International Journal of Industrial Engineering & Production Research

September 2014, Volume 25, Number 3
pp. 215-224

http://IJIEPR.iust.ac.ir/

pISSN: 2008-4889

# Multi Responses Optimization Through Game Theory Approach

**H.R. Navidi, A. Amiri*, R. Kamranrad**

***Hamidreza Navidi,*** *Associate Professor, Department of Applied Mathematics, Shahed University, Tehran, Iran, navidi@shahed.ac.ir*
***Amirhossein Amiri,*** *Assistant Professor, Industrial Engineering Department, Shahed University, Tehran, Iran, amiri@shahed.ac.ir*
***Reza Kamranrad,*** *Ph.D. student, Industrial Engineering Department, Shahed University, Tehran, Iran, r.kamranrad@shahed.ac.ir*

## KEYWORDS

Game theory;
Multi responses;
Signal to noise ratio;
Nash equilibrium;
VIKOR

## ABSTRACT

*In this paper, a new game theoretic-based approach is proposed for multi-response optimization problem. Game theory is a useful tool for decision making in the conflict of interests between intelligent players in order to select the best joint strategy for them through selecting the best joint desirability. Present research uses the game theory approach via definition of each response as each player and factors as strategies of each player. This approach can determine the best predictor factor sets in order to obtain the best joint desirability of responses. For this aim, the signal to noise ratio(SN) index for each response have been calculated with considering the joint values of strategies; then obtained SN ratios for each strategy is modeled in the game theory table. Finally, using Nash Equilibrium, the best strategy which is the best values of predictor factors is determined. A real case and a numerical example are given to show the efficiency of the proposed method. In addition, the performance of the proposed method is compared with the VIKOR method.*



## 1. Introduction

Game theory is a branch of operation research scope which is useful for decision making in the conflict of interests between intelligent players (decision makers). This method has wide applications in the social life, economy, policy, engineering sciences, biological sciences and so on (Navidi et al. [1] and Osborne [2]). There are many definitions of game theory; for example Osborne [2] believes that game is a description of the strategic interactions that occur between players.

Also Myerson says that game theory is a study of mathematical models of conflict and cooperation between wise players; for this reason, there are other titles for the game theory such as conflict analysis or interactive decision making theory [3]. In the game, there are at least two players with adverse ideas or targets which have similar or different strategies. Each player wants to achieve the maximum desirability. For example, if the results of the game are defined as winning or losing, then each player wish to maximize wining and minimize losing. For this problem, game theory can propose the useful method for obtaining the best joint desirability such that each player can receive his target (Navidi et al. [1] and Osborne [2]).

On the other hand, some of problems in the mentioned sciences have more than one response variables which are affected by one or more predictor factors. These cases are called multi response problems. Design of experiments (DOE) is a method which uses tools such as desirability function, Taguchi's loss function, Signal

* **Corresponding author: Amirhossein Amiri**
***Email:*** *amiri@shahed.ac.ir*
**Paper first received June 18, 2013, and in accepted form April, 21, 2014.**

to Noise ratio, response surface methodology and so on to analyze and optimize these problems.

The purpose of the DOE is to obtain the best set of predictor variables in order to determine the best value or rank of response variable (s) (Poroch-Seritan et al. [4]). This research is a new innovation in the optimization of multi response problems through game theory approach (GTA) in DOE. Note that, one of the most important differences between the DOE and the GTA is that the DOE approach has a general or global view to determine the optimum point, meanwhile the GTA searches equilibrium point (EP) with partial or local view.

This paper is organized as follows: in the next section previous related studies have been reviewed and discussed. In the third section, a novel approach has been proposed for optimization of multi response problems through using the GTA. In section 4, a real case as well as a numerical example has been illustrated to evaluate the performance of the proposed approach. Finally our concluding remarks have been presented in section 5.

## 2. Review of Literature

In recent years, some procedures have been developed to determine the best factor setting which optimizes multiple responses simultaneously. In many researches, experiment optimization approaches have been categorized into two general groups; approaches with considering and ignoring the correlation between responses. There are several methods in the first group which can be set in three categories. Some of these methods attempt to optimize the multi response problems using complicated mathematical models. Refer to Khuri and Conlon [5], Lagothetis and Haigh [6] and Tang and Su [7] for more information about these methods. Due to the complexity, these methods are not applicable for all multi responses problems. Other category of first group uses heuristic and Meta heuristic algorithms to optimize the mentioned problems.

For example, JeyaPaul et al. has been proposed an integrated approach to multi responses optimization using SN ratio and genetic algorithm [8]. Also Tang and Hsieh used neural network technique for their problem. These approaches also have drawbacks because they do not guarantee optimal solutions [9]. In the third category, at first all of responses are converted to a process performance index (PPI) and then the process has been optimized according to the calculated index. Pan et al. [10] and Haq et al. used the dark dependency analysis method to obtain the PPI [11]. Ramakrishnan and Karunamoorthy have been proposed the multi response SN ratio [12]. Tang et al. proposed the VIKOR (Vlse Kriterijumsk Optimizacija Kompromisno Resenje, a Serbian name), approach that is a contingency rating method to solve multi responses optimization [13]. Liao used the principal component analysis (PCA) technique to determine the PPI index [14]. Also, Datta et al. proposed the application of anthrophy measuring technique on the basis of Taguchi approach for solving the correlated optimization problems [15]. In their research, they used the PCA technique for eliminating the available correlation among responses and converting them into independent responses.

