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Condition Based Maintenance for Two-Component Systems with Reliability and Cost Considerations

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ABSTRACT

This paper studies a maintenance policy for a system composed of two components, which are subject to continuous deterioration and consequently stochastic failure. The failure of each component results in the failure of the system. The components are inspected periodically and their deterioration degrees are monitored. The components can be maintained using different maintenance actions (repair or replacement) with different costs. Using stochastic regenerative properties of the system, a stochastic model is developed in order to analyze the deterioration process and a novel approach is presented that simultaneously determines the time between two successive inspection periods and the appropriate maintenance action for each of the components based on the observed degrees of deterioration. This approach considers different criteria like reliability and long-run expected cost of the system. A numerical example is provided in order to illustrate the implementation of the proposed approach.

1. Introduction

A crucial issue in systems performing under stressful environmental conditions is to guarantee the satisfactory reliability of their performance. In most of these systems such as power plants and offshore structures, equipments are subject to random deteriorations [1]. These deteriorations can result in unexpected failures and consequent disastrous effects on safety and the economy. It implies that effective maintenance policies which prevent failures and increase the reliability of systems are of significant importance [2]. Traditional maintenance actions are performed just based on preventive purposes regardless of the deterioration degrees of equipments [3]. These types of maintenance can reduce high deteriorations and failure; however, they may lead to large system unavailability and unnecessary maintenance costs. In order to overcome this problem, condition based maintenance (CBM) is introduced. CBM is a dynamic preventive maintenance practice, in which the decisions of maintaining the system is made based on the observed condition of the system [4]. It has been proven that CBM is very effective in practice, since it can provide satisfactory levels of reliability and save resources by avoiding unnecessary maintenances [5]. As a result, numerous mathematical models have been developed in the area of CBM [6].

Generally CBM models can be divided into two categories, models studying completely observable systems and models considering partially observable systems. In a completely observable system it is possible to identify the state of system entirely. There is a lot of research that investigates CBM models for completely observable systems. Grall et al. [7] focus on the analytical modeling of a condition based inspection/replacement policy for a stochastically and

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continuously deteriorating single component system. They consider both the replacement threshold and the inspection schedule as decision variables. Their model minimizes the long run expected cost per component time by using the stationary law for the system state.

Amari and McLaughlin [8] utilize a Markov chain to describe the CBM model for a deteriorating system subject to periodic inspection. In their study, the optimal inspection frequency and maintenance threshold are found in order to maximize the system availability.

Castanier et al. [3] consider a two components system which can be maintained by good as new preventive or corrective replacements. They develop a stochastic model based on the semi-regenerative properties of the maintained system state and apply the associated cost model to optimize the performance of the maintenance model. Barata et al. [9] use Monte-Carlo simulation to model the continuously monitored deteriorating system. They assume that after each maintenance action a random level of improvement is made on the state of the system which is independent of current system state. Then they determine thresholds of maintenance to minimize the total expected cost of system.

Dieulle et al. [10] study a continuously deteriorating system which is inspected at random times. They assume that deterioration follows a gamma distribution and the system fails if its condition lies upper than a pre-specified threshold. In their model two types of replacement can be done depending on whether the system is failed or the condition of the system exceeds a critical threshold.

Van der Weide et al. [11] present a conceptually comprehensive derivation of formulas for computing the discounted cost associated with a maintenance policy combining both condition-based and age-based criteria for preventive maintenance. Liao et al. [12] consider a condition-based maintenance model for continuously degrading systems under continuous monitoring.

After maintenance, the states of the system are randomly distributed with residual damage. They examine a realistic maintenance policy, referred to as condition-based availability limit policy, which achieves the maximum availability level of such a system. The optimum maintenance threshold of their model is determined using a search algorithm.

There has been some research that examines CBM models for partially observable system in which the state of the system is not identified completely. Barbera et al. [13] propose a CBM model which assumes that failure rate of the system depends on the variables of the system state and fixed inspection periods. In their model the maintenance action is optimized so that the long term cost of maintenance actions and failures are minimized. Marseguerra et al. [2] investigate a continuously monitored multi-component system and use a generic algorithm for determining the optimal degradation level beyond which Preventive Maintenance is to be performed. Kumar and Westberg [14] suggest an approach based on reliability where inspection periods and maintenance thresholds are estimated in order to minimize the global cost per component time. Wang and Christer [15] consider a stochastic dynamic system subject to random deterioration, with regular condition monitoring and preventive maintenance. They propose the model that determines what maintenance action to take based upon the condition monitoring and preventive maintenance information obtained to date. A general assumption adopted in their paper is that the performance of the system concerned cannot be described directly by the monitored information, but is correlated with it stochastically.

Chen and Trivedi [16] have built the semi-Markov decision process for the maintenance policy optimization of condition based preventive maintenance problems and present an approach for joint optimization of inspection rates and maintenance policies.

