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A Game Theory Approach to Multi-Period Planning of Pricing, Ordering, and Inventory Decisions for a Make-to-Order Manufacturing Supply Chain

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Nash game;
Make-to-order;
Production planning.

ABSTRACT

Supply chain members coordinate with each other in order to obtain more profit. The major mechanisms for coordination among supply chain echelons are pricing, inventory management, and ordering decisions. This paper concerns these mechanisms in a multi-echelon supply chain consisting of multiple suppliers, one manufacturer, and multiple retailers in order to study the price and lead-time competition, where the make-to-order production mode is employed and consumers are sensitive to retail price and lead time. In the current study, a novel inventory model is presented, where the manufacturer has an exclusive supplier for every required component of its final product. The interactions and decisions of the firms are observed in multiple time periods. Moreover, each supply chain member has equal power and makes their decisions simultaneously. The proposed model considers the relationships among three-echelon supply chain members based on a non-cooperative Nash game with pricing and inventory decisions. An iterative solution algorithm is proposed to determine the Nash equilibrium point of the game. An example is presented to study the application of the model as well as the effectiveness of the algorithm.

1. Introduction

Make-to-order (hereafter MTO) is a production trend that typically allows consumers to purchase products that are customized to their specifications. This trend allows for greater product variety and flexibility. MTO is also referred to as a process in which the production of an item begins only after a confirmed customer order is received.

The development of MTO stems from the need for variety and flexibility, leading to higher customer satisfaction. Thus, numerous firms implement MTO production strategies.

These firms compete by offering short lead times as well as appealing prices. An MTO manufacturer quotes a delivery lead time to satisfy consumers' demands. In recent years, game theory has been used to study interactions among rational firms in a supply chain. Game theory is rooted in mathematics ***and is*** the formal ***study of*** decision-making, where at least two firms must make choices that potentially affect the interests of the other firms.

The main departure of this study from the current literature is in modeling an MTO supply chain as a three-echelon non-cooperative game with multiple retailers, a single manufacturer, and

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multiple suppliers using a Nash game to optimize the strategic decisions of the players across multiple time periods.

Similar to Qin [2], the manufacturer purchases multiple components according to their needs from multiple suppliers, produces the finished items, and wholesales them to its retailers. Therefore, this supply chain has three echelons of members: the suppliers of raw materials, the manufacturer, and the retailers. The main decisions of the suppliers are the prices of the raw materials and quantity of raw materials. The manufacturer determines the common production interval, wholesale prices, and the required amount of components to optimize his net profit. Finally, the retailers buy each product from the manufacturer with specific replenishment.

The suppliers compete at the bottom level with each other. Simultaneously, they play with the manufacturer in another game. Moreover, the retailers formulate non-cooperative game and, at the same time, compete with the manufacturer as a whole game, too. We have the Nash game, in which no player has anything to gain by changing only its own strategy. In other words, if each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged, then the current set of strategies is a Nash equilibrium. It further extends the literature in this area by regarding the problem as an M/M/1 model in queuing theory. These features of the proposed model would be useful for realistic MTO supply chains in helping them optimize their practical supply chain decisions. The initial framework whose work is based upon was presented by Qin [2].

The rest of this paper is organized as follows. In Section 2, the related researches are briefly reviewed. The assumptions and the notations used for the mathematical model are presented in Section 3. Section 4 describes the analytical and computational methods for solving the models. In Section 5, a numerical example is reported. Eventually, conclusion and Future Research are included in Section 6.

2. Literature Review

Today, firms do not compete individually on the market; however, they deliver their goods or services to customers as members of a supply chain. Supply chains are complex systems that plan, implement, and control the efficient, effective forward and reverse flow as well as storage of goods, activities, and information through all the members in order to meet customers' requirements and reduce the total delivered cost to customers. The members of the supply chain make their decisions such as price and quantity decisions, creating the highest value for the entire supply chain network[3].

This means that a supply chain is dependent on its members to improve ***its*** overall performance. Game theory has become an essential tool in the analysis of supply chains with multiple members or decision makers, often with conflicting objectives.

