



DEA with Common Set of Weights Based on a Multi Objective Fractional Programming problem

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ABSTRACT

Data envelopment analysis operates as a tool to appraise the relative efficiency of a set of homogenous decision making units. DEA allows each DMU to take its optimal weight in comparison to other DMUs while a similar condition is considered for other units. This feature threatens the comparability of different units because different weighting schemes are used for different DMUs. In this paper, a model is presented to determine a common set of weights to calculate DMUs efficiency. This model is developed based on a multi objective fractional linear programming model that considers the original DEA's results as ideal solution and seeks a set of common weights to evaluate DMUs and increases the model's discrimination power. A numerical example is solved and the proposed method's results are compared to some previous methods. This Comparison has shown the proposed method's advantages in ranking DMUs.

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1. Introduction

Data envelopment analysis is a nonparametric method to evaluate the relative efficiency of a group of homogenous units. Charnes, Cooper and Rhodes first originated the data envelopment analysis in 1978 and presented the basic CCR model [1]. Surveys of Emrouznejad et al. [2] and Liu et al. [3] illustrated that thousands of projects and studies are done based on different DEA models in its thirty-year duration after emersion. The DEA model is used to evaluate the relative efficiency of a group of n homo generous units (DMUs) which use m inputs to produce s outputs. Many scholars of science philosophy, following Popper's philosophy, believe that revocability is one of the most important attributes a real scientific finding

need. As a scientific model, DEA isn't an exception. DEA weakness in discrimination among efficient units has attracted strong criticism. Most efficiency measures in DEA estimate the distance between DMUs from an efficient frontier as their relative efficiency. By experience, however, it is found that many DMUs are classified as efficient (with identical efficiency score) and, hence, there is no discrimination among them. In particular, this problem is more significant when the number of DMUs regarding the number of inputs and outputs are small. This problem brings about the common set of weights (CSW) problem. The CSW problem seeks a common set of weights for factors used to evaluate DMUs' efficiency. Roll et al. [4] and Roll and Golany [5] were the first who considered factor weights in DEA. Many researchers developed various models for CSW problems which are mostly based on multiple objective programming as Kornbluth [6] initially accentuated DEA as a multi objective fractional programming problem.

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Andersen and Peterson [7] proposed an important model, AP model, for ranking efficient units by omitting the form of possibility production set. Then, Mehrabian et al. [8] improved AP model. In this course, Hossein zadeh Lotfi et al. [9] developed a multiple objective model to determine the CSW. Despotis [10] introduced the global efficiency approach to improve DEA discriminating power. Kao and Hung [11] proposed a compromising solution to determine the CSW based on minimum distance from weights of standard models like CCR and BCC. Jahanshahloo et al. [12] proposed a method which determines the CSW based on a single model. Kuosmanen et al. [13] introduced the law of one price in DEA that takes the same price for inputs and outputs of all firms. Makui et al. [14] developed a multi objective linear programming to determine the CSW to increase the discrimination power of standard DEA models.

Liu and Peng [15] developed a method to determine the CSW based on optimization of the group efficiency. Jahanshahloo et al. [16] developed two methods based on an ideal line and a special line that measure a new efficiency score for efficient DMUs. Tavakkoli-Moghaddam and Mahmoodi [17] applied the idea of fuzzy entropy in finding DEA common set of weights. Chiang et al. [18] proposed a separation method to locate a common set of weights. Wang et al. [19] proposed a method based on regression analysis to determine the CSWs. Saati et al. [20] also proposed a two-phase algorithm to determine the CSWs in which an ideal decision making unit is defined first, and then the efficiency of DMUs is determined. Davoodi and Zhiani Rezai [21] proposed a linear programming based model to find a common set of weights for DMUs and then to rank them. Ramon et al. [22] obtained a common set of weights to rank DMUs by minimizing the deviations of the CSW from the DEA weights profiles.

In this paper, a multi objective linear fractional programming (MOLFP) is proposed to rank the efficient DMUs based on a common set of weights and is solved based on fuzzy membership functions. The present paper is organized as follows. In section 2, DEA is discussed. Multi objective linear fractional programming (MOLFP) is briefly reviewed in section 3. The proposed MOLFP method to find CSWs is illustrated in section 4. A numerical example that is illustrated in Kao and Hung [11] is solved with proposed method and its results are compared to original results and the results obtained by Makui et al. [14] in section 5. Finally, the conclusion is presented in section 6.

