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THE EFFECT OF MATRIX BOND FAILURE ON STRESS DISTRIBUTION IN SHORT AND LONG FIBERS OF A HYBRID COMPOSITE LAMINA

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Abstract*: The effect of a bond failure and its extent is studied on stress concentration in long fibers as well as stress distribution in short fibers and their surrounding matrix bays. The material is assumed to be a finite width hybrid composite lamina which is subjected to a tensile load of magnitude "P" at infinity. The surrounding matrix is assumed to take only shear (shear-lag theory). The bay adjacent to the first intact filament is allowed to experience a bond failure of size 2δ. This failure is due to excessive shear load in the matrix which exceeds the fiber-matrix bond strength. The matrix at this zone may or may not experience yielding. The short fibers are simulated by assuming two successive breaks along each filament. The effect of bond failure length on short fiber load bearing capability, as well as stress concentration in the first intact filament is fully investigated. The effect of hybridization, in presence of bond failure is also examined on short fiber load bearing behavior.*

Keywords*: Hybrid, Short fiber load, Matrix bond failure, Stress concentration.*

1. Introduction

Structures fabricated from fine filaments of various types have found many industrial applications. One of the necessary factors in rational design of such structures is a vast knowledge of stress behavior in the vicinity of any local discontinuity which may be present in form of a hole, crack, fiber-matrix bond failure, and so on. Composites that are composed of more than one filament are called hybrids.

This arrangement of fibers is due to a need for any improvement in a deficiency present in a single type fiber composite.

In general, the use of the second type fiber could be to improve the weight of the overall structure, its mechanical property, or a reduction in cost of production. Since the presence of the second type fiber influences the stress distribution within the material, then, knowledge of this behavior will enable one to use these materials efficiently.

Many attempts have been made to better understand any stress field variation caused from the presence of local defects. To accomplish this task the material has to be modeled properly. One of the models available is based on shear lag theory, where in, all fibers are assumed to take axial load and the matrix sustains only shear.

The load transfer mechanism from any broken fiber to its adjacent filament is through shear stress in the matrix.

It is shown that [1]–[4] shear-lag model gives relatively accurate results on normal stresses developed in composites with a low extensional stiffness in the matrix.

The effect of inter-fiber spacing and matrix crack on stress concentration factor has also been examined by Sirivedin S. et. al, in reference [5]. The effect of fiber cross sectional shape on mechanical behavior was further discussed by Bond Ian et al [6].

Several authors have also studied the stress distribution and fracture behavior of hybrid composites [7]–[10]. Stress- strain behavior in initial stage of short fiber reinforced metal matrix composites was studied by Ding and his co-authors [11]. In references [11]-[15], short fiber reinforced composites were studied to determine the effect of fiber volume fraction and its length on tensile properties as well as stress distribution in overall material. Most of the research on composites with matrix plasticity has focused on materials with single type filament [16] –[18]. Several authors have tried to investigate bond failure strength on overall behavior of different type composites through experiments and analytical solutions and modeling [19]–[25]. Due to the complexity of stress distribution in short fiber composites, stress distribution in these materials have still many unresolved questions which yet have to be answered. In this paper, a try is made to understand the effect of bond failure and its extent on any stress

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concentration produced within a single type fiber and a hybrid composite lamina.
Also, stress distribution within any short fiber and the effect of debonded fiber-matrix interface on short fiber load bearing capability is fully investigated.

2. Derivation of Formulas

To obtain the necessary relations, a finite width composite lamina with N=2q+1 fibers is considered as shown in Figure 1.

It is assumed that all the fibers are aligned in parallel, and the spacing between them, namely "h", is equal to the fiber's diameter "d". Furthermore, it is assumed that all fibers will only take extensional load, and the matrix sustains only shear. This is a good assumption for most composites with a phenolic resin or weak in tension.

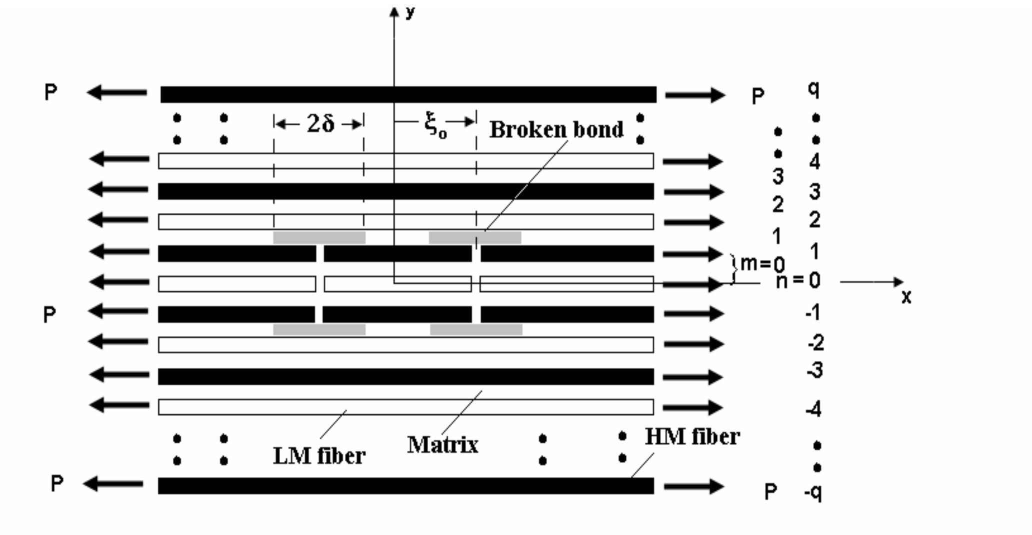

Fig 1. Fiber arrangement in a hybrid lamina with double cuts and broken bond zones of size 2δ.

The high modulus fibers (HM), as well as the low modulus fibers (LM), are assumed to have the same diameter and act as linear elastic materials up to the point of fracture. Two successive breaks are considered along each filament to simulate a short fiber. A perfect bond is assumed to exist between all fibers and matrix bays except those bonding the crack tip. In this region, there is a bond failure of size 2δ between the fibers bonding the crack tip and their neighboring matrix bay. This failure may exist due to the presence of excessive shear stress developed within the matrix. The stress in this zone may (or may not) have reached its limiting yield value. The lamina is subjected to a tensile load of magnitude P applied at infinity. Due to symmetry, only the right portion of the lamina is considered. To obtain field equations, the right portion of the lamina is divided into four regions (see Figure 2). In regions one and four there is a perfect bond between matrix and fibers while in regions two and three, a bond failure zone of size 2δ exists between the fibers and matrix bays bonding the crack tips. The matrix in this zone may or may not be assumed to be yielded. Two successive breaks are assumed along broken fibers to simulate a short fiber.
In all equations, an asterisk is used to distinguish those properties associated with LM fibers. Equilibrium equations in each region may be written by considering a volume element containing two successive fibers (one HM and one LM fiber), and their surrounded matrix bay as shown in Figure 3.