Jamali et al. [17] have developed a joint optimal periodic and conditional maintenance strategy for CBM problems under budgetary constraints. Wang [18] applies a stochastic recursive control model for CBM optimization based on the assumptions that the item monitored follows a two-period failure process with the first period of normal life and the second period of potential failure.

In this problem a stochastic recursive filtering model is used to predict the residual, and then a decision model is considered to recommend the optimal maintenance actions. Also, the optimal condition monitoring intervals are determined by a hybrid of simulation and analytical analysis. Goode et al. [19] study a model that provides the necessary basis to optimize condition monitoring intervals.

In order to formulate the CBM problems, typically two main types of modeling methods have been used in the literature. The first type formulates the problem using Markov decision process [2, 8, 20, 21, 22] in which the deteriorating process is considered as a multi state system. The other type of modeling applies renewal models to formulate the maintenance policies [3, 7, 10, 13, 23, 24, 25, 26, 27], where the system deterioration is considered as continuous and stochastic [28, 29, 30].

Most CBM models in the literature optimize the maintenance policies by minimizing the long run cost per time component [29].

However, in the real world there are critical systems such as power plants, aircrafts, submarines, military systems, and nuclear systems, in which failure during actual operation may lead to disastrous events [6]. In such systems, minimizing failure probability of the system seems more important than minimizing the long run cost. This implies that developing CBM models considering the failure probability for these systems deserves more attention. Also, most of the CBM

models are derived for single-component systems; still, there is more need for developing CBM models in respect of multi-components systems.

In this study a novel CBM model is presented that can be used in many real applications such as hydraulic structures [31], cutting tools [32], airplane engine compressor blades, brake linings [33], consumption, corroding pipe lines [34] and power plants [7]. Specifically, this paper considers a CBM problem for a critical system composed of two components that are subject to gradual deterioration and consequent random failure.

Failure of either or both components will result in failure of the system. In order to prevent failure, the system is inspected in periodic times and the levels of deterioration for components are monitored. Based on the deterioration levels of components, appropriate maintenance actions may be performed. The possible maintenance actions for such system are assumed to be preventive maintenance and preventive replacement. A preventive maintenance for each component is an imperfect (partial) repair action performed on it prior to a failure, while preventive replacement is a complete renewal of the component. In other words, preventive maintenance action reduces the deterioration level of the component without returning it to its initial state as well as new state, whereas preventive replacement renews the component to its initial condition. These kinds of maintenance actions are common in the literature and can be found in several papers such as [3, 4, 7, 10, 13].

The problem lies in determining maintenance policies that includes when components must be inspected, and when maintenance actions must be performed. In order to determine maintenance policies, a stochastic model based on the regenerative properties of the system is developed followed by a novel approach. In the proposed approach decision on the maintenance policy is made in two phases.

The first phase obtains the policies ensuring that the failure probability of the system does not exceed a pre-specified value. The second phase calculates the long-run expected cost of the system for each of the obtained policies in the first phase. Then, the policy which leads to the minimum long-run expected cost of the system is selected as the desired maintenance policy.

The reason for using this two phase method is that it considers not only long-run expected cost of the system, but also takes account of failure probability of the system. Thus, this work differs from earlier papers in two main directions. First, the proposed model in this article does not ignore the failure probability as a crucial criterion in critical systems. In fact, the failure probability of the system as well as the long-run expected cost of the system is considered closely in order to determine maintenance policies. Furthermore, in this study there is no restrictive assumption that the system is composed of only single component. The remainder of the paper is organized as follows. Section 2 states the assumptions of the problem, illustrates maintenance strategies and formulates the model. The approach for determining the desired maintenance policy is described in section 3. Section 4 provides numerical example in order to illustrate the implementation of the proposed approach. Finally, the last section concludes the paper along with suggestions for future research.

2. Modeling Framework

In this section, we provide a modeling frame work for a problem stated in section 1.

2.1. Notation

Following notation will be used throughout this paper:

i : index of components ($i = 1,2$);

t : index of inspection periods ($t = 1,2,3,...$);

X_t^i : state (deterioration level) of the component i at the end of inspection period t ;

Z_t^i : state (deterioration level) of the component i at the beginning of inspection period t ;

Y_t^i : deterioration level of the component i occurred in period t ;

ξ_1 :preventive maintenance (repair) threshold for (decision variable);

ξ_2 : preventive replacement threshold (decision variable);

T :the time between two successive inspection periods (decision variable);

C_{pm} : the cost incurred by preventive maintenance action;

C_{pr} : the cost incurred by preventive replacement action;

$E(C_\infty)$: the long-run expected cost of system;

$\pi(x_1, x_2)$: stationary law of the deterioration process;

$\lambda(X_t^i)$: failure rate of component i ;

