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Using Imperialist Competitive Algorithm in Optimization of Nonlinear Multiple Responses

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ABSTRACT

The quality of manufactured products is characterized by many controllable quality factors. These factors should be optimized to reach high quality products. In this paper we try to find the controllable factor's levels with minimum deviation from their target and with a least variation. To solve such problems a simple aggregation function is used to aggregate the multiple response functions followed by an imperialist competitive algorithm used to find the best level of each controllable variable. Moreover the problem has been better analyzed by Pareto optimal solution to release the aggregation function. Then the proposed multiple response imperialist competitive algorithm (MRICA) has been compared with Multiple objective Genetic Algorithm (MOGA). The experimental results show efficiency of the proposed approach in both aggregation and non aggregation methods for optimization of the nonlinear multi-response programming.

1. Introduction

Multi Response Optimization (MRO) methods can be classified into three general categories:

- Desirability viewpoints; in this method which is proposed by Derringer and Suich [1], response variables are aggregated into one function called desirability function. Then this single function should be optimized using an optimization method. Also a score between zero and one is assigned to each response. The score value of zero indicates that the predicted response value is completely undesirable and the value of one indicates that the corresponding response has reached its desired target value.
- Priority based techniques; in this category response variables have different importance according to the decision maker opinion. So these techniques optimize the responses based on their priority factors or importance weights.
- Loss function; in this case all response values are aggregated and converted to a single one based on Taguchi function considering the location and dispersion effects.

It is obvious that when a single objective function optimization method is desired; an aggregation function should be used in most of MRO problems. Some recent works in these three areas are reviewed as follows:

Saurav et al [2] have proposed a hybrid of Taguchi method and principal component analysis to solve the problem. Also they have compared the proposed approach with Grey-Taguchi method. Bashiri and Hejazi [3] have converted the MRO problem to a single response optimization problem in order to

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analyze the robust experimental design by implementing some Multiple Attribute Decision Making (MADM) techniques such as VIKOR, TOPSIS and PROMETHEE. They showed that their proposed method decreases the statistical error. Furthermore, Chang et al [4] have generalized the Taguchi method in order to use it in different situations. The proposed model is presented using a weighted convex loss function.

Alvarez et al. [5] have applied Response Surface Methodology (RSM) in designing and optimizing of a capacitive accelerometer. In their research a faced-centered cube design is implemented in the experimentation. Hsieh [6] has used neural networks to estimate the relation between control variables and responses.

Onur koksoy [7] and Lorenz Imhof [8], have proposed exact solution methods in order to find the optimal parameter settings of multi-response problems. Also Wu and Yeh [9] have proposed multiple polynomial regression models to minimize the total quality loss of dynamic multiple response systems. Chang [10] has introduced an artificial neural network (ANN) approach to solve a dynamic multi-response problem. However, these methods can obtain the best solution among the specified controllable factor levels. In other words in presence of continuous values of controllable factors, the mentioned methods are unable to achieve the real optimal factor combination. So it seems that introducing new heuristic and meta-heuristics which have abilities to find the real optimal solution in a reasonable computational time, is interesting. Recently many meta-heuristic solution approaches have been proposed to solve the MRO problems. Correia et al. [11] have compared the RSM and GA in optimization of welding process.

Ozcelik and Erzurmlu [12] have proposed an RSM and genetic integrated optimization method to minimize the bend of slim layer of plastic. Suresh et al. [13] have proposed second level model for the expecting roughness degree of steel parts. Their research goal was the optimization of roughness degree of the parts using a genetic algorithm. Oktem et al. [14] have presented a genetic algorithm and RSM. In their proposed procedure first RSM provides a model to determine the level of parameters followed by a GA which optimizes these levels. It can be understood from the literature that a few researches have been conducted with focus on heuristic and meta-heuristic approaches implemented to find the best levels of controllable variables.