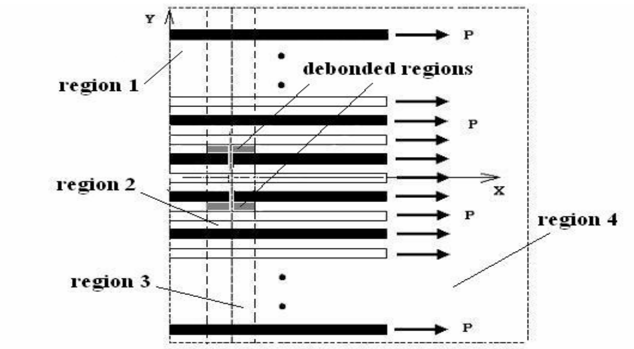

Fig 2. Fiber arrangements for the right hand portion of the lamina.

The displacement of each fiber in regions one and four is shown by u where for regions two and three parameter $\bar{u}$ is used instead. Application of force equilibrium equation along x on the mth volume element in regions one and four reveals that;

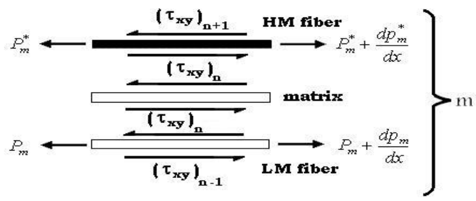

Fig 3. Force equilibrium on the mth volume element with intact matrix.

$$E_f A_f \frac{d^2u_m}{dx^2} + \frac{Gh}{d}(u_m^* - 2u_m + u_{m-1}^*) = 0 \qquad (2.1)$$

$$E_f^* A_f^* \frac{d^2u_m^*}{dx^2} + \frac{Gh}{d}(u_m - 2u_m^* + u_{m-1}) = 0 \qquad (2.2)$$

According to Figure 4, in regions two and three, for those fibers located at the crack tips, the equilibrium equation of fibers reduce into:

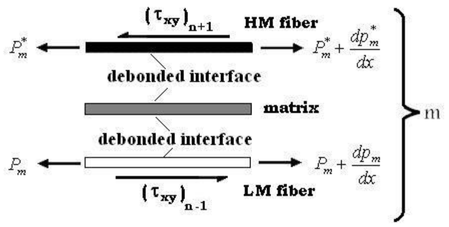

Fig 4. Force equilibrium on the m^th volume element at the crack tip in a zone with debonded matrix.

$$E_f A_f \frac{d^2\bar{u}_m}{dx^2} + \frac{Gh}{d}(\bar{u}_{m-1}^* - \bar{u}_m) = 0 \qquad (2.3)$$

$$E_f^* A_f^* \frac{d^2\bar{u}_m^*}{dx^2} + \frac{Gh}{d}(\bar{u}_{m+1} - \bar{u}_m^*) = 0 \qquad (2.4)$$

For other fibers and matrix bays surrounded in regions two and three the equilibrium equations may be expressed as;

$$E_f A_f \frac{d^2\bar{u}_m}{dx^2} + \frac{Gh}{d}(\bar{u}_m^* - 2\bar{u}_m + \bar{u}_{m-1}^*) = 0 \qquad (2.5)$$

$$E_f^* A_f^* \frac{d^2\bar{u}_m^*}{dx^2} + \frac{Gh}{d}(\bar{u}_m - 2\bar{u}_m^* + \bar{u}_{m-1}) = 0 \qquad (2.6)$$

Assuming the lamina ends in a HM fiber at the edge, then the equilibrium equations for the edge fibers in regions one

$$E_f A_f \frac{d^2u_n}{dx^2} + \frac{Gh}{d}(u_{n-1}^* - u_n) = 0 \qquad n = q \qquad (2.7)$$

$$E_f A_f \frac{d^2u_n}{dx^2} + \frac{Gh}{d}(u_{n+1}^* - u_n) = 0 \qquad n = -q \qquad (2.8)$$

And for regions two and three;

$$E_f A_f \frac{d^2\bar{u}_n}{dx^2} + \frac{Gh}{d}(\bar{u}_{n-1}^* - \bar{u}_n) = 0 \qquad n = q \qquad (2.9)$$

$$E_f A_f \frac{d^2\bar{u}_n}{dx^2} + \frac{Gh}{d}(\bar{u}_{n+1}^* - \bar{u}_n) = 0 \qquad n = -q \qquad (2.10)$$

For simplicity, equilibrium equations (2.1) through (2.10) are written in a non-dimensional form as follows.

$$\frac{d^2U_m}{d\xi^2} + (U_m^* - 2U_m + U_{m-1}^*) = 0 \qquad (2.11)$$

(regionsoneandfour,HMfibers)

$$R\frac{d^2U_m^*}{d\xi^2} + (U_m - 2U_m^* + U_{m-1}) = 0 \qquad (2.12)$$

(regions one and four, LM fibers)

$$\frac{d^2U_n}{d\xi^2} + (U_{n-1}^* - U_n^*) = 0 \qquad (2.13)$$

n = q (regionsone and four)

$$\frac{d^2U_n}{d\xi^2} + (U_{n+1}^* - U_n) = 0 \qquad (2.14)$$

n = -q (regions one and four)

$$\frac{d^2\bar{U}_m}{d\xi^2} + (\bar{U}_{m-1}^* - \bar{U}_m) = 0 \qquad (2.15)$$

(regions two and three, HM fibers)

$$R\frac{d^2\bar{U}_m^*}{d\xi^2} + (\bar{U}_{m+1} - \bar{U}_m^*) = 0 \qquad (2.16)$$

(regions two and three, LM fibers)

$$\frac{d^2\bar{U}_m}{d\xi^2} + (\bar{U}_m^* - 2\bar{U}_m + \bar{U}_{m-1}^*) = 0 \qquad (2.17)$$

(regions two and three, HM fibers)

$$R\frac{d^2\bar{U}_m^*}{d\xi^2} + (\bar{U}_m - 2\bar{U}_m^* + \bar{U}_{m-1}) = 0 \qquad (2.18)$$

(regions two and three, LM fibers)

$$\frac{d^2\bar{U}_n}{d\xi^2} + (\bar{U}_{n-1}^* - \bar{U}_n) = 0 \qquad (2.19)$$

n = q (regions two and three)

$$\frac{d^2\bar{U}_n}{d\xi^2} + (\bar{U}_{n+1}^* - \bar{U}_n) = 0 \qquad (2.20)$$

n = -q (regions two and three)

Similar expressions to those of (2.17) through (2.20) may be written if the edges end in a LM fiber.
In above equations, it has been assumed that a LM fiber bonds the crack tip and the breaks are symmetric with respect to n = 0 fiber. Similar expressions may be written if a HM fiber bonds the crack tip.

3. Displacement and Load Distribution Fields

(a). Regions 1 and 4
In these two regions, equations (2.11) through (2.14) may be written in a matrix notation as:

$$\mathbf{L_1U}^{''} - \mathbf{L_2U} = \mathbf{0} \tag{3.1}$$

Where $\mathbf{L_1}$ and $\mathbf{L_2}$ are coefficient matrices and $\mathbf{U}^{''}$ corresponds to the second derivative of $\mathbf{U}$ with respect to ξ. Hence, the solution to the differential-difference equation (3.1) may be written as follows.