$f(x)$: probability density function of the deterioration occurring during one period;

$f^{(T)}(x)$: T th convolution of $f(x)$;

p : maximum allowed failure probability determined by decision maker;

$R_i(t)$: Reliability of the component i in period t ;

$R(t)$: Reliability of the system in period t ;

2.2. Assumptions

In this problem the deterioration level of each component is measured on a continuous scale. The deterioration level is strictly increasing which means that the components worsen with time because of

ageing and accumulated wear or damage. It is assumed that the time to failure of each component follows a non homogeneous exponential distribution. The failure rate, $\lambda(X_t^i)$, is considered as an increasing linear function of the deterioration level of the component. It means that the failure rate of each component is variable and increases with an increase in the deterioration level of the component. Therefore, the increase in deterioration levels of components raises the probability of failure. The reason for assuming proposed failure distribution is the ability of such distribution to properly describe a period of steady damage accumulation of machinery for continuous production or a hydraulic structure subject to erosion [35, 36]. This failure distribution, also, has been used in several published works for continuously deteriorating systems [3, 4, 7, 10, 35, 37]. It is assumed that the system is inspected at equidistant periods. At the end of each period a decision is taken to determine whether each component needs preventive maintenance or preventive replacement. The decision is made based on the states of components and the maintenance thresholds (ξ_1 and ξ_2). Figure 1 demonstrates how this decision is taken according to the observed states of the components. Note that without loose of the generality, we assume the two components are the same.

If $X_t^i > \xi_2$	$\rightarrow$ Preventive replacement action is needed.
If $\xi_1 < X_t^i < \xi_2$	$\rightarrow$ Preventive maintenance action is needed.
If $X_t^i < \xi_1$	$\rightarrow$ No maintenance action is needed.

Fig. 1.The needed maintenance actions according to deterioration levels of the components.

It follows from figure 1 that the preventive replacement action is performed on each component *i* at the end of period *t*, if its deterioration level exceeds threshold ξ_2 (when $X_t^i > \xi_2$).

However, the preventive maintenance action is carried out when the deterioration level of component exceeds threshold ξ_1. Specifically, preventive maintenance action is carried out when $\xi_1 < X_t^i < \xi_2$ (it is obvious that $\xi_2 > \xi_1$). If the deterioration level of the component is less than threshold ξ_1 (when $X_t^i < \xi_1$), no maintenance action is performed. Within this structure, the thresholds ξ_1 and ξ_2 define the maintenance policy. These thresholds have to be tuned by the decision maker in order to optimize the performance of the policy [4]. It should be noted that the maintenance actions improve the states of components. In case that preventive replacement action is performed on a component, the component is replaced by the new one. As a result, the deterioration level of the component at the beginning of the next period will be zero. On the other hand, it is assumed that after each preventive maintenance action, the component state improves partially and the percent of improvement is a random variable. Thus, the state of the component at the beginning of the next period will be $\alpha_t X_t^i$ where $\alpha_t \in [0,1)$ are independent random variables following a normal distribution. If α denotes expected value of α_t, the expected value of the state of component after preventive maintenance will be αX_t^i. This demonstrates that the state of the component after preventive maintenance is directly proportional to its state before the maintenance action. This is a reasonable assumption, since for a large level of deterioration, it is expected that the state of restored component to be large.

2.3. Semi-Regenerative Property of the System

As mentioned in the previous section, after preventive replacement action the component is as good as new and its deterioration level is equal to zero. Also, after preventive maintenance action, the state of the component is directly proportional to its state before the maintenance action. Thus, after maintenance actions the component evolution depends only on the deterioration level before the maintenance action. In other words, conditional to the state of the component before maintenance action, the component state until the next maintenance action can be completely characterized independent of the past events. Therefore, the stochastic process created by the deterioration levels of the components, X^i, is a semi-regenerative process in which the semi-regeneration (or Markov renewal) points are the starting times of every maintenance action [4]. Ross [38] shows that as time increases a semi-regenerative process converges to a steady state distribution. So, we can infer that X_t^i converges to steady state distribution.

2.4. Maintenance Scenarios on a Markov Renewal Cycle and Stationary Probability

The construction of the stationary probability density of the system state at the beginning of maintenance actions, relies on an enumeration and description of its different possible behaviors from the beginning to the end of a Markov renewal cycle [4]. Let x_1 and x_2 respectively denotes the observed states of components 1 and 2 at the end of a period (before performing any maintenance actions). Also, let y_1 and y_2 denotes the states of the components 1 and 2 at the end of the next period (before performing any maintenance actions). Then, if no failure occurs until the next period, possible scenarios on a Markov renewal cycle are as follows:

Scenario 1: $x_1 < \xi_1, x_2 < \xi_1$. The two components are left as they are and no maintenance action is performed

on them. The transition probability density function from x_i to y_i for each component can be gained by $f^{(T)}(y_i - x_i)$.

