International Journal of Industrial Engineering & Production Research September 2024 Vol. 35, No. 3: 1-20
DOI: 10.22068/ijiepr.35.3.1706

RESEARCH PAPER

Joint Pricing and Inventory Control for Non-Instantaneous Deteriorating Items When Trade Credit is Linked to Order Quantity

Hamed Salehi Mourkani*[1], Anwar Mahmoodi[2] & Isa Nakhai Kamalabadi[3]

Received 28 January 2023; Revised 7 July 2024; Accepted 18 august 2024;

ABSTRACT
In today's highly competitive business market, the issues of pricing and inventory control are very important, and they should be jointly optimized, especially in the case of deteriorating products. In addition, some strategies should be considered that increase the profit of the supplier and the retailer simultaneously. Trade credit based on order quantity is a strategy by which the supplier can encourage the retailer to order larger quantities. On the other hand, the retailer can significantly increase his profit by taking advantage of more trade credit periods. This research investigates the problem of joint inventory control and pricing for non-instantaneous deteriorating products; while, the quantity dependent trade credit is allowed. It is observed here that the buyer order amount is equal or more than the amount specified by the seller. The Shortage is not permitted in the system. It is aimed in present study to find a procedure for achieving the optimal selling price and replenish cycle and to be able to maximize the system's profit. To do so, first, the system's total profit function is derived. Then, the uniqueness of the optimal replenishment cycle for a given price is proved. Next, the concavity of the total profit function concerning the price is revealed, depending on the trade credit policy. Thereafter, an algorithm is provided to fulfill the optimal solution and eventually a dual-purpose numerical analysis was carried out both to show the model performance and to evaluate the sensitivity of the main parameters.

KEYWORDS: *Pricing; Inventory control; Quantity dependent trade credit; Non-instantaneous deteriorating.*

1. Introduction

Inventory control and pricing are two critical decisions to be reached in each organization. In recent years, researchers have considered these two items together for deteriorating products. The products that dry, evaporate, diminish, or become out of date over the time are determined as deteriorating products. Therefore, the products such as drugs, blood, evaporating liquids, and electrical appliances are some examples of deteriorating products.

Instantaneous deteriorating items rot away upon (or instantly after) entering the retailer inventory system such as Gasoline; while, the items that do not instantly deteriorate upon entering the retailer inventory system are called non-instantaneous deteriorating products (NIDP). A "Period with no deterioration" is a duration considered for these products. In other words, within this period, the products preserve their original quality, and afterwards, the deterioration starts. Fruits, vegetables are among such products.

The managers and suppliers apply various encouragement techniques in order to convince customers for purchasing their products. One of these techniques is a specific time interval that is offered for the delay payment; thus, the buyer is not required to pay instantly. This period is called trade credit. The quantity dependent trade credit (QDTC) is provided by the seller or supplier when the buyer orders an amount equal to or greater than a specific quantity identified by the seller or supplier so that the customers would enjoy the advantages of this trade credit.

Regarding products that are deteriorated non-instantaneously, Wu et al. [1] studied an optimum replenishment policy of NIDP with demand that is stock-dependent and shortage that backlogged

* Corresponding author: Hamed Salehi Mourkani
ha.salehi993@gmail.com

1. Department of Industrial Engineering, Faculty of Engineering, University of Kurdistan, Sanandaj, Iran.
2. Department of Industrial Engineering, Faculty of Engineering, University of Kurdistan, Sanandaj, Iran.
3. Department of Industrial Engineering, Faculty of Engineering, University of Kurdistan, Sanandaj, Iran.

partially. Also, Yan et al. [2] considered optimum ordering policies of the retailer and pricing for NIDP with demand that is price-dependent and shortage that backlogged partially. An inventory model of NIDP with permissible delay in payments (PDP) presented by Ouyang et al. [3]. Geetha and Uthayakumar [4] surveyed economic design for an inventory policy of NIDP under PDP. Maihami and Nakhai [5] presumed dynamic pricing and ordering policy of NIDP. In their model, the shortage is allowable and backlogged partially when the rate of backlogging is varied and is dependent on the next replenishment's waiting time. Maihami et al. [6] presented pricing and inventory planning of NIDP. They considered a greening investment in the beef industry in their study. Rezagholifam et al. [7] studied optimal pricing and ordering strategy for NIDP. In their model, the demand was price and stock sensitive, and the capacity was constrained. Sundararajan et al. [8] devised an inventory system for NIDP supposing backlogging and time discounting. Chakraborty et al. [9] developed Multi-warehouse partial backlogging inventory system considering inflation for NIDP under an imprecise environment. He et al. [10] presented preservation, replenishment and pricing technology investment decisions for NIDP. Barman et al. [11] presumed an analysis of optimal inventory scheduling policy and pricing strategy for NIDP. Their model was considered for a two-layer supply chain. Tashakkor et al. [12] devised joint optimization of replenishment cycle and dynamic pricing of NIDP. They considered stock-dependent demand and variable non-instantaneous deterioration in their study. Khan et al. [13] presumed inventory management with hybrid cash-advance payment for non-instantaneous deterioration, time-varying holding cost and time-dependent demand under non-terminating situations and backordering. Mahdavisharif et al. [14] developed pricing and inventory policy for NIDP in vendor-managed inventory systems. They considered a Stackelberg game theory approach in their model. Almathkour and Benkherouf [15] presented optimal policies for a finite horizon model with backlogging, non-instantaneous deterioration and time-varying demand rate. Udayakumar et al. [16] developed an EOQ model for NIDP. In their model, the demand was time-dependent and backlogged partially. Tavassoli et al. [17] studied a lot-sizing model for NIDP. They proposed non-linear partial backlogging and advance payment in their model. Chandramohan et al. [18] surveyed comprehensive inventory management system of NIDP. They considered the model in supplier-retailer-customer supply chains. Priyamvada et al. [19] examined optimal inventory strategies of deteriorating products under price-sensitive investment in preservation technology.