(a.1). Region 1

As observed in Figure 2, this region is confined in region $0 \le \xi \le (\xi_\circ - \delta)$. All eigenvalues associated with this region are distinct. Hence, due to finiteness of this length, the solution to equations (3.1) may be written in terms of eigenvalues λ_ι and eigenvectors $\mathbf{R}^{(i)}$ as;

$$U_n^{(1)} = \xi + \sum_{i=1}^{2q+1}(A_i R_{(q-n+1)}^{(i)} e^{\lambda_i \xi} + B_i R_{(q-n+1)}^{(i)} e^{-\lambda_i \xi}) \tag{3.2}$$

$$P_n^{(1)} = 1 + \sum_{i=1}^{2q+1}(A_i\lambda_i e^{\lambda_i \xi} - B_i\lambda_i e^{-\lambda_i \xi}) \times R_{(q-n+1)}^{(i)} \quad \text{(for HM fibers)} \tag{3.3}$$

$$P_n^{*(1)} = R\{(1 + \sum_{i=1}^{2q+1}(A_i\lambda_i e^{\lambda_i \xi} - B_i\lambda_i e^{-\lambda_i \xi}) \times R_{(q-n+1)}^{(i)} \ \} \quad \text{(for LM fibers)} \tag{3.4}$$

In above equations, $R_{(q-n+1)}^{(i)}$ is a value associated with the (q-n+1)th row of the ith eigenvector. The superscript (1) corresponds to properties associated with region 1.

(a.2). Region 4

This region is defined between the limits $(\xi_\circ + \delta) \le \xi \le \infty$. All the eigenvalues associated with this region are distinct. Hence due to the boundness condition defined in equation (3.5), the solution to equations (3.1) may be written in terms of expressions (3.6) to (3.8).

$$\begin{cases} P_n = 1 \\ P_n^* = R \end{cases} \quad \text{as} \quad \xi \to \infty \tag{3.5}$$

$$U_n^{(4)} = \xi + \sum_{i=1}^{2q+1}(N_i R_{(q-n+1)}^{(i)} e^{-\lambda_i \xi}) \tag{3.6}$$

$$P_n^{(4)} = 1 + \sum_{i=1}^{2q+1}(-N_i\lambda_i e^{-\lambda_i \xi}) \ R_{(q-n+1)}^{(i)} \quad \text{(for HM fibers)} \tag{3.7}$$

$$P_n^{*(4)} = R\{\ 1 + \sum_{i=1}^{2q+1}(-N_i\lambda_i e^{-\lambda_i \xi}) \times R_{(q-n+1)}^{(i)} \ \} \quad \text{(for LM fibers)} \tag{3.8}$$

In Equations (3.6)-(3.8), positive values of λ_i are discarded due to the boundness conditions expressed in (3.5). In equations (3.2) - (3.8), A_i, B_i, and N_i, are constants yet to be defined from boundary and continuity conditions and superscript (4) corresponds to properties associated with region 4. One must realize that the non-dimensional load in each fiber is expressed as:

$$P_n = \frac{dU_n}{d\xi} \tag{3.9}$$

(b). Regions 2 and 3

In these two regions, equations (2.15) through (2.20) may be written in a matrix notation as:

$$\mathbf{L}_1\overline{\mathbf{U}}^{''} - \mathbf{L}_3\overline{\mathbf{U}} = \mathbf{0} \tag{3.10}$$

$\mathbf{L}_1$ and $\mathbf{L}_3$ are coefficient matrices. The solution for load and displacement in each region may be written as;

(b-1). Region 2

The field equations associated with this region may be written as:

$$\overline{U}_n^{(2)} = \xi + \sum_{i=1}^{2q+1}(C_i \xi^k R_{(q-n+1)}^{(i)} e^{\lambda_i \xi} + D_i \xi^k R_{(q-n+1)}^{(i)} e^{-\lambda_i \xi}) \tag{3.11}$$

$$\overline{P}_n^{(2)} = 1 + \sum_{i=1}^{2q+1}(C_i\lambda_i k \xi^{k-1} e^{\lambda_i \xi} - D_i k \xi^{k-1}\lambda_i e^{-\lambda_i \xi}) \ R_{(q-n+1)}^{(i)} \quad \text{(for HM fibers)} \tag{3.12}$$

$$\overline{P}_n^{(2)*} = R\{\ 1 + \sum_{i=1}^{2q+1}(C_i\lambda_i k \xi^{k-1} e^{\lambda_i \xi} - D_i k \xi^{k-1}\lambda_i e^{-\lambda_i \xi}) \ R_{(q-n+1)}^{(i)} \ \} \quad \text{(for LM fibers)} \tag{3.13}$$

In above equations, k is the number of occurrence of an eigenvalue. These equations may be re-written as:

$$\overline{U}_n^{(2)} = \xi + \sum_{i=1}^{2q+1}(C_i^{'} R_{(q-n+1)}^{(i)} \xi^k \sinh(\lambda_i \xi) + D_i^{'} R_{(q-n+1)}^{(i)} \xi^k \cosh(-\lambda_i \xi))$$

$$\overline{P}_n^{(2)} = 1 + \sum_{i=1}^{2q+1}(C_i^{'}\lambda_i (k) \xi^{k-1} \cosh(\lambda_i \xi) + D_i^{'}\lambda_i (k) \xi^{k-1} \sinh(-\lambda_i \xi)) \ R_{(q-n+1)}^{(i)} \quad \text{(for HM fibers)}$$

$$\overline{P}_n^{*(2)} = R\{\ 1 + \sum_{i=1}^{2q+1}(C_i^{'}\lambda_i (k) \xi^{k-1} \cosh(\lambda_i \xi) + D_i^{'}\lambda_i (k) \xi^{k-1} \sinh(-\lambda_i \xi)) \ R_{(q-n+1)}^{(i)} \ \} \quad \text{(for LM fibers)}$$

(b-2). Region 3

In this region, the expressions for displacements and loads in each fiber may be expressed as:

$$\overline{U}_n^{(3)} = \xi + \sum_{i=1}^{2q+1}(E_i \xi^k R_{(q-n+1)}^{(i)} e^{\lambda_i \xi} + M_i \xi^k R_{(q-n+1)}^{(i)} e^{-\lambda_i \xi}) \tag{3.14}$$

$$\overline{P}_n^{(3)} = 1 + \sum_{i=1}^{2q+1} (E_i \lambda_i k \xi^{k-1} e^{\lambda_i \xi} - M_i \lambda_i k \xi^{k-1} e^{-\lambda_i \xi})\ R_{(q-n+1)}^{(i)}$$

(for HM fibers) (3.15)

$$\overline{P}_n^{(3)^*} = R\{\ 1 + \sum_{i=1}^{2q+1} (E_i \lambda_i k \xi^{k-1} e^{\lambda_i \xi} - M_i \lambda_i k \xi^{k-1} e^{-\lambda_i \xi})\ R_{(q-n+1)}^{(i)}\ \}$$

(for LM fibers) (3.16)