One of the other statistical tools for analyzing the correlated multi responses problem is the multivariate analysis of variance (MANOVA). The second group has been addressed by Bashiri and Hejazi in which multi response optimization problem has been studied through the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) approach [16]. Pal & Gauri examined the efficiency of different continuous methods for optimizing the independent multi-responses problem using multinomial regression [17].

He et al. proposed a robust desirability function method to simultaneously optimize multiple responses with considering the uncertainty associated with the fitted response surface model. Their method takes into account all values in the confidence interval rather than a single predicted value for each response and then defines the robustness measure for the traditional desirability function using the worst case strategy. Also, they developed a hybrid genetic algorithm to find the robust optima [18].

Hejazi et al. assessed the quality engineering problems in which several quality characteristics and factors are to be analyzed through a simultaneous equations system. Besides, they considered several probabilistic covariates in the proposed model. Finally, they could identify interrelations among exogenous and endogenous variables, which give important insight for systematic improvements of quality [19]. Zadbood et al. proposed a Harmony search meta-heuristic algorithm to optimize the multi response surface problem.

This algorithm can find the best set of control variables which optimize the multiple responses. Finally, they indicated that the efficiency and performance of their proposed algorithm is better than some of the well known meta-heuristic algorithm [20]. Bashiri and Bagheri proposed an Imperialist Competitive Meta-heuristic algorithm to optimize the non-linear multi responses in which the best controllable factor's levels are obtained such that minimum deviations of the responses means from their targets are achieved. Moreover, they used the Pareto optimal solution to release the aggregative function. Then, they compared the multiple response imperialist competitive algorithm with multiple objective genetic algorithm. The results showed the efficiency of the proposed approach in both aggregative and non aggregative methods for optimization of the nonlinear multi response programming [21].

Zolgharnein et al. applied the Taguchi design and principle component analysis in multi responses optimization to find the effective parameters for achieving a higher adsorption capacity and removal percentage of the binary mixture containing alizarin red and yellow colors [22]. Shi et al. proposed a solution framework based on discrete-event simulation, sequential bifurcation and response surface methodology to analyze a multi responses optimization problem inherent in an auto parts supply chain. They identified the most efficient operating setting that would maximize the logistics performance after the expansion of the assembly plant's capacity due to market growth.

Then, they applied the sequential bifurcation to identify the most important factors that influence system performance. Also, in order to determine the optimal levels of these key factors, they employed the response surface methodology to develop Derringer–Suich's Meta models that best describe the relationship between key decision variables and the multiple system responses. Finally, the results of this paper showed that the proposed method enables the greatest improvement on system performance [23]. Also, there are some researches in multi-objective scope as the game optimization. For example, Li et al. proposed the game optimization theory (GOT) to stability optimization analysis, Distance Entropy Multi-Objective Particle Swarm Optimization and Fuzzy Multi-weights Decision-making Method.

They believed that the GOT is a comprehensive system not only handle multi-objective optimization problems effectively but also could offset the disadvantages of traditional optimization theories, such as lack of framework and the insufficient consideration of relevant elements. So, they used the GOT for solving the distribution systems planning issue by implementing distributed generation. Their proposed model integrates costs, losses, and voltage index to achieve optimal size and site of distributed generation. The model allows minimizing total system costs, system power losses and maximizing voltage improvement [24].

Rao presented a concept of Pareto-optimal solution in the context of a multi-objective structural optimization problem. They used the graphical interpretations of the non-cooperative and cooperative game theory approaches for a two-criteria optimization problem. Finally, they described the relationship between Pareto-optimal solutions and game theory optimization [25].

Zamarripa et al. applied the game theory optimization-based tool for supply chain planning in cooperative/competitive environments. Their proposed model could improve the tactical decision-making of a supply chain under an uncertain competition scenario through the use of different optimization criteria. Also, They solved the multi-objective optimization problem using the $\varepsilon$-constraint method in order to approximate the Pareto space of non-dominated solutions while a framework based on game theory is used as a reactive decision making support tool to deal with the uncertainty of the competitive scenario [26].

In this paper, a new approach based on conflicting between responses as the "Game theory" has been proposed to optimize the multi responses problem.

## 3. Problem Definition, Methods and Equations

In this paper, a new generalized game theory concept has been proposed to multi responses problem optimization. Optimization concept in the DOE means that the best set of predictor variables is selected in order to determine the best value or rank of response variable(s). For this purpose, the mentioned methods in the previous section are used. Note that in this paper, two concepts of the DOE and the GTA have been composed such that the joint desirability (or each joint optimization index) of each response (each player) is calculated using the DOE tools and then these desirability values are modeled in the game theory table.

For this aim, at first the desirability of each strategy from each player with considering the strategy of other player(s) should be calculated; in other words, desirability of each strategy is affected from the opposite player.

After calculating the joint desirability of each player, the game theory model is proposed as desirability table and then an EP of each row and column of the game model is selected using the NE. The EP of each row and column in the game table is the best desirability according to the target of related player. If the best desirability from the strategy of first player and other player(s) are occurred in the joint cell of the game theory table, then this point is a NE point.