Several researchers have considered the trade credit technique in their models. Shinn and Park [20] addressed the problem of joint inventory control and Pricing with the payment delay, and added the shipment discount to their models. Hwang and Shinn [21] considered the joint inventory control pricing and of deteriorating products by exponential deterioration and PDP. Maihami and Nakhai [22] studied the joint inventory control and pricing for NIDP under partial backlogging and PDP. In another research project, they also surveyed the joint inventory control and pricing for NIDP under partial backlogging and PDP and demand depending on the time and price [23]. Aliabadi et al. [24] survived an inventory model for NIDP with credit period and carbon emission sensitive demand considering a signomial geometric programming approach. Udayakumar et al. [25] studied economic ordering policy for NIDP. Their model was under permissible delay in payment under inflation and the demand was advertisement and price dependent. Das et al. [26] survived an inventory model for NIDP. This model was under preservation technology and multiple credit periods-based trade credit financing via particle swarm optimization. Rapolu and Kandpal [27] proposed joint advertisement, pricing, inventory policies and preservation technology investment for NIDP under trade credit. However, none of mentioned researchers has focused on the QDTC in his/her model. Tripathy et al. [28] studied EOQ inventory model of NIDP and added constant demand under progressive financial trade credit facility to the model. Choudhury and Mahata [29] considered inventory models of NIDP with fixed lifetime products under trade credit and hybrid partial prepayment. Viswanath and Thilagavathi [30] proposed two warehouse inventory dynamism of NIDP under trade credit policy and Weibull demand. Singh and Goel [31] assumed warehouse inventory model of perishable goods under trade credit policy and hybrid demand. Momena et al. [32] developed two-storage inventory model under quantity discounts. They added trade credit and time-varying holding cost to their model. Mashud et al. [33] supposed an inventory model for NIDP. They considered the model under preservation technology, the joined effect of trade-credit, and advertisement policy. Mondal et al. [34] devised two-warehouse inventory model of deteriorating products with trade credit policies and partially backlogged demand.

With respect to the QDTC, Chang et al. [35] devised an EOQ model for deteriorating products under the QDTC. Chung et al. [36] proposed an optimal inventory policy in the presence of the QDTC. Chang et al. [37] developed the optimum ordering policy and pricing of an integrated inventory model in the presence of QDTC. Chen et al. [38] studied economic order quantity of the Retailer in a situation where the supplier conditionally offers the QDTC. Chung et al. [39] presented a model of inventory with receipt that is non-instantaneous and products that deteriorate exponentially for an integrated three-tier supply chain system with two levels of trade credit. Ouyang et al. [40] proposed an optimized approach to the joint problem of ordering and pricing in an integrated inventory system with QDTC. Yang et al. [41] studied the retailer's credit policies and optimal order amount when a supplier presents either a delay payment or cash discount based on the order quantity. Shah et al. [42] represented the optimum ordering policy and pricing for an integrated model of inventory with demand that is quadratic in the presence of QDTC. Shah and Barron [43] developed retailer's decisions model for credit policies and ordering of deteriorating products when a supplier presents a cash discount or QDTC. Chang et al. [44] proposed optimal ordering policies and pricing for NIDP under QDTC. Shah et al. [45] developed an inventory model for stock-price-advertisement dependent probabilistic demand and trade credit financing. Their inventory model is a non-instantaneous controlled deteriorating model. Mahato et al. [46] assumed an inventory models for NIDP under carbon tax policy. They considered a bi-level trade credit policy under pricing and preservation technology in their model. Tiwari et al. [47] supposed retailer's inventory decisions and credit for deteriorating and imperfect quality items under two-level of trade credit. Soni and Chauhan [48] considered joint preservation, promotion, and inventory decisions for deteriorating items with stochastic demand and maximum lifetime under two-level partial trade credit. Das et al. [49] surveyed a dual-channel supply chain inventory model under two-warehouse setting. They added partial backordering, order volume-linked trade credit, and all-units discount to the model.

Upon reviewing the joint inventory control and pricing literature of NIDP and taking all the applied techniques into account, it was inferred that the trade credit technique related to the quantity in pricing and joint inventory control of NIDP have not been considered. Of course, researchers have added this technique to some models, but it has not been assumed in the issue of joint pricing and inventory control for NIDP. Applying this strategy, the supplier can encourage the retailer to order larger quantities. On the other hand, the retailer can significantly increase his/her profit by taking advantage of more trade credit periods. In the present study a pricing model and joint inventory control of NIDP is proposed under the QDTC technique.

The rest of this study includes the following sections. In section 2, the model is described and the assumptions and notations are presented. In section 3, the mathematical formulation for the model is provided. In section 4, the algorithm used to solve the problem will be discussed. In section 5, a numerical study is presented to show the efficiency of proposed model as well as the sensitivity analyses of the main parameter. Finally, in section 6, a summary and suggestions will be presented for future works.

