(t)ec},$$

$$\tilde{d}_{l_{int}jo}^{(t)ec} \text{ integer}, l_{int}^{(t)ec} \in O_{int}^{(t)ec},$$

Social member

$$\sum_{j=1}^n \gamma_j^t d_{lj}^{(t)so} = \tilde{d}_{lj}^{(t)so}, t = 1, \dots, T,$$

$$d_{lj}^{(t)so} = \tilde{d}_{lj}^{(t)so}, t = 1, \dots, T,$$

$$\tilde{d}_{l_{int}jo}^{(t)so} \text{ integer}, l_{int}^{(t)so} \in B_{int}^{(t)so},$$

- o In the same way, the following constraints are added to Model (4), where the item <sup>$l$</sup>  shows the integer undesirable output of the economic aspect and the integer desirable output of the social aspect:

Economic member

$$\sum_{j=1}^n \lambda_j^t d_{lj}^{(t)ec} = \tilde{d}_{lj}^{(t)ec}, t = 1, \dots, T,$$

$$d_{lj}^{(t)ec} = \tilde{d}_{lj}^{(t)ec}, t = 1, \dots, T,$$

$$\tilde{d}_{l_{int}jo}^{(t)ec} \text{ integer}, l_{int}^{(t)ec} \in B_{int}^{(t)ec},$$

Social member

$$\sum_{j=1}^n \gamma_j^t d_{lj}^{(t)so} \geq \tilde{d}_{ljo}^{(t)so}, \quad t=1, \dots, T,$$

$$\tilde{d}_{ljo}^{(t)so} \geq \theta_o^{t, social} d_{rjo}^{(t)so}, \quad t=1, \dots, T,$$

$$\tilde{d}_{l_{int}, jo}^{(t)so} \text{ integer}, l_{int}^{(t)so} \in O_{int}^{(t)so},$$

Note that, analogous to the above, environmental and economic issues and also social and environmental issues can be addressed. Moreover, these cases can conveniently be extended for situations that these items are bounded.

### 3-1 Performance changes

Apart from the sustainability performance analysis, computing performance changes between the two periods is a considerable issue. Thus, the following formulations are applied for calculating the performance changes of the overall sustainability and dimensions:

$$OS^{t, t+h} = \frac{\alpha_o^{*t+h}}{\alpha_o^{*t}}, \quad (5)$$

$$DS^{t, t+h, d} = \frac{\theta_o^{*t+h, d}}{\theta_o^{*td}} \quad \forall d \in D, \quad (6)$$

in which  $OS^{t, t+h}$  shows the change of the sustainability performance of  $DMU_o$  from period  $t$  to  $t+h$ .  $DS^{t, t+h, d}$  indicates the performance change of  $DMU_o$  for each

dimension  $d$  from period  $t$  to  $t+h$ . Optimal values  $\alpha_o^{*t+h}$  and  $\alpha_o^{*t}$  are sustainability performance scores of  $DMU_o$  for periods  $t+h$  and  $t$ , respectively. Furthermore, optimal values  $\theta_o^{*t+h, d}$  and  $\theta_o^{*td}$  are accordingly the dimension performance  $d$  of  $DMU_o$  for periods  $t+h$  and  $t$ .

If  $OS^{t, t+h}$  is obtained greater than 1, the performance is shown to have improved. If  $OS^{t, t+h} < 1$ , it has worsened. Moreover, the performance is without change when  $OS^{t, t+h} = 1$ . There is a similar interpretation about  $DS^{t, t+h, d}$ .

Notice that performance changes can be calculated using findings obtained from both approaches (3) and (4).

## 4. An Example and an Application of Gas Companies

### 4-1. An example

In this section, it is supposed that the sustainability performance of 7 manufacturers in three years is analyzed while there are some bounded and integer measures. Input and output factors and their dimensions are shown in Table 2. Data values can be found in Table 3.

Tab. 2. Data Description

| Economic (EC)                     | Role               | Type    |
|-----------------------------------|--------------------|---------|
| Labor (L)                         | Input              | Integer |
| Capital (C)                       | Input              | Real    |
| On-time delivery (OT)             | Desirable output   | Bounded |
| Environmental (EN)                | Role               | Type    |
| Water consumption (WC)            | Input              | Real    |
| The number of green products (GP) | Desirable output   | Integer |
| Waste (W)                         | Undesirable output | Real    |
| Social (SO)                       | Role               | Type    |
| Employee training cost (ET)       | Input              | Real    |
| Customer satisfaction (CS)        | Desirable output   | Bounded |

**Tab. 3. Data**

| Period 1 ( $t^1$ ) |     |      |    |     |     |    |      |    |
|--------------------|-----|------|----|-----|-----|----|------|----|
| #                  | L   | C    | OT | WC  | GP  | W  | ET   | CS |
| 1                  | 122 | 2473 | 95 | 121 | 994 | 47 | 1560 | 36 |
| 2                  | 173 | 2621 | 80 | 50  | 540 | 54 | 1800 | 49 |
| 3                  | 152 | 3712 | 79 | 68  | 979 | 28 | 1507 | 34 |
| 4                  | 153 | 3004 | 75 | 127 | 621 | 27 | 1058 | 61 |
| 5                  | 165 | 3775 | 69 | 86  | 531 | 21 | 1942 | 29 |
| 6                  | 120 | 1330 | 83 | 140 | 744 | 50 | 1831 | 31 |
| 7                  | 106 | 3752 | 65 | 135 | 538 | 31 | 1008 | 47 |
| Period 2 ( $t^2$ ) |     |      |    |     |     |    |      |    |
| #                  | L   | C    | OT | WC  | GP  | W  | ET   | CS |
| 1                  | 255 | 1263 | 34 | 97  | 410 | 69 | 1551 | 59 |
| 2                  | 207 | 2341 | 27 | 85  | 625 | 20 | 1333 | 58 |
| 3                  | 186 | 3039 | 74 | 119 | 940 | 68 | 1129 | 50 |
| 4                  | 165 | 2327 | 30 | 84  | 775 | 22 | 1655 | 83 |
| 5                  | 131 | 2331 | 6  | 93  | 814 | 26 | 1079 | 73 |
| 6                  | 209 | 4191 | 72 | 144 | 785 | 78 | 1663 | 52 |
| 7                  | 280 | 3571 | 2  | 100 | 834 | 73 | 1438 | 51 |
| Period 3 ( $t^3$ ) |     |      |    |     |     |    |      |    |
| #                  | L   | C    | OT | WC  | GP  | W  | ET   | CS |
| 1                  | 222 | 1790 | 72 | 141 | 944 | 54 | 1889 | 47 |
| 2                  | 188 | 3508 | 31 | 87  | 624 | 41 | 1378 | 65 |
| 3                  | 166 | 4780 | 49 | 144 | 983 | 54 | 1235 | 81 |
| 4                  | 186 | 4837 | 92 | 90  | 603 | 88 | 1589 | 78 |
| 5                  | 110 | 4274 | 94 | 114 | 633 | 73 | 1945 | 45 |
| 6                  | 193 | 3545 | 30 | 96  | 939 | 63 | 1729 | 30 |
| 7                  | 176 | 2150 | 91 | 73  | 721 | 88 | 1892 | 60 |

