International Journal of Industrial Engineering & Production Research

September 2011, Volume 22, Number 3
pp. 199-212

http://IJIEPR.iust.ac.ir/

ISSN: 2008-4889

A Comprehensive Mathematical Model for the Design of a Dynamic Cellular Manufacturing System Integrated with Production Planning and Several Manufacturing Attributes

F. Khaksar-Haghani, R. Kia, N. Javadian*, R. Tavakkoli-Moghaddam & A. Baboli

F. Khaksar-Haghani *received her master degree in Department of Industrial Engineering from Mazandaran University of Science & Technology, Babol, Iran*
R. Kia *is a faculty member in Department of Industrial Engineering, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran.*
N. Javadian *is an assistant professor in Department of Industrial Engineering, Mazandaran University of Science & Technology, Babol, Iran*
R. Tavakkoli-Moghaddam *is a professor in Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.*
A. Baboli *is an associate professor in DISP Laboratory, Université de Lyon, INSA-Lyon, F-69621, France.*

KEYWORDS

Dynamic cellular manufacturing systems,
Mixed-integer non-linear programming,
Production planning,
Manufacturing attributes

ABSTRACT

This paper presents a novel mixed-integer non-linear programming model for the design of a dynamic cellular manufacturing system (DCMS) based on production planning (PP) decisions and several manufacturing attributes. Such an integrated DCMS model with an extensive coverage of important design features has not been proposed yet and incorporates several manufacturing attributes including alternative process routings, operation sequence, processing time, production volume of parts, purchasing machine, duplicate machines, machine depot, machine capacity, lot splitting, material flow conservation equations, inflation coefficient, cell workload balancing, budget constraints for cell construction and machine procurement, varying number of formed cells, worker capacity, holding inventories and backorders, outsourcing part-operations, warehouse capacity, and cell reconfiguration. The objective of the integrated model is to minimize the total costs of cell construction, cell unemployment, machine overhead and machine processing, part-operations setup and production, outsourcing, backorders, inventory holding, material handling between system and warehouse, intra-cell and inter-cell movements, purchasing new machines, and machine relocation/installation/uninstallation. A comprehensive numerical example taken from the literature is solved by the Lingo software to illustrate the performance of the proposed model in handling the PP decisions and to investigate the incorporated manufacturing attributes in an integrated DCMS.

1. Introduction

Cellular manufacturing (CM), which is an innovative manufacturing strategy derived from a group technology (GT) concept, is an approach that can be used to improve both flexibility and efficiency in today's modern competitive manufacturing environments, such as flexible manufacturing systems (FMS) and just-in-time (JIT) production. The design of a cellular manufacturing system (CMS) involves in 1) cell formation (CF) (i.e., grouping parts with similar processing requirements into part families and corresponding machines into machine cells), 2) group layout (GL) (i.e., laying out machines within each cell, called intra-cell layout, and cells with regard to one another, called inter-cell layout), 3) group scheduling (GS) (i.e., scheduling part families), and 4) resource

* Corresponding author. **N. Javadian**
Email: *nijavadian@ustmb.ac.ir*
Paper first received Jan. 25, 2010, and in revised form Sep. 13, 2011.

allocation (i.e., assigning tools and human and material resources) [12].

An increasingly significant issue in CM is shorter product life cycles. Ignoring new products incoming at future imposes subsequent unplanned changes to the CMS and causes production disruptions and unexpected costs.

Hence, product life cycle changes should be incorporated in the design of cells. This type of model is called the dynamic cellular manufacturing system (DCMS) [7]. The DCMS is related to cell reconfiguration involving relocation of the existing machines between cells, purchasing and adding new machines to cells, removing the idle machines from cells and transferring machines between cells and machine depot.

Mungwattana [6] proposed a mathematical model and a solution approach for designing CMSs under dynamic and stochastic production environments with the routing flexibility. Schaller et al., [10] proposed a mathematical model showing how inventory planning can be integrated with cell formation to handle varying demands in a dynamic environment. They mentioned certain strategies that can be used to balance the effects of short-term demand variability in a CMS. These include combining cells to even out load variations, routing parts through alternative cells, allowing inter-cell movements, holding inventories, and backorders. Chen and Cao [2] integrated production planning and CMS in order to minimize the sum of costs of inter-cell material handling, setting up manufacturing cells, holding the finished items over the planning horizon, setting up the system to process different parts in different time periods, and machine operating.

Defersha and Chen [3] proposed the comprehensive mathematical model incorporating dynamic cell configuration, alternative routings, lot splitting, sequence of operations, multiple units of identical machines, machine capacity, workload balancing among cells, operation cost, subcontracting cost, tool consumption cost, setup cost, cell size limits, and machine adjacency constraints. Defersha and Chen [4] developed a comprehensive mathematical model for dynamic manufacturing cell formation with a multi-item and multi-level lot sizing aspects and the impact of lot size on product quality. They formulated a model incorporating a number of manufacturing features, such as dynamic system configuration, alternative routings, sequence of operations, machine capacity constraint, workload balancing, cell size limit and machine closeness requirements. Saidi-Mehrabad and Safaei [9] presented the dynamic cell formation model, in which the number of formed cells at each period can be different that minimizes the machine cost, relocation, and inter-cell movement costs. Ahkioon et al., [1] formulated the integrated approach to CMS design as the non-linear mixed-integer programming model incorporating production planning and system reconfiguration decisions with the presence of alternate process routings, operation sequence, duplicate machines, machine capacity, and lot splitting.

Tavakkoli-Moghaddam et al., [11] considered two kinds of cells in the CF, namely (1) common cells able to manufacture all kind of parts and (2) specific cells able to manufacture a specific type of product. They considered two kinds of capital constraints, namely (1) Capital constraints to form cells and (2) capital constraint to procure required equipment. In their research, three simultaneous goals were taken into account to be minimized three objectives, namely (1) the total cost of delay of delivering the part to the customers, (2) the costs of cell idleness in each period, and (3) the unused capital. Safaei and Tavakkoli-Moghaddam [8] integrated the multi-period cell formation and production planning in a dynamic CMS in order to minimize the costs of machine, inter/intra-cell movement, reconfiguration, subcontracting, and inventory holding. They investigated the effect of the trade-off between production and outsourcing costs on the cell reconfiguration.

Mahdavi et al., [5] presented an integer nonlinear mathematical programming model for the design of DCMSs by considering multi-period production planning, dynamic system reconfiguration, operation time, production volume of parts, machine capacity, alternative workers, available time of workers, hiring and firing of workers, and worker assignment. The objective of their presented model is to minimize the total costs of holding and backorder, inter-cell material handling, machine and reconfiguration, and hiring, firing and salary.