### 3-1. The Used Multi Responses Optimization Method

As mentioned in the previous section, there are many efficient methods for optimization of multi responses problem. One of these methods is the Taguchi's signal to noise ratio which can show importance or effect of the responses. The Taguchi method of experimental design is a widely used technique to accomplish the task for optimization of process parameters. Taguchi used orthogonal arrays to perform experiments and applied the signal to noise (SN) ratio as the quality measurement index, with simultaneous consideration of the mean and variability of the quality characteristic, to determine the optimal setting of process parameters (Wu & Yeh [27]).

This method has different formulas for any type of responses such as smaller the better (STB), nominal the best (NTB) and larger the better (LTB). The following equations are used to compute three mentioned types of

SN ratio ($\eta_{dB}$). Note that in this paper, two response variables from types of STB and NTB are studied.

$$\eta_{dB} = -10\log(\frac{1}{n}\sum_{i=1}^{n} y_i^2) \quad (STB type), \qquad (1)$$

$$\eta_{dB} = 10\log(\frac{\bar{y}^2}{s^2} - \frac{1}{n}) \quad (NTB type), \qquad (2)$$

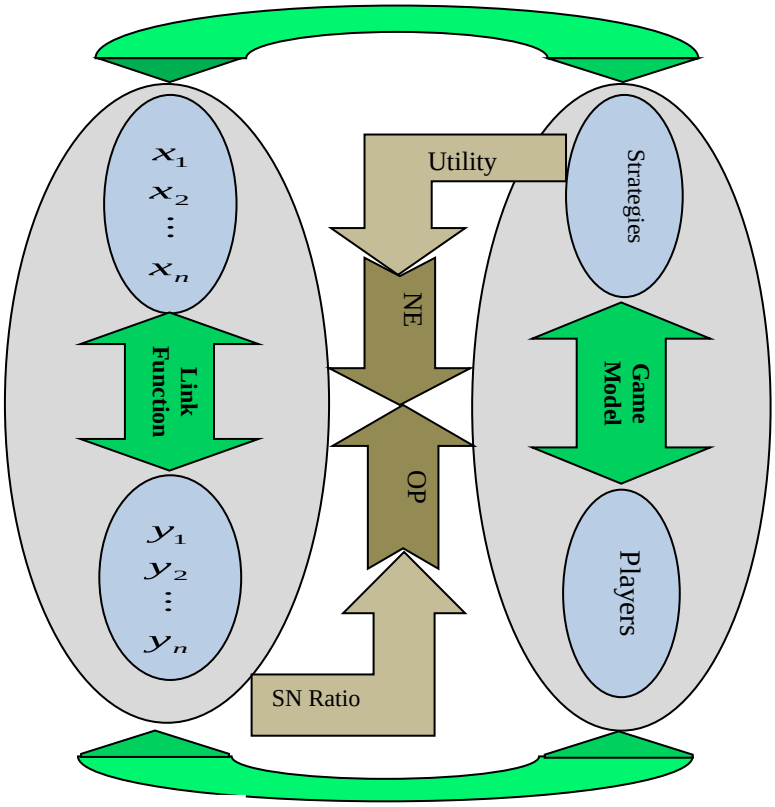
$$\eta_{dB} = -10\log(\frac{1}{n}\sum_{i=1}^{n}(\frac{1}{y_i^2})) \quad (LTB type), \qquad (3)$$

where y is the value of quality charasteristic, $\bar{y}$ is the samples mean of $n$ replicates and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ is the unbiased estimator of process variance Wu & Yeh [27].

**3-2. Proposed Method: Game Theory Approach for Multi Responses Optimization (GTA-MRO)**

In this section, GTA-MRO has been proposed to select the EP (optimum strategy). In the proposed method, the number of responses determines the number of players. In other words, each player is equivalent with each response. Also, the independent factors as well as their levels determine the strategies in the game theory model. Based on this definition, we provide the game theory table. Then, the joint SN ratio for each player with definite strategy is calculated according to other player(s) strategy(s) and is put in the game theory table. Finally, the EP is obtained using the NE.

The NE is the most common solution for solving the strategic games. The output of the NE is a stable state of strategic interaction for all of players such that if players behave based on the NE, then they do not have any motivation for deviation from their decisions. In other words, the NE strategy of each player is the best answer to the NE strategies of other players. Figure 1 shows the relationship between the DOE and the GTA schematically.




OP: Optimal Point

**Fig. 1. Schematic relationship of the DOE and the GTA**

The NE equation to determine the EP is given in equation (4).

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall s_i \in S_i, \forall i = 1,...,n \qquad (4)$$

where:

$$G = \langle N, (S_i)_{i\in N}, (u_i)_{i\in N} \rangle$$
$$N = \{1, 2, ..., n\}$$
$$S = S_1 \times S_2 \times ... \times S_n$$
$$s = (s_1, s_2, ..., s_n) \in S$$

$$S_{-i} = S_1 \times ... \times S_{i-1} \times S_{i+1} \times ... \times S_n$$
$$s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$$
$$u_i : S_1 \times S_2 \times ... \times S_n \rightarrow R \quad ,$$

such that:
G: Static game
N: Number of players
S: Set of all strategies of each player
s: Selected strategy by player
$S_{-i}$: Set of all strategies which is not included the ith set of strategy
$s_{-i}$: Selected strategies by players except ith strategy
$u_i$: Utility of selected joint strategies.