To measure the multi-period sustainability performance of manufacturers while bounded and integer-valued measures are presented, the input-oriented version is used. Results are shown in Table 4.

In addition, the performance changes between the years have been calculated and shown in

Table 4. To illustrate, the performance changes for the overall sustainability and dimensions are estimated through Expressions (5) and (6).

Findings show that Manufacturer 3 with a score of 0.782 is generally more sustainable than other manufacturers.

**Tab. 4. Results**

| # | EC    |       |       | EN    |       |       | SO    |       |       |
|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|   | $t^1$ | $t^2$ | $t^3$ | $t^1$ | $t^2$ | $t^3$ | $t^1$ | $t^2$ | $t^3$ |
| 1 | 1     | 1     | 0.950 | 0.605 | 0.505 | 0.969 | 0.400 | 0.562 | 0.379 |
| 2 | 0.636 | 0.464 | 0.277 | 1     | 0.887 | 0.915 | 0.472 | 0.643 | 0.719 |
| 3 | 0.671 | 1     | 0.408 | 1     | 0.914 | 1     | 0.391 | 0.655 | 1     |
| 4 | 0.636 | 0.526 | 0.715 | 0.658 | 1     | 0.681 | 1     | 0.741 | 0.748 |
| 5 | 0.539 | 0.122 | 1     | 0.723 | 0.952 | 0.578 | 0.259 | 1     | 0.353 |
| 6 | 1     | 0.866 | 0.264 | 0.426 | 0.627 | 1     | 0.294 | 0.462 | 0.265 |
| 7 | 0.792 | 0.023 | 1     | 0.496 | 1     | 1     | 0.809 | 0.524 | 0.484 |

| #              | $DS^{EC}$ |          |          | $DS^{EN}$  |          |          | $DS^{So}$ |          |          |
|----------------|-----------|----------|----------|------------|----------|----------|-----------|----------|----------|
|                | $t1, t2$  | $t1, t3$ | $t2, t3$ | $t1, t2$   | $t1, t3$ | $t2, t3$ | $t1, t2$  | $t1, t3$ | $t2, t3$ |
| 1              | 1         | 0.950    | 0.950    | 0.835      | 1.602    | 1.919    | 1.405     | 0.948    | 0.674    |
| 2              | 0.730     | 0.436    | 0.597    | 0.887      | 0.915    | 1.032    | 1.362     | 1.523    | 1.118    |
| 3              | 1.490     | 0.608    | 0.408    | 0.914      | 1        | 1.094    | 1.675     | 2.558    | 1.527    |
| 4              | 0.827     | 1.124    | 1.359    | 1.520      | 1.035    | 0.681    | 0.741     | 0.748    | 1.009    |
| 5              | 0.226     | 1.855    | 8.197    | 1.317      | 0.799    | 0.607    | 3.861     | 1.363    | 0.353    |
| 6              | 0.866     | 0.264    | 0.305    | 1.472      | 2.347    | 1.595    | 1.571     | 0.901    | 0.574    |
| 7              | 0.029     | 1.263    | 43.478   | 2.016      | 2.016    | 1        | 0.648     | 0.598    | 0.924    |
| Sustainability |           |          |          | $\Omega^*$ |          |          | $OS$      |          |          |
|                | $t1$      | $t2$     | $t3$     |            |          |          | $t1, t2$  | $t1, t3$ | $t2, t3$ |
| 1              | 0.668     | 0.689    | 0.766    | 0.708      |          |          |           | 1.031    | 1.147    |
| 2              | 0.703     | 0.665    | 0.637    | 0.668      |          |          |           | 0.946    | 0.906    |
| 3              | 0.687     | 0.856    | 0.803    | 0.782      |          |          |           | 1.246    | 1.169    |
| 4              | 0.765     | 0.756    | 0.715    | 0.745      |          |          |           | 0.988    | 0.935    |
| 5              | 0.507     | 0.691    | 0.643    | 0.614      |          |          |           | 1.363    | 1.268    |
| 6              | 0.573     | 0.652    | 0.510    | 0.578      |          |          |           | 1.138    | 0.890    |
| 7              | 0.699     | 0.516    | 0.828    | 0.681      |          |          |           | 0.738    | 1.147    |

Manufacturer 6 with a score of 0.578 is subject to lower sustainability than other manufacturers. Performance changes computed from the social perspective between Periods 1 and 2 indicate that the performance has improved for five manufacturers of 1, 2, 3, 5, and 6 and has worsened in other manufacturers. Similarly, the performance changes can be analyzed in other terms, too.

Estimation of sustainability for different years shows that Manufacturer 3 is more sustainable in the second year with a score of 0.856. Manufacturers 4 and 7 are more sustainable in periods 1 and 3, respectively. In addition, A comparison of the sustainability between two years 2 and 3 reveals that the performance of two manufacturers, 1 and 7, has improved while the performance of others has worsened. Furthermore, sustainability performance between Periods 1 and 3 has improved in Manufacturers 1, 3, 5, and 7, while Manufacturers 1, 3, 5, and 6 have progressed between periods 1 and 2.

Notice that the performances of manufacturers that are equal to one are shown to be sustainable in that dimension and that year. For instance, Manufacturers 1 and 6 are sustainable from an economic

point of view in the year 1, as shown in Table 4. In addition, Manufacturers 2 and 3 are environmentally sustainable in the year 1. In the next stage, the projection points of manufacturers are calculated.

Table 5 shows target points of labor as integral elements. Projection points of other integer and bounded values are equal with each other according to the model orientation ad its structure.