2. Model Description and Notation

In the present paper, we study problem of joint inventory control and pricing of NIDP under QDTC for a retailer. The retailer will receive trade credit of M, if the order be equal to or higher than. In this problem, order function is $D(p,t)=(\alpha-\beta p)e^{\lambda t}$. This is a linear, continuous and descending relative to price. It is descending or ascending relative to time based on $\lambda<0$ or $\lambda>0$. Products which deteriorated during the time period are not fixed or replaced. Shortage is not allowed in the system. Lead time is considered zero, and rate of replenishment is considered infinite.

General Notations:

α: a parameter of demand function which shows the potential market size

β: a parameter of demand function which shows the price sensitivity

c: The purchase cost per unit

h: The cost of holding per unit per unit time excluding the cost of capital

s: The cost of backorder per unit per unit time

o: The cost of lost sales per unit

p: The price of selling per unit, where $p>c$

θ: The rate of deterioration

t_d : The length of time in which the product exhibits no deterioration

T: The duration of the replenishment cycle ($T>t_d$)

Q: The amount of order

p^*: The optimum price of selling per unit
T^*: The optimum length of the replenishment cycle time
Q^*: The optimum amount of order
$I_1(t)$: The level of inventory at time $t \in [0, t_d]$
$I_2(t)$: The level of inventory at time $t \in [t_d, T]$
I_0: The maximum level of inventory
I_e: The earned interest per dollar
I_p: The charged interest per dollar
M: The period of trade credit
Q_L : The minimum ordered quantity to be qualified to get trade credit
t_L: The length of the time in which quantity of Q_L will be depleted
$TP(p,T)$: The inventory system total profit per unit time
TP^*: The inventory system optimal total profit per unit time, that is $TP^* = TP(p^*, T^*)$

3. The Model Formulation

Upon delivering ordered amount at the beginning of each cycle, the level of inventory is I_0. During $[0, t_d]$ the inventory is reduced only due to fulfilling the demands. Then in $[t_d, T]$ inventory level is reduced due to both deterioration and demand. As you can see, figure 1 shows the inventory behavior of the system.

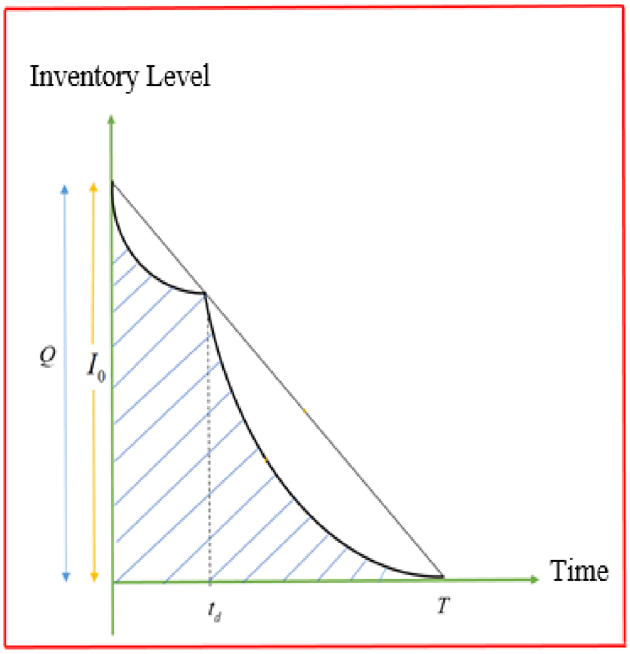

Fig. 1. Inventory level in a typical replenishment cycle

Therefore, during $[0, t_d]$ the stock level is derived from the following differential equation:

$$\frac{dI_1(t)}{dt} = -D(p,t) = -(\alpha - \beta p)e^{\lambda t}, \qquad 0 \le t \le t_d \qquad (1)$$

With the initial condition of $I_1(0) = I_0$, the equation could be solved as follows.

$$I_1(t) = \frac{(\alpha - \beta p)}{\lambda}\left[1 - e^{\lambda t}\right] + I_0, \qquad 0 \le t \le t_d \qquad (2)$$

Furthermore, during $[t_d, T]$ period, level of inventory is reduced due to both deterioration and demand. Subsequently the following differential equation shows the inventory behavior at this time interval.

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D(p,t), \qquad t_d \le t \le T \qquad (3)$$

As $I_2(T) = 0$, the level of inventory at period of $[t_d, T]$ would be as Equation (4).

$$I_2(t) = \frac{(\alpha - \beta p)e^{-\theta t}}{\lambda + \theta}\left[e^{(\lambda+\theta)T} - e^{(\lambda+\theta)t}\right], \qquad t_d \le t \le T \qquad (4)$$

As it can be seen in Figure 1, $I_1(t_d) = I_2(t_d)$. Consequently, the maximum level of inventory can be obtained as follows:

International Journal of Industrial Engineering & Production Research September 2024 Vol. 35, No. 3: 1-20
DOI: 10.22068/ijiepr.35.3.1706

$$I_0 = \frac{(\alpha - \beta p)e^{-\theta t_d}}{\lambda + \theta}\left[e^{(\lambda+\theta)\mathrm{T}} - e^{(\lambda+\theta)t_d}\right] - \frac{(\alpha - \beta p)}{\lambda}\left[1 - e^{\lambda t_d}\right] \quad (5)$$