Within the context of a multi-period production planning, it is assumed that the demands for parts vary in each period in a deterministic way. This allows the model to manufacture more in a period so that this inventory can be used in future periods or to outsource parts when internal production is not practical either due to insufficient machine capacity or uneconomical consequences. The dynamic cell formation and production planning decisions are interrelated and may not be handled sequentially [4].

However, for more reality, we incorporate some of the PP attributes, such as facility and worker capacity, inventories holding, outsourcing of part operations and backorders to form the manufacturing cells. Then, the aim of this paper is to present a new mathematical model integrating CF and PP with an extensive coverage of important manufacturing attributes consisting of alternative process routings, operation sequence, processing time, production volume of parts, purchasing machine, duplicate machines, machine depot, machine capacity, lot splitting, material flow conservation equations, inflation coefficient, cell workload balancing, budget constraints for cell construction and machine procurement, varying number of formed cells, worker capacity, holding inventories and backorders, outsourcing part operations, warehouse capacity, and cell

reconfiguration. The presented model provides a larger coverage of the manufacturing attributes than the reviewed papers. Also, a wider range of input parameters, decision variables and cell formation criteria are incorporated than the models reviewed above. The remainder of this paper is organized as follows. In Section 2, a novel mathematical model integrating DCMS and PP with manufacturing attributes is presented. We present the numerical example in Section 3. Finally, conclusion is given in Section 4.

2. Mathematical Model and Problem Descriptions

2.1. Model Assumptions

In this section, the integrated model is formulated under the following assumptions:

1. Each part type has a number of operations that must be processed based on its operation sequence.
2. The demand for each part type in each period is known.
3. The capabilities of part-operations processing and processing time of part-operations for each machine type are known and unvarying over the planning horizon.
4. In this model we assume that in the first period, there are few machines available to utilize. Hence, in the first period we possibly have to purchase some machines to satisfy machine capacity constraint. In the next periods, if the present capacity of machines cannot satisfy the part demand, some machines can be purchased and added to the current utilized machines.
5. In each period that there is surplus capacity, we can remove idle machines from the cells and transfer to machine depot in order to decrease the overhead cost and provide empty locations in cells and whenever it is necessary to increase the machine capacity of system, we can return those machines to the cells.
6. Each machine type has a limited capacity expressed in hours during each time period and constant over the planning horizon.
7. Machines can have one or more identical duplicates to satisfy capacity requirements and reduce/eliminate inter-cell movements.
8. Cell reconfiguration involves transferring of the existing machines between cells, purchasing and adding new machines to cells, removing the idle machines from cells and transferring machines between cells and machine depot.
9. The transferring cost of each machine type is known. Even if a machine is removed from or returned to the cells, this transferring cost is incurred. All machine types can be moved to any cell. This cost is paid for several situations: to install a new purchased machine or a machine returned from depot, to uninstall a machine removed from a cell, and to transfer a machine between two cells or between a cell and machine depot.
10. The overhead cost of each machine type is known and implies maintenance and other overhead costs such as energy cost and general service. This cost is also considered for each machine in each period if that machine is utilized on the cells to process part-operations. So the idle machines removed from the cells do not impose any overhead costs.
11. The variable cost of each machine type implying the operating cost is depended on the workload assigned to the machine and is known.
12. Parts are transferred between and within cells, or between cells and warehouse. Inter-cell movement happens whenever successive operations of a part type are carried out in different cells. Also, the intra-cell movement happens whenever successive operations of a part type are processed on different machines in the same cell. Moreover, movement between cells and warehouse happens whenever whole or partial of operations of a part type are transferred from cells to warehouse in order to continue processing in the next periods or transferred from warehouse to the cells in order to continue processing the remaining operations in the current period.
13. Material handling devices moving the parts between machines are assumed to carry only one part at a time.
14. Inter-cell and intra-cell movements and movements between cells and warehouse based on the part types have different costs regardless of distances.
15. Set up batches related to the operations of each part types have different sizes, costs and times.
16. The maximum and minimum of the cell size is known in advance. The presence of too many machines in a cell generates cluttered flows in a cell due to too many routes and reduces monitoring machines.
17. All machine types are assumed to be multi-purposed ones, which are capable to perform one or more operations. In the same manner, each operation of a part type can be performed on different machine types with different processing times. This feature providing the flexibility to the process plan of parts is known as alternative process routings that can be utilized to obtain a better cell design.
18. A part operation can be distributed between several machines within the same cell or even in different cells (lot splitting).
19. The maximum number of cells formed in each

period must be specified in advance.
20. Depending on the demand volume and total costs of meeting that demand, the system can produce some surplus parts in a period, hold between successive periods and use in future planning periods. Backorders, another PP strategy, are allowed. Also, due to limited machine capacities, outsourcing can be used to provide some of the required parts to meet the market demand. Furthermore, in this model holding inventory of semi-finished parts and outsourcing of the operations of a part type are allowed. The time-gap between releasing and receiving orders (i.e., lead time) is known in advance.
21. The workload of the cells should be balanced.
22. Capital available for the formation of manufacturing cells as well as capital available for the purchasing of machines is limited.
23. The working time of workers in each period is known.
24. The number of workers allotted for processing each operation of a part type on each corresponding machine is known in advance.
25. Cell idleness incurs cost.
26. The capacity of warehouse based on the maximum volume of parts which can be stored is known.
27. Inflation increase machine purchase cost and outsourcing cost in the successive periods based on inflation rate.

Sets:

$t = \{1,2,\ldots,T\}$	index set of time periods
$p = \{1,2,\ldots,P\}$	index set of part types
$m = \{1,2,\ldots,M\}$	index set of machine types
$c = \{1,2,\ldots,C\}$	index set of cells
$R(m)=\{1, \ldots, NA_m\}$	index set of machine numbers
$K(p) = \{1,2,\ldots,Kp\}$	index set of operations indices for part type p

Model Parameters:

BN	a big number
qc	balancing factor for the workload of a cell being as low as $qc \times 100\%$ from the average workload per cell
B_U	upper cell size limit
B_L	lower cell size limit
C	maximum number of cells that can be formed in each period
FCt	cost of forming a cell in period t
BMt	budget available to purchase machines in period t
BC_t	budget available to form cells in period t
CU	cost of a unit idle time for each cell
BS_{pkm}	batch size to set up operation k of part type p on machine type m
a_{pkm}	1 if operation k of part type p can be processed on machine type m; 0 otherwise
t_{pkm}	processing time of operation k of part type p on machine type m
HR_{pkm}	number of workers required to process operation k of part type p on machine type m
CHR	available working time of workers in hours
μ_{pkm}	setup cost for operation k of part type p on machine type m
DM_{pt}	demand for part type p in period t
NA_m	maximum number of machine type m to be available in system (cells and machine depot)
PC_{mt}	purchase cost of machine type m in period t
IND_m	number of machine type m available in machine depot before period 1
INS_m	cost of installing one machine of type m
$UINS_m$	cost of uninstalling one machine of type m
γ_m	cost of transferring one machine of type m between machine depot and cells
δ_m	cost of transferring one machine of type m between two cells
TM_m	capacity of one unit of machine type m in hours
a_m	overhead cost of machine type m
β_m	variable cost of machine type m per unit time
ε_{pk}	production cost per operation k of part type p
IA_p	intra-cell material handling cost per part type p
IE_p	inter-cell material handling cost per part type p
OC_{pkt}	outsourcing cost per operation k of part type p in period t
NO_{pkt}	number of operation k of part type p outsourced before current planning horizon and received in period 1
LT_{pk}	lead time between outsourcing operation k of part type p and receiving that operation
HC_{pk}	inventory holding cost per operation k of part type p
SWC_p	material handling cost between cells and warehouse per part type p
CW	warehouse capacity
WP_p	volume of one unit of part type p in warehouse
ISF_{pk1}	number of operation k of part type p processed before current planning horizon and hold in warehouse

θ_p backorder cost per unit part type p

BP_{p1} number of part type p backordered before current planning horizon

Decision Variables:

X_{pkmct} number of operation k of part type p processed on machine type m in cell c in period t

Y_{ct} 1 if cell c formed in period t; 0 otherwise

NP_{mt} number of machine type m purchased in period t

N_{mct} number of machine type m assigned to cell c in period t

N^{+}_{mct} number of machine type m added to cell c in period t

N^{-}_{mct} number of machine type m removed from cell c in period t

RS_{mt} number of machine type m transferred between cells in period t

D_{mt} number of machine type m available in machine depot at the end of period t

SD_{mt} number of machine type m removed from cells and transferred to machine depot in period t

DS_{mt} number of machine type m removed from machine depot and transferred to cells in period t

BP_{pt} number of part type p backordered in period t

NO_{pkt} number of operation k of part type p outsourced ago and received in period t

WS_{pkt} number of operation k of part type p processed and hold in warehouse ago and transferred to cells in period t

SW_{pkt} number of operation k of part type p processed and transferred from cells to warehouse in period t

ISF_{pkt} number of operation k of part type p hold in warehouse in the beginning of period t

IP_{pkt} number of part type p whose operations k and k+1 are processed internally in cells in period t

ICP_{pkct} number of part type p whose operations k and k+1 are processed in the same cell c in period t

NR_{pkmc} number of part type p whose operations k and k+1 are processed on the same machine type m in cell c in period t

IFC_{pkct} total intra-cell movements of operation k of part type p in cell c in period t

ECP_{pkt} total inter-cell movements of operation k of part type p in period t

Assistant Variable:

P_{pkt} number of operation k of part type p processed in period t

2.2 Objective Function and Constraints

The proposed comprehensive DCMS model is now formulated as a non-linear mixed integer program:

$$\min \quad Z = \sum_{t}^{T}\sum_{c}^{C} FC_t . Y_{ct} \tag{1-1}$$

$$+\sum_{t}^{T}\sum_{c}^{C} CU_t \left(\sum_{m}^{M} T_m . N_{mct} - \sum_{p}^{P}\sum_{k}^{K_p}\sum_{m}^{M} t_{pkm} . X_{pkmct}\right) \tag{1-2}$$

$$+\sum_{t}^{T}\sum_{c}^{C}\sum_{m}^{M} \alpha_m . N_{mct} \tag{1-3}$$

$$+\sum_{t}^{T}\sum_{p}^{P}\sum_{k}^{K_p}\sum_{c}^{C}\sum_{m}^{M} \beta_m . t_{pkm} . X_{pkmct} \tag{1-4}$$

$$+\sum_{t}^{T}\sum_{p}^{P}\sum_{k}^{K_p}\sum_{c}^{C}\sum_{m}^{M} \varepsilon_{pk} . X_{pkmct} \tag{1-5}$$

$$+\sum_{t}^{T}\sum_{p}^{P}\sum_{k}^{K_p}\sum_{c}^{C}\sum_{m}^{M} \mu_{pk} . \left\lceil \frac{X_{pkmct}}{BS_{pkm}} \right\rceil \tag{1-6}$$

$$+\sum_{t=2}^{T}\sum_{p}^{P}\sum_{k}^{K_p} OC_{pk(t-LT_{pk})} . NO_{pkt} \tag{1-7}$$

$$+\sum_{t}^{T-1}\sum_{p}^{P} \theta_p . BP_{pt} \tag{1-8}$$

$$+\sum_{t}^{T}\sum_{p}^{P}\sum_{k}^{K_p} HC_{pk} . ISF_{pkt} \tag{1-9}$$

$$+\sum_{t}^{T}\sum_{p}^{P} SWC_p . \left(\sum_{k}^{K_p-1} WS_{pkt} + \sum_{k}^{K_p} SW_{pkt}\right) \tag{1-10}$$

$$+\sum_{t}^{T}\sum_{p}^{P}\sum_{k}^{K_p-1}\sum_{c}^{C} IA_p . IFC_{pkct} \tag{1-11}$$

$$+\sum_{t}^{T}\sum_{p}^{P}\sum_{k}^{K_p-1} IE_p.ECP_{pkt} \quad (1\text{-}12)$$

$$+\sum_{t}^{T}\sum_{m}^{M}(PC_{mt}+INS_m)NP_{mt} \quad (1\text{-}13)$$

$$+\sum_{t}^{T}\sum_{m}^{M}(\gamma_m+INS_m)DS_{mt} \quad (1\text{-}14)$$

$$+\sum_{t=2}^{T}\sum_{m}^{M}(UINS_m+\gamma_m)SD_{mt} \quad (1\text{-}15)$$

$$+\sum_{t=2}^{T}\sum_{m}^{M}(UINS_m+\delta_m+INS_m)RS_{mt} \quad (1\text{-}16)$$