Now, Jablonsky's approach [14] is computed (the input-oriented model) considering all items as continuous and unbounded (and without incorporating sustainability dimensions) to show the validation of the proposed method. By solving the problem, all manufacturers are determined to be efficient with a score of 1. Therefore, the performance scores cannot be distinguished, while the suggested approach can identify the performance in a rational way and is more informative and accurate. Furthermore, integer-valued targets are obtained for integer items by the proposed approach. In the next subsection, the introduced method is utilized to analyze the sustainability performance of gas companies in several years while integer and bounded items appear.

Tab. 5. Target points of labor

| # | Labor |       |       |
|---|-------|-------|-------|
|   | $t_1$ | $t_2$ | $t_3$ |
| 1 | 122   | 255   | 139   |
| 2 | 110   | 96    | 52    |
| 3 | 102   | 186   | 67    |
| 4 | 97    | 87    | 133   |
| 5 | 89    | 16    | 110   |
| 6 | 120   | 181   | 50    |
| 7 | 84    | 6     | 176   |

#### 4-2. An application

Now, the offered approach is used to analyze the sustainability performance of 29 Iranian gas companies located in 29 provinces. The investigation was performed over the years 2013-2015. Performance measures and their dimensions are detailed in Table 6. Moreover, a statistical description of the performance data of gas companies over 2013-2015 is provided in Table 7. To address sustainability performance, Model (4) is utilized. Overall sustainability and the sustainability performance obtained for each year under evaluation are shown in Columns 3-6 of Table

8. Furthermore, the performance changes of sustainability over 2013-2015 are displayed in Columns 7-9. As can be seen, Kohgiluyeh and Boyer-Ahmad is a multi-period sustainable item, overall. It is implied that it is efficient in each dimension and each period. Thus, the performance is without any change. Kerman with a score of 0.371 has the least sustainability level. Furthermore, the sustainability level of Ardabil has only regressed between the years 2013-2014 and 2013-2015, whilst the performance of 25 companies has worsened between 2014-2015.

Tab. 6. Performance data description

| Role                | Factor   | Type    | Dimension       |
|---------------------|--|---------|-----------------|
| Input               | Operational cost: Expenses related to the system operation       | Real    | Economic        |
|                     | Asset: All benefits belong to gas companies                      | Real    | Economic        |
|                     | Volume of natural gas received: The devoted gas to gas companies | Real    | Environmental   |
|                     | Employee: The number of staff (E)                                | Integer | Social          |
| Desirable output    | Number of installed branches (NIB)                               | Integer | Economic        |
|                     | Gas subscriptions: The number of subscribers (GS)                | Integer | Economic        |
|                     | Income   | Real    | Social-economic |
|                     | Replacing gas in preference for petroleum                        | Real    | Environmental   |
|                     | Influence factor of outfitted cities and villages (IF)           | Bounded | Social          |
| Undesirable outputs | Outstanding debts (OD)   | Bounded | Economic        |
|                     | Environmental pollution resulting from gas leak emissions        | Real    | Environmental   |

**Tab. 7. Statistical representation of data**

| Variable  | Year | Max        | Min       | Mean      | Standard deviation |
|---|------|------------|-----------|-----------|--------------------|
| Operational cost  | 2013 | 4035879    | 1847.269  | 877517.92 | 868343.36          |
|   | 2014 | 5892482    | 2446.617  | 1290421.2 | 1303805.7          |
|   | 2015 | 6342750    | 3247.697  | 1575454   | 1535995            |
| Asset   | 2013 | 17712118   | 1386132   | 5176525.7 | 4268731.9          |
|   | 2014 | 20488410   | 972393    | 6108664.6 | 5045065.8          |
|   | 2015 | 24415670   | 294597    | 6923816.3 | 5859170.9          |
| Volume of natural gas received                            | 2013 | 27797      | 409       | 5321.3241 | 5756.5887          |
|   | 2014 | 25911      | 455       | 5729.7931 | 5896.2142          |
|   | 2015 | 26956      | 500       | 5985.4345 | 6040.4403          |
| Employee  | 2013 | 3462       | 126       | 853.68966 | 820.51966          |
|   | 2014 | 3145       | 123       | 843.62069 | 773.18665          |
|   | 2015 | 2620       | 144       | 889.03448 | 725.14814          |
| Number of installed branches                              | 2013 | 41577      | 1890      | 15515.964 | 11284.29           |
|   | 2014 | 29546      | 1121      | 13292.655 | 8368.2253          |
|   | 2015 | 46721      | 1552      | 14645.31  | 10907.48           |
| Gas subscriptions   | 2013 | 277425     | 1052      | 49109.103 | 53415.905          |
|   | 2014 | 278533     | 1165      | 45065.931 | 52436.259          |
|   | 2015 | 208987     | 1125      | 39967.138 | 40818.777          |
| Income  | 2013 | 14426760   | 171430    | 2343164.1 | 3203315.9          |
|   | 2014 | 20778626   | 312674    | 3534333.4 | 5148778.4          |
|   | 2015 | 20208906   | 600479    | 4917151.3 | 5482697.9          |
| Replacing gas in preference for petroleum                 | 2013 | 1003       | 30.9      | 253.66897 | 235.70204          |
|   | 2014 | 1042       | 33.3      | 245.38621 | 225.41584          |
|   | 2015 | 1064       | 41        | 253.89655 | 232.53801          |
| Influential factor of outfitted cities and villages       | 2013 | 100        | 1.4       | 74.72931  | 19.546851          |
|   | 2014 | 97.4       | 2.6       | 76.098276 | 18.559992          |
|   | 2015 | 98.8       | 6.85      | 81.300345 | 17.954088          |
| Outstanding debts   | 2013 | 84         | 0.01      | 19.806207 | 17.205086          |
|   | 2014 | 83         | 0.9       | 22.798276 | 14.618542          |
|   | 2015 | 76         | 9         | 26.625517 | 2078368.5          |
| Environmental pollution resulting from gas leak emissions | 2013 | 99314971.1 | 205872.35 | 21018574  | 20834724           |
|   | 2014 | 104647129  | 261617.96 | 22064810  | 21832350           |
|   | 2015 | 108589111  | 286360.54 | 23148796  | 22492401           |

