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An Integrated Queuing Model for Site Selection and Inventory Storage Planning of a Distribution Center with Customer Loss Consideration

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ABSTRACT

The distribution center location problem is a crucial question for logistics decision makers. The optimization of these decisions needs careful attention to the fixed facility costs, inventory costs, transportation costs and customer responsiveness. In this paper we study the location selection of a distribution center which satisfies demands with a M/M/1 finite queueing system plus balking and reneging. The distribution center uses one for one inventory policy, where each arrival demand orders a unit of product to the distribution center and the distribution center refers this demand to its supplier. The matrix geometric method is applied to model the queueing system in order to obtain the steady-state probabilities and evaluate some performance measures. A cost model is developed to determine the best location for the distribution center and its optimal storage capacity and a numerical example is presented to determine the computability of the results derived in this study.

1. Introduction

Total logistics costs (inventory plus transportation) are important parts of a product cost in many countries. According to costly and difficult to reverse nature of facility location problems (FLPs) and its long time horizon impact, there is a critical management decision in the design of efficient logistics systems, which discuss about the choice of locations for distribution centers (DCs) to enhance operation efficiency and logistics performance. Distribution center is defined as an entity that links an enterprise with its suppliers and customers. Identification of a distribution center location should be based on expenses as inventory costs, transportation costs, construction costs, operating costs and in some cases the service level costs. There are many articles of analytic study on FLP, where locating distribution centers is one of the main motivations for them. In an article by Aikens (1985), nine basic location models where all of them are to minimize the fixed investment costs and transportation costs are surveyed. Many applications and methods for facility location problems and location models for distribution systems are surveyed in Dresner (1995) and Klose and Drexl (2005), respectively. Moreover, Syam (2002) investigated a FLP model and some methodologies considering logistical components.

Some models considered the demand uncertainty effects on DCs optimal location. A notable work in such topic is the published article by Cole (1995) who considers Normal distribution for demand and a required safety stock for a specific customer service level in order to identify the DC location and customer allocation. Also, simple dynamic and stochastic location models are developed for considering the dynamic nature of FLP and the stochastic nature of demand by Owen and Daskin (1998). In a related article Song (2006), proposes a model where he minimizes the sum of all the associated costs of a

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supply chain network, subject to a variety of related constraints.

Inventory costs should be considered jointly with other facility transportation and operation costs in determining the optimal locations for DCs. An instance for mentioned topic is Nozick and Turnquist (1998) article where they describe a method including inventory costs within a fixed charge facility location model for developing an optimal system design. Furthermore, they present a case study involving the distribution of finished vehicles by an automotive manufacturer in another article which provides an integrated view with a careful attention to facility costs, transportation costs, inventory and customer responsiveness costs (Nozick and Turnquist 2001).

Applying queueing theory in FLPs, first was discussed by Larson (1974) where he analyzed problems of vehicle location-allocation and response district design in emergency response services that operate in the server to customer mode. Following Larson (1974) study, providing probabilistic models that consider queuing theory in FLPs has been developed by many researchers. Instances of such models are described by Batta (1989), Marianov and Serra (2002), Snyder (2006) and Marianov (2008).

The distribution center location problem that is considered as a basic model in this paper is composed of a supplier with stochastic replenishment lead time, a distribution center with stochastic service time and an infinite source of customers (who can balk and renege) with stochastic demand arrival times. We deal with the $M/M/1$ queue with finite capacity of impatient customers. The behavior of impatient customers which upon arrival may or may not go in the queue for service depending on the number of customers in the system and those which on going to the queue depart the queue without being served, is investigated in this study. The queueing systems with balking and reneging have been discussed in many articles. Examples of such studies can be seen in Wang et al. (2007), Yue and Sun (2008) and Al-Seedy et al. (2009).