In this paper the relation function between response variables and controllable factors are estimated. Then an aggregated single objective function has been optimized using an adaptive ICA. Also a new multi-response Imperialist Competitive Algorithm (MRICA) is proposed in order to find the best settings of controllable factors. The rest of presented paper is organized as follows:

In section 2, the RSM will be discussed in details. The proposed solution method will be introduced in section 3. Experimental results comparing to other existing solution approaches have been illustrated in section 4 followed by conclusion in section 5.

2. Problem Description

Relation function between a response and controllable factors can be formulated as equation (1).

$$y_i = f\ (x_1, x_2, ... x_n) + \varepsilon \qquad (1)$$

This equation illustrates the relation function between the controllable factors $(x_1, x_2, ... x_n)$ and the *i*-th process response (y_i). The first step in the RSM is finding the best approximated relation function between a response and a set of independent controllable variables. If there is a curvature in the total system, the second-order or higher polynomial models should be implemented. A second-order relation function taken from [15] can be modeled as equation (2).

$$y(x, z) = \beta_0 + x'\beta + x'Bx + z'\gamma + x'\Delta z + \varepsilon \qquad (2)$$

Where $y(x, z)$ denotes the response variable. x and z denote the vector of controllable and noise factors, respectively. In this model β_0 is the intercept of the regression function. β is the vector of linear effects of control variables. B is the matrix which its main diagonals denote the regression pure factors quadratic coefficients and its off-diagonals denote the coefficient of half quadratic factors interactions. γ is the vector of coefficients for the linear effects in the noise variables and Δ is a matrix which shows the interactions between the controllable variables and noise factors, followed by ε which is the random error with $NID(0, \sigma_\varepsilon^2)$.

In most of RSM related studies, the problem is investigated considering a single response variable. However in practice there are more than one responses and a practitioner should optimize all of these factors, simultaneously. In this paper we assume that the responses are independent or uncorrelated. During the quality control process, to create a robust design with optimal level of factors, the mean and variance of system should be considered.

This procedure is called Dual-Response System (DRS). Many effective methods for DRS have been proposed recently ([16], [17], [18]). One of the popular approaches to the DRS optimization is the MSE criterion proposed by Lin and Tu[15]. They have suggested three basic approaches of MSE for DRS optimization which All of them should be minimized. The first is used when the smaller value of a response is preferable.

$$MSE_1 = (\hat{\mu}_z[\hat{y}(x)])^2 + \hat{\sigma}_z^2[\hat{y}(x)] \quad (3)$$

The second approach implies that the response value should be maximized:

$$MSE_2 = -\{(\hat{\mu}_z[\hat{y}(x)])^2 + \hat{\sigma}_z^2[\hat{y}(x)]\} \quad (4)$$

And the last approach is useful when the target value of the response variable is desirable:

$$MSE_3 = (\hat{\mu}_z[\hat{y}(x)] - T)^2 + \hat{\sigma}_z^2[\hat{y}(x)] \quad (5)$$

In Eq (5), *T* is the target value of the response.

3. Solution Approaches

Imperialist competitive algorithm is a new meta-heuristic evolutionary algorithm that starts with a population of answers, any of them called a country [19]. These countries are divided into two basic groups. The best answers or the powerful countries become the imperialists and the rest of the solutions will be treated as the colonies of imperialists. In the main algorithm there are two types of inner and outer competitions which will be discussed in the following parts.

3-1. Inner Competition:

In each empire it is preferable that the colonies move towards the imperialist and by this strategy, we can explore the search space more efficiently.

3-2. Outer Competition:

The basic competition in an iteration of the algorithm is between the empires. Based on this competition each imperialist wants to take the possession of other imperialists' colonies. And after running the algorithm in the next iterations the weak imperialists will collapsed and at the end of the algorithm only the powerful empire with the best objective function value will be remained which is the optimal or at least close to optimal solution. In each iteration, the weakest empire is selected and its weakest country is assigned to the more powerful empires.