**Tab. 8. Sustainability performance and changes**

| # | Province        | Overall Sustainability | 2013 sustainability | 2014 Sustainability | 2015 Sustainability | 2013, 2014 | 2013, 2015 | 2014, 2015 |
|---|-----------------|------------------------|---------------------|---------------------|---------------------|------------|------------|------------|
| 1 | East Azerbaijan | 0.586                  | 0.375               | 0.847               | 0.789               | 2.259      | 2.104      | 0.932      |
| 2 | West Azerbaijan | 0.716                  | 0.497               | 1                   | 0.85                | 2.012      | 1.710      | 0.850      |
| 3 | Ardabil         | 0.848                  | 0.883               | 0.825               | 0.839               | 0.934      | 0.950      | 1.017      |
| 4 | Isfahan         | 0.421                  | 0.263               | 0.550               | 0.667               | 2.091      | 2.536      | 1.213      |
| 5 | Ilam            | 0.636                  | 0.481               | 0.831               | 0.698               | 1.728      | 1.451      | 0.840      |
| 6 | Bushehr         | 0.448                  | 0.339               | 0.513               | 0.553               | 1.513      | 1.631      | 1.078      |

|    |                            |       |       |       |       |       |       |       |
|----|----------------------------|-------|-------|-------|-------|-------|-------|-------|
| 7  | Tehran & Alborz            | 0.54  | 0.340 | 0.764 | 0.769 | 2.247 | 2.262 | 1.007 |
| 8  | Chaharmahal and Bakhtiari  | 0.551 | 0.404 | 0.729 | 0.626 | 1.804 | 1.550 | 0.859 |
| 9  | South Khorasan             | 0.394 | 0.246 | 0.594 | 0.534 | 2.415 | 2.171 | 0.899 |
| 10 | Razavi Khorasan            | 0.589 | 0.378 | 0.845 | 0.795 | 2.235 | 2.103 | 0.941 |
| 11 | North Khorasan             | 0.584 | 0.385 | 0.782 | 0.792 | 2.031 | 2.057 | 1.013 |
| 12 | Khuzestan                  | 0.5   | 0.327 | 0.716 | 0.645 | 2.190 | 1.972 | 0.901 |
| 13 | Zanjan                     | 0.621 | 0.409 | 0.776 | 0.912 | 1.897 | 2.230 | 1.175 |
| 14 | Semnan                     | 0.627 | 0.440 | 0.867 | 0.734 | 1.970 | 1.668 | 0.847 |
| 15 | Fars                       | 0.598 | 0.383 | 0.843 | 0.822 | 2.201 | 2.146 | 0.975 |
| 16 | Qazvin                     | 0.697 | 0.461 | 0.904 | 0.974 | 1.961 | 2.113 | 1.077 |
| 17 | Qom                        | 0.68  | 0.470 | 0.932 | 0.827 | 1.983 | 1.760 | 0.887 |
| 18 | Kurdistan                  | 0.543 | 0.366 | 0.750 | 0.685 | 2.049 | 1.872 | 0.913 |
| 19 | Kermanshah                 | 0.612 | 0.432 | 0.890 | 0.683 | 2.060 | 1.581 | 0.767 |
| 20 | Kerman                     | 0.371 | 0.244 | 0.497 | 0.504 | 2.037 | 2.066 | 1.014 |
| 21 | Kohgiluyeh and Boyer-Ahmad | 1     | 1     | 1     | 1     | 1     | 1     | 1     |
| 22 | Golestan                   | 0.628 | 0.423 | 0.931 | 0.745 | 2.201 | 1.761 | 0.800 |
| 23 | Gilan                      | 0.551 | 0.335 | 0.822 | 0.806 | 2.454 | 2.406 | 0.981 |
| 24 | Lorestan                   | 0.623 | 0.464 | 0.767 | 0.737 | 1.653 | 1.588 | 0.961 |
| 25 | Mazandaran                 | 0.595 | 0.382 | 0.840 | 0.808 | 2.199 | 2.115 | 0.962 |
| 26 | Markazi                    | 0.404 | 0.267 | 0.547 | 0.542 | 2.049 | 2.030 | 0.991 |
| 27 | Hormozgan                  | 0.691 | 0.426 | 1     | 1     | 2.347 | 2.347 | 1     |
| 28 | Hamadan                    | 0.468 | 0.303 | 0.685 | 0.606 | 2.261 | 2.000 | 0.885 |
| 29 | Yazd                       | 0.394 | 0.248 | 0.533 | 0.585 | 2.149 | 2.359 | 1.098 |

In order to study the findings more accurately, Table 9 shows the efficiency values of any dimension of sustainability for the years 2013-2015. As can be seen, 4 provinces, i.e., Bushehr, North Khorasan, Fars, and Mazandaran, are sustainable in economic-social terms. Moreover, their changes are indicated in Table 10. The last three rows of Table 10 represent general statistics of the performance changes of dimensions. Table 11 exhibits target points of integer and bounded measures. As shown, the projecting points of integer-valued measures are obtained as integer values. Moreover, targets of bounded variables are established between the defined ranges. Notice that the projection points of outstanding debts and the number of employees are equal with each other; thus, they are not stated here.

To demonstrate the advantages of the suggested approach, Model (2) is calculated. To illustrate, all measures are considered as continuous and unbounded and also without including sustainability dimensions. The results of efficiency scores are provided in Table 12. As can be observed, approximately 73% of companies were determined to be totally efficient. Moreover, almost 76%, 90%, and 83% of companies were identified as efficient in the years 2013, 2014, and 2015, respectively. Therefore, as depicted in Figure 1, the overall performance scores of the presented approach are more distinctive and informative in comparison to those of Model (2). What's more, the target points of integer and bounded items are displayed in Table 13. It can be seen that non-integer targets are obtained for some companies. Moreover,

projection points of the influence factor (IF) for some companies such as Company 1 over the years 2013 and 2015 are not determined within the specified boundaries.

Therefore, detections show that the proposed approach can estimate the multi-period sustainability performance and the projection points of integer and/or bounded items in a reliable and accurate manner.