Giri and Sharma [4] studied a two-echelon supply chain comprising one manufacturer and two competing retailers with advertising cost-dependent demand using the Stackelberg game models.

On a larger scale, Croxton et al. [3] studied a three-level supply chain system in which market demand correlates with product green degree using the Stackelberg and cooperative game models.

Azari Khojasteh et al. [5] developed a model for a real-world case problem as a price competition model between two leader-follower supply chains where each of them consists of a manufacturer and a retailer. They explored the effect of varying the level of substitutability coefficient of two products on the profits of the leader and follower supply chains.

Esmaeili et al. [6] considered advertising, pricing, and service decisions simultaneously to coordinate the supply chain with a manufacturer and a retailer. Three well-known approaches to game theory, including the Nash, the Stackelberg-retailer, and the cooperative game, are exploited to study the effects of these policies on the supply chain.

Rasti-Barzoki et al. [7] considered a dual-channel supply chain containing one manufacturer and two retailers. They assumed that the manufacturer and retailers have the same decision powers. They established a Nash model to obtain the equilibrium decisions in the decentralized case. Then, they developed a centralized model to maximize the total profit of the whole system. Finally, the equilibrium decisions are discussed, and some managerial insights are revealed.

There are several types of researches that have considered coordination of inventory and production decisions in a multi-echelon supply chain. Sana et al. [8] proposed an integrated production-inventory model for a three-echelon supply chain and determined optimal order size

of raw materials, production rate and unit production cost, and idle times.

Jiang et al. [9] presented a new model for a multi-echelon supply chain where both pricing and inventory decisions were determined in each echelon. Aust and Buscher [10] investigated production lot-size problem for perfect and imperfect products in a three-layer supply chain.

Based on Chung et al. [11] review, the superior supply chain component is a bilateral monopoly consisting of one manufacturer and one retailer, while only few papers have studied the interaction of more than two players. Huang et al. [12] proposed a three-echelon model for a pricing game and presented mathematical models for three contract mechanisms. In addition, it was mentioned that multi-echelon models combined with the analysis of more than two decision variables, e.g., advertising, pricing, and quality, could be a promising research area.

For simultaneous games, the Nash equilibrium is the most widely accepted solution approach. Nash equilibrium for analyzing the game in supply chains is presented in many articles to show the independence of supply chain members and improve the profits.

Giri and Sharma [4] explained Bertrand and Cournot models, which are some examples of simple models that consider the Nash equilibrium, i.e., each individual producer chooses its output to maximize its profit given that its rivals' outputs are fixed. Nash equilibrium was also used by Zhang and Liu [13], in which one supplier and one retailer determined the pricing strategies simultaneously. They demonstrated that the price discount contracts outperformed compared to the non-contract scenarios.

Ordering decisions, such as order price and order quantity, are the most important coordination mechanisms in the supply chain and are applied to improve the profit of both of the supply chain and individual firms. This paper investigates a supply chain with multiple suppliers, one manufacturer, and multiple retailers who are involved in supplying raw materials, producing, and selling finished products, respectively.

The manufacturer purchases multiple components from multiple suppliers and produces the finished products and wholesales them to the retailers, who receive the order of products and, finally, sell the products to end customers. This supply chain, therefore, has three echelons of members, component suppliers, a manufacturer, and retailers [14].

There are studies on supply chains that assume the market demand is a linear or non-linear function of price and lead time. Quoted lead time has become an important dimension of competition in different supply chains. Xiao et al. [15] argued that a major decision variable in supply chain models is delivery lead time. They studied the price and lead-time competition between a supply chain and an integrated rival.

Penalties are placed when the delivery time exceeds the specified time to keep the demand [2].

According to many studies in an MTO environment, delivery lead time is often an important factor in winning orders besides retail price, i.e., market demand is sensitive to both price and lead time. A few studies have adopted the game-theoretic approach in an MTO supply chain.

3. Problem Description

A three-echelon supply chain consisting of multiple suppliers, a single manufacturer, and multiple retailers was presented, where suppliers sell components to the manufacturer. The manufacturer produces products and wholesales them to retailers and, finally, sell the products to end customers. Fig. 1 shows the relations among the supply chain members.