2. Data Envelopment Analysis

Data Envelopment Analysis (DEA) measures relative efficiency of a set of decision making units (DMUs) that consume multiple inputs and produce

multiple outputs. Original DEA models were formulated by Charnes, Cooper, and Rhodes [1] and DEA models have become one of the main modeling tools for efficiency analysis since 1980's. In fact, DEA is a multi-factor productivity analysis model to measure the relative efficiencies of a homogenous set of DMUs. The efficiency score in the presence of multiple inputs and outputs is defined as below:

$$\text{Efficiency} = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}} \quad (1)$$

The basic multiplier form of CCR model can be illustrated as follows:

$$\text{Max } u^t y_0$$

S.T.

$$v^t x_0 = 1 \quad (2)$$

$$u^t y_j - v^t x_j \leq 0$$

$$u^t, v^t \geq 0$$

Where, there are n DMUs under evaluation, $j = 1, \dots, n$. Every DMU used m -dimensional input vector $x_j = [x_{1j}, \dots, x_{mj}]$ to produce an s -dimensional output vector $y_j = [y_{1j}, \dots, y_{sj}]$. The m -dimensional vector $u^t = [u_1, \dots, u_m]$ is the weight of input variables and the s -dimensional vector $v^t = [v_1, \dots, v_s]$ is the weight of output variables that are applied to determine the relative efficiency of under evaluation unit. DEA model is run for each DMU and determined the optimal values of u^t and v^t to measure the relative efficiency of units.

This model is called input oriented CCR model under constant return to scale. There are many extensions of this initial model with different assumptions like variable return to scale, output/ input orientation, additive models etc. There is a wide range of publications that examine and identify different DEA models, among which interested readers can refer to Charnes [23], Ray [24], and Cooper et al. [25].

3. Multi Objective Linear Fractional Programming

A multi objective linear fractional programming (MOLFP) model can be defined as follows:

$$\max \left(\frac{N_1(x)}{D_1(x)}, \frac{N_2(x)}{D_2(x)}, \dots, \frac{N_p(x)}{D_p(x)} \right) \quad (3)$$

S.T.

$$x \in X$$

$N_i(x)$ and $D_i(x), i = 1, 2, \dots, p$ are linear functions and the problem is to simultaneously maximize $N_i(x)/D_i(x), i = 1, 2, \dots, p$ ratios.

Kornbluth and Steuer [26], Luhadjula [27], and Dutta et al. [28] examined MOLFP problems and proposed some methods to solve such problems. In this paper, the Dutta et al.'s [28] model is used to solve CSWs problem. According to Dutta et al. [28], the following membership functions— $\forall i = 1, 2, \dots, p$ can be defined for nominators and denominators of objective functions in Eq. (3).

$$C^{N_i}(x) = \begin{cases} 0 & \text{if } N_i(x) < p_i \\ \frac{N_i(x) - p_i}{N_i^0 - p_i} & \text{if } p_i \leq N_i(x) \leq N_i^0 \\ 0 & \text{if } N_i(x) > N_i^0 \end{cases} \quad (4)$$

$$C^{D_i}(x) = \begin{cases} 0 & \text{if } D_i(x) > s_i \\ \frac{s_i - D_i(x)}{s_i - D_i^0} & \text{if } D_i^0 \leq D_i(x) \leq s_i \\ 0 & \text{if } D_i(x) < D_i^0 \end{cases} \quad (5)$$

Where N_i^0 and $D_i^0 (\forall i = 1, 2, \dots, p)$ represent the maximal value of nominator $N_i(x)$ and the minimal value of denominator $D_i(x)$ on the set X, while p_i, s_i are the thresholds beginning with which values $N_i(x)$ and $D_i(x)$ are acceptable.