By substituting Equation (5) into Equation (2), we have:

$$I_1(t) = \frac{(\alpha - \beta p)}{\lambda}\left[1 - e^{\lambda t}\right] + \frac{(\alpha - \beta p)e^{-\theta t_d}}{\lambda + \theta}\left[e^{(\lambda+\theta)\mathrm{T}} - e^{(\lambda+\theta)t_d}\right] - \frac{(\alpha - \beta p)}{\lambda}\left[1 - e^{\lambda t_d}\right], \qquad 0 \le t \le t_d \quad (6)$$

The total order quantity equals to the maximum of inventory level, that is

$$Q = I_0 = \frac{(\alpha - \beta p)e^{-\theta t_d}}{\lambda + \theta}\left[e^{(\lambda+\theta)\mathrm{T}} - e^{(\lambda+\theta)t_d}\right] - \frac{(\alpha - \beta p)}{\lambda}\left[1 - e^{\lambda t_d}\right] \quad (7)$$

After obtaining inventory level, the system's costs and revenues could be calculated. The costs include Inventory holding cost, fixed ordering cost, purchasing cost, interest payables, and the revenues include selling revenue and interest earned. The formulation of these costs and revenues is provided at the following to derive the profit function.

A: fixed ordering cost in a replenishment cycle

Let *HQ* denotes the cost of inventory holding per replenishment cycle. This cost is obtained by multiplying the cost of holding per unit per time unit to the mean of the system inventory. Therefore,

$$HQ = h\left[\int_0^{t_d} I_1(t)dt + \int_{t_d}^{T} I_2(t)dt\right] = h\left[\frac{(\alpha - \beta p)\left(e^{t_d\lambda}\theta + e^{-t_d\theta + T(\theta+\lambda)}\lambda - e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)} + \frac{e^{-t_d\theta}(\alpha - \beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta + e^{T(\theta+\lambda)}t_d\lambda - \frac{e^{t_d\theta}\left(-1 + e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\right] \quad (8)$$

Furthermore, Let *PC* denotes the purchasing cost at a replenishment cycle. It is obtained by multiplying the unit cost to the order quantity per cycles.

$$PC = cQ = cI_0 = C\left[\frac{(\alpha - \beta p)e^{-\theta t_d}}{\lambda + \theta}\left[e^{(\lambda+\theta)\mathrm{T}} - e^{(\lambda+\theta)t_d}\right] - \frac{(\alpha - \beta p)}{\lambda}\left[1 - e^{\lambda t_d}\right]\right] \quad (9)$$

Now let *SR* denotes the selling revenue during a replenishment cycle, which equals to multiplication of the unit selling price to the amount of sold items.

$$SR = p\left[\int_0^T D(p,t)dt\right] = \frac{p(\alpha - \beta p)\left(-1 + e^{T\lambda}\right)}{\lambda} \quad (10)$$

3.1. Interest payable

Other elements of profit function are interest payable (denoted by *IP*), and interest earned (denoted by *IE*). We apply the approach applied in Maihami and Kamalabadi [20]. When the retailer pays for the purchased goods, opportunity cost is considered for the paid money. This is due to the fact that if he had bought the goods by credit, he could receive interest for his money until due time. In fact, *IP* is the opportunity cost of the money paid for the inventory. To obtain *IP*, we should consider four cases separately, as follows.

Case 1: $0 < T < T_L$

In the case 1, the retailer's order (*Q*) is less than Q_L ($T < T_L$). Therefore, the retailer should pay instantly after receiving goods. Thus the opportunity cost is

International Journal of Industrial Engineering & Production Research September 2024 Vol. 35, No. 3: 1-20
DOI: 10.22068/ijiepr.35.3.1706

$$IP_1 = CI_p\left(\int_0^{t_d} I_1(t)dt + \int_{t_d}^{T} I_2(t)dt\right) =$$

$$CI_P\left(\begin{array}{l}\dfrac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta + e^{-t_d\theta+T(\theta+\lambda)}\lambda - e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)} + \\ \dfrac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta + e^{T(\theta+\lambda)}t_d\lambda - \dfrac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\end{array}\right)$$

Case 2: $T_L \le T \le M$

In the case 2, the amount of retailer's order (Q) is equal to or greater than Q_L $(T_L \le T)$, and therefore, the credit period M would be provided. Also the credit period is expired after depletion of inventory or replenishment cycle $(T_L \le M)$. Hence, no opportunity cost would be occurred.

$IP_2 = 0$

Case 3: $T_L \le T$ and $M < t_d$

In the case 3, the amount of retailers order (Q) is equal to or greater than Q_L. However, the credit period would be finished before t_d. Consequently, the opportunity cost will be

$$IP_3 = CI_p\left(\int_M^{t_d} I_1(t)dt + \int_{t_d}^{T} I_2(t)dt\right) =$$

$$CI_P\left(\begin{array}{l}\dfrac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta + e^{-t_d\theta+T(\theta+\lambda)}\lambda - e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)} + \\ \left(\dfrac{e^{-t_d\theta}(\alpha-\beta p)}{\lambda^2(\theta+\lambda)}\right) * \\ \left(-e^{T(\theta+\lambda)}(M-t_d)\lambda^2 + e^{t_d\theta+M\lambda}(\theta+\lambda) + e^{t_d(\theta+\lambda)}\left(-\lambda+\theta(-1-M\lambda+t_d\lambda)\right)\right)\end{array}\right)$$

Case 4: $T_L \le T$, $t_d < M \le T$

In the case 4, the amount of retailers order (Q) is equal to or greater than Q_L, and credit period would be finished between t_d and T. Thus the opportunity cost is

$$IP_4 = CI_p\int_M^T I_2(t)dt = \frac{CI_P(\alpha-\beta p)\left(e^{M\lambda}\theta + e^{-M\theta+T(\theta+\lambda)}\lambda - e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}$$

3.2. Interest earned

Another component of profit function is Interest earned. We apply the approach applied in Maihami and Kamalabadi [20]. When the retailer receives trade credit, he can put off the payment to a specific time. In this case, during trade credit period, he can obtain interest for the money earned by selling goods. Received interest is obtain in four cases as follows.