Subject to:

$$\sum_{p}^{P}\sum_{k}^{K_p}\sum_{c}^{C}\sum_{m}^{M} HR_{pkm}.t_{pkm}.X_{pkmct} \le CHR \qquad \forall t \in T \quad (2)$$

$$\sum_{p}^{P}\sum_{k}^{K_p}\sum_{m}^{M} t_{pkm}.X_{pkmct} \ge \frac{qc}{\sum_c^C Y_{ct}}.\sum_{p}^{P}\sum_{k}^{K_p}\sum_{c}^{C}\sum_{m}^{M} t_{pkm}.X_{pkmct} \qquad \forall c \in C, \forall t \in T \quad (3)$$

$$\sum_{c}^{C} FC_t.Y_{ct} \le BC_t \qquad \forall t \in T \quad (4)$$

$$\sum_{m}^{M} PC_{mt}.NP_{mt} \le BM_t \qquad \forall t \in T \quad (5)$$

$$\sum_{c}^{C}\sum_{m}^{M} a_{pkm}.X_{pkmct} = P_{pkt} \forall p \in P, \forall k \in K_p, \forall t \in T \quad (6)$$

$$P_{pkt} + NO_{pkt} + WS_{pkt} - SW_{pkt} = P_{p(k+1)t} + NO_{p(k+1)(t+LT_{p(k+1)})} \qquad \forall p \in P, \forall k \in K_p - 1, \forall t \in T \quad (7)$$

$$P_{pK_pt} + NO_{pK_pt} + WS_{pK_pt} - SW_{pK_pt} = DM_{pt} + BP_{p(t-1)} - BP_{pt} \qquad \forall p \in P, \forall t \in T \quad (8)$$

$$IP_{pkt} = min\{P_{p(k+1)t}, P_{pkt}\} \quad \forall p \in P, \forall k \in K_p - 1, \forall t \in T \quad (9)$$

$$NR_{pkmct} = min\{X_{p(k+1)mct}, X_{pkmct}\} \qquad \forall p \in P, \forall k \in K_p - 1, \forall m \in M, \forall c \in C, \forall t \in T \quad (10)$$

$$ICP_{pkct} = min\left\{\sum_{m}^{M} X_{p(k+1)mct}, \sum_{m}^{M} X_{pkmct}\right\} \qquad \forall p \in P, \forall k \in K_p - 1, \forall c \in C, \forall t \in T \quad (11)$$

$$IFC_{pkct} = ICP_{pkct} - \sum_{m}^{M} NR_{pkmct} \qquad \forall p \in P, \forall k \in K_p - 1, \forall c \in C, \forall t \in T \quad (12)$$

$$ECP_{pkt} = IP_{pkt} - \sum_{c}^{C} ICP_{pkct} \forall p \in P, \forall k \in K_p - 1, \forall t \in T \quad (13)$$

$$\sum_{p}^{P}\sum_{k}^{K_p} WP_p.ISF_{pkt} \le CW \qquad \forall t \in T \quad (14)$$

$$ISF_{pkt} - WS_{pkt} + SW_{pkt} = ISF_{pk(t+1)} \qquad \forall p \in P, \forall k \in K_p, \forall t \in T-1 \qquad (15)$$

$$WS_{pkT} = ISF_{pkT} \qquad \forall p \in P, \forall k \in K_p \qquad (16)$$

$$BL.Y_{ct} \leq \sum_{m}^{M} N_{mct} \leq BU.Y_{ct} \qquad \forall c \in C, \forall t \in T \qquad (17)$$

$$Y_{(c+1)t} \leq Y_{ct} \qquad \forall c \in C-1, \forall t \in T \qquad (18)$$

$$IND_m - DS_{m1} = D_{m1} \qquad \forall m \in M \qquad (19)$$

$$D_{m(t-1)} + SD_{mt} - DS_{mt} = D_{mt} \qquad \forall m \in M, \forall t-1 \in T \qquad (20)$$

$$\sum_{p}^{P}\sum_{k}^{K_p} t_{pkm}.X_{pkmct} \leq TM_m.N_{mct} \qquad \forall m \in M, \forall c \in C, \forall t \in T \qquad (21)$$

$$\sum_{c}^{C} N_{mcT} + D_{mT} \leq NA_m \forall m \in M, \forall t \in T \qquad (22)$$

$$NP_{m1} + DS_{m1} = \sum_{c}^{C} N_{mc1} \qquad \forall m \in M \qquad (23)$$

$$N_{mc(t-1)} + N^{+}_{mct} - N^{-}_{mct} = N_{mct} \qquad \forall m \in M, \forall c \in C, \forall t-1 \in T \qquad (24)$$

$$\sum_{c}^{C} N^{+}_{mct} - \sum_{c}^{C} N^{-}_{mct} = NP_{mt} + DS_{mt} - SD_{mt} \qquad \forall m \in M, \forall t-1 \in T \qquad (25)$$

$$RS_{mt} = min\left\{\sum_{c}^{C} N^{+}_{mct}, \sum_{c}^{C} N^{-}_{mct}\right\} \qquad \forall m \in M, \forall c \in C, \forall t-1 \in T \qquad (26)$$

$$Y_{ct} \ \ is\ binary \qquad \forall c \in C, \forall t \in T \qquad (27)$$

$$RS_{mt}, SD_{mt} \geq 0 \ \ and \ \ intger \qquad \forall m \in M, \forall t-1 \in T \qquad (28)$$

$$DS_{mt}, NP_{mt} \geq 0 \ \ and \ \ intger \qquad \forall m \in M, \forall t \in T \qquad (29)$$

$$BP_{pt} \geq 0 \ \ and \ \ intger \qquad \forall p \in P, \forall t \in T-1 \qquad (30)$$

$$N_{mct} \geq 0 \ \ and \ \ intger \qquad \forall m \in M, \forall c \in C, \forall t \in T \qquad (31)$$

$$N^{+}_{mct}, N^{-}_{mct} \geq 0 \ \ and \ \ intger \ \forall m \in M, \forall c \in C, \forall t-1 \in T \qquad (32)$$

$$WS_{pkt}, P_{pkt} \geq 0 \ \ and \ \ intger \qquad \forall p \in P, \forall k \in K_p, \forall t \in T \qquad (33)$$

$$NO_{pkt}, ISF_{pkt} \geq 0 and \ intger \forall p \in P, \forall k \in K_p, \forall t-1 \in T \qquad (34)$$

$$SW_{pkt} \geq 0 \ \ and \ \ intger \ \forall p \in P, \forall k \in K_p, \forall t \in T-1 \qquad (35)$$

$$IP_{pkt}, ECP_{pkt} \geq 0 \text{ and } intger \forall p \in P, \forall k \in K_p - 1, \forall t \in T \quad (36)$$

$$ICP_{pkct}, IFC_{pkct} \geq 0 \text{ and } intger \quad \forall p \in P, \forall k \in K_p - 1, \forall m \in M, \forall c \in C, \forall t \in T \quad (37)$$

$$NR_{pkmct} \geq 0 \text{ and } intger \quad \forall p \in P, \forall k \in K_p - 1, \forall m \in M, \forall c \in C, \forall t \in T \quad (38)$$

$$X_{pkmct} \geq 0 \text{ and intger} \quad \forall p \in P, \forall k \in K_p, \forall m \in M, \forall c \in C, \forall t \in T \quad (39)$$

There are 16 iterms in the considered objective function as follows. Term (1.1) is the total cost of forming cells. Term (1.2) is the total cost of idleness of cells. Term (1.3) incorporates the overhead cost for all machines utilized in the manufacturing cells during planning horizon.