As the membership function of the goal i is induced by the objective function $N_i(x)/D_i(x)$, the function $\mu_i(x)$ is chosen as follows:

$$\mu_i(x) = w_i C^{N_i}(x) + w'_i C^{D_i}(x), \forall i = 1, 2, \dots, p \quad (6)$$

Where w_i and w'_i are the weights indicating the relative importance given by decision makers to the criteria so that they can verify the condition $\sum_{i=1}^p (w_i + w'_i) = 1$. The following problem is then used to obtain the solution of problem (3).

$$\begin{aligned} \max V(\mu) &= \sum_{i=1}^p (w_i \mu_i^{N_i} + w'_i \mu_i^{D_i}) \\ \text{S.T.} & \\ \mu_i^{N_i} &= C^{N_i}(x), \mu_i^{D_i} = C^{D_i}(x) \\ 0 \leq \mu_i^{N_i} &\leq 1, \quad 0 \leq \mu_i^{D_i} \leq 1, \forall i = 1, 2, \dots, p \\ x &\in X \end{aligned} \quad (7)$$

Dutta et al. [28], and then Stanco-Minasian and Pop [29] proved the efficiency of solution obtained by solving model (7) for the problem (3).

4. MOLFP for Determination of CSWs

Suppose that there are n decision making units, in which each DMU let's $DMU_j (j = 1, 2, \dots, n)$, use inputs vector $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ to produce the output vector $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$. The efficiency of DMUs then is evaluated based on original CCR model. Now, DMUs are classified into two sets: efficient units (set E), and inefficient units (set I). DMUs in set r can be ranked based on their initial CCR scores. The problem here is to rank the DMUs in set E. This ranking procedure can be taken into account as finding a common set of weights that efficient DMUs are evaluated against with these weights. According to Eq. (1), efficiency of each DMU can be shown as:

$$E_0 = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \quad (8)$$

The optimum weights of $u_r^* (r = 1, 2, \dots, s)$ and $v_i^* (i = 1, 2, \dots, m)$ are determined based on Eq. (2) so that E_0 takes its maximum value. The following MOLFP model determines a set of CSWs for efficient unit's output variables.

$$\begin{aligned} \max_j & \left\{ E_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m x_i y_j} \right\} \\ E_j & \leq 1 \\ u_r & \geq 0 \quad r = 1, 2, \dots, s \\ v_i & \geq 0 \quad i = 1, 2, \dots, m \end{aligned} \quad (9)$$

According to Dutta et al. [28], a major point to solve the model (4) is to determine the nominator and denominator of E_j membership functions. For efficient units, i.e. $j \in E$, the ideal values of nominator (N_i^0) and denominator (D_i^0) can be easily obtained as one, which is the input oriented and output oriented solution for efficient units. On the other hand, decision maker wants that common efficiency scores of efficient units not be worse than the inefficient unit with the highest efficiency that is:

$$\theta^+ = \max_{j \in I} (E_j) \quad (10)$$

That is $p_i = \theta^+$ and $s_i = 1/\theta^+$. Based on these conditions, the membership functions of Eq. (9) objectives are defined as follows:

$$C_E^{N_i}(x) = \begin{cases} 0 & \text{if } N_i(x) < \theta^+ \\ \frac{N_i(x) - \theta^+}{1 - \theta^+} & \text{if } \theta^+ \leq N_i(x) \leq 1, \forall i \in E \\ 0 & \text{if } N_i(x) > 1 \end{cases} \quad (11)$$

$$C_E^{D_i}(x) = \begin{cases} 0 & \text{if } D_i(x) < 1 \\ \frac{1/\theta^+ - D_i(x)}{1/\theta^+ - 1} & \text{if } 1 \leq D_i(x) \leq 1/\theta^+, \forall i \in E \\ 0 & \text{if } D_i(x) > 1/\theta^+ \end{cases} \quad (12)$$

Similarly, these values can be defined for inefficient units as $N_i^0 = \theta^+$, $D_i^0 = 1/\theta^+$, $p_i = \theta^-$ and $s_i = 1/\theta^-$ where:

$$\theta^- = \min_{j \in I} (E_j) \quad (13)$$

Therefore, the membership functions for inefficient units will be as follows:

$$C_I^{N_i}(x) = \begin{cases} 0 & \text{if } N_i(x) < \theta^- \\ \frac{N_i(x) - \theta^-}{\theta^+ - \theta^-} & \text{if } \theta^- \leq N_i(x) \leq \theta^+, \forall i \in I \\ 0 & \text{if } N_i(x) > \theta^+ \end{cases} \quad (14)$$

$$C_I^{D_i}(x) = \begin{cases} 0 & \text{if } D_i(x) < 1/\theta^+ \\ \frac{1/\theta^- - D_i(x)}{1/\theta^- - 1/\theta^+} & \text{if } 1/\theta^+ \leq D_i(x) \leq 1/\theta^-, \forall i \in I \\ 0 & \text{if } D_i(x) > 1/\theta^- \end{cases} \quad (15)$$