**Tab. 9. Sustainability dimensions performance**

| #  | Province                   | Economic |       |       | Environmental |       |       | Social |       |       |
|----|----------------------------|----------|-------|-------|---------------|-------|-------|--------|-------|-------|
|    |                            | 2013     | 2014  | 2015  | 2013          | 2014  | 2015  | 2013   | 2014  | 2015  |
| 1  | East Azerbaijan            | 0.598    | 0.783 | 0.682 | 0.192         | 0.858 | 0.789 | EC 13  | EC 14 | EC 15 |
| 2  | West Azerbaijan            | 1        | 1     | 0.936 | 0.279         | 1     | 1     | 0.687  | 1     | 0.685 |
| 3  | Ardabil                    | 0.87     | 0.98  | 0.826 | 1             | 0.703 | 0.82  | 0.8    | 0.838 | 0.873 |
| 4  | Isfahan                    | 0.474    | 0.542 | 0.852 | 0.121         | 0.388 | 0.435 | 0.943  | 0.974 | 0.973 |
| 5  | Ilam                       | 1        | 1     | 0.607 | 0.236         | 0.62  | 0.669 | 1      | 1     | 0.868 |
| 6  | Bushehr                    | 1        | 1     | 1     | 0.146         | 0.26  | 0.292 | 1      | 1     | 1     |
| 7  | Tehran & Alborz            | 1        | 1     | 1     | 0.147         | 0.519 | 0.526 | 0.948  | 1     | 1     |
| 8  | Chaharmahal and Bakhtiari  | 0.65     | 0.799 | 0.522 | 0.21          | 0.575 | 0.56  | 0.881  | 0.892 | 0.915 |
| 9  | South Khorasan             | 1        | 0.987 | 0.561 | 0.1           | 0.36  | 0.383 | 0.806  | 0.793 | 0.82  |
| 10 | Razavi Khorasan            | 1        | 1     | 0.911 | 0.168         | 0.644 | 0.674 | 1      | 1     | 0.839 |
| 11 | North Khorasan             | 1        | 1     | 1     | 0.173         | 0.545 | 0.56  | 1      | 1     | 1     |
| 12 | Khuzestan                  | 0.719    | 0.967 | 0.701 | 0.156         | 0.472 | 0.505 | 0.739  | 0.967 | 0.805 |
| 13 | Zanjan                     | 0.699    | 0.753 | 1     | 0.221         | 0.809 | 0.775 | 0.726  | 0.769 | 1     |
| 14 | Semnan                     | 1        | 0.991 | 0.62  | 0.208         | 0.695 | 0.71  | 1      | 0.991 | 0.941 |
| 15 | Fars                       | 1        | 1     | 1     | 0.171         | 0.641 | 0.605 | 1      | 1     | 1     |
| 16 | Qazvin                     | 0.797    | 0.863 | 1     | 0.25          | 1     | 0.926 | 0.797  | 0.863 | 1     |
| 17 | Qom                        | 1        | 1     | 0.748 | 0.228         | 0.821 | 0.826 | 1      | 1     | 0.926 |
| 18 | Kurdistan                  | 0.795    | 0.822 | 0.653 | 0.182         | 0.698 | 0.59  | 0.687  | 0.738 | 0.869 |
| 19 | Kermanshah                 | 1        | 1     | 0.554 | 0.202         | 0.73  | 0.828 | 1      | 1     | 0.725 |
| 20 | Kerman                     | 0.476    | 0.701 | 0.553 | 0.123         | 0.378 | 0.419 | 0.495  | 0.51  | 0.57  |
| 21 | Kohgiluyeh and Boyer-Ahmad | 1        | 1     | 1     | 1             | 1     | 1     | 1      | 1     | 1     |
| 22 | Golestan                   | 1        | 1     | 0.575 | 0.196         | 0.819 | 0.815 | 1      | 1     | 0.945 |
| 23 | Gilan                      | 0.929    | 1     | 1     | 0.149         | 0.606 | 0.581 | 0.87   | 1     | 1     |
| 24 | Lorestan                   | 0.915    | 0.993 | 0.759 | 0.254         | 0.633 | 0.662 | 0.702  | 0.756 | 0.805 |
| 25 | Mazandaran                 | 1        | 1     | 1     | 0.171         | 0.637 | 0.584 | 1      | 1     | 1     |
| 26 | Markazi                    | 0.855    | 0.584 | 0.511 | 0.111         | 0.422 | 0.391 | 0.932  | 0.713 | 0.978 |
| 27 | Hormozgan                  | 0.331    | 1     | 1     | 1             | 1     | 1     | 0.331  | 1     | 1     |
| 28 | Hamadan                    | 0.68     | 0.669 | 0.484 | 0.136         | 0.541 | 0.534 | 0.952  | 0.967 | 0.988 |
| 29 | Yazd                       | 0.493    | 0.545 | 0.735 | 0.116         | 0.408 | 0.392 | 0.695  | 0.745 | 0.825 |