Case 1: $0 < T < T_L$.

In the case 1, the retailer's order (Q) is less than Q_L. Therefore, there would be no credit, and

$IE_1 = 0$

Case 2: $T_L \le T \le M$

In the case 2, the interest earned could be obtained as

$$IE_2 = pI_e\left[\int_0^T D(\mathrm{p},t)\,\mathrm{t}\,dt + (\mathrm{M}-T)\int_0^T D(\mathrm{p},t)dt\right] =$$

$$pI_e\left(\frac{\left(-1+e^{T\lambda}\right)(\alpha-\beta p)(M-T)}{\lambda} + \frac{(\alpha-\beta p)\left(1+e^{T\lambda}(-1+T\lambda)\right)}{\lambda^2}\right)$$

Case 3: $T_L \le T$ and $M < t_d$

$$IE_3 = pI_e \int_0^M D(p,t)\,t\,dt = \frac{pI_e(\alpha-\beta p)\left(1+e^{M\lambda}\left(-1+M\lambda\right)\right)}{\lambda^2}$$

Case 4: $T_L \le T$, $t_d < M \le T$

$$IE_4 = pI_e \int_0^M D(p,t)\,t\,dt = \frac{pI_e(\alpha-\beta p)\left(1+e^{M\lambda}\left(-1+M\lambda\right)\right)}{\lambda^2}$$

Now that all costs and revenues of the inventory system are obtained, the system total profit could be derived as

$$TP_{(p,T)} = \begin{cases} TP_1(p,T) = \dfrac{SR - A - HC - PC - IP_1 + IE_1}{T} & ; \quad 0 < T \le T_L \\ TP_2(p,T) = \dfrac{SR - A - HC - PC - IP_2 + IE_2}{T} & ; \quad T_L < T \le M \\ TP_3(p,T) = \dfrac{SR - A - HC - PC - IP_3 + IE_3}{T} & ; \quad M < t_d \text{ and } T_L \le T \\ TP_4(p,T) = \dfrac{SR - A - HC - PC - IP_4 + IE_4}{T} & ; \quad t_d < M < T \text{ and } T_L \le T \end{cases}$$

Where

$$TP_1(p,T) = \frac{1}{T}\left[\left(\begin{array}{l} \dfrac{p(\alpha-\beta p)\left(-1+e^{T\lambda}\right)}{\lambda} - A \\ -h\left[\dfrac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta + e^{-t_d\theta+T(\theta+\lambda)}\lambda - e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)} + \dfrac{e^{-t_d\theta}(\alpha-\beta p)\left(\begin{array}{l} e^{t_d(\theta+\lambda)}t_d\theta + e^{T(\theta+\lambda)}t_d\lambda - \\ \dfrac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\end{array}\right)}{\lambda(\theta+\lambda)}\right] \\ -c\left[\dfrac{(\alpha-\beta p)e^{-\theta t_d}}{\lambda+\theta}\left[e^{(\lambda+\theta)T} - e^{(\lambda+\theta)t_d}\right] - \dfrac{(\alpha-\beta p)}{\lambda}\left[1-e^{\lambda t_d}\right]\right] \\ -CI_p\left(\dfrac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta + e^{-t_d\theta+T(\theta+\lambda)}\lambda - e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)} + \dfrac{e^{-t_d\theta}(\alpha-\beta p)\left(\begin{array}{l} e^{t_d(\theta+\lambda)}t_d\theta + e^{T(\theta+\lambda)}t_d\lambda - \\ \dfrac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\end{array}\right)}{\lambda(\theta+\lambda)}\right) \end{array}\right)\right] \quad (11)$$

$$TP_2(p,T)=\left[\frac{1}{T}\left(\begin{aligned}&\frac{p(\alpha-\beta p)\left(-1+e^{T\lambda}\right)}{\lambda}-A\\&-h\left[\frac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+\frac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\right]\\&-c\left[\frac{(\alpha-\beta p)e^{-\theta t_d}}{\lambda+\theta}\left[e^{(\lambda+\theta)T}-e^{(\lambda+\theta)t_d}\right]-\frac{(\alpha-\beta p)}{\lambda}\left[1-e^{\lambda t_d}\right]\right]\\&+pI_e\left(\frac{\left(-1+e^{T\lambda}\right)(\alpha-\beta p)(M-T)}{\lambda}+\frac{(\alpha-\beta p)\left(1+e^{T\lambda}(-1+T\lambda)\right)}{\lambda^2}\right)\end{aligned}\right)\right] \tag{12}$$