If $(s_1^*, s_2^*, ..., s_n^*) \in S$ is a NE, deviation of each player from the NE leads to worse utility for that player (Navidi et al. [1] and Osborne [2]). Figure 2 demonstrates the implementation of the proposed approach (GTA-MRO). Note that, the proposed approach always has at least one NE for finite experimental designs. Because, Von Neumann and Morgenstern [28, pp. 138-140] showed that each finite matrix game has at least one NE. Since, all of the experimental designs with finite number of the treatments and the response variables are defined as the finite matrix game. Hence, the proposed approach can be used to analyze all multi response optimization problems.

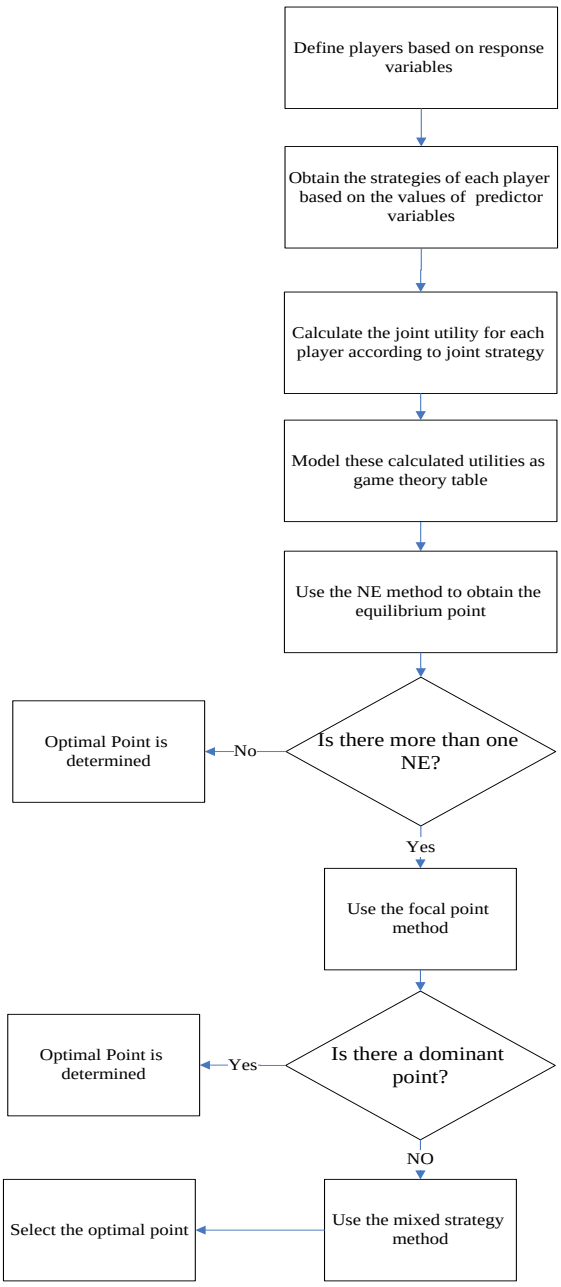



**Fig. 2. Multi responses optimization by the GTA**

## 4. Performance Evaluation and Comparison

### 4-1. Example 1: A Real Case

Phadke (1989) introduced a case on the optimization of a polysilicon deposition process (Wu & Yeh [27]). The experiment data of two quality characteristics including surface defect and thickness are used to illustrate the GTA for optimization of multi responses problem. This example contains three predictor variables called deposition temperature (A), deposition pressure (B) and silence flow (C) which have 2, 2 and 3 levels, respectively; hence, total numbers of treatments for this example are 12. The level of predictor variables in experimental design and two response variables data are summarized in Table 1.

**Tab. 1. Levels of predictor variables, and surface defect and thickness response values**

| Experiment No. | Predictor Variables | | | Surface Defect Data | | | Thickness Data | | |
|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | | | | | | |
| **1** | 1 | 1 | 1 | 0 | 1 | 2 | 1975 | 1961 | 1975 |
| **2** | 1 | 2 | 2 | 2 | 8 | 180 | 5191 | 5242 | 5201 |
| **3** | 1 | 1 | 3 | 35 | 106 | 360 | 5894 | 5874 | 6152 |
| **4** | 2 | 2 | 1 | 15 | 6 | 17 | 2109 | 2099 | 2140 |
| **5** | 2 | 2 | 2 | 1980 | 2000 | 487 | 4152 | 4174 | 4556 |
| **6** | 2 | 1 | 3 | 360 | 1620 | 2430 | 2932 | 2913 | 2833 |
| **7** | 1 | 2 | 1 | 0 | 0 | 3 | 3205 | 3242 | 3468 |
| **8** | 1 | 1 | 2 | 0 | 1 | 5 | 2499 | 2499 | 2576 |
| **9** | 1 | 2 | 3 | 1620 | 90 | 216 | 5766 | 5844 | 5780 |
| **10** | 2 | 1 | 1 | 25 | 270 | 810 | 2752 | 2716 | 2684 |
| **11** | 2 | 1 | 2 | 21 | 162 | 90 | 2835 | 2859 | 2829 |
| **12** | 2 | 2 | 3 | 6 | 40 | 54 | 3149 | 3124 | 3261 |

Here, it should be noted that levels of A and B factors are as the surface defect response (second player) strategies and C factor levels are the other response (first player) strategies. In other words, the strategies of the second player are as pairs.