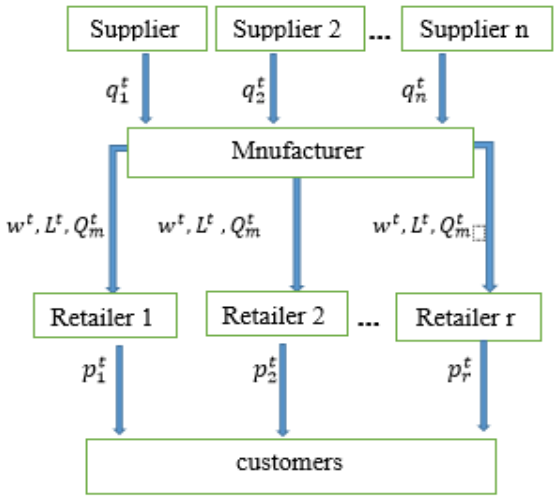

Fig. 1. Three-stage supply chain model

3-1. Model assumptions

The following assumptions are considered for the proposed model of this paper:

1. Demand is sensitive to price and lead time. It follows the non-linear market demand curve $D_r^t = \beta p_r^{-y} L^{-\gamma}$, as adopted by various works.
2. Shortage is not permitted. Therefore, the total production rate of products is greater than demand rate.
3. All parameters of supply chain members are deterministic and known in advance.
4. For each type of the component of each product that the manufacturer produces and wholesales it to retailers, at least, two suppliers are available. Each supplier is also distinct in a way that it only supplies one type of component to the manufacturer. In other words, the manufacturer buys each component from a supplier that specializes in that component.
5. The price at which suppliers purchase their raw materials is a decreasing function of the order quantity of the suppliers (i.e., a higher order quantity results in a lower price). This function is dictated by the suppliers' supplier.
6. In unit time, there are always the same orders for the customized products in a certain market; therefore, the manufacturer serves for the same number of customers at each time, and the problem can be regarded as the M/M/1 model in queuing theory. In the case of Assumption (6), similar to [2], it is assumed that the problem can be reduced as the M/M/1 model in the queuing theory, and the reach time between each batch follows a negative exponential distribution. Similarly, the negative exponential distribution with parameter θ is followed, and $1/\theta$ denotes the average time of customization performed by the manufacturer; thus, the density function is:

$$f(l^t) = \theta e^{-\theta l^t}, \qquad l^t > 0 \quad (\theta > 0) \tag{1}$$

The distribution function is

$$F(l^t) = 1 - e^{-\theta l^t}, (\theta > 0) \tag{2}$$

3-2. Sets

n Components of manufacturer n ∈ {1,2. . .,N}

t Time periods t ∈ {1,2. . .,T}

3-3. Model parameters

τ^t Lateness in lead time in period t.

C_m^t Unit cost of manufacturer in period t

L^t	The time of customization promised by manufacturer in period t.
I_n^t	Manufacturer's inventory level of component n in period t.
C_n^t	Purchasing cost of supplier n in period t.
F_n^t	Wholesale price of supplier n in period t.
λ_r^t	Unit penalty cost per duration of lateness paid for the retailer when the manufacturer cannot deliver customized product in period t.
λ_c^t	Unit penalty cost per duration of lateness paid for the customer when retailer r cannot deliver customized product in period t
l^t	The actual time of customization performed by manufacturer; $F(l^t)$ denotes the distribution function
Hm_n	holding cost of manufacturer for n[th] component
π_m	Profit function of manufacturer.
π_{R_r}	Profit function of retailer r.
π_{N_n}	Profit function of supplier n.
H_n	Unit holding cost of supplier n in period t.
C_P^t	Total production cost of manufacturer.
D_r^t	Market demand in period t ($D_r^t = \beta p_r^{-t} L^{-\gamma}$).
β	Price/Lead-time sensitivity of demand.
y	Retailer-*r*'s effectiveness of retail price.
γ	Retailer-*r*'s effectiveness of lead time.
v_n	Number of a certain type of raw material in one unit of component n.
$h(q_n^t)$	Supplier's decreasing function of price discount
k_n	Number of component n to produce one unit of manufacturer's output