This paper aims to study the impact of impatient customers and their demand uncertainties on the optimal location of a distribution center and the size of its storage capacity which is needed to be established. The primary objectives of the study are:

- Developing the steady-state solutions for the $M/M/1$ queueing system with finite capacity, reneging and balking.
- Developing a cost model to identify the optimal distribution center location in order to minimize the steady-state expected cost per unit time.
- Obtaining the optimal storage capacity in each candidate location site which minimizes the steady-state related inventory expected costs per unit time.

The remainder of the paper is organized as follows. In Section 2 we describe the system definition and more explicitly queueing relations. Section 3 is dedicated to calculating some performance measures and deriving the cost analysis is discussed in Section 4. We have shown the convexity of the expected total cost function by a numerical example in Section 5 and finally, Section 6 provides conclusions and directions for future research.

2. System Definition

We consider a distribution center with one server which satisfies the arrival demands using a queueing system with finite capacity. The arrival demands signal out from *i* demand points and the time points of these demands occurrences form a Poisson process with parameter λ_i $(i=1,2,...,a)$ for each demand point. So, the total rate of demands referring to the distribution center equals with $\lambda=\sum_{i=1}^{a}\lambda_i$. See Fig. 1 for a diagram depicting the model.

Each customer who comes into the distribution center has a demand and satisfying this demand needs an on hand inventory and a process (set up) must be done by distribution center, which takes some time. Each customer needs exactly a product of unit size and DC uses one for one inventory policy. Therefore, the on hand inventory plus in order inventory must be hold at a pre-specified level (*S*) so the demand pattern is transferred exactly to the supplier. The DCs' ordering quantity to supplier is of unit size and it occurs when a demand refers to it.

It is notable that the order satisfying times from the DC to the demand points are exponentially distributed with mean $1/\mu(>0)$ which are independent from the distances to the demand points. Also replenishment lead time from the supplier is exponentially distributed with mean $1/\nu(>0)$.

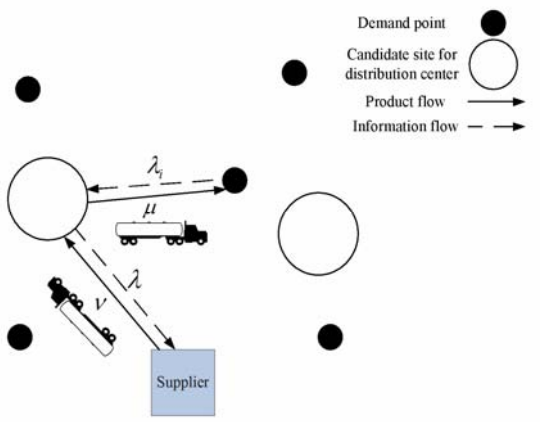

Fig. 1. Distribution center location problem with stochastic replenishment time, service time and demand

The certain time that a customer wait for service to begin before getting impatient, is random variable which is distributed as a negative exponential

distribution with parameter β. It is important to say there is no physical queue in the system and the arrival demands go to an order queue which does not need a physical line. The transportation process is performed using one vehicle.

The DC can serve just one customer at a time and the service process assumed to be independent of customer arrivals. The problem is to find the best location for distribution center and its optimal storage capacity according to defined system parameters.

2-1. The Markov Chain

The system can be described by a quasi birth-and-death Markov process with states (n,k). Consider the continuous Markov chain $\{(n,k),0\le n\le N,0\le k\le S\}$ where *n* is the number of customers in the system including the one being served and *k* is the number of products which are available in the distribution center storage. The state space with transition rates is depicted in Fig.2.