Terms (1.4) to (1.9) are machine operating cost, production cost of part-operations, setup cost, outsourcing cost, backorders cost, and inventory holding cost, respectively. Term (1.10) is material handling cost between cells and warehouse for all part types. Terms (1.11) and (1.12) are the total costs of intra-cell and inter-cell material handling, respectively. Term (1.13) is purchasing and installation costs of new machines to be added to cells. Term (1.14) is installation and transferring costs of machines to be removed from machine depot and added to cells. Term (1.15) is uninstallation and transferring costs of machines to be removed from cells and added to machine depot. Finally, Term (1.16) is uninstallation, installation and transferring costs of machines to be transferred between cells. Constraint (2) ensures that the workload assigned to workers in man-hours does not exceed from available capacity.

Constraint (3) enforces workload to be balanced among cells where the factor $qc_t \in [0,1\}$ is used to determine the degree of the workload balance. If qc_t is chosen close to 1.0, the allowable workload of each cell will be close to the average workload determined by $1/\sum_c^c Y_{ct} \times 100\%$ of the total workload. Constraints (4) and (5) are related to available budget to forming cells and purchasing machines, respectively. Constraint (6) is to determine the quantity of operation *k* of part type *p* processed internally in manufacturing cells. Constraint (7) is material flow conservation equation for operations of parts.

Constraint (7) implies that to processing operation *k* + 1 of part type *p* internally by manufacturing cells or externally by outsourcing in period *t*, a portion of previous operation *k* should be processed internally, a portion of previous operation would be outsourced parts which are received in period *t*, some processed parts are transferred to warehouse and the rest is received from warehouse. Equation (8) is demand satisfaction constraint at each period. To meet the demand of part type *p* in period *t* and aggregated backorders from previous periods, it enforces processing the final operation K_p internally by manufacturing cells or externally by outsourcing. Holding inventory, another PP strategy to manufacture in the previous periods by lower level of demands to meet demand in the subsequent periods by higher level of demands, is allowed. The unsatisfied demand is backordered to subsequent period.

Equation (9) is to determine the quantity of successive operations *k* and *k* + 1 of part type *p* processed internally by manufacturing cells. Equation (10) is to determine the quantity of successive operations *k* and *k* + 1 of part type *p* processed by a same machine type in a cell.

Equation (11) is to determine the quantity of successive operations *k* and *k* + 1 of part type *p* processed in a same cell. Equations (12) and (13) are to determine the quantity of intra-cell and inter-cell movements of successive operations *k* and *k* + 1 of part type *p*, respectively. Constraint (14) is related to warehouse capacity. Equation (15) is to determine the quantity of processed operations of each part type hold in the warehouse before each period. Equation (16) is to use up the processed operations of each part hold in the warehouse at the end of the last period *T*. Cell size limits are defined by Constraint (17). The order of forming cells is determined by Constraint (18). The number of machines available in a machine depot at the end of period 1 and the subsequent periods are calculated by Constraints (19) and (20), respectively.

Inequality (21) is machine time capacity constraint. Inequality (22) guarantees that the number of machines of type *m* utilized in manufacturing cells or available in machine depot does not exceed from the maximum number of machines of type *m* to be available in system.

Equation (23) is to guaranty that the machines purchased or transferred from depot to cells in the first period are assigned to manufacturing cells. Equation (24) says that the number of machines of type *m* utilized in cell *c* in the current period, *t*, is equal to the number of machines of type *m* utilized in cell *c* in the previous period, *t* - 1, plus the number of machines of type *m* added to cell *c*, or minus the number of machines of type *m* removed from cell *c*. Equation (25) describes that the deviation between the number of machines of type *m* added to cells and the number of

those removed from cells is equal to the number of machines of type *m* purchased, plus those transferred from depot to cells, or minus those transferred from cells to depot. Equation (26) is to determine the number of machines of each type transferred between different cells. Finally, Constraints (27) to (39) provide the logical binary and non-negativity integer necessities for the decision variables.

2.3. Linearization of the Proposed Model

The proposed model is a nonlinear mixed-integer programming model because of Min function in Equations (9), (10), (11) and (26) and Ceiling function in Equation (1.6). To linearize Equation (9), binary variable IP^{+}_{pkt} is introduced and the following constraints are added to the main model.

$$IP_{pkt} + BN.IP^{+}_{pkt} \geq P_{p(k+1)t}$$
$$IP_{pkt} + BN.(1 - IP^{+}_{pkt}) \geq P_{pkt}$$

To linearize Equation (10), adding the following constraints to the main model is sufficient.

$$NR_{pkmct} \leq X_{p(k+1)mct}$$
$$NR_{pkmct} \leq X_{pkmct}$$

Similar to linearization of Equation (10), adding the following constraints to the main model is enough to linearize Equation (11).

$$ICP_{pkct} \leq \sum_{m}^{M} X_{p(k+1)mct}$$
$$ICP_{pkct} \leq \sum_{m}^{M} X_{pkmct}$$

To linearize Equation (26), the following constraints must be added to the main model.

$$RS_{mt} = \frac{\sum_c^C N^{+}_{mct} + \sum_c^C N^{-}_{mct} - |\sum_c^C N^{+}_{mct} - \sum_c^C N^{-}_{mct}|}{2}$$
$$RS_{mt} = \frac{\sum_c^C N^{+}_{mct} + \sum_c^C N^{-}_{mct} - (NP_{mt} + DS_{mt} + SD_{mt})}{2}$$

To linearize the Ceiling term in Equation (1.6), non-negative variable XS_{pkmct} is introduced and the Ceiling term is rewritten by:

$$\left\lceil \frac{X_{pkmct}}{BS_{pkm}} \right\rceil = XS_{pkmct}$$

where the following constraint should be added to the main model.

$$\frac{X_{pkmct}}{BS_{pkm}} \leq XS_{pkmct} < \frac{X_{pkmct}}{BS_{pkm}} + 1$$

3. Computational Results

To validate the proposed model and illustrate its various features, a numerical examples with randomly generated data taken from the literature is solved by a branch-and-bound (B&B) method under the Lingo 8.0 software on an Intel® Core™2.5 GHz Personal Computer with 4 GB RAM. The information related to this example is given in Tables 1 to 4.