Now, using the data set in Table 1 and equations (1) and (2), the joint SN ratios for each treatments of the above experimental design are calculated and shown in Table 2.

**Tab. 2. Joint SN ratios for first and second player**

| Experiment No. | Predictor Variables | | | SN Ratios | |
|---|---|---|---|---|---|
| | A | B | C | First player (STB) | Second Player (NTB) |
| **1** | 1 | 1 | 1 | -2.21849 | 47.73942 |
| **2** | 1 | 2 | 2 | -40.3433 | 45.70373 |
| **3** | 1 | 1 | 3 | -46.7535 | 31.71377 |
| **4** | 2 | 2 | 1 | -22.6324 | 39.911 |
| **5** | 2 | 2 | 2 | -64.3444 | 25.52635 |
| **6** | 2 | 1 | 3 | -64.6034 | 34.81587 |
| **7** | 1 | 2 | 1 | -4.77121 | 27.31242 |
| **8** | 1 | 1 | 2 | -9.37852 | 35.08503 |
| **9** | 1 | 2 | 3 | -59.5088 | 42.8847 |
| **10** | 2 | 1 | 1 | -53.8598 | 38.04805 |
| **11** | 2 | 1 | 2 | -40.6427 | 45.05537 |
| **12** | 2 | 2 | 3 | -31.8108 | 32.78075 |

According to Table 2, the game theory table is modeled in Table 3 as follows:

**Tab.3. Game theory model using joint SN ratios**

| | | Player 2 with strategies of (A,B) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Strategy | **(1,1)** | | **(1,2)** | | **(2,1)** | | **(2,2)** | |
| Player 1 with strategy of (C) | **1** | -2.22 | 47.74 | -4.77 | 27.31 | -53.86 | 38.05 | -22.63 | 39.91 |
| | **2** | -9.38 | 35.09 | -40.34 | 45.70 | -40.64 | 45.06 | -64.34 | 25.53 |
| | **3** | -46.75 | 31.71 | -59.51 | 42.88 | -64.60 | 34.82 | -31.81 | 32.78 |

The set of all NE's for game "G" is shown as N (G), in fact the NE is based on the best solution. The steps of determining the NE are as follows:

1. If player 1 selects the strategy number 1; player 2 selects the (1, 1) strategy: maximum joint SN ratio for NTB type; then player 1 will select the

strategy of 1 in the column (1, 1): minimum joint SN ratio for STB type. Since the selected strategies of each player for first treatment are occurred in common point, so NE for this treatment is [1, (1, 1)].

2. If player 1 selects the strategy number 2; player 2 selects the (1, 2) strategy; then, player 1 will select the strategy of 1 in the column of (1, 2). Since the selected strategies of each player for second treatment are not occurred in a common point, so there is not NE for this treatment.
3. If player 1 selects the strategy number 3; player 2 selects the (1, 2) strategy; then Player 1 will select the strategy of 1 in the column of (1, 2). Since the selected strategies of each player for third treatment are not occurred in common point, there is not NE for this treatment.

According to these steps, the only EP of this game is (1, (1, 1)). In other words, the NE of this game is N (surface defect data and thickness data) = (1, (1, 1)).

It is possible that some games have more than one NE. A question arises in such games is "which NE is better to behave?"

Schelling suggests using the concept of focal point effect to solve this problem [29]. Using this concept, it is possible that selecting expectation of one NE is more than different NE's of other players. This case of NE is called as Focal Equilibrium (FE). There is another method that is extension of the NE referred to as Pareto optimal Nash equilibrium (PONE). PONE of a strategic game (G) is the NE of $s^* = (s_1^*, s_2^*, ..., s_n^*)$ such that there is no any equilibrium like $s'^* = (s_1'^*, s_2'^*, ..., s_n'^*)$ in the N (G) which satisfies the following condition (Navidi et al. [1] and Osborne [2]):

$$u_i(s_i^*) < u_i(s_i'^*) \qquad \forall i \in N \tag{5}$$

The attractiveness of Pareto optimal Nash equilibrium in strategic games is that players not as one-sided and not as coordinate are reluctant to deviate from this equilibrium point. The set of all PONE's for game "G" is shown by $N_{po}$ (G). Another case that may happen is that one of the NE is not dominant over the other points such that the best EP cannot be determined by Pareto optimal; hence, the mixed strategy method should be used to solve this problem. For more information refer to Navidi et al. [1] and Osborne [2]. Following example has been illustrated for this case.

### 4-2. Example 2: A Numerical Example with Two NE Points

Consider previous real case; in this section the data related to first and the second responses have been changed in order to show the two NE points and select the dominate point in game theory model. The data related to two responses are shown in Table 4.