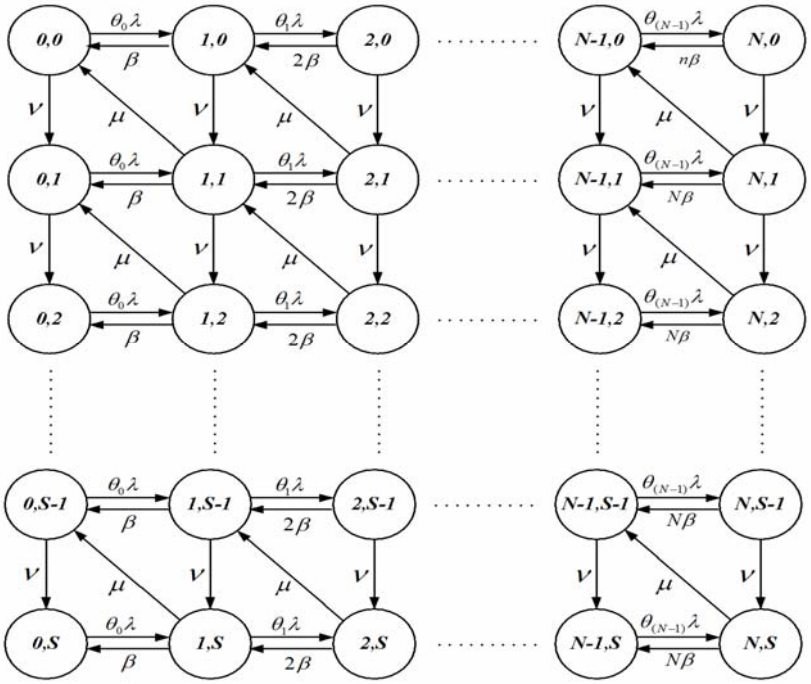

Fig. 2. State transition rates diagram

2-2. Steady state results

We described the state of the system by the pairs $\{(n,k),0\le n\le N,0\le k\le S\}$. Now, let θ_n denote the probability that a customer enters the queue when there are *n* customers in the system. θ_n is defined as follows (Bhat 2008):

$$\theta_n=\begin{cases}1 & n=0,\\ e^{-n/\mu} & 1\le n\le N-1,\\ 0 & n\ge N\end{cases}$$

Since the waiting time before getting impatient is a random variable which follows an exponential distribution with mean $1/\beta$ and customers decisions are independent of each other, the average renege rate is $n\beta$.

In order to develop the steady-state probabilities $\pi_{(n,k)},0\le n\le N,0\le k\le S$, we get help from the matrix geometric method which was first introduced by Neuts (1981). The generator matrix of the under study Markov chain is given as:

$$Q=\begin{bmatrix}B_0 & A_0 & & & \\ C_1 & B_1 & A_1 & & \\ & \ddots & \ddots & \ddots & \\ & & C_{N-1} & B_{N-1} & A_{N-1}\\ & & & C_N & B_N\end{bmatrix}$$

Where A_n, B_n and C_n are block square matrixes of order $S+1$ which are displayed in Appendix A. According to the finite capacity of the queue, there is no need to check the stability condition for the system under consideration. It is notable that A_n, $B_n\,(n\neq 0)$ and C_n giving the rate at which the number of the customer orders in the system increase by one, stay at

the same level, or decrease by one. B_0 is the matrix rate at which the customer orders in the system moves from zero to one.

We define the steady-state probability vector $\pi = [\pi_0, \pi_1, ..., \pi_N]$ for the Markov chain $\{(n,k), 0 \le n \le N, 0 \le k \le S\}$. Each π_n can be calculated using $\pi Q = 0$ and $\sum_{n=0}^{N} \pi_n = 1$, where $\pi_n = [\pi_{(n,0)}, \pi_{(n,1)}, ..., \pi_{(n,S)}]$ is a $1 \times (S+1)$ row vector. $\pi_{(n,k)}$ denotes the steady-state probability associated with the condition that there are *n* customers and *k* products in the system.