Now, the following ordinal linear programming model can be solved to obtain a common set of weights.

$$\begin{aligned} & \max \frac{1}{2} \sum_{j=1}^n [C^{N_j}(x) + C^{D_j}(x)] \\ & S.T. \\ & \sum_{j=1}^n u C_e^{N_j}(x) \leq 1 \quad \forall e \in E \\ & C_e^{D_j}(x) \leq 1 \quad \forall e \in E \\ & C_i^{N_j}(x) \leq \theta^+ \quad \forall i \in I \\ & C_i^{D_j}(x) \geq 1/\theta^+ \quad \forall i \in I \\ & u_r \geq 0 \quad r = 1, 2, \dots, s \\ & v_i \geq 0 \quad i = 1, 2, \dots, m \end{aligned} \quad (16)$$

By substituting the Eq. (11), (12), (14), and (15) in Eq. (16), the final model will be as follows:

$$\begin{aligned} & \max \frac{1}{2} \sum_{j=1}^n [C^{N_j}(x) + C^{D_j}(x)] \\ & S.T. \\ & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad \forall j \in E \quad (I) \\ & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq \theta^+ - 1/\theta^+, \quad \forall j \in I \quad (II) \\ & u_r \geq 0 \quad r = 1, 2, \dots, s \\ & v_i \geq 0 \quad i = 1, 2, \dots, m \end{aligned} \quad (17)$$

The model (16) is an ordinary linear programming model which can be solved by well-developed methods. Note that the model is designed so that none of inefficient DMUs in original DEA model gains a higher rank rather than the efficient units and also all DMUs can be ranked regarding a common set of weights, which was the main purpose of the model. Now, the following two important theorems are proved for proposed model.

Theorem 4.1. A DMU which is shown to be efficient in model (17) is essentially efficient in original CCR model.

Proof. Suppose that $s_j^*, j=1, 2, \dots, n$ shows the slack variables' values in optimal solution of model (17). There are 2 positions for these values:

- (1) If $s_j^* = 0, j \in E$, then regarding constraints (I), the $E_j^* = 1$ according to Eq. (8) and $DMU_j, j \in E$ will be an efficient unit based on model (17) that is also a CCR efficient unit, for $j \in E$.
- (2) If $s_j^* = 0, j \in I$, then regarding constraints (II), the $E_j^* < 1$ according to Eq. (8) and $DMU_j, j \in I$ will be an inefficient unit based on model (17) which is also a CCR inefficient unit, for $j \in I$.

The second theorem is based on Roll et al. [4] and Golany and Yu [30] who argued that a general requirement for CSWs problems is that at least one DMU must be recognized as efficient.

Theorem 4.2. There exists at least a $DMU_j, j=1, 2, \dots, n$ which is characterized as the efficient DMU by model (17).

Proof. Let: $(v^*, u^*) = (v_1^*, v_2^*, \dots, v_m^*, u_1^*, u_2^*, \dots, u_s^*)$ be an optimal solution of model (17), for which none of the efficient units, according to theorem 1, take 1 efficiency score in model (17). This means that $s_j^* > 0$ for $j \in E$. It can be found as real valued vectors ε_v and ε_u for which $(v^*, u^*) - (-\varepsilon_v, \varepsilon_u)$ is also a feasible solution that has an objective function value, which is greater than optimal

solution. This is in contradiction to optimality of $(v_1^*, v_2^*, \dots, v_m^*, u_1^*, u_2^*, \dots, u_s^*)$ and the proof is completed.

5. Numerical Example

Kao and Hung[11] illustrated their compromising solution approach for CSWs problem by expressing an example that is derived from Kao and Hung [11] and later Makui et al. [14] also examined this example by their goal programming method for finding CSWs and compared their results with Kao and Hung [11].