In above equations, k is the number of occurrence of an eigenvalue. The above equations may be re-written as:

$$\overline{U}_n^{(3)} = \xi + \sum_{i=1}^{2q+1} (E_i^{'} R_{(q-n+1)}^{(i)} \xi^k \sinh(\lambda_i \xi) + M_i^{'} R_{(q-n+1)}^{(i)} \xi^k \cosh(-\lambda_i \xi))$$

$$\overline{P}_n^{(3)} = 1 + \sum_{i=1}^{2q+1} (E_i^{'} \lambda_i (k) \xi^{k-1} \cosh(\lambda_i \xi) + M_i^{'} \lambda_i (k) \xi^{k-1} \sinh(-\lambda_i \xi))\ R_{(q-n+1)}^{(i)}$$

(for HM fibers)

$$\overline{P}_n^{*(3)} = R\{\ 1 + \sum_{i=1}^{2q+1} (E_i^{'} \lambda_i (k) \xi^{k-1} \cosh(\lambda_i \xi) + M_i^{'} \lambda_i (k) \xi^{k-1} \sinh(-\lambda_i \xi))\ R_{(q-n+1)}^{(i)}\ \}$$

(for LM fibers)

In equations (3.11) to (3.16), C_i, D_i, E_i, and M_i are constants yet to be defined from boundary conditions and continuity equations.

4. Boundary Conditions and Continuity Equations

Upon the application of following boundary conditions and continuity equations, one may solve for the constants introduced in displacement fields of regions 1 to 4.

$$U_n^{(1)}(0) = 0 \qquad -2q - 1 \le n \le 2q + 1 \qquad (4.1)$$

$$U_n^{(1)^*}(0) = 0 \qquad -2q - 1 \le n \le 2q + 1 \qquad (4.2)$$

either $\overline{U}_n^{*(3)}(\xi_o + \delta) = U_n^{*(4)}(\xi_o + \delta)$ $\quad -2q - 1 \le n \le 2q + 1$

$$\overline{P}_n^{*(3)}(\xi_o + \delta) = P_n^{*(4)}(\xi_o + \delta) \qquad -2q - 1 \le n \le 2q + 1 \qquad (4.3)$$

or $\overline{U}_n^{(3)}(\xi_o + \delta) = U_n^{(4)}(\xi_o + \delta)$ $\quad -2q - 1 \le n \le 2q + 1$

$$\overline{P}_n^{(3)}(\xi_o + \delta) = P_n^{(4)}(\xi_o + \delta) \qquad -2q - 1 \le n \le 2q + 1 \qquad (4.4)$$

$$\overline{U}_n^{(2)}(\xi_o) = \overline{U}_n^{(3)}(\xi_o) \qquad \begin{matrix} (r+1)/2 \prec n \le 2q+1 \\ -2q - 1 \le n \le -(r+1)/2 \end{matrix} \qquad (4.5)$$

$$\overline{U}_n^{*(2)}(\xi_o) = \overline{U}_n^{*(3)}(\xi_o) \qquad \begin{matrix} (r+1)/2 \prec n \le 2q+1 \\ -2q - 1 \le n \le -(r+1)/2 \end{matrix} \qquad (4.6)$$

$$\overline{P}_n^{(2)}(\xi_o) = \overline{P}_n^{(3)}(\xi_o) \qquad \begin{matrix} (r+1)/2 \prec n \le 2q+1 \\ -2q - 1 \le n \le -(r+1)/2 \end{matrix} \qquad (4.7)$$

$$\overline{P}_n^{(2)}(\xi_o) = \overline{P}_n^{(3)}(\xi_o) = 0 \qquad -(r-1)/2 \le n \le (r-1)/2 \qquad (4.8)$$

$$\overline{P}_n^{*(2)}(\xi_o) = \overline{P}_n^{*(3)}(\xi_o) \qquad \begin{matrix} (r+1)/2 \prec n \le 2q+1 \\ -2q - 1 \le n \le -(r+1)/2 \end{matrix} \qquad (4.9)$$

$$\overline{P}_n^{*(2)}(\xi_o) = \overline{P}_n^{*(2)}(\xi_o) = 0 \qquad -(r-1)/2 \le n \le (r-1)/2 \qquad (4.10)$$

Applying above boundary conditions and continuity equations one can solve for 3n unknowns present in equilibrium equations.

5. Results and Discussion

In order to investigate the effect of bond failure on stress distribution in a finite width regular composite lamina, it was first assumed that all fibers are of the same type (R=1), and each damaged filament bears two successive cuts along its length such that the portion of fiber caught in between forms a short fiber (see Figure 1). Figure 5 shows the effect of bond failure length on stress concentration in the first intact filament bonding the crack tip. The results of an infinite sheet with no bond failure are superimposed for further comparison.

As realized, for ξ_o=2, an increase in length of the debonded region reduces the peak normal stress concentration in the filament. The reduction appears to be more pronounced for larger cracks (higher values of "r"). According to Figure 6, for any specific value of crack size (here r = 3), the magnitude of stress concentration is barely a function of $2\xi_o$(the distance between any two successive cuts). An increase in ξ_o would result in higher values of K_r, maximum of which happens to be in the first intact filament bonding the crack tip. As δincreases from zero to 0.4, the percentage decrease in K_r appears to be almost the same ($\approx 12\%$) for all values of ξ_o. The effect of hybridization on peak stress concentration is studied in Figure 7 for R = 0.33. This value simulates the boron-graphite epoxy hybrid composite. Two values of $\delta = 0.1$ and $\delta = 0.3$ are selected for the half length of debonded region. For further comparison, the results of an infinite sheet with R=1 and no bond failure are superimposed.

The first intact filament at the crack tip happens to be a LM fiber. According to this figure, hybridization causes LM fibers to experience higher values of stresses compared to a case where all fibers are of the same type. The initiation of a bond failure at the fiber-matrix interface causes a reduction in K_r. The effect is more pronounced for higher values of crack size (larger number of broken fibers). As δ increases from zero (no

bond failure) to 0.3, the magnitude of stress concentration reduces from 2.94 to 2.25 (at r = 5).
This corresponds to a reduction of 23.5% in K_r. Similar study on HM fibers is shown in Figure 8. According to this figure, a HM fiber experiences smaller value of stresses, once the lamina is hybridized. As a bond failure occurs and grows in size, the values of K_r are further decreased. This reduction is about 9% at r = 7 (values of K_r is reduced from 2 to 1.88). Hence, from the results in Figures 7 and 8, one can conclude that the effect of bond failure seems to be more pronounced on LM fiber stresses. Figure 9 compares the results of K_r for three different cases indicated on the figure. The values of stress concentration for the yielded matrix were deduced from reference [18] wherein it was assumed that the yielded zone has the same size as the debonded region and the matrix bay at the crack tip has kept its grip with its neighboring fibers.
According to this figure, a yielded matrix causes a higher stress reduction in intact filaments compared to a debonded region of the same size. For example, at r = 7, a yielded zone size of 0.4 forces the values of K_r to be lowered from 2.45 (for no bond failure) to 1.89, while, if the fiber-matrix interface at the crack tip is debonded up to the same size, this value is only reduced to 2.1. The effect of δ on peak normal loads in the first short fiber adjacent to the crack tip is shown in Figure 10. According to the results, for values of $\delta << \xi_o$the effect of the debonded region on $(P_s)_{max}$ becomes negligible.
According to this figure, for ξ_o=2, the growth of δ from zero (no bond failure) to 0.5, causes a 28.8% in $(P_s)_{max}$ while this decrease is about 3% at $\xi_o = 7$.
As the lamina is hybridized, the LM short fibers experience more loads (compared to a single type composite lamina) as "R" is reduced from 1 to 0.33. This is shown in Figure 11 for $\delta = 0.3$. Similar analysis on peak shear stress in the matrix bay bonding the crack tip is performed and shown in Figures 12 and 13. According to Figure 12, an increase in δ, lowers the value of peak shear stress. This effect seems to be larger for smaller values of ξ_o. For further comparison, the results of no bond failure are superimposed. According to Figure 13, assuming a HM fiber at the crack tip, an increase in "R", lowers the value of peak shear stress in the matrix. The reduction appears to be 25.8% for $\delta = 0.3$ at ξ_o=2.7, as R is reduced from 1 to 0.33.