Referring to the state transition diagram for the finite $M/M/1$ queueing system with balking and reneging which is shown in Fig. 2, the following balance equations are derived:

$$(\theta_n\lambda + \nu)\pi_{(n,k)} = \beta\pi_{(n+1,k)} + \mu\pi_{(n+1,k+1)} \qquad n = 0, k = 0$$

$$(\theta_n\lambda + \nu)\pi_{(n,k)} = \beta\pi_{(n+1,k)} + \mu\pi_{(n+1,k+1)} + \nu\pi_{(n,k-1)} \qquad n = 0, 1 \le k \le S-1$$

$$\theta_n\lambda\pi_{(n,k)} = \beta\pi_{(n+1,k)} + \nu\pi_{(n,k-1)} \qquad n = 0, k = S$$

$$(\theta_n\lambda + n\beta + \nu)\pi_{(n,k)} = (n+1)\beta\pi_{(n+1,k)} + \mu\pi_{(n+1,k+1)} + \theta_{n-1}\lambda\pi_{(n-1,k)} \qquad 1 \le n \le N-1, k = 0$$

$$(\theta_n\lambda + n\beta + \mu + \nu)\pi_{(n,k)} = (n+1)\beta\pi_{(n+1,k)} + \mu\pi_{(n+1,k+1)} + \theta_{n-1}\lambda\pi_{(n-1,k)} + \nu\pi_{(n,k-1)} \qquad 1 \le n \le N-1, 1 \le k \le S-1$$

$$(\theta_n\lambda + n\beta + \mu)\pi_{(n,k)} = (n+1)\beta\pi_{(n+1,k)} + \theta_{n-1}\lambda\pi_{(n-1,k)} + \nu\pi_{(n,k-1)} \qquad 1 \le n \le N-1, k = S$$

$$(n\beta + \nu)\pi_{(n,k)} = \theta_{n-1}\lambda\pi_{(n-1,k)} \qquad n = N, k = 0$$

$$(n\beta + \mu + \nu)\pi_{(n,k)} = \theta_{n-1}\lambda\pi_{(n-1,k)} + \nu\pi_{(n,k-1)} \qquad n = N, 1 \le k \le S-1$$

$$(n\beta + \mu)\pi_{(n,k)} = \theta_{n-1}\lambda\pi_{(n-1,k)} + \nu\pi_{(n,k-1)} \qquad n = N, k = S$$

In order to solve $\pi Q = 0$, it is not possible to define a constant rate matrix *R* such that $\pi_n = \pi_{n-1}R = \pi_1 R^{n-1}$ as discussed in Neuts (1981), because of the asymmetric structure of *Q*s' sub-matrixes.

So, due to the finite number of sub-matrixes in generator matrix *Q*, balance equations are solved directly by MATLAB 7.1 (the language of technical computing), in order to calculate the steady-state probabilities.

3. System Performance Measures

In this section we derive a number of performance measures of the system under consideration in the steady-state.

3.1. Mean Inventory Level

Let $E(I)$ represent the average inventory of products in the steady state. Then we have

$$E(I) = \sum_{n=0}^{N}\sum_{k=1}^{S} k\pi_{(n,k)}$$

3.2. Mean Backorder Level

Let $E(B)$ denote the mean number of backorders in the steady-state. Then we have $E(B) = \sum_{n=1}^{N} n\pi_{(n,0)}$

3.3. Mean Customer Order Fulfillment Delay

Let $E(W)$ represent the average customer order fulfillment delay in the steady-state. In order to calculating $E(W)$ we have to obtain the mean customer orders in the system $E(L)$ that is achievable as follows $E(L) = \sum_{k=0}^{S}\sum_{n=1}^{N} n\pi_{(n,k)}$