Tab. 10. Performance changes of sustainability dimensions

| #           | Province                   | DS     |       |       |       |       |       |       |       |       |
|-------------|----------------------------|--------|-------|-------|-------|-------|-------|-------|-------|-------|
|             |                            | 13,14* | 13,15 | 14,15 | 13,14 | 13,15 | 14,15 | 13,14 | 13,15 | 14,15 |
|             |                            | EC     | EC    | EC    | EN    | EN    | EN    | SO    | SO    | SO    |
| 1           | East Azerbaijan            | 1.309  | 1.140 | 0.871 | 4.469 | 4.109 | 0.920 | 1.024 | 1.052 | 1.027 |
| 2           | West Azerbaijan            | 1      | 0.936 | 0.936 | 3.584 | 3.584 | 1     | 1.456 | 0.997 | 0.685 |
| 3           | Ardabil                    | 1.126  | 0.949 | 0.843 | 0.703 | 0.820 | 1.166 | 1.048 | 1.091 | 1.042 |
| 4           | Isfahan                    | 1.143  | 1.797 | 1.572 | 3.207 | 3.595 | 1.121 | 1.033 | 1.032 | 0.999 |
| 5           | Ilam                       | 1      | 0.607 | 0.607 | 2.627 | 2.835 | 1.079 | 1     | 0.868 | 0.868 |
| 6           | Bushehr                    | 1      | 1     | 1     | 1.781 | 2.000 | 1.123 | 1     | 1     | 1     |
| 7           | Tehran & Alborz            | 1      | 1     | 1     | 3.531 | 3.578 | 1.013 | 1.055 | 1.055 | 1     |
| 8           | Chaharmahal and Bakhtiari  | 1.229  | 0.803 | 0.653 | 2.738 | 2.667 | 0.974 | 1.012 | 1.039 | 1.026 |
| 9           | South Khorasan             | 0.987  | 0.561 | 0.568 | 3.600 | 3.830 | 1.064 | 0.984 | 1.017 | 1.034 |
| 10          | Razavi Khorasan            | 1      | 0.911 | 0.911 | 3.833 | 4.012 | 1.047 | 1     | 0.839 | 0.839 |
| 11          | North Khorasan             | 1      | 1     | 1     | 3.150 | 3.237 | 1.028 | 1     | 1     | 1     |
| 12          | Khuzestan                  | 1.345  | 0.975 | 0.725 | 3.026 | 3.237 | 1.070 | 1.309 | 1.089 | 0.832 |
| 13          | Zanjan                     | 1.077  | 1.431 | 1.328 | 3.661 | 3.507 | 0.958 | 1.059 | 1.377 | 1.300 |
| 14          | Semnan                     | 0.991  | 0.620 | 0.626 | 3.341 | 3.413 | 1.022 | 0.991 | 0.941 | 0.950 |
| 15          | Fars                       | 1      | 1     | 1     | 3.749 | 3.538 | 0.944 | 1     | 1     | 1     |
| 16          | Qazvin                     | 1.083  | 1.255 | 1.159 | 4.000 | 3.704 | 0.926 | 1.083 | 1.255 | 1.159 |
| 17          | Qom                        | 1      | 0.748 | 0.748 | 3.601 | 3.623 | 1.006 | 1     | 0.926 | 0.926 |
| 18          | Kurdistan                  | 1.034  | 0.821 | 0.794 | 3.835 | 3.242 | 0.845 | 1.074 | 1.265 | 1.178 |
| 19          | Kermanshah                 | 1      | 0.554 | 0.554 | 3.614 | 4.099 | 1.134 | 1     | 0.725 | 0.725 |
| 20          | Kerman                     | 1.473  | 1.162 | 0.789 | 3.073 | 3.407 | 1.108 | 1.030 | 1.152 | 1.118 |
| 21          | Kohgiluyeh and Boyer-Ahmad | 1      | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     |
| 22          | Golestan                   | 1      | 0.575 | 0.575 | 4.179 | 4.158 | 0.995 | 1     | 0.945 | 0.945 |
| 23          | Gilan                      | 1.076  | 1.076 | 1     | 4.067 | 3.899 | 0.959 | 1.149 | 1.149 | 1     |
| 24          | Lorestan                   | 1.085  | 0.830 | 0.764 | 2.492 | 2.606 | 1.046 | 1.077 | 1.147 | 1.065 |
| 25          | Mazandaran                 | 1      | 1     | 1     | 3.725 | 3.415 | 0.917 | 1     | 1     | 1     |
| 26          | Markazi                    | 0.683  | 0.598 | 0.875 | 3.802 | 3.523 | 0.927 | 0.765 | 1.049 | 1.372 |
| 27          | Hormozgan                  | 3.021  | 3.021 | 1     | 1     | 1     | 1     | 3.021 | 3.021 | 1     |
| 28          | Hamadan                    | 0.984  | 0.712 | 0.723 | 3.978 | 3.926 | 0.987 | 1.016 | 1.038 | 1.022 |
| 29          | Yazd                       | 1.105  | 1.491 | 1.349 | 3.517 | 3.379 | 0.961 | 1.072 | 1.187 | 1.107 |
| Progress    |                            | 13     | 8     | 4     | 26    | 26    | 14    | 16    | 17    | 12    |
| No. Regress |                            | 4      | 15    | 17    | 1     | 1     | 12    | 3     | 7     | 9     |
| Fixed       |                            | 12     | 6     | 8     | 2     | 2     | 3     | 10    | 5     | 8     |

\*In this table, years 2013, 2014, and 2015 are denoted by 13, 14, and 15, respectively.

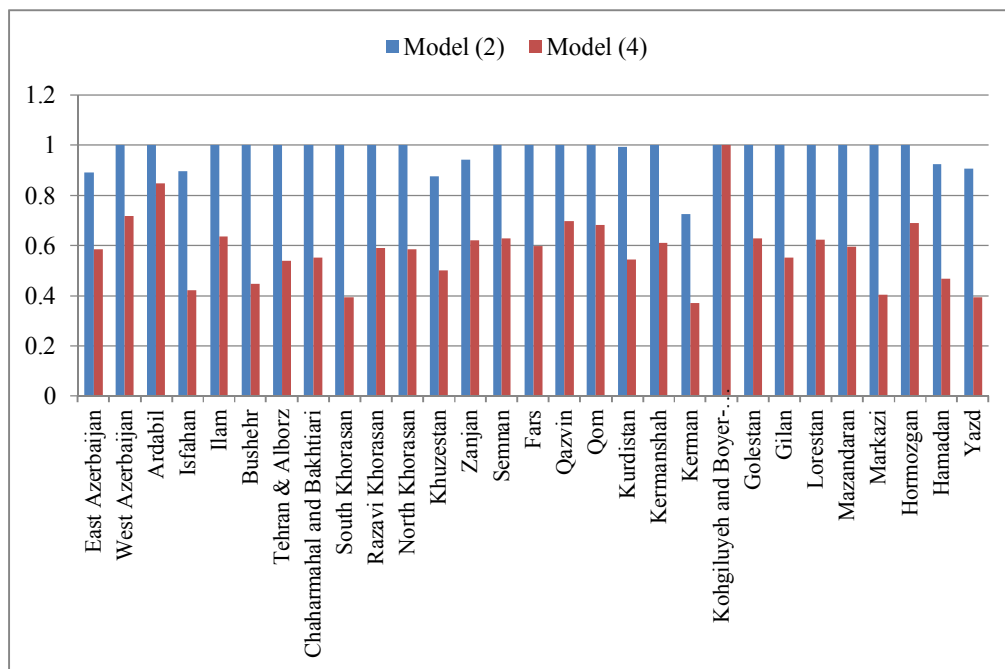
**Tab. 11. Projection points of bounded and integer measures**