**Tab. 4. Levels of predictor variables, other surface defect and thickness data**

| Experiment No. | Predictor Variables | | | $Y_1$ | | | $Y_2$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | | | | | | |
| **1** | 1 | 1 | 1 | 14 | 7 | 12 | 1956 | 1961 | 1975 |
| **2** | 1 | 2 | 2 | 2 | 8 | 180 | 5391 | 5442 | 5432 |
| **3** | 1 | 1 | 3 | 35 | 106 | 360 | 5894 | 5874 | 6152 |
| **4** | 2 | 2 | 1 | 15 | 12 | 107 | 2109 | 2099 | 2140 |
| **5** | 2 | 2 | 2 | 1980 | 2000 | 487 | 4152 | 4174 | 4556 |
| **6** | 2 | 1 | 3 | 360 | 1620 | 2430 | 2932 | 2913 | 2833 |
| **7** | 1 | 2 | 1 | 1 | 5 | 16 | 3205 | 3242 | 3468 |
| **8** | 1 | 1 | 2 | 6 | 12 | 15 | 2499 | 2499 | 2576 |
| **9** | 1 | 2 | 3 | 1620 | 90 | 216 | 5766 | 5844 | 5780 |
| **10** | 2 | 1 | 1 | 25 | 270 | 810 | 2752 | 2716 | 2684 |
| **11** | 2 | 1 | 2 | 21 | 143 | 90 | 2835 | 2859 | 2829 |
| **12** | 2 | 2 | 3 | 4 | 10 | 14 | 4349 | 4324 | 4361 |

For this example, the joint SN ratios are calculated and put in the game theory table that is shown in Table 5.

**Tab. 5. Game theory model using joint SN ratios with two NE points**

| Player 1 with strategy of (C) | Player 2 with strategies of (A,B) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Strategy | **(1,1)** | | **(1,2)** | | **(2,1)** | | **(2,2)** | |
| **1** | -21.13 | 46.00 | -19.73 | 27.31 | -53.86 | 38.05 | -35.95 | 39.91 |
| **2** | -21.30 | 35.09 | -40.34 | 46.05 | -39.85 | 45.06 | -64.34 | 25.53 |
| **3** | -46.75 | 31.71 | -59.51 | 42.88 | -64.60 | 34.82 | -20.17 | 47.24 |

As shown in Table 5, there are two NE of (1, (1, 1)) and (3, (2, 2)) or N (G) = {(1, (1, 1)), (3, (2, 2))} for the second example. Dominated point should be selected using the following instruction: since the joint SN ratios of each player in (3, (2, 2)) strategy are better than the (1, (1, 1)) strategy, so in this example the

PONE is [3, (2, 2)]. In other words, $N(G)=\{(1,(1,1));\ (3,(2,2))\}$. Hence, the Pareto optimal NE is $N_{po}(G)=(3,(2,2))$.

### 4-3. Comparison the Proposed Approach with the VIKOR Method

In this section, a decision making method called VIKOR has been used to accumulate the SN ratios of responses. This comparison can determine the efficiency of the proposed GTA for MRO. Note that the best levels of controllable factors are determined such that the best treatment has minimum value of Q. The VIKOR index is given in equation (8) (Tang et al. [13]).

$$Q_i=\nu\left[\frac{Z_i-Z^+}{Z^--Z^+}\right]+(1-\nu)\left[\frac{R_i-R^+}{R^--R^+}\right], \quad (8)$$

where the variables of the equation (8) are defined in Table 6 as follows:

**Tab. 6. Variables of the VIKOR index**

| Variable | Title | Variable | Title |
|---|---|---|---|
| $Q_i$ | Accumulation Index | $Z^+$ | Minimum $Z_i$ |
| $\upsilon$ | Weight Gain Group | $Z^-$ | Maximum $Z_i$ |
| $Z_i$ | Utility measure | $R^+$ | Minimum $R_i$ |
| $R_i$ | Regret measure | $R^-$ | Maximum $R_i$ |

We assumed the value of 0.5 for parameter υ. Also, the $Z_i$ and the $R_i$ indices in the equation (8) are computed by using equations (9) and (10), respectively:

$$Z_i=\frac{\sum_{j=1}^{n} w_j\,(NSN_j^+-NSN_{ij})}{NSN_j^+-NSN_j^-}, \quad (9)$$

$$R_i=\max\left[\frac{w_j\,(NSN_j^+-NSN_{ij})}{NSN_j^+-NSN_j^-}\right], \quad (10)$$

where $NSN_{ij}$ is the Normalized S/N ratio of the *j*th response in the *i*th treatment, the $NSN_j^+$ and the $NSN_j^-$ are the maximum and minimum $NSN_{ij}$, respectively for the *j*th response[13].

In this section, the $Q_i$ indices for the real case and the numerical example are calculated and shown in Table 7.

**Tab. 7. Accumulation index variables for the examples 1 and 2**

| Experiment No. | Example 1 | | | Example 2 | | |
|---|---|---|---|---|---|---|
| | $Z_i$ | $R_i$ | $Q_i$ | $Z_i$ | $R_i$ | $Q_i$ |
| **1** | 0.000 | 0.000 | 0.000 | 0.180 | 0.114 | 0.045 |
| **2** | 1.189 | 0.978 | 0.359 | 0.861 | 0.735 | 0.262 |
| **3** | 2.294 | 1.356 | 0.593 | 2.074 | 1.144 | 0.519 |
| **4** | 1.508 | 1.014 | 0.413 | 1.593 | 1.121 | 0.444 |
| **5** | 3.391 | 2.191 | 0.911 | 3.387 | 2.187 | 0.910 |
| **6** | 2.780 | 1.500 | 0.691 | 2.759 | 1.500 | 0.688 |
| **7** | 2.740 | 2.667 | 0.904 | 2.662 | 2.662 | 0.893 |
| **8** | 0.993 | 0.798 | 0.296 | 0.843 | 0.784 | 0.269 |
| **9** | 2.484 | 2.112 | 0.762 | 2.381 | 2.039 | 0.733 |
| **10** | 1.682 | 1.159 | 0.465 | 1.573 | 1.065 | 0.430 |
| **11** | 1.535 | 1.293 | 0.469 | 1.142 | 0.942 | 0.343 |
| **12** | 1.532 | 1.010 | 0.415 | 0.011 | 0.011 | 0.000 |

As mentioned the treatment which has a minimum of $Q_i$ value is the best treatment that includes the best levels of predictor variables. As shown in Table 7, the minimum value of $Q_i$ index for the real case is zero; and this value is related to the first treatment which has levels of "1, 1, 1" for each predictor variable.