Then via Little's law we have $E(W) = E(L)/\lambda'$

$$\lambda' = (1 - \sum_{k=0}^{S} \pi_{(N,k)})\lambda$$

3.4. Mean Rate of Customer Loss

Let *E(BA), E(RE)* and *E(LO)* denote the average balking rate, the average reneging rate and the average rate of customer loss. Using the concept of Ancker and Gafarian (1963) these average rates are obtained as follows

$$E(BA) = \sum_{k=0}^{S}\sum_{n=1}^{N} (1-\theta_n)\lambda\pi_{(n,k)}$$

$$E(RE) = \sum_{k=0}^{S}\sum_{n=1}^{N} n\beta\pi_{(n,k)}$$

$$E(LO) = E(BA) + E(RE)$$

4. Optimal DC Location and Storage Capacity

The expected total cost in the steady-state for the considered logistic model is defined to be:

$$TC(j, S_j) = F_j + \sum_{i=1}^{a} cr_{ij}\lambda_i(\frac{\lambda - E(LO)}{\lambda}) + cr_{sj}\nu + k_j c_{cj}$$
$$+ c_b E(B) + h_j E(I) + c_w E(W) + (c_l \times E(LO))$$

where

j	Index of the distribution center candidate sites
S_j	Maximum storage capacity which can be established in candidate site j
F_j	Fixed cost of opening a distribution center in candidate site j
c	Constant coefficient for transforming distance to cost
r_{ij}	Euclidian distance between demand point i and *j* th candidate site
r_{sj}	Euclidian distance between supplier and *j* th candidate site
c_{cj}	The cost of establishing product storage capacity per unit product per unit time in candidate site j
c_b	The fixed backorder cost per order per unit time
h_j	The holding cost per unit product per unit time in candidate site j
c_w	The cost of order fulfillment delay per unit product per unit time
c_l	The loss cost of one customer (renege or balk) per unit time

It is notable that the constant $(\frac{\lambda - E(LO)}{\lambda})$ denotes the percentage of satisfied orders for each demand point. Replacing the values of mean performance measures, we get the following expected total cost function.

$$TC(j, S_j) = F_j + \sum_{i=1}^{a} cr_{ij}\lambda_i(\frac{\lambda - \sum_{k=0}^{S}\sum_{n=1}^{N}(1-\theta_n)\lambda\pi_{(n,k)} + n\beta\pi_{(n,k)}}{\lambda}) + cr_{sj}\nu + k_j c_{cj} + c_b\sum_{n=1}^{N} n\pi_{(n,0)}$$
$$+ h_j\sum_{n=0}^{N}\sum_{k=1}^{S} k\pi_{(n,k)} + c_w(\frac{\sum_{k=0}^{S}\sum_{n=1}^{N} n\pi_{(n,k)}}{\lambda}) + (c_l \times (\sum_{k=0}^{S}\sum_{n=1}^{N}(1-\theta_n)\lambda\pi_{(n,k)} + n\beta\pi_{(n,k)}))$$

According to recursive computation of the π's, it is quite difficult to show the convexity of the expected total cost function. However we present a numerical example to prove the computability of the results derived in this study.

5. Numerical Example

For a distribution center with a queueing system, we set the system capacity $N = 8$. We assume the following parameter values: $\mu = 35$ products per month, $\nu = 33$ unit replenishments per month, $c = 20$ dollars per kilometer, $c_b = 35$ dollars per order per month, $c_w = 70$ dollars per unit product per month and $c_l = 100$ dollars per customer per month. The waiting time before getting impatient is a random negative exponential distributed variable with parameter $\beta = 0.3$. The replenishment supplier is located in coordinates (4,1) which is expressed in kilometers. Other necessary information about demand points and candidate sites for locating the distribution center are provided in Table 1 and Table 2.

Tab. 1. Demand points information

Demand point	Demand point coordinates (Kilometers)	Demand arrival rate (Order per month)
1	(1,3)	4
2	(3,1)	5
3	(4,4)	3
4	(6,7)	6
5	(7,4)	3
6	(2,8)	6
7	(5,3)	5

Tab. 2. Candidate sites information

Candidate site	Candidate site coordinates (Kilometers)	Fixed cost of opening a distribution center (1000 dollars)	(Dollars per unit c_c product per month)	(Dollars per unit h product per month)	Maximum possible storage capacity
1	(3,3)	10	45	8	7
2	(4,4)	9	36	7	9
3	(5,4)	9.5	47	10	6

The values of expected total costs are given in Table 3. The optimal storage capacity is shown in bold for each distribution center candidate site. The numerical values shown that expected total cost function is convex in k. The optimum value which represents the minimum possible cost for the distribution center is obtained in candidate site 1 with 3 products storage capacity.