This strategy will lead to increase in the searching diversity. In this research two main solution approaches have been considered. In the first one an aggregated single function is optimized by an adaptive ICA and as the second approach the Pareto frontier is constructed and analyzed using the proposed MRICA.

3-3. The First Approach: Aggregated Function

One of the basic contributions of this research is proposing of an adaptive ICA in the optimization of an aggregated function of MRO problem.

Suppose that we have two response functions based on MSE criteria, so we define the following statement as the single objective function:

$$R = \alpha_1 MSE_1 + \alpha_2 MSE_2 \quad (6)$$

Where

$\alpha_1 + \alpha_2 = 1.$

The pseudo code for the proposed ICA method in optimization of the aggregated function can be written as follow:

1. *Start*

2. *Create the initial countries with special controllable variables values randomly.*

3. *Select the predefined number of best strings as the imperialists and construct the empires.*

4. *Divide the rest of the strings (countries) between the empires based on their normalized power.*

5. *If only one empire remains, go to step 8.*

6. *Start the inner competition:*
   -Apply some of heuristic operators to each empire.

7. *Start the outer competition:*
   -Find the weakest empire based on the total normalized power of the empires.
   -Find the weakest country in that empire and release it from its empire.
   -Form the vector "P" consist of the possession probability of each empire
   -Form the vector "E" with the same size of "P" vector and create the elements of "E" between zero and one randomly.
   -Form the vector R as R=P-E;
   -Select the empire which has the maximum value of R.
   -Assign the released country to the selected empire.
   -Go to step 2;

8. *End;*

3-4. The Second Approach: MRICA

To reach the non dominated solutions in the multiple responses problem a new algorithm which is called Multiple Response Imperialist Competitive Algorithm is proposed in this section. The main structure of the MRICA has been depicted in figure 1.

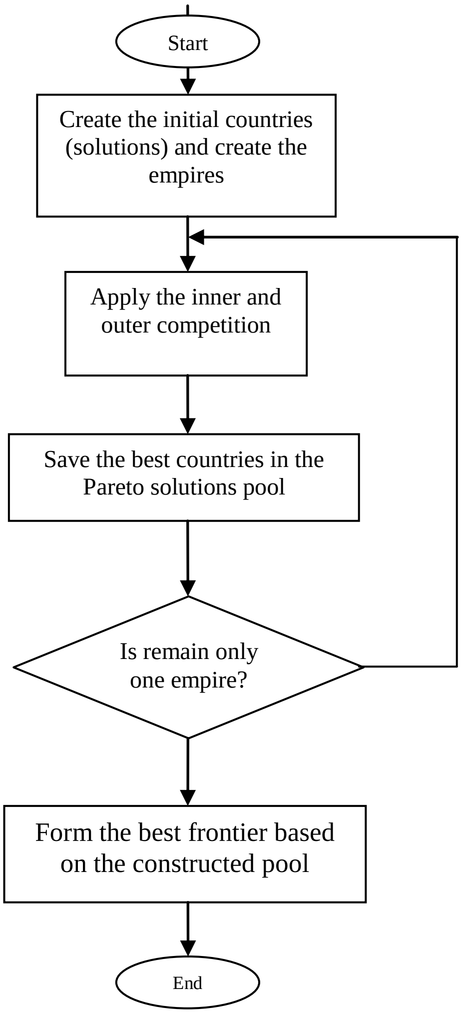

Fig. 1. Main steps of the proposed MRICA

The overall purpose is finding the efficient solutions for MRO problem solving in a situation that the aggregated function can't be considered. However the inner and outer competitions remain unaltered according to the basic ICA.

In the next section the proposed approaches will be discussed in details using a numerical example taken from the literature.

4. Numerical Example

In this paper we use the example proposed by Onur koksoy [5]. In mentioned industrial case study there are three controllable variables named x_1, x_2, x_3 and also two noise factors named z_1, z_2. Table 1 reports the levels of various factors. Two basic responses y_1, y_2 have been introduced. In this example it is assumed that there is no correlation between responses.