This example consists of four part types, four machine types and three periods, in which each part type is assumed to have three operations that must be processed, sequentially. Each operation can be processed on two alternative machines.

The first part of Table 1 consists of the information related to cost of a unit idle time for each cell, warehouse capacity, available working time of workers in hours and the inflation rate only affecting on purchase cost of machines and outsourcing cost of part-operations.

The second part consists of the information related to cost of forming a cell, budget available to forming cells and budget available to purchasing machines in three periods.

Tab. 1. Cost parameters

parameter	value	parameter	value		
			T1	T2	T3
CU	10	FC_t	5000	8000	9000
CW	500	BC_t	5000	8000	9000
CHR	3500	BM_t	2000	2400	2880

The first part of Table 2 presents demand for each part type in each period. Backorder cost per unit of each part type in each period and volume of one unit of each part type are two last columns in Table 2.

Tab. 2. Part information

	DM_{pt}			θ_p	WP_p
	T1	T2	T3		
Part 1			100	191	1
Part 2	200	100		196	2
Part 3	200	150	50	220	1
Part 4	250		100	176	1

The first part of Table 3 contains the information related to inventory holding cost, production cost, number of part-operations outsourced before current planning horizon and received in the first period, and number of part-operations processed before current planning horizon and hold in warehouse.

The second part presents the outsourcing cost of part-operations influenced by inflation rate 1.2 at period 2 and afterward. The third part presents the processing time of each operation for all part types. The fourth part presents setup costs for part-operations on each machine type.

The fifth part presents the number of workers required to process part-operations on each machine type. Furthermore, 100 units of part type 4 are backorders before current planning horizon.

For more simplicity, it is assumed that lead time for each operation of a part type, batch size to set up each operation of a part type on each machine type, intra-cell and inter-cell material handling cost per each part type in each period, and material handling cost between cells and warehouse per each part type in each period are equal to 1, 50, 2, 4 and 5, respectively. The outsourcing cost is influenced by inflation rate at period 2 and afterward.

Table 4 related to machine information consists of the machine time capacity in hours, number of each machine type available in machine depot before the first period, purchase cost of each machine type, machine variable cost, machine overhead cost, machine transferring cost between two cells, machine transferring cost between machine depot and a cell, and machine installing/uninstalling cost. The purchase cost of each machine type is influenced by inflation rate 1.2 at period 2 and afterward.

Tab. 3. Part information

		HC_{pk}	ε_{pk}	INO_{pk}	$IISF_{pk}$	OC_{pkt}			t_{pkm}				μ_{pkm}				HR_{pkm}			
						T1	T2	T3	M1	M2	M3	M4	M1	M2	M3	M4	M1	M2	M3	M4
Part 1	O1	6	25		10	241	289	347	10		7		5		6		1		1	
	O2	12	24			243	291	349		5	1			4	3			1	1	
	O3	18	30			201	241	289	4		8		2		5		2		2	
Part 2	O1	8	36		50	235	282	338	1			2	3			5	1			1
	O2	15	27	50		218	261	313	6			2	9			8	1			1
	O3	22	36	50	50	226	271	325	5		2		6		5		2		1	
Part 3	O1	8	36	100		219	262	315		1		1		3		5		1		2
	O2	16	35			236	283	340	2		5		2		3		1		1	
	O3	23	32			240	288	346	4	1			5	6			2	2		
Part 4	O1	6	28			230	276	331	4		4		7		9		2		2	
	O2	12	23	50		208	250	300	1			4	7			5	1			1
	O3	17	26			202	242	291		5	6			4	5			1	1	

Tab. 4. Machine information

	TM_m	IND_m	PC_m	β_m	α_m	δ_m	γ_m	INS_m	$UINS_m$
M1	200	1	1500	9	75	27	40	120	75
M2	200	0	1300	7	65	23	34	100	65
M3	200	1	1100	5	55	21	31	90	55
M4	200	2	1400	9	70	24	36	95	70
$PC_{mt} = (1.2)PC_{m(t-1)}$									

All the machines are to be grouped into maximum three fairly independent cells with the lower and upper sizes of 2 and 5, respectively. Maximum number of each machine type to be available in system is four. The balancing factor for the workload of a cell is considered as $qc = 0.3$.

Reconfiguration can be implemented at the beginning of the second period to respond to the changing demand volume of production parts with fewer costs. The solution obtained with our model presented in this paper on the explained example is detailed out in the rest of this section.

Table 5 shows the machine assignments to cells for three periods. As can be seen, in the first period, cells 1 and 2 are formed to process the part-operations. For instance, one unit of machine types 1, 2, 3 and 4 are assigned to cell 1 in period 1 and fixed in that cell for successive periods. In the second period, cell 3 is also formed to increase manufacturing capacity of system. In the third period, no cell is added to system because the present capacity is enough to process the part-operations.

Before period 1, one unit of machine types 1 and 3 and two units of machine type 4 are available in machine depot. In period 1, all machines available in depot are transferred to cells. Furthermore, one unit of machine types 1 and 2 and two units of machine type 3 are purchased and assigned to cells. In period 2, one unit of machine types 1 and 3 and three units of machine type 2 are purchased and assigned to cells.

Tab. 5. Machine assignments to cells

	Cell 1	Cell 2	Cell 3	Machine depot
Before Period1				M_1 M_3 M_4 M_4
Period 1	M_1 M_2 M_3 M_4	M_1 M_3 M_4 M_3		
Period 2	M_1 M_2 M_3 M_4	M_1 M_3 M_4 M_2 M_2	M_1 M_2 M_3 M_3	
Period 3	M_1 M_2 M_3 M_4	M_1 M_3 M_4 M_2 M_2	M_1 M_2 M_3 M_3 M_1	

M: Purchased machine
M: Transferred machine from depot to cells
M: Transferred machine between two cell
M: Fixed machine in a cell

Tab. 6. Objective function value and cost components

OFV	**320136**	**CPU Time**	**02:51':52''**
Forming cells	19840	Inventory holding	520
Idleness of Cells	80	Material handling between cells and a warehouse	850
Overhead	3205	Intra-cell material handling	2740
Processing machine	50560	Inter-cell material handling	800
Production	84270	Purchasing a machine	15990
Set up	257	Transferring a machine from a depot to cells	151
Outsourcing part-operations	78730	Transferring a machine from cells to a depot	0
Backorders	61600	Transferring a machine between cells	543

Furthermore, one unit of machine type 3 is transferred from cell 2 to cell 3. Finally, one unit of machine type 1 is purchased and assigned to cell 3 in period 3. The objective function values obtained in this paper cannot be compared to the previous studies because of the different cost components of objective function and manufacturing attributes involved.