Here, this example is analyzed with proposed method and its results are compared with Kao and Hung [11] and Makui et al. [14] results. Thus, the study is related to evaluation of 17 forest districts. Four inputs: budget

(in US dollars), initial stocking (in cubic meters), labor (in number of employees) and land (in hectares), and three outputs: main product (in cubic meters), soil conservation (in cubic meters), and recreation (in number of visits) are considered to measure the efficiency. Table 1 contains the original data. Table 2 shows the results of solving this DEA problem. Numbers that are appeared in parentheses show the rank of DMUs. Second column of table 2 shows the results obtained from original CCR model that is run for data. Based on CCR model results, there are 9 efficient DMUs among which the model cannot discriminate, and all efficient DMUs are categorized as efficient.

Tab. 1. Input and output data of the 17 forest districts in Taiwan

District	Input				Outputs		
	Budget (\$)	Initial Stocking (m ³)	Labor (persons)	Land (ha)	Main product (m ³)	Soil conservation (m ³)	Recreation (visits)
1	51.62	11.23	49.22	33.52	40.49	14.89	3155.71
2	85.78	123.98	55.13	108.46	43.51	173.93	6.45
3	66.65	104.18	257.09	13.65	139.74	115.96	0
4	27.87	107.6	14	146.43	25.47	131.79	0
5	51.28	117.51	32.07	84.5	46.2	144.99	0
6	36.05	193.32	59.52	8.23	46.88	190.77	822.92
7	25.83	105.8	9.51	227.2	19.4	120.09	0
8	123.02	82.44	87.35	98.8	43.33	125.84	404.69
9	61.95	99.77	33	86.37	45.43	79.6	1252.6
10	80.33	104.65	53.3	79.06	27.28	132.49	42.67
11	250.62	183.49	144.1	59.66	14.09	196.29	16.15
12	82.09	104.94	46.51	127.28	44.87	108.53	0
13	202.21	187.74	149.39	93.65	44.97	184.77	0
14	67.55	82.83	44.37	60.85	26.04	85	23.95
15	72.6	132.73	44.67	173.48	5.55	135.65	24.13
16	84.83	104.28	159.12	171.11	11.53	110.22	49.09
17	71.77	88.16	69.19	123.14	44.83	74.54	6.14

Columns 3 - 5 of table 2 that are labeled as MAD, MSE, and MAX show the results of Kao and Hung [11]. Their model is developed based on an L_p -metric measure that MAD is based on $p = 1$, MSE based on $p = 2$, and MAX $p = \infty$.

According to these results, MAD model reduces the number of efficient units from 9 to 4. On the other hand, MSE model reduces this number to 2 while in MAX model, there are 3 efficient units. It is clear that the discrimination power of DEA model is improved significantly. Examination of Makui et al. [14] results, column 6, show that in their model 5 efficient units are identified. The last column shows the results of proposed model. In solving proposed model, note that for CCR efficient units (DMU1 - DMU9), following settings are used:

$$N_i^0 = D_i^0 = 1; p_i = 0.9403; s_i = 1.0635 \tag{17}$$

These settings for inefficient units (DMU10 - DMU17) are as follows:

$$N_i^0 = 0.9403; D_i^0 = 1.0635; p_i = 0.6873; s_i = 1.455 \tag{18}$$

Then, the proposed model's results are obtained by solving the model (16). The CSWs obtained from model (17) are shown in table 3. The results of the proposed method in column 7 show that only DMUs 1

and 6 are classified as efficient. Also, the proposed model results are consistent with MSE model of Kao and Hung [11], which specifies two DMUs as efficient. Table 4 shows the results of the relationship between

different model results with original CCR model based on Spearman rank correlation. These results show that the proposed method has the highest correlation with original CCR model.