Also it is assumed that the first response should be minimized while the second should reach the target (*T=1*). Table 2 illustrates the Central Composite Design (CCD) and experimental data for the example [5].

Tab. 1. Levels of factors for the example

Factors \ Levels	**-1**	**0**	**1**
x_1	15	30	45
x_2	8	11	14
x_3	7	9	11
z_1	-1.5	0	1.5
z_2	-.25	0	.25

Tab. 2. The experimental results of Central Composite Design for the numerical example

x1	x2	x3	z1	z2	y1	y2
-1	-1	-1	-1	1	1.810	1.10
-1	-1	-1	1	-1	1.690	1.11
-1	-1	1	-1	-1	1.900	1.07
-1	-1	1	1	1	1.780	1.07
-1	1	-1	-1	-1	1.800	1.47
-1	1	-1	1	1	1.630	1.18
-1	1	1	-1	1	1.920	1.41
-1	1	1	1	-1	1.780	1.58
1	-1	-1	-1	-1	1.360	1.57
1	-1	-1	1	1	1.220	2.03
1	-1	1	-1	1	1.480	1.38
1	-1	1	1	-1	1.440	1.68
1	1	-1	-1	1	0.693	3.37
1	1	-1	1	-1	0.616	3.75
1	1	1	-1	-1	0.950	2.81
1	1	1	1	1	0.817	2.83
-1	0	0	0	0	1.790	1.24
1	0	0	0	0	1.030	2.46
0	-1	0	0	0	1.530	1.23
0	1	0	0	0	1.220	1.73
0	0	-1	0	0	1.300	1.63
0	0	1	0	0	1.440	1.67
0	0	0	0	0	1.380	1.73
0	0	0	0	0	1.390	1.74
0	0	0	0	0	1.400	1.74

The estimated regression models for two responses using the Minitab 16 can be stated as follows:

$$\hat{y}_1(x,z) = 1.38 - .361x_1 - .155x_2 + .0771x_3 - .148x_1x_2 + .0218x_1x_3 + .0130x_2x_3 + .0481x_1^2 - .0588z_1 - .0116z_2 + .0100x_1z_1 \quad (7)$$

$$\hat{y}_2(x,z) = 1.64 + .592x_1 + .438x_2 - .0950x_3 + .301x_1x_2 - .143x_1x_3 + .201x_1^2 + .0794x_1z_1 \quad (8)$$

The mean and variance of responses can be obtained as following equations:

$$\hat{\mu}_1[y_1] = 1.38 - .361x_1 - .155x_2 + .0771x_3 - .148x_1x_2 + .0218x_1x_3 + .0130x_2x_3 + .0481x_1^2 \quad (9)$$

$$\hat{\mu}_2[y_2] = 1.64 - .592x_1 - .438x_2 - .0950x_3 + .301x_1x_2 - .143x_1x_3 + .201x_1^2 \quad (10)$$

$$\hat{\sigma}_1^2[y1] = (-.0588 + .01x_1)^2 \sigma_{z1}^2 + (-.0116)^2 \sigma_{z2}^2 + s_1^2 \qquad (11)$$

$$\hat{\sigma}_2^2[y2] = (.0794x_1)^2 \sigma_{z1}^2 + s_2^2 \qquad (12)$$

Eq(3) and Eq(5) are implemented to this example with respect to the MSE criteria.

Firstly we use the aggregated single objective for optimization according to equation (6) with predefined weights given in Table 3. This table also illustrates a solutions obtained by ICA.
The results are compared with GRG algorithm proposed by Onur koksoy[5].