Optimal cell configurations, part-operation allocations to the machines and production planning decisions for three periods are presented in Table 6. To illustrate the material flow conservation equations for the proposed model, we also depicted the material flow between machines on directed arcs in Table 6. For instance, to show the outsourcing feature in the proposed model we investigate the material flow of part type 4 in three periods.

Before period 1, the quantity of backorders is 100 units and the demand is equal to 250 units in period 1. 20 units of operations 1 and 2 are processed on machine types 3 and 4 in cell 1 and another 20 units of operations 1 and 2 are also processed on machine types 3 and 4 in cell 2. Finally, 40 units of operation 3 are processed on machine type 3 in cell 2. By considering 100 units of backorders and 250 units of demand in period 1, we can understand that producing only 40 units in period 1 results in 310 units of backorders for period 2. In the second period, 10 units of operation 1 are processed on machine type 3 in cell 2 and 210 units of operation 1 outsourced in period 1 are received in period 2.

All these 220 units are processed by operations 2 and 3 internally in manufacturing cells. Furthermore, 50 units of operation 3 outsourced in period 1 are received in period 2.

By considering 310 units of backorders and no demand in period 2, producing 270 units (220 units internally and 50 units externally) results in 40 units of backorders for period 3. In the third period, 140 units of operations 1, 2 and 3 are processed internally in manufacturing cells. By considering 40 units of backorders and 100 units of demand in period 3, producing 140 units results in satisfaction of demand for the third period.

To illustrate the inventory holding feature in the proposed model, we investigate the material flow of part type 1 in three periods. In period 1, there are 10 units of operation 1 in warehouse and there is no demand. In period 2, 30 units of operation 1 outsourced ago are received from which 5 units are processed by operations 2 and 3 internally. Then, 35 units of operation 1 and 5 units of operation 3 are hold for the next period. In period 3, 35 units of operation 1 available in warehouse are processed by operations 2 and 3 internally. Another 60 units are processed by all three operations in cells 2 and 3. Totally, these 95 units processed in period 3 and 5 units of operation 3 hold in warehouse meet 100 units of demand in period 3.

In the above configuration, parts can be produced on the different machines assigned to multiple cells (i.e., lot splitting). This is also shown through directed arcs in Table 7 representing the selected routings for all part types in three periods. A routing for a part is defined in terms of both the sequence of operations required and the machines visited to process operations, sequentially. For example, consider the material flow of part type 4. In the second period, operation 2 is processed by machine type 1 in cell 1 with quantity of 40 units from which 30 units are processed by machine 2 and the remaining 10 units are processed by machine 3 in cell 1 to complete processing operation 3. Then a form of lot splitting is done by dividing the production batch between two machines in a same cell.

In the third period, operation 2 is processed by machine type 4 in cell 1 with quantity of 50 units from which 40 units are processed by machine 2 in cell 1 and the remaining 10 units are processed by machine 2 in cell 2

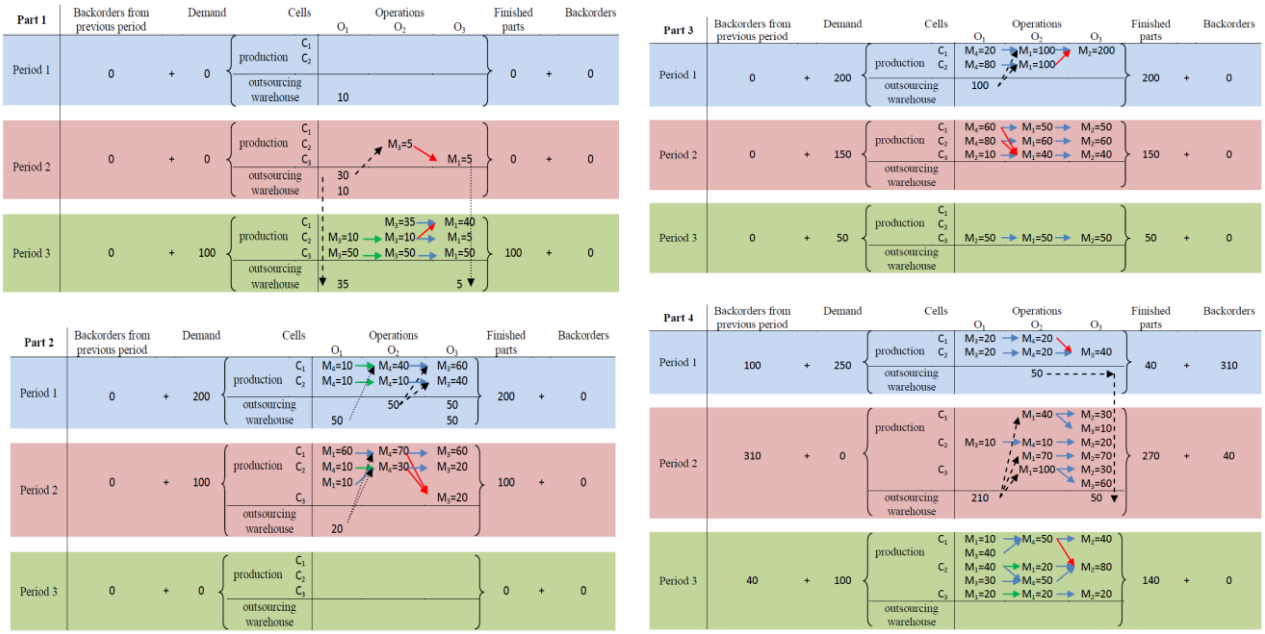
to complete processing operation 3. Then a form of lot splitting is done by dividing the production batch between two different cells. All operations of 50 units of part type 3 are entirely processed in cell 3 during period 3.

Hence, this batch of part 3 is processed without incurring the inter-cell material handling cost, but processing operations 2 and 3 of this batch on machines 1 and 2 incurs the intra-cell material handling cost.

Table 8 shows how part demands are satisfied for part types 1 to 4 through internal production, inventory holding and external outsourcing during the three planning periods.

Furthermore, unsatisfied demands are carried to next periods as backorders. Since the option of holding inventory is considered, the system can leverage the excess capacity of capable machines to start production of part types 1 to 4 during periods 2 and 3. By simultaneously considering all of the four strategies of production planning to satisfy the demand for all four part types, the model in this paper presenting the optimal production plan given in Table 8 shows a higher flexibility in satisfying the part demand in compare to previous studies.