$$TP_3(p,T)=\left[\frac{1}{T}\left(\begin{aligned}&\frac{p(\alpha-\beta p)\left(-1+e^{T\lambda}\right)}{\lambda}-A\\&-h\left[\frac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+\frac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\right]\\&-c\left[\frac{(\alpha-\beta p)e^{-\theta t_d}}{\lambda+\theta}\left[e^{(\lambda+\theta)T}-e^{(\lambda+\theta)t_d}\right]-\frac{(\alpha-\beta p)}{\lambda}\left[1-e^{\lambda t_d}\right]\right]\\&-CI_P\left(\begin{aligned}&\frac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+\\&\frac{e^{-t_d\theta}(\alpha-\beta p)\left(-e^{T(\theta+\lambda)}(M-t_d)\lambda^2+e^{t_d\theta+M\lambda}(\theta+\lambda)+e^{t_d(\theta+\lambda)}\left(-\lambda+\theta(-1-M\lambda+t_d\lambda)\right)\right)}{\lambda^2(\theta+\lambda)}\end{aligned}\right)\\&+\frac{pI_e(\alpha-\beta p)\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}\end{aligned}\right)\right] \tag{13}$$

$$TP_4(p,T)=\left[\frac{1}{T}\left(\begin{aligned}&\frac{p(\alpha-\beta p)\left(-1+e^{T\lambda}\right)}{\lambda}-A\\&-h\left[\frac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+\frac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\right]\\&-c\left[\frac{(\alpha-\beta p)e^{-\theta t_d}}{\lambda+\theta}\left[e^{(\lambda+\theta)T}-e^{(\lambda+\theta)t_d}\right]-\frac{(\alpha-\beta p)}{\lambda}\left[1-e^{\lambda t_d}\right]\right]\\&-\frac{CI_P(\alpha-\beta p)\left(e^{M\lambda}\theta+e^{-M\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+\frac{pI_e(\alpha-\beta p)\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}\end{aligned}\right)\right] \tag{14}$$

4. Solution Procedure

The purpose of this paper is to present the optimal selling price and inventory policy to maximize the profit. To do this, at first, the optimal T for a given p is provided as a function of p. Next, by substituting the obtained optimal function in the profit function, the optimal p would be presented.

Since the profit function is piecewise function, the approach is discussed for all 4 cases.

Case1. $0<T\leq T_L$

is a continuous, non-linear function in P and T. Thus for a given P, the necessary condition to maximize profit function is

$$\frac{\partial TP_1(p,T)}{\partial T}=0 \qquad -\frac{1}{T^2}\left(\begin{array}{l} -A+\dfrac{\left(-1+e^{T\lambda}\right)p(\alpha-\beta p)}{\lambda}-C\left(-\dfrac{\left(1-e^{t_d\lambda}\right)(\alpha-\beta p)}{\lambda}+\dfrac{e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)(\alpha-\beta p)}{\theta+\lambda}\right)- \\ CI_p\left(\begin{array}{l}\dfrac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+ \\ \dfrac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\dfrac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\end{array}\right)- \\ h\left(\begin{array}{l}\dfrac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+ \\ \dfrac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\dfrac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\end{array}\right)\end{array}\right)+ \tag{15}$$

$$\frac{\partial TP_1(p,T)}{\partial T}= \qquad \frac{1}{T}\left(\begin{array}{l} -Ce^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)+e^{T\lambda}p(\alpha-\beta p)- \\ CI_p\left(e^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)t_d+\dfrac{(\alpha-\beta p)\left(-e^{T\lambda}\lambda(\theta+\lambda)+e^{-t_d\theta+T(\theta+\lambda)}\lambda(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}\right)- \\ h\left(e^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)t_d+\dfrac{(\alpha-\beta p)\left(-e^{T\lambda}\lambda(\theta+\lambda)+e^{-t_d\theta+T(\theta+\lambda)}\lambda(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}\right)\end{array}\right)$$

Conjecture 1: for a given p, Equation (15) has a unique solution, which satisfies the condition of second degree to become global maximum.

Argumentation: Due to the complexity of the model, the straightforward proof of the above conjecture is so complicated that we cannot prove it mathematically. However, solution of (15) satisfies the condition of second degree for the maximum and also, a simpler version of this conjecture has been proved in relevant literature for example in Ouyang et al. [3], Geetha and Uthayakumar [4] and also Maihami and Kamalabadi [20]. Therefore, we consider this conjecture as a correct expression and use it to present our solution procedure.

To obtain the optimum amount of selling price for a given T^*, the necessary condition is

$$\frac{\partial TP_{1(p,T^*)}}{\partial p}=0$$

$$\frac{\partial TP_{1(p,T^*)}}{\partial p}=\frac{1}{T}\left(\begin{array}{l}-\frac{\beta\left(-1+e^{T\lambda}\right)p}{\lambda}+\frac{\left(-1+e^{T\lambda}\right)(\alpha-\beta p)}{\lambda}-C\left(\frac{\beta\left(1-e^{t_d\lambda}\right)}{\lambda}-\frac{\beta e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)}{\theta+\lambda}\right)-\\ CI_p\left(-\frac{\beta\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}-\frac{\beta e^{-t_d\theta}\left(\begin{array}{l}e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\\ \frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\end{array}\right)}{\lambda(\theta+\lambda)}\right)-\\ h\left(-\frac{\beta\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}-\frac{\beta e^{-t_d\theta}\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\right)\end{array}\right) \quad (16)$$

With assumptions $\lambda<0,|\lambda|>\theta,\beta p<\alpha<2\beta p$, Equation (16) would have solution if $\beta c-\beta p+(\alpha-\beta p)<0$. Moreover the second derivative of $TP_1(p,T^*)$ with respect to p is always less than or equal to zero:

$$\frac{\partial^2 TP_{1(p,T^*)}}{\partial p^2}=\frac{2\left(\frac{\beta\left(1-e^{t_d\lambda}\right)}{\lambda}-\frac{\beta e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)}{\theta+\lambda}\right)}{T}\le 0$$