The calculated accumulation indices of second example show that last treatment of this example has minimum value of $Q_i$ which has included the levels of "2, 2, 3" for predictor variables, respectively. Note that the first $Q_i$ value of second example has a very low gap from last $Q_i$.

This case can be better interpreted with the GTA. Through looking at Table 5, it can be understood that in addition to the (3, (2, 2)) NE, the (1, (1, 1)) strategy also is NE; but using the Pareto optimal, the (3, (2, 2)) is the best strategy of this game. The accuracy of the GTA is determined by comparing the results of Tables 3 with 7 and 5 with 7.

### 4-4. Validation of the Proposed Game Theory-Based Approach

In this subsection, we validate the proposed game theory-based approach (heuristic approach) through simulation studies. For this purpose, we compare the results of the proposed approach and the VIKOR method under 10,000 simulated problems. The simulation steps are given as follows:

**Step 1**: Generate the three independent factor levels with specified lower and upper bound using random numbers between 1 and 4 in Matlab software and then generate the correlated residuals of the regression

models using multivariate normal distribution with $\boldsymbol{\mu} = [0,0]$ and $\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$, where $\sigma_1^2$ and $\sigma_2^2$ are the variances of two responses and $\sigma_{12}$ is the covariance between the responses. In our simulation, we use the 1,1 and 0.1 respectively for variances of the responses and the covariance between responses.

**Step 2**: Calculate the values of two responses using the following hypothetical regression models:

$$y_1 = 0.3x_1 + 0.2x_2 + 0.5x_3 + \varepsilon_1$$

$$y_2 = 0.5x_1 + 0.8x_2 + 0.2x_3 + \varepsilon_2$$

**Step 3**: Determine the S/N ratio of these responses and design the game theory model to select the NE point.

**Step 4**: Determine the VIKOR index and select the optimal setting.

The simulation showed the identical results of the proposed approach and VIKOR method in 91% of cases. Hence, the proposed approach performs efficient in different problems.

## 5. Conclusion and Future Researches

In this paper, a new method was proposed to multi responses optimization based on the game theory approach. GTA is a useful method for decision making in the conflict of interests between intelligent players in order to select the best joint strategy for them through selecting the best joint desirability. Many experiments have more than one response variable which may have conflicting objectives such as LTB, NTB and STB. This paper considered two responses with the LTB and NTB type in the real case and the numerical example. First, SN ratios of each response (or each player) were calculated and modeled as game theory table. Then, using the NE and PONE, the PE of those examples were determined which were the best joint utilities for selected strategies of each player.

Also, the VIKOR method was used to evaluate the efficiency of the proposed GTA. The obtained results represented the accuracy of the proposed approach. Hence, the GTA can be used efficiently for multi responses optimization problems. Generally, the proposed approach is a novel heuristic approach which can be easily applied instead of some complicated multi response optimization methods and get the same results.

As a future research, multi responses optimization problem with more than two responses which represents the games by three and more players can be investigated. Also, in some situations, the response variables are linguistic and characterized by fuzzy random numbers. In these cases, it seems the fuzzy game theory model should be developed to solve the multi response optimization problem which is a fruitful area for research.

## 6. Acknowledgement

The authors would like to thank the anonymous reviewers for his valuable and constructive comments which led to improvement in the paper.

## References

[1] Navidi, H., Ketabchi, S., Messi, M., An Introduction to Game Theory, (in Farsi), Shahed University, 2011.

[2] Osborne, M.J., A Course in Game Theory, Oxford University Press, 2003.

[3] Myerson, R.B., Game Theory: Analysis of Conflict, The MIT Press, 1997.

[4] Poroch-Seritan, M., Gutt, S., Gutt, G., Crestescu, I., Cojocaru, C., Severin, T., "*Design of Experiments for Statistical Modeling and Multi-Response Optimization of Nickel Electroplating Process"*, Chemical Engineering Research and Design, Vol. 89, 2011, pp. 136-147.

[5] Khuri, A.I., Conlon, M., "*Simultaneous Optimization of Multiple Responses Represented by Polynomial Regression Functions*", Technometrics, Vol. 23, 1981, pp. 363–75.

[6] Logothetis, N., Haigh, A., "*Characterizing and Optimizing Multi-Response Processes by the Taguchi Method*", Quality and Reliability Engineering International, Vol. 4, 1988, pp. 159–69.

[7] Tong, L.I., Su, C.T., "*Optimizing Multi-Response Problems in the Taguchi Method by Fuzzy Multiple Attribute Decision Making*", Quality and Reliability Engineering International, Vol. 13, 1997, pp. 25-34.