Tab. 3. Numerical example results

Established storage capacity (k)	Expected total cost for each candidate site (Dollars)		
	1	2	3
1	14180	14998	14200
2	14079	14888	14089
3	**14050**	14850	**14056**
4	14057	**14848**	14061
5	14082	14862	14087
6	14114	14885	14121
7	14154	14915	-
8	-	14947	-
9	-	14981	-

Using the presented model, we can select the near optimal decisions if the optimal one can not be performed. More explicitly, if we can not open the DC in candidate site 1, we know the next optimal solution, which is establishing the DC in candidate site 3 with 3 products storage capacity.

6. Conclusion

A distribution center location problem with uncertain demand and impatient customers is studied. Product replenishment time in DC, demand satisfying time and the time between demand arrivals follow an

exponential distribution. The service procedure is modeled as a finite queueing system with customer loss (renege and balk). The matrix geometric method is used to model the queueing system. This paper generalizes a method to obtain an optimal location for a distribution center among some candidate sites and its optimal storage capacity, applying an integrated total cost function. Using this method, decision maker enables to select near optimal solutions simultaneously, if the optimal one can not be performed.

Analyzing the problem discussed in this article assuming the queueing system has no finite capacity and customers can jockey between more than one distribution centers would be a good topic for future research. Another interesting extension could be made by relaxing the assumptions of exponentially distributed replenishment time, service time and the time between demand arrivals.

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Appendix A

$$B_n = \begin{bmatrix} B_{n_{0,0}} & B_{n_{0,1}} & & & \\ & B_{n_{1,1}} & B_{n_{1,2}} & & \\ & & \ddots & \ddots & \\ & & & B_{n_{S-1,S-1}} & B_{n_{S-1,S}} \\ & & & & B_{n_{S,S}} \end{bmatrix} \tag{A.1}$$

$$\left.\begin{aligned} B_{n_{k,k}} &= \begin{cases} -(\nu+\theta_n\lambda) & 0\le k\le S-1 \\ -\theta_n\lambda & k=S \end{cases} \\ B_{n_{k,k+1}} &= \nu \qquad 0\le k\le S-1 \end{aligned}\right\} \quad n=0 \tag{A.2}$$

$$\left.\begin{aligned} B_{n_{k,k}} &= \begin{cases} -(\nu+\theta_n\lambda+n\beta) & k=0 \\ -(\nu+\theta_n\lambda+n\beta+\mu) & 1\le k\le S-1 \\ -(\theta_n\lambda+n\beta+\mu) & k=S \end{cases} \\ B_{n_{k,k+1}} &= \nu \qquad 0\le k\le S-1 \end{aligned}\right\} \quad 1\le n\le N-1 \tag{A.3}$$

$$\left.\begin{aligned} B_{n_{k,k}} &= \begin{cases} -(\nu+n\beta) & k=0 \\ -(\nu+n\beta+\mu) & 1\le k\le S-1 \\ -(n\beta+\mu) & k=S \end{cases} \\ B_{n_{k,k+1}} &= \nu \qquad 0\le k\le S-1 \end{aligned}\right\} \quad n=N \tag{A.4}$$

$$A_n = \theta_n\lambda\mathbf{I}_{k\times k}, \quad 0\le n\le N-1 \tag{A.5}$$

$$C_n = \begin{bmatrix} n\beta & & & \\ \mu & n\beta & & \\ & \ddots & \ddots & \\ & & \mu & n\beta \\ & & & \mu \quad n\beta \end{bmatrix}, \qquad 1\le n\le N \tag{A.6}$$