Consequently, for each T^*, $TP_1(p,T^*)$ is a concave function with respect to *p*. So the obtained quantity of *p* from Equation (16) will be unique

Case 2: $T_L<T\le M$

$TP_2(p,T)$ is a continuous, non-linear function in *p* and *T*. Thus for each *p*, the necessary condition to maximize the profit function with respect to *T* is

$$\frac{\partial TP_2(p,T)}{\partial T}=0$$

$$\frac{\partial TP_2(p,T)}{\partial T}=-\frac{1}{T^2}\left(\begin{array}{l}-A+\frac{\left(-1+e^{T\lambda}\right)p(\alpha-\beta p)}{\lambda}-C\left(-\frac{\left(1-e^{t_d\lambda}\right)(\alpha-\beta p)}{\lambda}+\frac{e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)(\alpha-\beta p)}{\theta+\lambda}\right)-\\ h\left(\begin{array}{l}\frac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+\\ \frac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\end{array}\right)+\\ I_e p\left(\frac{\left(-1+e^{T\lambda}\right)(\alpha-\beta p)(M-T)}{\lambda}+\frac{(\alpha-\beta p)\left(1+e^{T\lambda}(-1+T\lambda)\right)}{\lambda^2}\right)\end{array}\right)+$$

$$\frac{1}{T}\left(\begin{array}{l}-Ce^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)+e^{T\lambda}p(\alpha-\beta p)-\\ h\left(e^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)t_d+\frac{(\alpha-\beta p)\left(-e^{T\lambda}\lambda(\theta+\lambda)+e^{-t_d\theta+T(\theta+\lambda)}\lambda(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}\right)+\\ I_e p\left(e^{T\lambda}(\alpha-\beta p)(M-T)-\frac{\left(-1+e^{T\lambda}\right)(\alpha-\beta p)}{\lambda}+\frac{(\alpha-\beta p)\left(e^{T\lambda}\lambda+e^{T\lambda}\lambda(-1+T\lambda)\right)}{\lambda^2}\right)\end{array}\right) \quad (17)$$

Conjecture 2: for a given *p*, Equation (17) has a unique solution, which satisfies the condition of second degree to become global maximum.

Argumentation: Please refer to Conjecture 1.

Now for a given T^*, the optimum value of the selling price should be obtained. For a given T^*, the necessary condition to obtain the optimal value of *p* is

$$\frac{\partial TP_{2(p,T^*)}}{\partial p}=0$$

$$\frac{\partial TP_{2(p,T^*)}}{\partial p}=\frac{1}{T}\left(\begin{array}{l}-\frac{\beta\left(-1+e^{T\lambda}\right)p}{\lambda}+\frac{\left(-1+e^{T\lambda}\right)(a-bp)}{\lambda}-C\left(\frac{\beta\left(1-e^{t_d\lambda}\right)}{\lambda}-\frac{\beta e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)}{\theta+\lambda}\right)-\\ h\left(-\frac{\beta\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}-\frac{\beta e^{-t_d\theta}\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\right)+\\ pI_e\left(-\frac{\beta\left(-1+e^{T\lambda}\right)(M-T)}{\lambda}-\frac{\beta\left(1+e^{T\lambda}(-1+T\lambda)\right)}{\lambda^2}\right)+I_e\left(\begin{array}{l}\frac{\left(-1+e^{T\lambda}\right)(\alpha-\beta p)(M-T)}{\lambda}+\\ \frac{(\alpha-\beta p)\left(1+e^{T\lambda}(-1+T\lambda)\right)}{\lambda^2}\end{array}\right)\end{array}\right) \quad (18)$$

Now by assuming $\lambda<0,|\lambda|>\theta,\beta p<\alpha<2\beta p$, it is clear that if $\beta c-\beta p+(\alpha-\beta p)<0$ Equation (18) would have an answer. Moreover, the second derivative of $TP_2(p,T^*)$ with respect to *p* is always less than or equal to zero:

$$\frac{\partial^2 TP_{2(p,T^*)}}{\partial p^2}=\frac{2\left(\frac{\beta\left(1-e^{t_d\lambda}\right)}{\lambda}-\frac{\beta e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)}{\theta+\lambda}\right)+2I_e\left(-\frac{\beta\left(-1+e^{T\lambda}\right)(M-T)}{\lambda}-\frac{\beta\left(1+e^{T\lambda}(-1+T\lambda)\right)}{\lambda^2}\right)}{T}\leq 0$$

Consequently, $TP_2(p,T^*)$, for each T^* is a concave function of p. So the obtained quantity of p from Equation (18) will be unique.