|    |                            | IF    |       |       | NIB   |       |       | GS     |        |        |
|----|----------------------------|-------|-------|-------|-------|-------|-------|--------|--------|--------|
|    | Province                   | 2013  | 2014  | 2015  | 2013  | 2014  | 2015  | 2013   | 2014   | 2015   |
| 1  | East Azerbaijan            | 100   | 100   | 100   | 34619 | 27429 | 31337 | 135049 | 116654 | 104091 |
| 2  | West Azerbaijan            | 95.17 | 67    | 100   | 14830 | 12642 | 19240 | 49213  | 49806  | 51948  |
| 3  | Ardabil                    | 100   | 100   | 100   | 13269 | 12128 | 15326 | 37933  | 29369  | 33267  |
| 4  | Isfahan                    | 100   | 100   | 100   | 58768 | 48387 | 23245 | 182239 | 132699 | 73622  |
| 5  | Ilam                       | 59.5  | 62    | 85.26 | 9346  | 8450  | 13176 | 12142  | 14364  | 16861  |
| 6  | Bushehr                    | 57    | 67.5  | 79    | 22893 | 18599 | 21366 | 21765  | 19208  | 21050  |
| 7  | Tehran & Alborz            | 96.57 | 94.5  | 94    | 14045 | 24610 | 16204 | 277425 | 278533 | 208987 |
| 8  | Chaharmahal and Bakhtiari  | 100   | 100   | 100   | 6943  | 6580  | 11224 | 25285  | 17334  | 25511  |
| 9  | South Khorasan             | 65.73 | 69.36 | 69.81 | 9607  | 7992  | 14096 | 24123  | 14680  | 25077  |
| 10 | Razavi Khorasan            | 77.9  | 83.35 | 100   | 41577 | 28266 | 18891 | 143870 | 115723 | 90244  |
| 11 | North Khorasan             | 78.25 | 81.6  | 84.4  | 8861  | 9053  | 10575 | 21444  | 17617  | 15213  |
| 12 | Khuzestan                  | 100   | 76.38 | 100   | 48241 | 24577 | 36410 | 86594  | 59919  | 70665  |
| 13 | Zanjan                     | 100   | 100   | 80.2  | 11628 | 12018 | 12416 | 31867  | 26289  | 20867  |
| 14 | Semnan                     | 85.5  | 87.27 | 92.49 | 4912  | 3034  | 4258  | 18208  | 11584  | 15105  |
| 15 | Fars                       | 72.05 | 72.05 | 75.3  | 30511 | 19159 | 41201 | 81048  | 74680  | 88526  |
| 16 | Qazvin                     | 98    | 93.86 | 85.5  | 7545  | 7914  | 6721  | 30688  | 25683  | 18667  |
| 17 | Qom                        | 95.4  | 90.1  | 100   | 2422  | 3801  | 2726  | 22879  | 19034  | 20824  |
| 18 | Kurdistan                  | 100   | 100   | 100   | 13296 | 15248 | 14097 | 42350  | 33314  | 38801  |
| 19 | Kermanshah                 | 63.5  | 65.5  | 100   | 24078 | 21004 | 23385 | 35858  | 42928  | 55347  |
| 20 | Kerman                     | 100   | 100   | 100   | 22774 | 10541 | 33186 | 75280  | 60513  | 59946  |
| 21 | Kohgiluyeh and Boyer-Ahmad | 70.9  | 74.25 | 82    | 5818  | 5590  | 5603  | 12094  | 9630   | 9590   |
| 22 | Golestan                   | 86.6  | 88.9  | 100   | 13721 | 8016  | 13091 | 36383  | 29151  | 42311  |
| 23 | Gilan                      | 100   | 87.5  | 92.5  | 29801 | 29546 | 46721 | 65642  | 64223  | 67328  |
| 24 | Lorestan                   | 100   | 100   | 100   | 11114 | 11561 | 14963 | 34618  | 30923  | 38233  |
| 25 | Mazandaran                 | 100   | 92.45 | 94.1  | 39802 | 24580 | 32539 | 86548  | 65252  | 84916  |
| 26 | Markazi                    | 100   | 100   | 100   | 13605 | 15343 | 11681 | 37142  | 39853  | 47767  |
| 27 | Hormozgan                  | 4.23  | 2.6   | 6.85  | 5704  | 1121  | 1552  | 3175   | 1165   | 1125   |
| 28 | Hamadan                    | 100   | 100   | 100   | 16442 | 12446 | 19510 | 55329  | 42874  | 49904  |
| 29 | Yazd                       | 100   | 100   | 100   | 17540 | 12771 | 19064 | 46531  | 39105  | 37622  |

**Tab. 12. Results of Model (2)**

| # | Province        | Efficiency |       |       |         |
|---|-----------------|------------|-------|-------|---------|
|   |                 | 2013       | 2014  | 2015  | Overall |
| 1 | East Azerbaijan | 0.792      | 1     | 0.912 | 0.893   |
| 2 | West Azerbaijan | 1          | 1     | 1     | 1       |
| 3 | Ardabil         | 1          | 1     | 1     | 1       |
| 4 | Isfahan         | 1          | 0.744 | 1     | 0.897   |
| 5 | Ilam            | 1          | 1     | 1     | 1       |
| 6 | Bushehr         | 1          | 1     | 1     | 1       |

|    |                            |       |           |       |       |
|----|----------------------------|-------|-----------|-------|-------|
| 7  | Tehran & Alborz            | 1     | 1         | 1     | 1     |
| 8  | Chaharmahal and Bakhtiari  | 1     | 1         | 1     | 1     |
| 9  | South Khorasan             | 1     | 1         | 1     | 1     |
| 10 | Razavi Khorasan            | 1     | 1         | 1     | 1     |
| 11 | North Khorasan             | 1     | 1         | 1     | 1     |
| 12 | Khuzestan                  | 0.862 | 1         | 0.792 | 0.876 |
| 13 | Zanjan                     | 0.845 | 1         | 1     | 0.943 |
| 14 | Semnan                     | 1     | 1         | 1     | 1     |
| 15 | Fars                       | 1     | 1         | 1     | 1     |
| 16 | Qazvin                     | 1     | 1         | 1     | 1     |
| 17 | Qom                        | 1     | 1         | 1     | 1     |
| 18 | Kurdistan                  | 0.978 | 1         | 1     | 0.993 |
| 19 | Kermanshah                 | 1     | 1         | 1     | 1     |
| 20 | Kerman                     | 0.594 | 0.861     | 0.776 | 0.726 |
| 21 | Kohgiluyeh and Boyer-Ahmad | 1     | 1         | 1     | 1     |
| 22 | Golestan                   | 1     | 1         | 1     | 1     |
| 23 | Gilan                      | 1     | 1         | 1     | 1     |
| 24 | Lorestan                   | 1     | 1         | 1     | 1     |
| 25 | Mazandaran                 | 1     | 1         | 1     | 1     |
| 26 | Markazi                    | 1     | 1         | 1     | 1     |
| 27 | Hormozgan                  | 1     | 1         | 1     | 1     |
| 28 | Hamadan                    | 0.944 | 0.9009821 | 0.932 | 0.925 |
| 29 | Yazd                       | 0.795 | 1         | 0.952 | 0.907 |