Scenario 2: $x_1 < \xi_1,\ \xi_1 < x_2 < \xi_2$ or $x_2 < \xi_1,\ \xi_1 < x_1 < \xi_2$. The component for which $\xi_1 < x_i < \xi_2$ is preventively maintained and its transition probability density function from x_i to y_i can be obtained by $f^{(T)}(y_i - \alpha x_i)$. The other one is not maintained and its transition probability density function from x_i to y_i can be gained by $f^{(T)}(y_i - x_i)$.

Scenario 3: $x_1 < \xi_1,\ \xi_2 < x_2 < \infty$ or $x_2 < \xi_1,\ \xi_2 < x_1 < \infty$. The component for which $\xi_2 < x_i < \infty$ is replaced and its transition probability density function from x_i to y_i can be obtained by $f^{(T)}(y_i)$. The other one is not maintained and its transition probability density function from x_i to y_i can be gained by $f^{(T)}(y_i - x_i)$.

Scenario 4: $\xi_1 < x_1 < \xi_2,\ \xi_2 < x_2 < \infty$ or $\xi_1 < x_2 < \xi_2,\ \xi_2 < x_1 < \infty$. The component for which $\xi_1 < x_i < \xi_2$ is preventively maintained and its transition probability density function from x_i to y_i is gained by $f^{(T)}(y_i - \alpha x_i)$. The other component is replaced and its transition probability density function from x_i to y_i is obtained by $f^{(T)}(y_i)$.

Scenario 5: $\xi_1 < x_1 < \xi_2,\ \xi_1 < x_2 < \xi_2$. The two components are preventively maintained and the transition probability density functions from x_i to y_i for each component is calculated by $f^{(T)}(y_i - \alpha x_i)$.

Scenario 6: $\xi_2 < x_1,\ \xi_2 < x_2$. The two components are replaced and the transition probability density functions from x_i to y_i for each component is obtained by $f^{(T)}(y_i)$.

Let $\pi(y_1, y_2)$ denotes the stationary probability density for the deterioration process at inspection times. Furthermore, $F(y_1, y_2 | x_1, x_2)$ denotes the transition probability density functions from (x_1, x_2) to (y_1, y_2). Using conditional probability theory [38], we will have:

$$\pi(y_1, y_2) = \pi(x_1, x_2) F(y_1, y_2 | x_1, x_2) \tag{1}$$

Considering (1), the scenarios and their transition probability density functions listed above, the stationary probability density of the system state can be written as follows:

$$\begin{aligned}
\pi(y_1,y_2) = {} & \underbrace{\left(\int_0^{\xi_1}\int_0^{\xi_1} \pi(x_1,x_2)dx_1dx_2 f^{(T)}(y_1-x_1) f^{(T)}(y_2-x_2)dx_1dx_2\right)}_{\text{Scenario 1}} \\
& + \underbrace{\left(\int_0^{\xi_1}\left(\int_{\xi_1}^{\xi_2} \pi(x_1,x_2) f^{(T)}(y_1-\alpha x_1)dx_1\right) f^{(T)}(y_2-x_2)dx_2\right) +}_{} \\
& \underbrace{\left(\int_0^{\xi_1}\left(\int_{\xi_1}^{\xi_2} \pi(x_1,x_2) f^{(T)}(y_2-\alpha x_2)dx_2\right) f^{(T)}(y_1-x_1)dx_1\right)}_{\text{Scenario 2}} \\
& + \underbrace{\left(\int_0^{\xi_1}\left(\int_{\xi_2}^{\infty} \pi(x_1,x_2) f^{(T)}(y_1)dx_1\right) f^{(T)}(y_2-x_2)dx_2\right) +}_{} \\
& \underbrace{\left(\int_0^{\xi_1}\left(\int_{\xi_2}^{\infty} \pi(x_1,x_2) f^{(T)}(y_2)dx_2\right) f^{(T)}(y_1-x_1)dx_1\right)}_{\text{Scenario 3}} \\
& + \underbrace{\left(\int_{\xi_1}^{\xi_2}\left(\int_{\xi_2}^{\infty} \pi(x_1,x_2) f^{(T)}(y_1)dx_1\right)(y_2-\alpha x_2)dx_2\right) +}_{} \\
& \underbrace{\left(\int_{\xi_1}^{\xi_2}\left(\int_{\xi_2}^{\infty} \pi(x_1,x_2) f^{(T)}(y_2)dx_2\right) f^{(T)}(y_1-\alpha x_1)dx_1\right)}_{\text{Scenario 4}} \\
& + \underbrace{\left(\int_{\xi_1}^{\xi_2}\int_{\xi_1}^{\xi_2} \pi(x_1,x_2)dx_1dx_2 f^{(T)}(y_1-\alpha x_1) f^{(T)}(y_2-\alpha x_2)dx_1dx_2\right)}_{\text{Scenario 5}} \\
& + \underbrace{\left(\int_{\xi_1}^{\xi_2}\int_{\xi_1}^{\xi_2} \pi(x_1,x_2)dx_1dx_2 f^{(T)}(y_1) f^{(T)}(y_2)dx_1dx_2\right)}_{\text{Scenario 6}}
\end{aligned} \tag{2}$$