Tab. 3. The solution comparison between ICA and GRG method

Response coefficient	X_1	X_2	X_3	MSE_1 (ICA)	MSE_2 (ICA)	MSE_1 (GRG)	MSE_2 (GRG)	Efficiency of MSE_1	Efficiency of MSE_2
α_1 =0.91	0.0768	0.3470	0.995	0.11693	2.6809	0.12	2.44	%100	90%

As the GRG method is an exact solution approach it can only reach an optimal solution by restricting one of the controllable variables into a fixed value. The proposed ICA solves the problem with respect to all variables in a reasonable computational time. The reported results in Table 3 show that the proposed ICA can find the results reached by the GRG method considering proper weights. It is worth to mention that the proposed solution approach is independent of the restrictions of exact approaches.
In the next stage the proposed MRICA was applied for the numerical example to find the Pareto frontier. To show the efficiency of the proposed MRICA, it is compared to the multi-objective Genetic Algorithm. It is clear that the algorithm efficiency can be considered by some measures related to the extracted non dominated solutions (NDS).
Number of extracted non dominated solutions is a basic measure and its higher value is desirable. Moreover the diversity of solutions in the determined non dominated solutions can be computed as another quality measure which is known as crowding distance and the algorithm with higher crowding distance value is more efficient. It counts the distances between two solutions in both sides of a solution. This factor can be calculated using equations 13 and 14.

$$CD_{im} = \frac{f_m(x_{i+1}) - f_m(x_{i-1})}{f_m(x_{\max}) - f_m(x_{\min})} \qquad (13)$$

$$CD_T = \sum_{i=1}^{n} \sum_{m}^{M} CD_{im} \qquad (14)$$

Where
CD_{im} is the crowding distance of *i'th* solution considering the objective *m*.
$f_m(x_i)$ is the *m'th* objective function value of *i'th* solution.
CD_T is the total crowding distance.

In order to reach a robust algorithm design the essential parameters, should be adjusted into their appropriate levels using the experimental design techniques. In this paper a Taguchi method has been used for parameter tuning of both algorithms of MRICA and MOGA. For instance the essential parameters of genetic algorithm are population size and total number of iterations. A three-level Taguchi design has been created with respect to the lower and upper bounds for each factor given in table 4.
The experimental results of parameter tuning for multiple objective Genetic Algorithm have been depicted in this table. In the experimental study of parameters tuning two responses are studied including the computational time and number of extracted non dominated solutions. The same experimental study has been done for the MRICA.

Tab. 4. The Taguchi design for implemented MOGA

Factor 1	Factor 2	Response 1	Response 2
Population size **Levels:** 100-200-300	Total number of algorithm iterations **Levels:** 1000-2000-3000	Computational time(s)	Number of obtained non dominated solutions
100	1000	1.62	27
100	2000	3.05	35
100	3000	4.62	31
…	…	…	…
300	1000	4.88	73
300	2000	10.04	52
300	3000	13.83	54

Figure 2 demonstrates the mean of signal to noise (SN) ratio based on two parameters. According to this figure the values of 300 and 3000 are selected as population size and total iteration numbers, respectively. For the second solution algorithm (MRICA) the essential parameters are population size and number of imperialists. By running the Taguchi method the values of 300 and 5 are obtained as population size and number of imperialists, respectively.

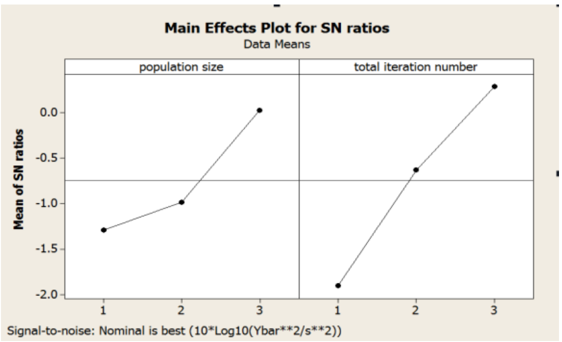

Fig 2. The mean SN ratios for population size and total iteration number based on Taguchi method

Then the problem was solved by both tuned algorithms. The comparison results of both MRICA and MOGA algorithms are reported in Table 5. These results are obtained after 10 algorithms replications.