Tab. 2. Results of analyzing data with different CSWs methods

DMU	CCR	Kao and Hung (2005)			Makui et al. (2008)	Proposed model
		MAD	MSE	MAX		
1	1.0000(1)	1.0000(1)	1.0000(1)	1.0000(1)	1.0000(1)	1.0000(1)
2	1.0000(1)	1.0000(1)	1.0000(1)	1.0000(1)	1.0000(1)	0.3739(8)
3	1.0000(1)	1.0000(1)	0.9989(3)	0.7231(11)	1.0000(1)	0.7200(3)
4	1.0000(1)	1.0000(1)	0.9927(4)	0.8987(4)	1.0000(1)	0.5211(4)
5	1.0000(1)	0.9747(5)	0.9866(5)	1.0000(1)	1.0000(1)	0.5072(6)
6	1.0000(1)	0.8534(9)	0.9123(6)	0.8692(7)	0.9654(6)	1.0000(1)
7	1.0000(1)	0.9244(6)	0.8849(7)	0.7432(9)	0.8743(8)	0.4028(7)
8	1.0000(1)	0.8954(7)	0.8707(9)	0.8939(5)	0.8469(9)	0.2707(9)
9	1.0000(1)	0.6619(14)	0.6690(14)	0.7230(12)	0.6783(13)	0.5202(5)
10	0.9403(10)	0.8721(8)	0.8768(8)	0.8761(6)	0.8779(7)	0.3059(10)
11	0.9346(11)	0.6398(15)	0.6518(15)	0.6577(13)	0.6526(15)	0.1377(17)
12	0.8290(12)	0.7456(10)	0.7282(10)	0.7594(8)	0.7175(11)	0.2829(11)
13	0.7997(13)	0.6229(17)	0.6260(16)	0.6453(14)	0.6227(16)	0.1888(16)
14	0.7733(14)	0.7140(12)	0.7142(12)	0.7406(10)	0.7126(12)	0.2631(12)
15	0.7627(15)	0.7245(11)	0.7210(11)	0.6410(15)	0.7215(10)	0.2364(14)
16	0.7435(16)	0.6996(13)	0.6811(13)	0.4665(17)	0.6696(14)	0.1965(15)
17	0.6873(17)	0.6310(16)	0.6068(17)	0.5908(16)	.05925(17)	0.2589(13)

Tab. 3. CSWs for inputs and outputs based on proposed model

	Input			Outputs			
	Budget (\$)	Initial Stocking (m ³)	Labor (persons)	Land (ha)	Main product (m ³)	Soil conservation (m ³)	Recreation (visits)
CSW	0.017578	0.001803	0	0.002159	0.004615	0.003062	0.000242

Tab. 4. Pair wise Spearman rank correlation between different CSWs models

	MAD	MSE	MAX	Makui et al. (2008)	Proposed model
CCR	0.74	0.746	0.753	0.763	0.834

Tab. 5. CSWs for inputs and outputs with weights restrictions

	Input			Outputs			
	Budget (\$)	Initial Stocking (m ³)	Labor (persons)	Land (ha)	Main product (m ³)	Soil conservation (m ³)	Recreation (visits)
CSW	0.017367	0.003473	0.000695	0.000905	0.004607	0.003072	0.000241

As it can be seen in table 3, the 3rd input variable, labor, takes a weight equal to zero and it appears that this variable is inactive in efficiency appraisal. To solve this problem, a set of restrictions can be imposed over weights. Suppose that the following restrictions are added to model (17).

$$1 \leq u_1/u_2 \leq 1.5, 3 \leq u_2/u_3 \leq 5$$

$$3 \leq v_1/v_2 \leq 5, 5 \leq v_2/v_3 \leq 10$$

$$0.5 \leq v_3/v_4 \leq 1.5$$

Now, if the model (17) is solved with these additional constraints, table 5 shows the resulted CSWs. It indicates that neither inputs nor outputs weights become zero with weight restrictions taken into account

6. Conclusion

DEA models can be considered as classification models that classify DMUs into two efficient and inefficient groups. One criticism against this approach is that there is no discrimination among efficient units.

In this paper, a multi objective linear fractional model is developed for calculation of CSWs in DEA problems. The proposed method is then solved based on fuzzy membership functions of nominators and denominators. The results of the proposed model provide a full ranking of DMUs. The main differences between Kornbluth [6] and the proposed model are that (1) Kornbluth formulated and solved the problem as a multi objective linear vector maximization problem while in this paper, CSW is formulated as a multi objective linear fractional programming problem, and (2) the Kornbluth model can be considered as a priori analysis of the efficiency evaluation problem without any relation to classic DEA models while the proposed model is a posteriori analysis of classic DEA models result.