**Fig. 1. The comparison of results obtained from Models (2) and (4)**

Tab. 13. Targets of integer and bounded variables obtained from Model (2)

| #  | Province                   | IF      |         |         | NIB      |          |          | GS        |          |          |
|----|----------------------------|---------|---------|---------|----------|----------|----------|-----------|----------|----------|
|    |                            | 2013    | 2014    | 2015    | 2013     | 2014     | 2015     | 2013      | 2014     | 2015     |
| 1  | East Azerbaijan            | 112.220 | 91.000  | 102.510 | 26153.89 | 21489    | 23416.27 | 102025.29 | 91392    | 77781.62 |
| 2  | West Azerbaijan            | 65.350  | 67.000  | 68.500  | 14830    | 12642    | 18000    | 49213     | 49806    | 48601    |
| 3  | Ardabil                    | 80.000  | 83.750  | 87.250  | 11550    | 11890    | 12664    | 33018     | 28793    | 27489    |
| 4  | Isfahan                    | 94.350  | 130.880 | 97.300  | 27850    | 35236.82 | 19813    | 86364     | 96635.22 | 62752    |
| 5  | Ilam                       | 59.500  | 62.000  | 74.000  | 9346     | 8450     | 8000     | 12142     | 14364    | 10237    |
| 6  | Bushehr                    | 57.000  | 67.500  | 79.000  | 22893    | 18599    | 21366    | 21765     | 19208    | 21050    |
| 7  | Tehran & Alborz            | 91.500  | 94.500  | 94.000  | 14045    | 24610    | 16204    | 277425    | 278533   | 208987   |
| 8  | Chaharmahal and Bakhtiari  | 88.100  | 89.200  | 91.500  | 4513     | 5256     | 5863     | 16436     | 13846    | 13326    |
| 9  | South Khorasan             | 53.000  | 55.000  | 57.250  | 9607     | 7891     | 7910     | 24123     | 14493    | 14072    |
| 10 | Razavi Khorasan            | 77.900  | 83.350  | 83.850  | 41577    | 28266    | 17211    | 143870    | 115723   | 82218    |
| 11 | North Khorasan             | 78.250  | 81.600  | 84.400  | 8861     | 9053     | 10575    | 21444     | 17617    | 15213    |
| 12 | Khuzestan                  | 85.730  | 73.850  | 101.670 | 40218.14 | 23764    | 32226.42 | 72191.74  | 57937    | 62544.47 |
| 13 | Zanjan                     | 85.930  | 76.900  | 80.200  | 9620.85  | 9044     | 12416    | 26365.01  | 19783    | 20867    |
| 14 | Semnan                     | 85.500  | 86.500  | 87.000  | 4912     | 3007     | 2638     | 18208     | 11482    | 9360     |
| 15 | Fars                       | 72.050  | 72.050  | 75.300  | 30511    | 19159    | 41201    | 81048     | 74680    | 88526    |
| 16 | Qazvin                     | 78.100  | 81.000  | 85.500  | 6013     | 6830     | 6721     | 24456     | 22165    | 18667    |
| 17 | Qom                        | 95.400  | 90.100  | 92.590  | 2422     | 3801     | 2039     | 22879     | 19034    | 15579    |
| 18 | Kurdistan                  | 70.240  | 73.850  | 86.950  | 10804.15 | 12533    | 9202     | 34413.37  | 27381    | 25328    |
| 19 | Kermanshah                 | 63.500  | 65.500  | 72.500  | 24078    | 21004    | 12957    | 35858     | 42928    | 30667    |
| 20 | Kerman                     | 83.340  | 59.210  | 73.410  | 18263.68 | 8577.02  | 23625.13 | 60372.16  | 49238.89 | 42675.41 |
| 21 | Kohgiluyeh and Boyer-Ahmad | 70.900  | 74.250  | 82.000  | 5818     | 5590     | 5603     | 12094     | 9630     | 9590     |
| 22 | Golestan                   | 86.600  | 88.900  | 94.500  | 13721    | 8016     | 7523     | 36383     | 29151    | 24316    |
| 23 | Gilan                      | 87.000  | 87.500  | 92.500  | 27692    | 29546    | 46721    | 60995     | 64223    | 67328    |
| 24 | Lorestan                   | 70.250  | 75.600  | 80.550  | 10173    | 11483    | 11356    | 31685     | 30714    | 29017    |
| 25 | Mazandaran                 | 100.000 | 92.450  | 94.100  | 39802    | 24580    | 32539    | 86548     | 65252    | 84916    |
| 26 | Markazi                    | 93.150  | 71.250  | 97.820  | 11632    | 8964     | 5970     | 31756     | 23283    | 24414    |
| 27 | Hormozgan                  | 1.400   | 2.600   | 6.850   | 1890     | 1121     | 1552     | 1052      | 1165     | 1125     |
| 28 | Hamadan                    | 100.870 | 107.380 | 106.060 | 11840.1  | 9240.06  | 10138.29 | 39844.36  | 31832.42 | 25931.87 |
| 29 | Yazd                       | 87.450  | 74.500  | 86.640  | 10877.49 | 6962     | 14711.18 | 28855.23  | 21318    | 29030.63 |

## 5. Conclusions

Performance analysis of organizations in several periods of time and their sustainability performance measurement are notable points for making decisions and developing plans. Moreover, discrete and bounded performance measures are introduced in many areas of application. Accordingly, DEA-based approaches were developed in this paper to evaluate the overall multi-period sustainability performance while discrete and bounded factors were available. Furthermore, the efficiency of each sustainability dimension for each period was estimated. In addition, internal relationships of dimensions were investigated. Afterwards, performance changes over time

were addressed for sustainability and its dimensions. The suggested approach was clarified by a numerical example and a case study of gas companies. The current findings add to a growing body of literature concerning the performance analysis of sustainability with discrete and/or bounded elements over several periods of time.

The proposed approach can be extended to situations that Likert scales and imprecise data are presented. Further, multi-period sustainability performance measurement in the presence of negative data is an interesting topic for future research.

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