Analyzing the probability density function $\pi(y_1, y_2)$ is difficult and requires solving a bi-dimensional one sided integral equation. This kind of equation can be solved numerically, for which Monte-Carlo simulation techniques are the only solution [4, 9, 39]. Using $\pi(y_1, y_2)$, the expected failure probability and long run cost of the system are obtained in the following section.

3. Finding the Desired Maintenance Policy

In this section an efficient approach is presented to find the desired maintenance thresholds ξ_1, ξ_2 and the length of inspection periods *T*. This approach consists of two phases: The first phase finds different ξ_1, ξ_2 and *T* which cause the probability of system failure not to exceed the maximum allowed probability p. In the second phase, the long-run cost of system is calculated for all the policies provided by the first phase. The policy which results in the lowest cost is selected as the desired policy.

3.1. The First Phase

In order to determine ξ_1, ξ_2 and T, the reliability of the components are used in the first phase. The reliability of a component in the period t is the probability that the component will not fail by the end of period t [38]. Thus, the reliability of component i can be obtained by:

$$R_i(t) = e^{-\lambda(X_{t-1}^{i}+Y_t^{i})T} \tag{3}$$

Formula (3) calculates the probability that the time between failures is greater than T considering the fact that the time to failure of the component follows a non homogeneous exponential distribution. Since failures of either or both of the components will result in the failure of the system, the reliability of the system in the period t is the probability that both components will not fail by the end of the period t. Therefore, the reliability of system can be gained by:

$$R(t) = e^{\sum_{i=1}^{2}-\lambda(X_{t-1}^{i}+Y_t^{i})T} \tag{4}$$

The failure probability of the system in period t can be calculated based on the reliability of the system as follows:

$$P_f = 1-R(t) = 1-e^{\sum_{i=1}^{2}-\lambda(X_{t-1}^{i}+Y_t^{i})T} \tag{5}$$

It is assumed that preventing failure of this critical system is of great importance. As a result, the failure probability of the system at the steady state must be equal or less than maximum allowed probability p. In the first phase we obtain different thresholds ξ_1, ξ_2 and length of inspection periods T which cause that the failure probability of the system at the steady state not to exceed p.

Let triples (ξ_1, ξ_2, T) denote a combination of thresholds and inspection periods for the critical deteriorating system. First, different thresholds and inspection periods (ξ_1, ξ_2, T) are selected for simulation. Then, the failure probability of the simulated system at a steady state is calculated for all of these policies. Considering (5), the failure probability of the simulated system at steady state can be obtained as:

$$P_f = \int_{x_2=0}^{x_2=\infty}\int_{x_1=0}^{x_1=\infty}(1-e^{\sum_{i=1}^{2}-\lambda(X_t^{i}+Y_t^{i})T})\pi(x_1,x_2)dx_1dx_2 \tag{6}$$

The maintenance thresholds and inspection periods for which the failure probability is less than or equal to p are selected for the next phase of the solution approach. In the second phase, the desired value for ξ_1, ξ_2 and T are obtained by minimizing long run expected maintenance costs.

3.2. The Second Phase

In this phase, the long-run expected costs of the system are computed for all of the maintenance policies provided by the first phase. Among these maintenance policies, (ξ_1, ξ_2, T) having the minimum expected cost is chosen as a desired maintenance policy.

In order to calculate expected costs of the system for each policy, the scenarios defined in subsection 2.4 are considered. Table 1 demonstrates the probability of occurrence and cost of the system for each of the scenarios.

Tab. 1. Probability of occurrence and total cost for each scenario defined in section 2.