One of the properties of this proposed method is that when original CCR model classifies DMUs into two groups, the final ranking does not lead to an original inefficient unit which lies in a position higher than an original efficient unit while it is possible in other CSW models.

This method has a direct link with classic DEA model results which strengthen its application with underlying theories of DEA. Also, it is possible to add some constraints on weight restriction to prevent common weights to become zero. Two main properties of the proposed method are proved which guarantee the existence of an efficient unit as one of the required characteristics of DEA. The results of numerical example show that (1) at least one DMU gains an efficiency of 1 according to theorem 2; (2) the proposed method has the highest correlation with original CCR model; and (3) weights restrictions can remove the weights of inputs and outputs. The linear structure of proposed model is its main property that makes its solution much easier, compared to previous nonlinear models.

The obtained results from the proposed model have a meaningful implication for managers. Classic DEA models compute different weights for different inputs and outputs for each DMU while managers often seek a comprehensive and common base for evaluating and comparing them under supervision units. Therefore, the proposed model's results provide foundation for managerial decision making process regarding DMUs' performance.

As a clue for future studies, researchers can focus on finding common set of weights in uncertain environment, with fuzzy or stochastic information. Application of common set of weights concept in multi attribute decision making can be also considered as another field for future researches.

References

- [1] Charnes, A., Cooper, W., Rhodes, E., "Measuring Efficiency of Decision Making Units," European Journal of Operational Research, Vol.2, 1978, pp. 429-444.
- [2] Emrouznejad, A., Parker, B.R., Tavares, G., "Evaluation of Research in Efficiency and Productivity: A Survey and Analysis of the First 30 Years of Scholarly Literature in DEA," Socio-Economic Planning Sciences, Vol.2, 2008, pp.151-157.
- [3] Liu, S.J., Lu, L.Y.Y., Lu, W.M., Lin, B.J.Y., "Data Envelopment Analysis 1978–2010: A Citation-Based Literature Survey", Omega, Vol. 41, 2013, pp. 3-15.
- [4] Roll, Y., Cook, W.D., Golany, B., "Controlling Factor Weights in Data Envelopment Analysis," IIE Trans, Vol. 23, 1991, pp. 2-9.
- [5] Roll, Y., Golany, B., "Alternate Methods of Treating Factor Weights in DEA," Omega, Vol. 21, 1993, pp. 99-109.
- [6] Kornbluth, J.S.H., "Analyzing Policy Effectiveness using Cone Restricted Data Envelopment Analysis," Journal of the Operational Research Society, Vol. 42, 1991, pp. 1097-1104.
- [7] Andersen, P., Petersen, N.C., "A Procedure for Ranking Efficient Units in Data Envelopment Analysis," Management Science, Vol. 39, 1993, pp.1261-1264.
- [8] Mehrabian, S., Alirezaee, M.R., Jahanshahloo, G.R., "A Complete Efficiency Ranking of Decision Making Units in Data Envelopment Analysis," Computational Optimizations and Applications, Vol. 14, 1999, pp. 261–266.
- [9] Hosseinzadeh Lotfi, F., Jahanshahloo, G.R., Memariani, A., "A Method for Finding Common set of Weights by Multiple Objective Programming in Data Envelopment Analysis," Southwest Journal of Pure and Applied Mathematics, Vol. 1, 2000, pp. 44-54.
- [10] Despotis, D.K., "Improving the Discriminating Power of DEA: Focus on Globally Efficient Units," Journal of Operation Research Society, Vol. 53, 2002, pp. 314-323.
- [11] Kao, C., Hung, H.T., "Data Envelopment Analysis with Common Weights: the Compromise Solution Approach," Journal of Operation Research Society, Vol. 56, 2005, pp. 1196-1203.
- [12] Jahanshahloo, G.R., Memariani, A., Hosseinzadeh Lotfi, F., Rezaei, H.Z., "A Note on Some of DEA Models and Finding Efficiency and Complete Ranking using Common set of Weights," Applied Mathematics and Computation, Vol. 166, 2005, pp. 265-281.
- [13] Kuosmanen, T., Cherchye, L., Sipilainen, T., "The Law of One Price in Data Envelopment Analysis: Restricting Weight Flexibility Across Firms," European Journal of Operational Research, Vol. 170, 2006, pp. 735-757.
- [14] Makui, A., Alinezhad, A., KianiMavi, R., Zohrebandian, M., "A Goal Programming Method for