[8] Jeyapaul, R., Shahabudeen, P., Krishnaiah, K., "*Simultaneous Optimization of Multi-Response Problems in the Taguchi Method Using Genetic Algorithm*", The International Journal of Advanced Manufacturing Technology, Vol. 30, 2005, pp. 870-78.

[9] Tong, L.I., Hsieh, K.L., "*A novel means of applying artificial neural networks to optimize multi-response problem*", Quality Engineering, Vol. 13, 2000, pp. 11-18.

[10] Pan, L.K., Haq., "*Optimizing Multiple Quality Characteristics Via Taguchi Merthod-Based Grey Analysis*", Journal of Materials Processing Technology, Vol. 128, 2007, pp. 107-16.

[11] Haq, A.N., Marimuthu, P., and Jeyapaul, R., "*Multi Response Optimization of Machining Parameters of Drilling Al/SiC Metal Matrix Composite using Grey Relational Analysis in the Taguchi Method*", The International Journal of Advanced Manufacturing Technology, Vol. 37, 2007, pp. 250-55.

[12] Ramakrishnan, R., Karunamoorthy, L., "*Multi Response Optimization of Wire EDM Operations Using Robust Design of Experiments*", The International Journal of Advanced Manufacturing Technology, Vol. 29, 2006, pp. 105-12.

[13] Tong, L.I., Chen, C.C., Wang, and C.H., "*Optimization of Multi Response Processes Using the VIKOR Method*", The International Journal of Advanced Manufacturing Technology, Vol 31, 2007, pp. 1049-57.

[14] Liao, H.C., "*Multi-Response Optimization Using Weighted Principal Component*", The International Journal of Advanced Manufacturing Technology, Vol. 27, 2006, pp. 720-25.

[15] Datta, S., Nandi, G and Bandyopadhyay, A., "*Analyzing Experiments with Correlated Multiple Responses*", Journal of Manufacturing Systems, Vol. 28, 2009, pp. 55-63.

[16] Bashiri, M., Hejazi, T.H., "*An Extension of Multi Response Optimization in MADM View*", Journal of Applied Sciences, Vol. 9, 2009, pp. 1695-702.

[17] Pal, S., Gauri, S.K., "*Assessing Effectiveness of the Various Performance Metrics for Multi-Response Optimization Using Multiple Regressions*", Computers & Industrial Engineering, Vol. 59, 2010, pp. 976–985.

[18] He, Z., Zhu, P.F., Park, S.H., "*A Robust Desirability Function Method for Multi-Response Surface Optimization Considering Model Uncertainty*", European Journal of Operational Research, Vol. 221, 2012, pp. 241–247.

[19] Hejazi, T.H., Ramezani, M., Seyyed-Esfahani, M.M., Kimiagari, A.M., "*Multiple Response Optimization with Probabilistic Covariates Using Simultaneous Equation Systems*", International Journal of Industrial Engineering & Production Research, Vol. 24, 2013, pp. 113-121.

[20] Zadbood, A., Noghondarian, K.., Zadbood, Z., "*Multiresponse Surface Optimization Via Harmony Search Algorithm*", International Journal of Industrial Engineering & Production Research, Vol. 24, 2013, pp. 131-136.

[21] Bashiri, M., Bagheri, M., "*Using Imperialist Competitive Algorithm in Optimization of Nonlinear Multiple Responses*", International Journal of Industrial Engineering & Production Research, Vol. 24, 2013, pp. 229-235.

[22] Zolgharnein, J., Asanjrani, N., Bagtash, M., Azimi, GH., "*Multi-Response Optimization Using Taguchi Design and Principle Component Analysis for Removing Binary Mixture of Alizarin Red and Alizarin Yellow from Aqueous Solution by Nano γ-Alumina*", Spectrochimica Acta Part A: Molecular and Biomolecular Spectroscopy, Vol. 126, 2014, pp. 291–300.

[23] Shi, W., Shang, J., Liu, Z., Zuo, X., "*Optimal Design of the Auto Parts Supply Chain for JIT Operations: Sequential Bifurcation factor Screening and Multi-Response Surface Methodology*", European Journal of Operational Research, Vol. 236, 2014, pp. 664–676.

[24] Li, R., Ma, H., Wang, F., Wang, Y., Liu, Y., Li, Z., "*Game Optimization Theory and Application in Distribution System Expansion Planning, Including Distributed Generation*", Energies, Vol. 6, 2013, pp. 1101-1124.

[25] Rao, S.S., "*Game Theory Approach for Multi Objective Structural Optimization*", Computers & Structures, Vol. 25, 1987, pp. 119–127.

[26] Zamarripaa, M.A., Aguirre, A.M., Méndez, C.A., Espuñ, A., "*Mathematical Programming and Game Theory Optimization-Based Tool for Supply Chain Planning in Cooperative/Competitive Environments*", Chemical Engineering Research and Design, Vol. 91, 2013, pp. 1588–1600.

[27] Wu, F.C., Yeh, C.H.,"*A Comparative Study on Optimization Methods for Experiments with Ordered Categorical Data*", Computers &Industrial Engineering, Vol. 50, 2006, pp. 220–232.

[28] Von Neumann, J., Morgenstern, O., Theory of Games and Economic Behavior. Princeton University Press, 1944, page 138-140.

[29] Schelling, T.C., The strategy of conflict, Harvard University, 1960.

.