3-4. Decision variables manufacturer's decision variables

$\boldsymbol{Q_m^t}$	Production quantity in period t
$\boldsymbol{L^t}$	The time of customization promised by distributor
$\boldsymbol{w^t}$	Wholesale price in period t
	Supplier's decision Variables
$\boldsymbol{q_n^t}$	Order quantity of supplier n for component n in period t.
	Retailer's decision Variables
$\boldsymbol{p_r^t}$	Retail price of retailer r in period t

3-5. The retailers' model formulation

In the MTO production mode, there exists a delivery lead time, besides price. The market demand of retailer *r* is linear with respect to product price and promised delivery time, and demand decreases as price increases, so does promised delivery time. However, there is no relation between product price and promised delivery time. Thus, they are incorporated into our demand model. The demand of retailer *r* is given by the following:

$$D_r^t = \beta p_r^{-y} L^{-\gamma} \ , t, \gamma > 0 \quad (3)$$

Each retailer's main objective is to maximize his net profit by optimizing his decision variables including retail price. These sets of decision variables are known as strategy X_{R_r}. Thus, the retailer-r's net profit can be calculated as the total sales revenue of its product minus the purchasing cost from the manufacturer; the penalty cost ($\lambda_r^t \tau^t D_r^t$) is paid to the retailer by the manufacturer as a result of delayed delivery, and the penalty cost ($\lambda_c^t \tau^t D_r^t$) per product is paid to the customer by the manufacturer when the delivery is delayed, given as follows:

$$\begin{aligned} & max\, \pi_{R_r} = \sum_t [(p_r^t - w^t) D_r^t + \lambda_r^t \tau^t D_r^t - \lambda_c^t \tau^t D_r^t] \\ & \text{s.t.} \\ & D_r^t = \beta p_r^{-ty} L^{-t\gamma} \\ & Q_m^t = \sum_r D_r^t \\ & p_r^t \geq w^t, \quad \forall t, r \\ & \tau^t = \sum_t [\int_{L^t}^{\infty} (l^t - L^t) f(l^t)\, dl^t] \end{aligned} \quad (4)$$

3-6. The manufacturer's model formulation

It is assumed that the unit wholesale price w^t, $\boldsymbol{L^t}$, and Q_m^t are the manufacturer's decisions. Then, the strategy of the manufacturer is denoted by X_m. The manufacturer's net profit equals the wholesales revenue of products to all retailers minus the total production cost, and the penalty cost ($\lambda_r^t \tau^t \sum_r D_r^t$) is paid to the retailer by the manufacturer as a result of delayed delivery and holding costs with the inventory level of component n in period t, given as follows:

$$\begin{aligned} & max\, \pi_m = \sum_t (w^t - C_m^t) Q_m^t - \lambda_r^t \tau^t \sum_r D_r^t - \sum_n Hm_n (I_n^t) \\ & \text{s.t.} \\ & C_m^t = \sum_n k_n F_n^t + \frac{C_p^t}{Q_m^t}, \quad \forall t, n \\ & I_n^t = I_n^{t-1} + q_n^t - k_n Q_m^t \ \forall t, n \\ & D_r^t = \beta p_r^{-y} L^{-\gamma} \\ & Q_m^t = \sum_r D_r^t \\ & w^t \geq C_m^t, \quad \forall t \\ & \tau^t = \sum_t [\int_{T_c^t}^{\infty} (l^t - L^t) f(t_c^t)\, dt_c^t] \\ & Q_m^t, w^t, L^t \geq 0, \quad \forall t \end{aligned} \quad (5)$$

3-7. The supplier's model formulation

It is assumed that q_n^t is each supplier's decision. Then, this strategy of suppliers is denoted by X_{N_n}. Each supplier's net profit equals the wholesales revenue of products to all manufacturers minus the total production cost and holding costs with the inventory level of component n in period t: given as follows**:**

$$max\,\pi_{N_n} = \sum_t (F_n^t - \upsilon_n C_n^t) q_n^t - H_n(I_n^t)$$

s.t.