- Finding Common Weights in DEA with an Improved Discriminating Power for Efficiency,"* Journal of Industrial and Systems Engineering, Vol. 1, 2008, pp. 293-303
- [15] Liu, F.H.F., Peng, H.H. "Ranking of Units on the DEA Frontier with Common Weights," Computers & Operations Research, Vol. 35, 2008, pp. 1624-1637.
- [16] Jahanshahloo, G.R., Hosseinzadeh Lotfi, F., Khanmohammadi, M., Kazemimanesh, M., Rezaei, V., "Ranking of Units by Positive Ideal DMU with Common Weights," Expert Systems with Application, Vol. 37, 2010, pp. 7483-7488.
- [17] Tavakkoli-Moghaddam, R., Mahmoodi, S. "Finding a Common Set of Weights by the Fuzzy Entropy Compared with Data Envelopment Analysis – a Case Study," International Journal of Industrial Engineering & Production Research, Vol. 21, 2010, pp. 81-88.
- [18] Chiang, C.I., Hwang, M.J., Liu, Y.H., "Determining a Common Set of Weights in a DEA Problem Using a Separation Vector", Mathematical and Computer Modelling, Vol. 54, 2011, pp. 2464-2470.
- [19] Wang, Y.M., Luo, Y., Lan, Y.X., "Common Weights for Fully Ranking Decision Making Units by Regression Analysis," Expert Systems with Applications, Vol. 38, 2011, pp. 9122-9128.
- [20] Saati, S., Hatami-Marbini, A., Agrell, P.J., Tavana, M., "A Common Set of Weight Approach Using an Ideal Decision Making Unit in Data Envelopment Analysis," Journal of Industrial and Management Optimization, Vol. 8, 2012, pp. 623-637.
- [21] Davoodi, A., Zhiani Rezaei, H. "Common Set of Weights in Data Envelopment Analysis: a Linear Programming Problem," Central European Journal of Operations Research, Vol. 20, 2012, pp. 355-365.
- [22] Ramon, N., Ruiz, J.L., Sirvent, I. "Common Sets of Weights as Summaries of DEA Profiles of Weights: with an Application to the Ranking of Professional Tennis Players," Expert Systems with Applications, Vol. 39, 2012, pp. 4882-4889.
- [23] Charnes, A., Data Envelopment Analysis: Theory, Methodology and Applications, Kluwer Academic Publishers, New York, 1994.
- [24] Ray, S.C., Data Envelopment Analysis: Theory and Techniques for Economics and Operations Research, Cambridge University Press, Cambridge, 2004.
- [25] Cooper, W., Seiford, L.M., Tone, K., Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References, and DEA-Solver Software, Kluwer Academic Publishers, New York, 2006.
- [26] Kornbluth, J.H.S., Steuer, R.E., "Multiple Objective Linear Fractional Programming," Management Science, Vol. 27, 1981, pp. 1024-1039.
- [27] Luhandjula, M.K., "Fuzzy Approaches for Multiple Objective Linear Fractional Optimization," Fuzzy Sets and Systems, Vol. 13, 1984, pp. 11-23.
- [28] Dutta, D., Tiwari, R.N., Rao, J.R., "Multiple Objective Linear Fractional Programming Problem-a Fuzzy Set Theoretic Approach," Fuzzy Sets and Systems, Vol. 52, 1992, pp. 39-45.
- [29] Stanco-Minasian, I.M., Pop, B., "On a Fuzzy Set Approach to Solving Multiple Objective Linear Fractional Programming Problem," Fuzzy Sets Systems, Vol. 134, 2003, pp. 397-405.
- [30] Golany, B., Yu, G., "A Goal Programming-Discriminant Function Approach to the Estimation of an Empirical Production Function Based on DEA Results," Journal of Production Analysis, Vol. 6, 1995, pp. 171-186.