6. Conclusions

This paper presents the effect of a bond failure between a matrix and its neighboring fibers, and its extent, on peak stress concentration, load bearing capability and peak shear stress in a regular and hybrid composite lamina. According to the results, the growth of a bond failure noticeably lowers the magnitude of stress concentration in the fibers. This effect seems to be the same for all values of ξ_o.

Hybridization effect causes LM fibers to experience more stress, while the presence of a bond failure lowers this effect considerably. As the lamina is hybridized, HM fibers undergo lower stress values while initiation and growth of a bond failure lower these values even more. The effect of stress reduction as a result of a bond failure appears to be less in HM fibers. According to the results, a fiber-matrix bond failure lowers the values of stress concentrations less compared to a case where the matrix has yielded to the same extent as the size of the debonded zone, but has kept its grip with its neighboring fibers. For values of δ much smaller than ξ_o, the growth of the debonded region has a negligible effect on load bearing capability of short fibers. Hybridization does not seem to alter this behavior much. The growth of a debonded interface also affects the peak shear stress produced in the matrix. The trend appears to be similar to that described for load bearing capability of short fibers.

Nomenclature

A_f :	Cross sectional areas of HM fibers.
A_f^* :	Cross sectional areas of LM fibers.
d:	Fiber diameter, fiber spacing.
E_f :	Elastic Modulus of HM fibers.
E_f^*	Elastic Modulus of LM fibers.
G:	Shear modulus of the matrix.
h:	Thickness of the lamina.
K_r :	Stress concentration factor in each fiber $\left(K_r = \frac{p_n}{p}\right)$.
m:	Volume element containing one HM and one LM fiber.
n:	Filament number.
N:	Total number of fibers (N=2q+1).
p:	Normal Load applied to the lamina at infinity.
p_n :	Local normal load in each fiber
P:	Non-dimensional load in HM Fibers.
P^*:	Non dimensional load in LM Fibers.
P_s :	Non-dimensional load in short Fibers.
R :	Extensional stiffness ratio of LM fibers to HM fibers
r:	Total number of broken fibers.
(S_{xy}):	Non-dimensional shear stress in each matrix bay.
$u^{(i)}$:	Displacement of fibers in regions one and two (i = 1,2).
$\overline{u}^{(j)}$:	Displacement of fibers in regions two and three (j = 2,3).
$U^{(i)}$:	Non-dimensional displacement of fibers in regions one and two (i =1,2).
$\overline{U}^{(j)}$:	Non-dimensional displacement of fibers in regions two and three (j = 2,3).
x,y :	Coordinate system centered in the middle of the lamina.
2δ :	Non dimensional size of the debonded region.
λ_i :	Eigenvalue.
ξ :	Non dimensional coordinate along each filament.
$2\xi_o$:	Total length of each short fiber.

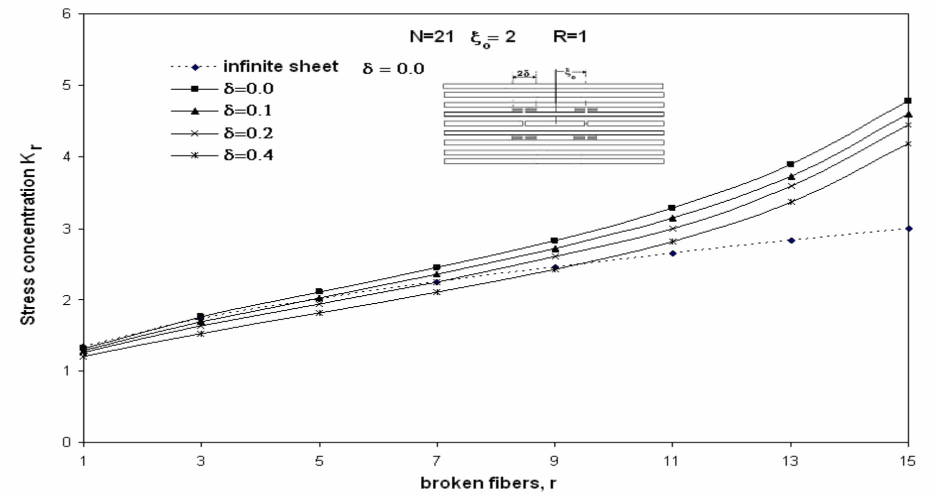

Fig 5. Effect of bond failure extent on stress concentration factors for various numbers of broken fibers

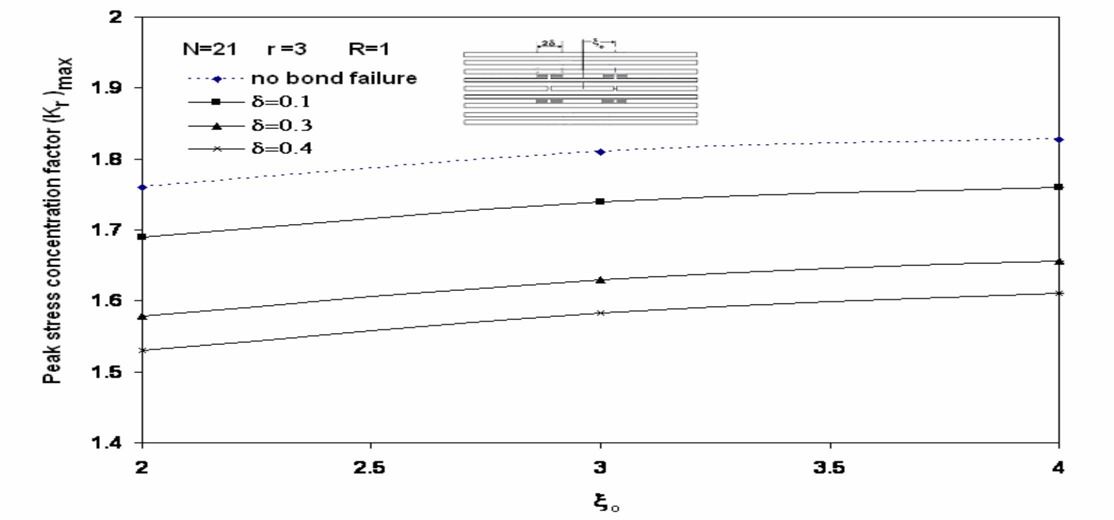

Fig 6. Effect of short fiber length on peak stress concentration produced in the lamina.