$$\begin{aligned} &I_n^t = I_n^{t-1} + q_n^t - \upsilon_n(q_n^t) \quad \forall t, s \\ &C_n^t = h(q_n^t) \quad \forall t, s \\ &F_n^t > 0 \\ &q_n^t = \{1,2,3,\dots\} \end{aligned} \tag{6}$$

4. Solution Method

The above formulation for a three-echelon supply chain is considered by using a non-cooperative game theory approach with S + 1 + R players in which there are S suppliers, one manufacturer, and R retailers. In this research, in order to analyze the strategies of suppliers, manufacturers, and retailers, the concept of Nash equilibrium is applied.

4-1. Nash game

Suppliers, manufacturer, and retailers' decision problems are solved separately in order to determine the Nash equilibrium. However, unfortunately, this model cannot be solved parametrically; therefore, a repetitive algorithm has been introduced. In addition, π_m is increasing in line with w^t, which means that the optimal value for w^t is p_r^t. However, $p_r^t > w^t$ cannot be equal to p_r^t; in other words, there would be no profit for both sides. A similar approach as previously suggested in Xie and Neyret [16] has been used to handle the problem. It is assumed here that each retailer will not sell the product if she does not obtain a minimum unit margin. The manufacturer's unit margin is taken as such minimum level, and the wholesale price constraint is replaced with:

$$\mu_{R_r} > \mu_m \rightarrow p_r^t - w^t > w^t \rightarrow w^t \leq \frac{p_r^t}{2} \tag{7}$$

where $\mu_{R_r} = p_r^t - w^t$, and μ_m is equal to w^t, and they are retailers' and manufacturer's unit margins, respectively. Thus, the optimal value of w^t is $\frac{p_r^t}{2}$. The first-order conditions for the suppliers, manufacturer, and the retailers are as follows:

$$\frac{\partial\,\pi_{N_n}}{\partial q_n^t} = (F_n^t - H_n(1-\upsilon_n) - 2h * q_n^t * \upsilon_n) = 0 \tag{8}$$

$$\frac{\partial \pi_m}{\partial Q_m^t} = -F_n^t k_n + Hm_n k_n + w^t > 0 \tag{9}$$

$$\frac{\partial \pi_m}{\partial w^t} = Q_m^t > 0 \tag{10}$$

$$\frac{\partial \pi_m}{\partial L^t} = -\beta L^{-t\Upsilon} p_r^{-ty} \lambda_r^t\, e^{-\theta L^t} + \beta L^{-t(\Upsilon+1)} p_r^{-ty} \lambda_r^t \Upsilon \int_L^{\infty} \theta e^{-\theta l^t} (l - L^t)\, dl = 0 \tag{11}$$

$$\frac{\partial \pi_{R_r}}{\partial p_r^t} = \beta L^{-ty} p_r^{-t\Upsilon} - \beta L^{-ty} p_r^{-t(1+\Upsilon)} (p_r^t - w^t)\Upsilon - \beta L^{-ty} p_r^{-t(1+\Upsilon)} (\lambda_r^t - \lambda_c^t)\tau^t \Upsilon = 0 \tag{12}$$

Therefore, by equating the first-order partial derivative of the player's profits to zero, regarding the relevant decision variables and, also, by solving all the derived equations simultaneously, one can obtain the following results from the Nash equilibrium.