Tab. 7. Optimal cell configurations, part-operation allocations to the machines and production planning decisions



Tab. 8. Optimal production plan

	Period 1						Period 2						Period 3					
	backorder from earlier	demand	operations	production	outsourcing	warehouse	backorder from earlier	demand	operations	production	outsourcing	warehouse	backorder from earlier	demand	operations	production	outsourcing	warehouse
Part 1	0	0	O_1			10	0	0	O_1		30	10	0	100	O_1	60		35
			O_2						O_2	5					O_2	95		
			O_3						O_3	5					O_3	95		5
Part 2	0	200	O_1	20		50	0	100	O_1	80		20	0	0	O_1			
			O_2	50	50				O_2	100					O_2			
			O_3	100	50	50			O_3	100					O_3			
Part 3	0	200	O_1	100	100		0	150	O_1	150			0	50	O_1	50		
			O_2	200					O_2	150					O_2	50		
			O_3	200					O_3	150					O_3	50		
Part 4	100	250	O_1	40			310	0	O_1	10	210		40	100	O_1	140		
			O_2	40	50				O_2	220					O_2	140		
			O_3	40					O_3	220	50				O_3	140		

For example, the demand for part type 2 in period 1 is satisfied by manufacturing 100 parts, outsourcing 50 parts and using the 50 parts kept in inventory. Similar to part type 2, part type 4 also has to be outsourced in the third period to satisfy the demand. This can be due to insufficient machine availabilities and capacities for the required operations.

Finally, it is worth to mention that considering the manufacturing attributes, such as alternative process routing, purchasing machine, duplicate machines, machine depot, lot splitting, varying number of formed cells and the production planning decisions (i.e., semi-finished and finished parts inventory holding, semi-finished and finished parts outsourcing), backorders and internal part production creates flexibility in the integrated DCMS model to respond changing part mix and demands.

4. Conclusions

In this paper, a comprehensive mathematical programming model for the dynamic cellular manufacturing system (DCMS) design integrated with production planning (PP) decisions and several manufacturing attributes is proposed. The model attempts to minimize the total costs related to cells, machines, part-operations, material handling, and PP decisions and incorporates a number of manufacturing attributes and practical constraints.

These include alternative process routings, operation sequence, processing time, production volume of parts, purchasing machine, duplicate machines, machine depot, machine capacity, lot splitting, material flow conservation equations, inflation coefficient, cell workload balancing, budget constraints for cell construction and machine procurement, varying number of formed cells, worker capacity, holding inventories and backorders, outsourcing part-operations, warehouse capacity, and cell reconfiguration.

Our presented model was capable to determine in each period over the planning horizon the following aspects: the optimal number of formed cells, the optimal number of each machine type purchased and assigned to each cell, the relocation of machines between two cells or between a cell and machine depot, the best processing route for each part type, the optimal production plan (PP decisions) for each part type, and the optimal material flow for each part type. Thus, with this work, we have demonstrated the effect of incorporating several manufacturing attributes in an integrated manner.

The performance of this model was illustrated by a numerical example.

The solution from this example for the integrated model has shown that additional CM structural and manufacturing design features and PP decisions that were not integrated in previous research can be used by researchers and practitioners.

The presented model is still opened for incorporating other features, such as material handling in batch, introducing uncertainty in a part demand, machine availability and cost coefficients, multi-objective optimization, multi-level lot sizing aspects, group scheduling and group layout issues, and the like that suggested for future research. Since, the proposed mixed-integer non-linear programming model is NP-hard, we are going to develop heuristic or meta-heuristic methods to efficiently solve the proposed model for large-sized problems and generate several near-optimal solutions.

References

[1] Ahkioon, S., Bulgak, A.A., Bektas, T., *"Integrated Cellular Manufacturing Systems Design with Production Planning and Dynamic System Reconfiguration*," Eur. J. of Operational Research, Vol. 192, 2009, pp. 414–428.

[2] Chen, M., Cao, D., *"Coordinating Production Planning in Cellular Manufacturing Environment using Tabu Search*," Computers & Industrial Engineering, Vol. 46, 2004, pp. 571–588.

[3] Defersha, F.M., Chen, M., "*A Comprehensive Mathematical Model for the Design of Cellular Manufacturing Systems,"* Int. J. of Production Economics, Vol. 103, 2006, pp. 767–783.

[4] Defersha, F.M., Chen, M., "*A Linear Programming Embedded Genetic Algorithm for an Integrated cell Formation and Lot Sizing Considering Product Quality*," Eur. J. of Operational Research, Vol. 187(1), 2007, pp. 46–69.

[5] Mahdavi, I., Aalaei, A., Paydar, M.M., Solimanpur, M., *"Designing a Mathematical Model for Dynamic Cellular Manufacturing Systems Considering Production Planning and Worker Assignment*," Computers and Mathematics with Applications, Vol. 60(4), 2010, pp. 1014–1025.

[6] Mungwattana, A., "*Design of Cellular Manufacturing Systems for Ddynamic and Uncertain Production Requirements with Presence of Routing Flexibility*," Ph.D. Thesis, Faculty of the Virginia Polytechnic Institute and State University, Blackburg, VA, 2000.

[7] Rheault, M., Drolet, J., Abdulnour, G., *"Physically Reconfigurable Virtual Cells: a Dynamic Model for a Highly Dynamic Environment*," Computers & Industrial Engineering, Vol. 29 (1–4), 1995, pp. 221–225.

[8] Safaei, N., Tavakkoli-Moghaddam, R., *"Integrated Multi-Period Cell Formation and Subcontracting Production Planning in Dynamic Cellular Manufacturing,"* Int. J. of Production Economics, Vol. 120 (2), 2009, pp. 301–314.

[9] Saidi-Mehrabad, M., Safaei, N., "*A New Model of Dynamic Cell Formation by a Neural Approach*," Int. J. of Advanced Manufacturing Technology, Vol. 33 (9–10), 2007, pp. 1001–1009.

[10] Schaller, J.E., Erengüç, S.S., Vakharia, A.J., *"A Methodology for Integrating Cell Formation and Production Planning in Cellular Manufacturing*," Annals of Operations Research, Vol. 77, 1998, pp. 1 – 21.

[11] Tavakkoli-Moghaddam, R., Rahimi-Vahed, A.R., Ghodratnama, A., Siadat, A., *"A Simulated Annealing Method for Solving a New Mathematical Model of a Multi-Criteria Cell Formation Problem with Capital Constraints*," Advances in Engineering Software, Vol. 40, 2009, pp. 268–273.

[12] Wemmerlov, U., Hyer, N.L., *"Procedures for the Part Family/Machine Group Identification Problem in Cellular Manufacture*," J. of Operations Management, Vol. 6, 1986, pp. 125–147.