Table5. The comparison between MRICA and MOGA

Algorithm	Population size	Mean of Computational time(s)	Mean of number of non dominated solutions	Mean of Total Crowding distance
MRICA	300	5.02	132	1.923
MOGA	300	9.78	51	1.574

According to the reported results of Table 5, it is clear that the proposed MRICA is more efficient than MOGA. It has better crowding distance and determined non dominated solutions in a predefined population and computation time. The reason can be described as follow; in imperialist competitive algorithm we have predefined number of imperialists in which the inner competition is running simultaneously and this process yields to obtain better solutions in lower computational time than the genetic algorithm. Also the outer competition quarantines the diversity in solution space by admitting an unrelated but new solution to an empire competition.

Also the results can be compared with other exact methods such as Branch & Bound technique using Lingo optimization package. This software can find only an optimal answer with respect to the various MSE criteria.

In this case two solutions 0.0028 and 2.41 are obtained for MSE1 and MSE2 criteria, respectively by considering the responses separately. This results show that the optimal solution corresponding to the MSE2 can't be found using branch and bound technique because better MSE2 values have been found by the proposed MRICA (Fig.3).

The computational time for the exact solution is 0.04(s). However the proposed meta-heuristic can find 132 non dominated solutions in only 5(s) containing the obtained solution by the exact approach. So the proposed approach can be used widely in multi response optimization problems.

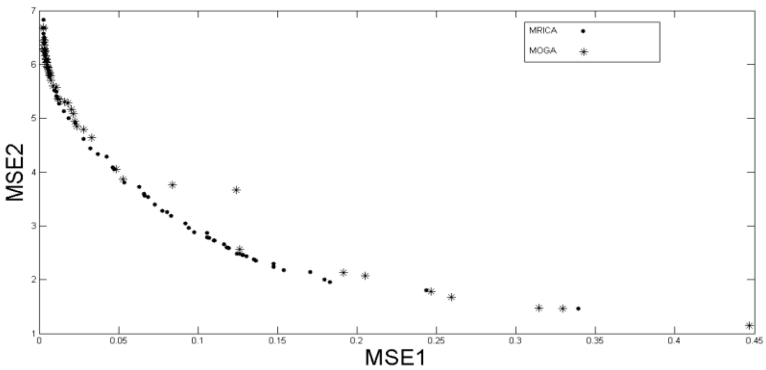

Fig. 3. The Pareto optimal frontier obtained by ICA and GA

Figure 3 demonstrates the Pareto frontier obtained by each algorithm. It is worth to be mentioned that the comparison has been done with an equal initial population size. However both of the MRICA and MOGA approaches can reach the optimal solutions of GRG and ICA methods in the reported non dominated solutions.

5. Conclusion

Imperialist competitive algorithm is a new meta-heuristic algorithm which tries to optimize the problems. Literally, many problems especially in industrial fields have many factors which contradict each other. The optimization of these models is difficult and need to a high value of computational time. In this research a new meta-heuristic based approach has been implemented to optimize a nonlinear multi-response programming model. The experimental results show that the efficiency of the proposed MRICA is comparable with those which obtain to an optimum solution. The proposed approach achieve to the optimum solution or at least near to optimum solution in a reasonable computational time. Furthermore the flexibility of this approach will permit the practitioner to choose the factor which is more important to be optimized. However the study of correlated responses will be worth as future studies.

References

[1] Derringer, G., Suich, R., "*Simultaneous Optimization of Several Response Variables*", Journal of Quality Technology 12, 1980, pp. 214–219.

[2] Saurav, D., Goutam, N., Bandyopadhyay, A., Pradip, K.P., "*Application of PCA-Based Hybrid Taguchi Method for Correlated multicriteria Optimization of Submerged Arc Weld: a Case Study*", International Journal of Advanced Manufacturing Technology. 45 (3–4), 2009, pp. 276–286.