Scenario	Probability of Occurrence	Cost of System
Scenario 1	$\int_{x_1=0}^{x_1=\xi_1}\int_{x_2=0}^{x_2=\xi_1}\pi(x_1,x_2)dx_1dx_2$	0
Scenario 2	$\int_{x_1=0}^{x_1=\xi_1}\int_{x_2=\xi_1}^{x_2=\xi_2}\pi(x_1,x_2)dx_1dx_2 + \int_{x_1=\xi_1}^{x_1=\xi_2}\int_{x_2=0}^{x_2=\xi_1}\pi(x_1,x_2)dx_1dx_2$	C_{pm}
Scenario 3	$\int_{x_1=0}^{x_1=\xi_1}\int_{x_2=\xi_2}^{x_2=\infty}\pi(x_1,x_2)dx_1dx_2 + \int_{x_1=\xi_2}^{x_1=\infty}\int_{x_2=0}^{x_2=\xi_1}\pi(x_1,x_2)dx_1dx_2$	C_{pr}
Scenario 4	$\int_{x_1=\xi_1}^{x_1=\xi_2}\int_{x_2=\xi_2}^{x_2=\infty}\pi(x_1,x_2)dx_1dx_2 + \int_{x_1=\xi_2}^{x_1=\infty}\int_{x_2=\xi_1}^{x_2=\xi_2}\pi(x_1,x_2)dx_1dx_2$	$C_{pm}+C_{pr}$
Scenario 5	$\int_{x_1=\xi_1}^{x_1=\xi_2}\int_{x_2=\xi_1}^{x_2=\xi_2}\pi(x_1,x_2)dx_1dx_2$	$2C_{pm}$
Scenario 6	$\int_{x_1=\xi_2}^{x_1=\infty}\int_{x_2=\xi_2}^{x_2=\infty}\pi(x_1,x_2)dx_1dx_2$	$2C_{pr}$

Considering Table 1, the expected cost of the system at a steady state can be obtained as follows:

$$E(C_\infty) = \sum_{k=1}^{6} C_k \times P_k$$

$$= 0 \times \left(\int_{x_1=0}^{x_1=\xi_1} \int_{x_2=0}^{x_2=\xi_1} \pi(x_1, x_2) dx_1 dx_2 \right)$$

$$+C_{pm} \times \left[\int_{x_1=0}^{x_1=\xi_1} \int_{x_2=\xi_1}^{x_2=\xi_2} \pi(x_1, x_2) dx_1 dx_2 + \int_{x_1=\xi_1}^{x_1=\xi_2} \int_{x_2=0}^{x_2=\xi_1} \pi(x_1, x_2) dx_1 dx_2 \right]$$

$$+C_{pr} \times \left[\int_{x_1=0}^{x_1=\xi_1} \int_{x_2=\xi_2}^{x_2=\infty} \pi(x_1, x_2) dx_1 dx_2 + \int_{x_1=\xi_2}^{x_1=\infty} \int_{x_2=0}^{x_2=\xi_1} \pi(x_1, x_2) dx_1 dx_2 \right]$$

$$+(C_{pm} + C_{pr}) \times \left[\int_{x_1=\xi_1}^{x_1=\xi_2} \int_{x_2=\xi_2}^{x_2=\infty} \pi(x_1, x_2) dx_1 dx_2 + \int_{x_1=\xi_2}^{x_1=\infty} \int_{x_2=\xi_1}^{x_2=\xi_2} \pi(x_1, x_2) dx_1 dx_2 \right]$$

$$+2C_{pm} \times \left[\int_{x_1=\xi_1}^{x_1=\xi_2} \int_{x_2=\xi_1}^{x_2=\xi_2} \pi(x_1, x_2) dx_1 dx_2 \right]$$

$$+2C_{pr} \times \int_{x_1=\xi_2}^{x_1=\infty} \int_{x_2=\xi_2}^{x_2=\infty} \pi(x_1, x_2) dx_1 dx_2 \qquad (7)$$

Where C_k and P_k denotes the probability of occurrence and cost of system for the scenario k ($k=1,\ldots, 6$).

4. Numerical Example

In this section a numerical example is presented in order to demonstrate the implementation of the proposed approach. Recall that this problem could only be studied through Monte-Carlo simulations due to the complexity of computation [4, 9, 39]. The deterioration process is simulated for 10000000 periods using C++ program for following data:

$$\mu = 1, \lambda = 0.3, p = 0.2, \Delta = 0.1, X_0^1 = 0, X_0^2 = 0.$$

In order to simulate a continuous deterioration process, the states of the components are considered discrete. In other words, it is assumed that for each t we have $X_t^i = Z_t^i + \Delta N$ where Δ denotes a small level of deterioration and N follows a Poisson distribution. The deterioration process is simulated for 10,000,000 periods. In the beginning of each period, if the component state appears to be Δn , then, +1 is added to the frequency of state variable in Δn . The frequency of each state is divided by 10,000,000 to obtain a probability distribution.