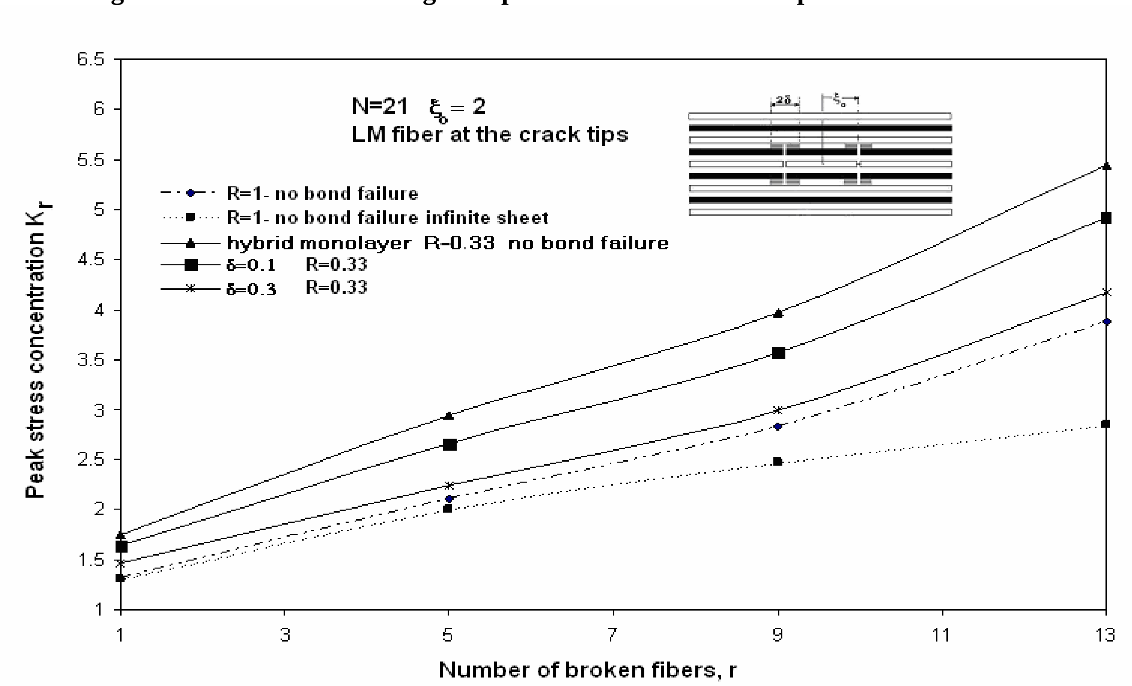

Fig 7. The effect of bond failure extent on stress concentration factor in LM fibers.

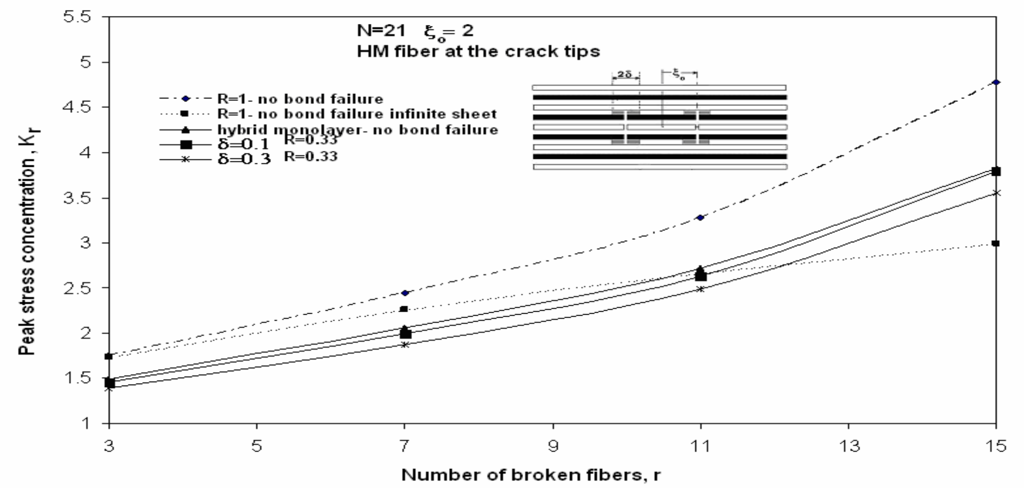

Fig 8. The effect of bond failure extent on stress concentration factor in LM fibers.

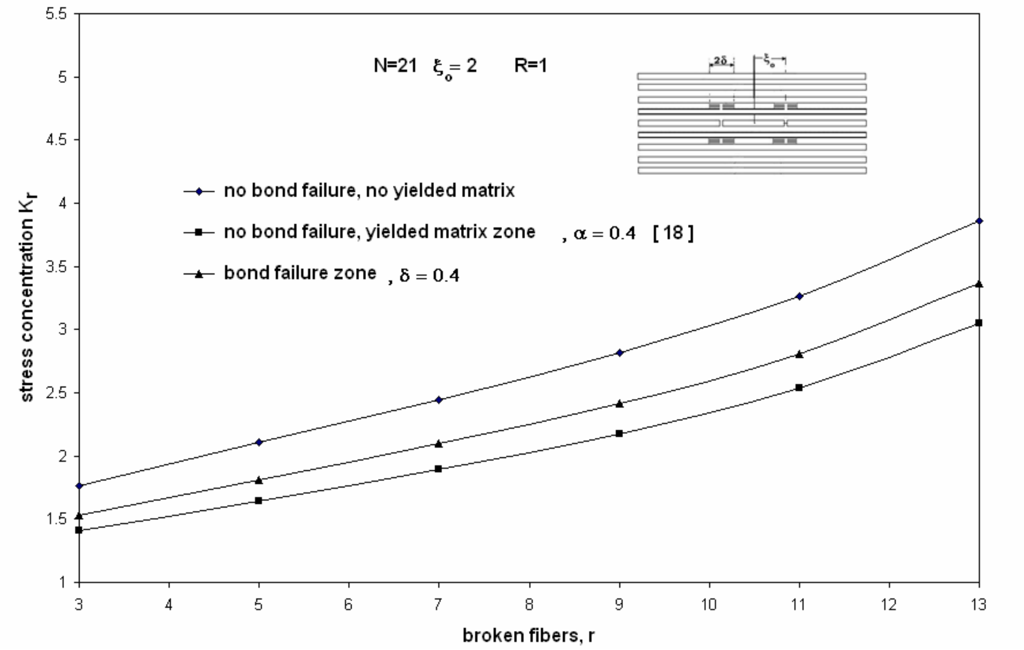

Fig 9. Comparison of stress concentration reduction due to yielded zone and debonded region in the matrix.