[3] Bashiri, M., Hejazi, T.H., "*An Extension of Multi Response Optimization in MADM view*", Journal of Applied Polymer Science. 9 (9), 2009, pp.1695–1702.

[4] Chang, Ch.Y., Ti, L.Ch., Liang, H.W., "*Optimization of Process Parameters using Weighted Convex Loss Functions*", European Journal of Operations Research, 196, 2009, pp. 752–763.

[5] Alvarez, M.J., Gil-Negrete, N., Ilzarbe, L., Tanco, M., Viles, E., Asensio, A., "*A Computer Experiment Application to the Design and Optimization of a Capacitive Accelerometer*", Applied Stochastic Models in Business and Industry, 25 (2), 2009, pp. 151–162.

[6] Hsieh, K.L., "*Parameter Optimization of a Multi-Response Process for Lead Frame Manufacturing by Employing Artificial Neural Networks*", International Journal of Advanced Manufacturing Technology, 28, 2006, pp. 584–591.

[7] Koksoy, O., "*A Nonlinear Programming Solution to Robust Multi-Response Quality Problem*", Applied mathematics and computation 196, 2008, pp. 603–612.

[8] Imhof, L., "*Optimum Designs for a Multi-Response Regression Model*", Journal of Multivariate Analysis 72, 2000, pp. 120-131.

[9] Wu, F.C., Yeh, C.H., "*Robust Design of Multiple Dynamic Quality Characteristics*", International Journal of Advanced Manufacturing Technology 25, 2005, pp. 579–588.

[10] Chang, H.H., "*Dynamic Multi-Response Experiments by Back Propagation Networks and Desirability Functions*", Journal of the Chinese Institute of Industrial Engineers 23(4), 2006, pp. 280–288.

[11] Correia, D.S., Goncoalves, C.V., Da Cunha, S.S., Ferraresi, V.A., "*Comparison Between Genetic Algorithm and Response Surface Methodology in GMAW Welding Optimization*", Journal of Materials Processing Technology 160, 2005, pp. 70–76.

[12] Ozcelik, B., Erzurmlu, T., "*Determination of Effecting Dimensional Parameters on War Page of Thin Shell Plastic Parts using Integrated Response Surface Method and Genetic Algorithm*", International Communications in Heat and Mass Transfer 32, 2005, pp. 1085–1094.

[13] Suresh, P.V.S., Rao, P.V., Deshmukh, S.G., "*A Genetic Algorithm Approach for Optimization of Surface Roughness Prediction Model*", International Journal of Machine Tools & Manufacture 42, 2002, pp. 675–680.

[14] Oktem, H., Erzurumlu, T., Kurtaran, H., "*Application of Response Surface Methodology in the Optimization of Cutting Conditions for Surface Roughness*", Journal of Materials Processing Technology 170, 2005, pp. 11–16.

[15] Lin, D.K.J., Tu, W., "*Dual Response Surface Optimization*", Journal of Quality Technology 27, 1995, pp. 34–39.

[16] Tang, L.C., Xu, K.A., "*Unified Approach for Dual Response Surface Optimization*", Journal of Quality Technology 34, 2002, pp. 437–447.

[17] Koksoy, O., Doganaksoy, N., "*Joint Optimization of Mean and Standard Deviation using Response Surface Methods*", Journal of Quality Technology 35, 2003, pp. 239–252.

[18] Robinson, T.J., Borror, C.M., Myers, R.H., "*Robust Parameter Design: a Review*", Quality and Reliability Engineering International 20, 2004, pp. 81–101.

[19] Atashpaz, E., lucas, C., "*Imperialist Competitive Algorithm: An Algorithm for Optimization Inspired by Imperialist Competition*", IEEE 2007, 1-4244-1340-0/07$25.00_c.