4.1 Implementing the First Phase

In the first phase, the failure probability of the system at the steady state is calculated for different values of ξ_1 , ξ_2 and T using (6). Figure 2-4 respectively illustrate the results for T = 2, T = 4 and T = 8. In each of these figures different values of ξ_2 are represented using different curves. Similarly, different values of ξ_1 are indicated by a horizontal axis. A vertical axis, also, represents the failure probability at the steady state corresponding with different pairs of ξ_1 and ξ_2 .For instance, the failure probability of the system at a steady state of ξ_1 = 3, ξ_1 = 8 and T = 8 is equal to 0.25. It follows from figures 2 to 4 that the policies which result in the failure probability less than $p = 0.2$ are as follows:

(1, 2, 2), (2, 2, 2), (1, 4, 2), (2, 4, 2), (3, 4, 2), (4, 4, 2), (1, 6, 2), (2, 6, 2), (3, 6, 2), (1, 8, 2), (2, 8, 2), (3, 8, 2), (1, 2, 4), (2, 2, 4), (1, 4, 4), (2, 4, 4), (3, 4, 4), (4, 4, 4), (1, 6, 4), (2, 6, 4), (1, 8, 4), (1, 2, 8), (2, 2, 8), (1, 4, 8), (2, 4, 8), (1, 6, 8)

where each of triples corresponds to (ξ_1, ξ_2, T). As a result, these policies are selected for the next step.

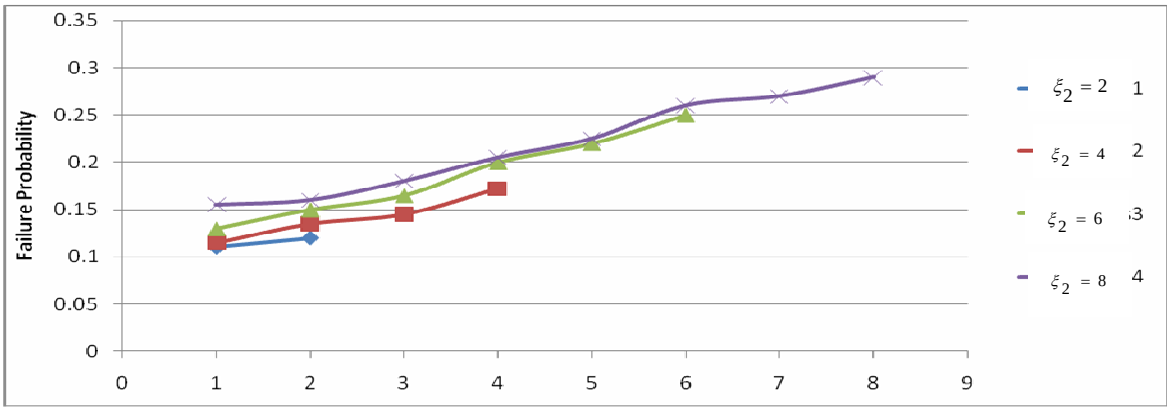

Fig. 2. Expected failure probability of the system at the steady state of T = 2.

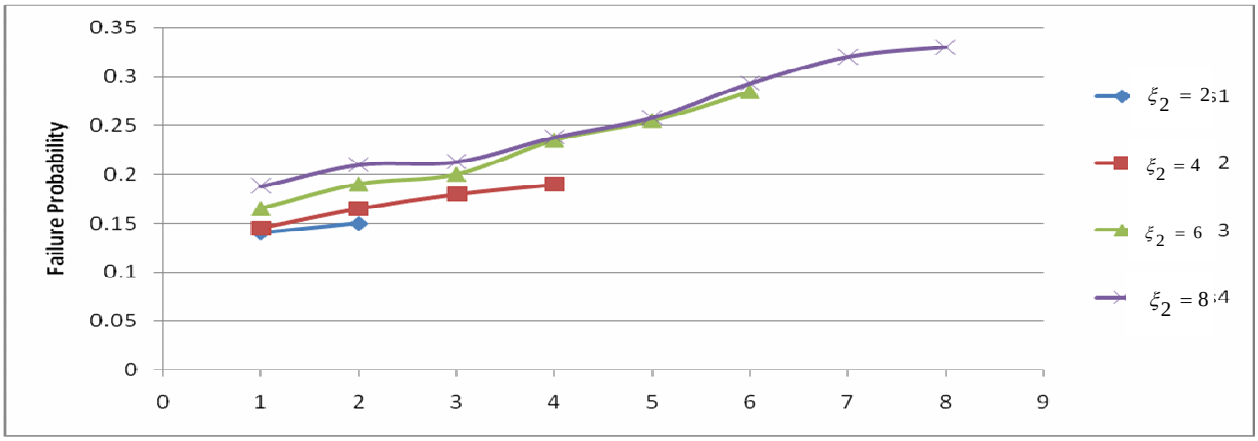

Fig. 3. Expected failure probability of the system at the steady state of T = 4.