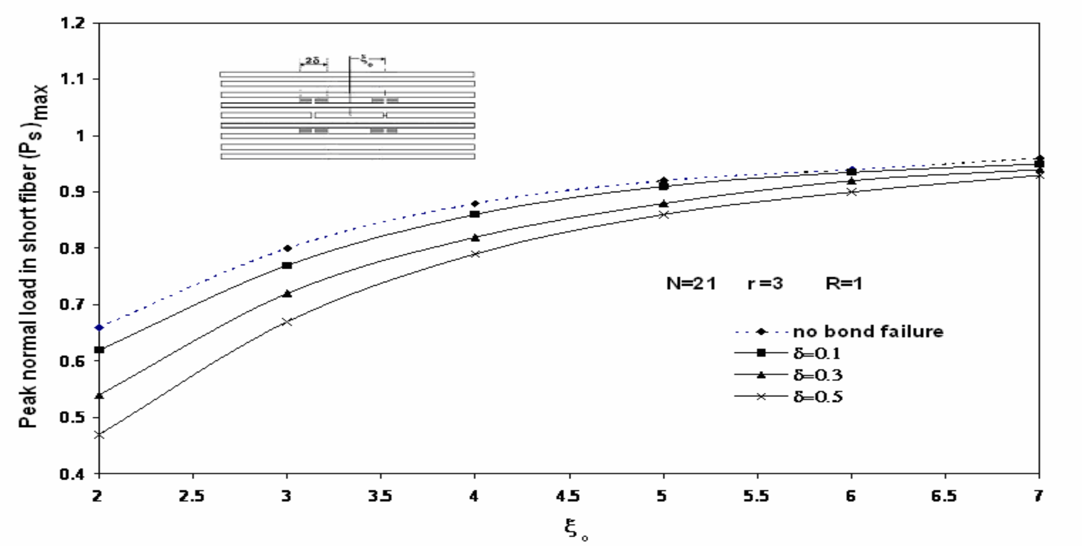

Fig 10. Variation of peak normal load in short fibers.

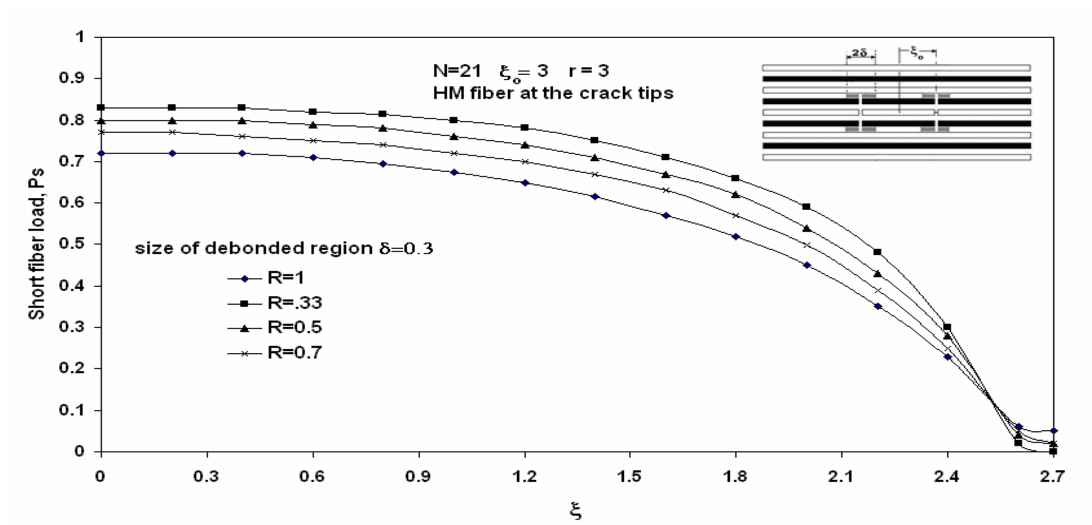

Fig 11. The effect of "R" on short fiber load distribution in presence of a demoded matrix zone of 0.3.

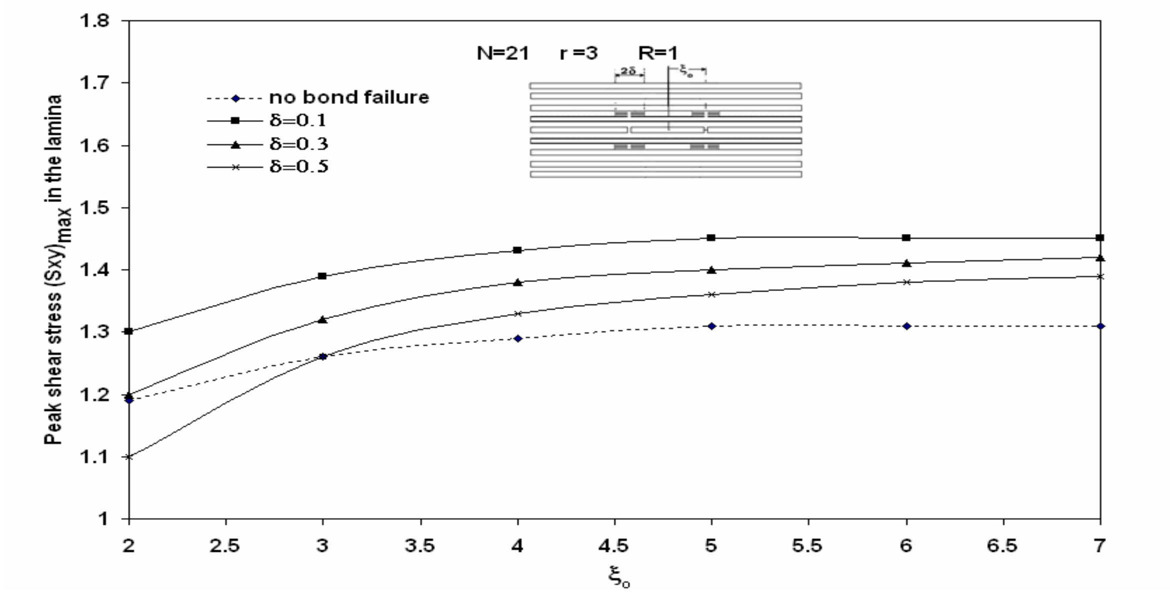

Fig 12. Variation of peak shear stress in the matrix as a function of short fiber length.