$$p_r^t = \frac{w^t\Upsilon - \lambda_r^t \tau \Upsilon + \lambda_c^t \tau \Upsilon}{-1 + \Upsilon} \tag{13}$$

$$q_n^t = \frac{F_n^t - H_n + H_n \upsilon_n}{2h\upsilon_n} \tag{14}$$

$$w^t = p_r^t/2 \tag{15}$$

$$Q_m^t = \sum_r \beta p_r^{-y} L^{-\gamma} \tag{16}$$

$$L^t \epsilon \; argmax_{L^t} \; \pi_m(L^t) \tag{17}$$

We failed to analytically solve Eqs. (13-17). For calculating the Nash equilibrium of a two-echelon closed-loop supply chain numerically, the following solution algorithm is presented based on Gauss-Seidel decomposition presented by Cai et al. [17], where X is denoted as the strategy set of the supply chain member. Thus, $X_{N_n}, X_m, and\ X_{R_r}$ are the strategy profile sets of the supplier n, manufacturer, and retailer r strategies, respectively. A measure for the completion of algorithm is introduced; if $\left|X_I^* - X_i^0\right|$ is lower than ε, algorithm is accomplished and available solution is close enough to the equations' solution. The following repetitive algorithm is used for solving the non-cooperative game model:

Step 0: Give the initial strategy profile for all members $X^0 = \left(X_{N_n}^0, X_m^0, X_{R_r}^0\right)$, where it is feasible for all members.

Step 1: In retailers' model, if strategies of manufacturer and suppliers achieved from Step 0 are fixed, then obtain the optimal responses of retailers $X_{R_r}^* = p_r^*$.

Step 2: At the manufacturer's level, if the strategies of retailers and suppliers respectively achieved from Steps 1 and 0 are fixed, then calculate the best response of manufacturer $X_m^* = (Q_m^*, w^*, L^*)$.

Step 3: Calculate the best responses of suppliers $X_{N_n}^* = (q_n^*)$ based on optimal strategy of manufacturer and retailers obtained from Step 2 and Step 1, respectively.

Step 4: For the whole supply chain, determine the following. If $\left|X_{N_n}^* - X_{N_n}^0\right| \le \varepsilon$, $|X_m^* - X_m^0| \le \varepsilon$, and $\left|X_{R_r}^* - X_{R_r}^0\right| \le \varepsilon$, the Nash Equilibrium is obtained. Output the optimal results and stop. If $X_{N_n}^0 = X_{N_n}^*$, $X_m^0 = X_m^*$, and $X_{R_r}^0 = X_{R_r}^*$, go to Step 0. (ε is a very small positive number).

Proposition: To prove the optimality of these solutions, the second-order derivatives are calculated.

Proof. The second-order derivatives are presented as follows:

$$\frac{\partial^2 \pi_{R_r}}{\partial {p_r^t}^2} - \beta L^{-ty} p_r^{t(-1-\Upsilon)} \Upsilon - \beta L^{-ty} p_r^{t(-2-\Upsilon)} (\lambda_r^t - \lambda_c^t) \tau (-1-\Upsilon) \Upsilon + \frac{1}{2} \beta L^{-ty} p_r^{t(-1-\Upsilon)} \Upsilon^2 \tag{18}$$

The function of π_{R_r} is a concave function of p_r^t

(since, $\frac{\partial^2 \pi_{R_r}}{\partial {p_r^t}^2} < 0$) .

$$H_{N_m}^t = \begin{bmatrix} \frac{\partial^2 \pi_m}{\partial {Q_m^t}^2} & \frac{\partial^2 \pi_m}{\partial w^t \partial Q_m^t} & \frac{\partial^2 \pi_m}{\partial L^t \partial Q_m^t} \\ \frac{\partial^2 \pi_m}{\partial Q_m^t \partial w^t} & \frac{\partial^2 \pi_m}{\partial {w^t}^2} & \frac{\partial^2 \pi_m}{\partial L^t \partial w^t} \\ \frac{\partial^2 \pi_m}{\partial Q_m^t \partial L^t} & \frac{\partial^2 \pi_m}{\partial w^t \partial L^t} & \frac{\partial^2 \pi_m}{\partial {L^t}^2} \end{bmatrix} \tag{19}$$