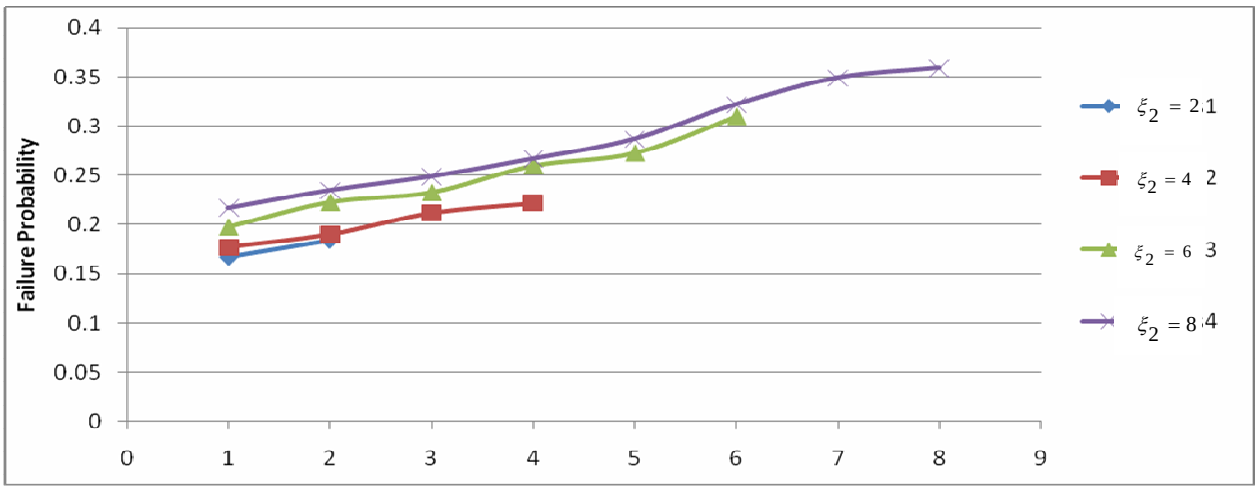

Fig. 4. Expected failure probability of the system at the steady state of T = 8.

4.2. Implementing the Second Phase

In this phase, the long-run expected cost of the system is calculated for all of the maintenance policies selected by the first phase. Recall that long-run expected costs are obtained by (7). Table 2 demonstrates the long-run expected cost for the chosen policies in the first phase of the optimization procedure. Recall that each policy has the failure probability less than $p = 0.2$. Table 2 reveals that the policy $(2, 4, 2)$ leads to the less long-run expected cost; consequently, it is selected as the optimum policy.

Tab.2. Expected long-run cost for the policies selected in the first phase.

Policies selected in the first phase	Expected long-run cost
(1, 2, 2)	33
(2, 2, 2)	35
(1, 4, 2)	32.5
(2, 4, 2)	31
(3, 4, 2)	34.5
(4, 4, 2)	38
(1, 6, 2)	32
(2, 6, 2)	32
(3, 6, 2)	34
(1, 8, 2)	34
(2, 8, 2)	35
(3, 8, 2)	36
(1, 2, 4)	37
(2, 2, 4)	39.5
(1, 4, 4)	38.5
(2, 4, 4)	36.5
(3, 4, 4)	39.5
(4, 4, 4)	42.5
(1, 6, 4)	37
(2, 6, 4)	37
(1, 8, 4)	39
(1, 2, 8)	41.5
(2, 2, 8)	45
(1, 4, 8)	41
(2, 4, 8)	42.5
(1, 6, 8)	43.5

5. Conclusions

In this paper, we have studied condition based maintenance (CBM) for a critical system composing of two components. The components deteriorate continuously over the time; as a result, the system is vulnerable to random failures. In order to overcome random failures, the system is inspected at equidistant periods and the deterioration levels (state) of the two components are monitored. Based on the observed state of the components, the preventive maintenance or replacement actions may be performed on them. In this paper a novel approach has been proposed that determines maintenance policies using regenerative property of the system. In fact, through two phases this approach determines when the components must be inspected and when they must be replaced or maintained preventively. The first phase of the approach finds the different maintenance policies for which failure probability of the system does not exceed maximum allowed probability. Then, among found policies in the first phase, the second phase selects a desired maintenance policy which results in minimum long-run expected cost of the system. The proposed model can be used in many real applications such as hydraulic structures, cutting tools, airplane engine compressor blades and power plants.

This research can be extended in some directions. For instance, it would be possible to extend CBM models for *n* component systems. In order to formulate the problem for these systems, the number of parameters and scenarios defined for the Markov renewal model must be increased; as a result, models will be more complex. Similarly, studying multi-components system where components are different would be useful. In this condition, hidden Markov renewal cycle [40] can be applied to model the more complicated conditions.

Also, developing the proposed model in the uncertain environment, where parameters can be fuzzy or stochastic, makes it more helpful in the practice. Finally, this research can be extended by coordinating production and maintenance decisions. As a case in point, in a flow shop and job shop scheduling problem typically it is assumed that machines are always available during the planning time horizon. However, this assumption does not hold for many real-world applications in which machines may be unavailable due to maintenance, breakdown, and repair. In these cases, it would be useful to optimize production decisions with regard to CBM effects on the production lines.

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