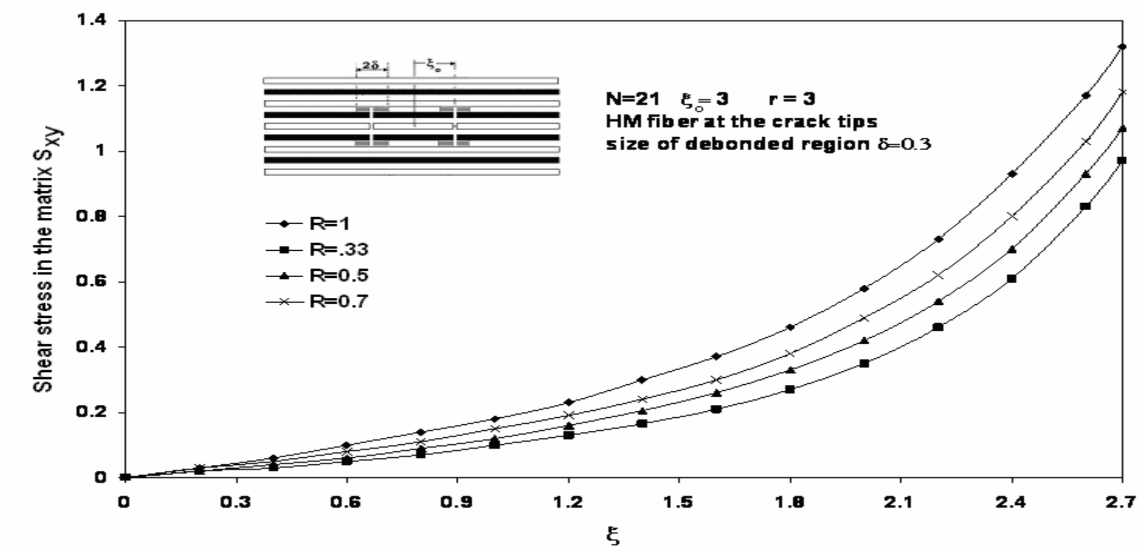

Fig 13. The effect of "R" on shear stress distribution in the matrix bay the crack tip with a debonded region of size 0.3.

References

[1] Hedgepeth, J.M., "*Stress concentrations in filamentary structures*", NASA-TND 882, May 1961.

[2] Reedy, E.D., *"Fiber stresses in cracked monolayer: comparison of shear-lag and 3-D finite element predictions*", Journal of Composite Materials, Vol. 18, 1984.

[3] Gao, X.L., &. Li, K., *"Shear-lag model for carbon nan tube-reinforced polymer composites'*, International Journal of Solids and Structures, Vol. 42, Issues 5-6, 2005.

[4] Xia, Z., Okabe, T., & Curtin W.A., "*Shear-lag versus finite element models for stress transfer in fiber-reinforced composites*", Composite Science and Technology, 62, 2002.

[5] Sirivedin, S., Fenner, D.N., Nath, R.B., & Galiotis, C., *"Effects of inter-fiber spacing and matrix cracks on stress amplification factors in carbon-fiber/epoxy matrix composites. Part I: planar array of fibers*", Journal of Composites, Part A: Applied Science and Manufacturing, Vol. 34, 2003.

[6] Bond, Ian., Hucker, M., Weaver, P., Bleay, St., & Haq, S., *"Mechanical behavior of circular and triangular glass fibers and their composites*", Journal of Composites Science and Technology, Vol. 62, 2002.

[7] Fukuda, H., and Chou, T.W., *"Stiffness and strength of hybrid composites",* Proc. Japan -US Conference, Tokyo, 1980.

[8] Fukuda, H., & Chou, T.W., *"Stress concentration in hybrid composite sheet*", Journal of Applied Mechanics, Vol. 50, Dec 1983.

[9] Dhararani, L.R., & Goree, J.G., *"Analysis of a hybrid uni-directional laminate with damage*", Mechanics of Composite Materials, Recent Advances, Pergamon Press, 1982.

[10] Dlouhy, I., Chlup, Z., Boccaccini, D.N., Atiq, S., & Boccaccini, A.R., *"Fracture behavior of hybrid glass matrix composites: thermal ageing effects*", Journal of Composites, Part A: Applied Science and Manufacturing, Vol. 34, 2003.

[11] Ding, X.D., Jiang, Z.H., Sun, J., Lian, J.s., & Xiao, L., "*Stress-strain behavior in initial yield stage of short fiber reinforced metal matrix composite*", Journal of Composites Science and Technology, Vol. 62, 2002.

[12] Fukuda, H., & Kawata, K., *"Stress and strain fields in short fiber reinforced composites*", Fiber Science and Technology, Vol. 7, 1974.

[13] Fu, S.Y., Lauke, B., Mäder, E., Yue, C.Y., & Hu, X., *"Tensile properties of short-glass-fiber and short-carbon- fiber reinforced polypropylene composites*", Journal of Composites, Part A: Applied Science and Manufacturing, Vol. 31, 2000.

[14] Shishesaz, M., *"The effect of mechanical and physical properties of polymers on stress amplification factor in composites with an internal damage*", Iranian Polymer Journal, Vol. 14, No. 5, Issue 59, 2005.

[15] Koss, D.A., Petrich, R.R., Kallas, M.N. & Hellmann, J.R. *"Interfacial shear and matrix plasticity during fiber push-out in a metal-matrix composite",* Composites Science and Technology, Vol. 51, Issue 1, 1994.

[16] Kang, G., & Gao, Q., "*Tensile properties of randomly oriented short* δ-Al_2O_3 *fiber reinforced aluminum alloy composites: II Finite element analysis for stress transfer, elastic modules and stress-strain curve",* Journal of Composites, part A 33, 2002.

[17] Miserez, A., Rossoll, A., & Mortensen, A., *"Investigation of crack-tip plasticity in high vol. fraction particulate metal matrix composites*", Engineering Fracture Mechanics, In Press, 2004.

[18] Shishesaz, M., *"The effect of matrix plasticity and duplicate cuts on stress distribution in short and long fibers of a hybrid composite lamina (Perfect bond model)",* Iranian journal of Science and Technology, Transaction B, Engineering Vol. 31, 2007, PP. 81-94.

[19] Owen, D.R.J., & Lyness, J.F., *"Investigation of bond failure in fiber-reinforced materials by the finite element method*", Fiber Science and Technology, Vol. 5, Issue 2, April 1972.

[20] Hoecker, F., Friedrich, K., Blumberg, H., & Karger-Kocsis, J., *"Effects of fiber/matrix adhesion on off-axis mechanical response in carbon-fiber/epoxy resin composites*", Composites Science and Technology Vol. 54, Issue 3, 1995.

[21] Zhandarov, S.F., & Pisanova, E.V., *"The local bond strength and its determination by fragmentation and pull-out tests*", Composites Science and Technology, Vol. 57, Issue 8, 1997.

[22] Harwell, M.G., Hirt, D.E., Edie, D.D., Popovska, N. & Emig, G., *"Investigation of bond strength and failure mode between SiC-coated mesophase ribbon fiber and an epoxy matrix",* Carbon, Vol. 38, Issue 8, 2000.

[23] Lin, G., Geubelle, P.H., & Sottos, N.R., *"Simulation of fiber decoding with friction in a model composite push out test*", International Journal of Solids and Structures, Vol. 38, Issues 46-47, November 2001.

[24] Vlasveld, D.P.N., Parlevliet, P.P., Bersee, H.E.N., & Picken, S.J., *"Fiber – matrix adhesion in glass-fiber reinforced polyamide - 6 silicate Nan composites*", Composites Part A: Applied Science and Manufacturing, Vol. 36, Issue 1, 2005.

[25] Zhandarov, S., & Mäder, E., *"Characterization of fiber/matrix interface strength: applicability of different tests, approaches and parameters"*, Composites Science and Technology, Vol., 65, Issue 1, 2005.