The second-order partial derivatives are as follows:

$$\frac{\partial^2 \pi_m}{\partial {Q_m^t}^2} = \frac{\partial^2 \pi_m}{\partial {w^t}^2} = \frac{\partial^2 \pi_m}{\partial Q_m^t \partial w^t} = \frac{\partial^2 \pi_m}{\partial Q_m^t \partial L^t} = \frac{\partial^2 \pi_m}{\partial w^t \partial L^t} = 0 \tag{20}$$

$$\left(\frac{\partial^2 \pi_m}{\partial {L^t}^2} = -\beta \theta e^{-\theta l^t} L^{t(1-\Upsilon)} . p_r^{-ty} \lambda_r^t + \beta L^{t(1-\Upsilon)} p_r^{-ty} \lambda_r^t \Upsilon \int_L^\infty -\theta e^{-\theta l^t} \, dl + \beta L^{t(-2-\Upsilon)} \, p_r^{-ty} \lambda_r^t (-1-\Upsilon) \Upsilon \int_L^\infty \theta e^{-\theta l^t} (l - L^t) \, dl^t < 0 \right. \tag{21}$$

Considering the second-order partial derivatives of π_m with respect to Q_m^t, L^t, and w^t, we obtain that the Hessian matrix of the total profit of the manufacturer is negative definite. Thus, π_m is a

concave function of Q_m^t, L^t, and w^t. The function of π_{N_n} is a concave function of q_r^t (since, $\frac{\partial^2 \pi_{N_n}}{\partial {q_r^t}^2} = -2hv_n < 0$).

5. Numerical Example

A supply chain consisting of two suppliers, one manufacturer, and two retailers are assumed where each supplier produces two components from two raw materials. It is supposed that the average time of customization performed by the manufacturer, l^t, follows the negative exponential distribution with Parameter **3**. All of the parameters are generated randomly from uniform distributions. Data for the example can be found in Appendix A.

Tab. 1. Results for the example

Objective (Profit):	
Manufacturer	112,412.7
Retailer 1	27,302.60
Retailer 2	
Supplier1	37,405.70
Supplier2	29,656.06
Suppliers' Decisions	
(q_1^1)	4
(q_2^1)	20
manufacturer's Decisions:	
(Q_m^1)	4800
(L^1)	10
(w^1)	37.6
Retailers' s Decisions:	
(p_1^t)	75.2
(p_2^t)	75.2

The example is solved by the proposed algorithm described in Section 4.1. The algorithm converges to the Nash equilibrium point after a few iterations in a reasonable amount of time. Furthermore, to validate the results, a Branch and Bound algorithm was applied as an alternative approach to solving the mixed integer non-linear sub problems of the suppliers described in Section 3.7. The Branch and Bound algorithm was coded in GAMS 24.1.2 software and solved by the SBB solver. The results show that both the proposed algorithm and the Branch and Bound algorithm converge to the same Nash equilibrium point.

6. Conclusion and Future Research

This paper presented an integrated framework for the use of a Nash game model to analyze the interaction between the suppliers, a manufacturer, and its retailers under an MTO supply chain. The proposed model is a mixed-integer non-linear programming (MINLP) problem indexed with multiple time periods and components.

Unlike several research studies that have considered MTO systems mainly focused on just a few parameters and adopted an additive form of demand function, this paper considered pricing, ordering, and even production decisions such as inventory levels in a three-echelon competitive supply chain including multiple retailers, one manufacturer, and multiple suppliers. An inventory model was formulated as a non-cooperative game.

The suppliers compete with each other and with the manufacturer simultaneously. At the same time, the retailers compete with the manufacturer. To obtain Nash equilibrium, an iterative solution algorithm was proposed. In order to interrogate the proposed model and solution algorithm, a numerical example was provided. For future research, it is interesting to extend the model to consider the competition for large-sized problems with more products. Cooperative game is also worth addressing.

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Appendix A: Collected Data for example

Parameter and Description	Example
C_m^1	17.2
I_1^1	1
I_2^1	0
C_1^1	10
C_2^1	10
F_1^1	27.7
F_2^1	12.6
λ_r^1	6
λ_c^1	8
C_P^1	1000
β	0.6
y	0.2
γ	0.4
k_1	0.5
k_2	0.25
(υ_1)	1
(υ_2)	1
$h(q_1^t)$	20
$h(q_2^t)$	30
Hm_1	5
Hm_2	10

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