Case 3. $M<t_d$ and $T_L\leq T$

$TP_3(p,T)$ is a non-linear and continuous function in *p* and *T*; thus for each *p*, the necessary condition to maximize profit function is

$$\frac{\partial TP_3(p,T)}{\partial T}=0$$

$$\frac{\partial TP_3(p,T)}{\partial T}=\frac{1}{T}\left(\begin{array}{l}-Ce^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)+e^{T\lambda}p(\alpha-\beta p)-\\ CI_p\left(-e^{-t_d\theta+T(\theta+\lambda)}(a-bp)(M-t_d)+\frac{(\alpha-\beta p)\left(-e^{T\lambda}\lambda(\theta+\lambda)+e^{-t_d\theta+T(\theta+\lambda)}\lambda(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}\right)-\\ h\left(e^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)t_d+\frac{(\alpha-\beta p)\left(-e^{T\lambda}\lambda(\theta+\lambda)+e^{-t_d\theta+T(\theta+\lambda)}\lambda(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}\right)\end{array}\right)-$$

$$\frac{1}{T^2}\left(\begin{array}{l}-A+\frac{\left(-1+e^{T\lambda}\right)p(\alpha-\beta p)}{\lambda}-C\left(-\frac{\left(1-e^{t_d\lambda}\right)(\alpha-\beta p)}{\lambda}+\frac{e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)(\alpha-\beta p)}{\theta+\lambda}\right)+\\ \frac{I_e p(\alpha-\beta p)\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}-\\ CI_p\left(\begin{array}{l}\frac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+\\ \frac{e^{-t_d\theta}(\alpha-\beta p)\left(-e^{T(\theta+\lambda)}(M-t_d)\lambda^2+e^{t_d\theta+M\lambda}(\theta+\lambda)+e^{t_d(\theta+\lambda)}\left(-\lambda+\theta(-1-M\lambda+t_d\lambda)\right)\right)}{\lambda^2(\theta+\lambda)}\end{array}\right)\\ -h\left(\begin{array}{l}\frac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+\\ \frac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\end{array}\right)\end{array}\right) \quad (19)$$

Conjecture 3: for a given p, Equation (19) has a unique solution, which satisfies the condition of second degree to become global maximum.
Argumentation: Please refer to Conjecture 1.
Also for each optimum quantity of T^*, the profit function could be changed into a single variable function based on the price. Thus to obtain the optimum amount of price, it is sufficient to consider the first derivative of profit function equal to zero. Therefore, we have

$$\frac{\partial TP_{3(p,T^*)}}{\partial p}=0\Rightarrow \frac{\partial TP_{3(p,T^*)}}{\partial p}= \frac{1}{T}\left(\begin{array}{l} -\frac{\beta\left(-1+e^{T\lambda}\right)p}{\lambda}+\frac{\left(-1+e^{T\lambda}\right)(\alpha-\beta p)}{\lambda}-C\left(\frac{\beta\left(1-e^{t_d\lambda}\right)}{\lambda}-\frac{\beta e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)}{\theta+\lambda}\right) \\ -\frac{\beta I_e p\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}+\frac{I_e(\alpha-\beta p)\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}- \\ h\left(-\frac{\beta\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}-\frac{\beta e^{-t_d\theta}\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\right)- \\ CI_p\left(\begin{array}{l}-\frac{\beta\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}- \\ \frac{\beta e^{-t_d\theta}\left(-e^{T(\theta+\lambda)}(M-t_d)\lambda^2+e^{t_d\theta+M\lambda}(\theta+\lambda)+e^{t_d(\theta+\lambda)}\left(-\lambda+\theta(-1-M\lambda+t_d\lambda)\right)\right)}{\lambda^2(\theta+\lambda)}\end{array}\right)\end{array}\right) \tag{20}$$

By assuming $\lambda<0,|\lambda|>\theta,\beta p<\alpha<2\beta p$ from Equation (20), it would be clear that if $\beta c-\beta p+(\alpha-\beta p)<0$, Equation (20) will have an answer. Moreover, the second derivative of $TP_3(p,T^*)$ with respect to p is always less than or equal to zero.

$$\frac{\partial^2 TP_{3(p,T^*)}}{\partial p^2}=\frac{2\left(\frac{\beta\left(1-e^{t_d\lambda}\right)}{\lambda}-\frac{\beta e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)}{\theta+\lambda}\right)-\frac{2\beta I_e\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}}{T}\le 0$$

Consequently, $TP_3(p,T^*)$, for each T^*, is a concave function of p, so the obtained quantity of p from Equation (20) will be unique.

Case 4. $t_d<M\le T$ and $T_L\le T$

$TP_4(p,T)$ is a non-linear and continuous function in p and T; thus for each p, the necessary condition to maximize profit function is:

$$\frac{\partial TP_4(p,T)}{\partial T}=0$$

$$\frac{\partial TP_4(p,T)}{\partial T}=\frac{1}{T^2}\left(\begin{array}{l} -A+\frac{\left(-1+e^{T\lambda}\right)p(\alpha-\beta p)}{\lambda}-C\left(-\frac{\left(1-e^{t_d\lambda}\right)(\alpha-\beta p)}{\lambda}+\frac{e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)(\alpha-\beta p)}{\theta+\lambda}\right)+ \\ \frac{I_e p(\alpha-\beta p)\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}-\frac{CI_p(\alpha-\beta p)\left(e^{M\lambda}\theta+e^{-M\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}- \\ h\left(\begin{array}{l}\frac{(\alpha-\beta p)\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}+ \\ \frac{e^{-t_d\theta}(\alpha-\beta p)\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\end{array}\right)\end{array}\right)+ \tag{21}$$

$$\frac{1}{T}\left(\begin{array}{l} -Ce^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)+e^{T\lambda}p(\alpha-\beta p)-\frac{CI_p(\alpha-\beta p)\left(-e^{T\lambda}\lambda(\theta+\lambda)+e^{-M\theta+T(\theta+\lambda)}\lambda(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)} \\ h\left(e^{-t_d\theta+T(\theta+\lambda)}(\alpha-\beta p)t_d+\frac{(\alpha-\beta p)\left(-e^{T\lambda}\lambda(\theta+\lambda)+e^{-t_d\theta+T(\theta+\lambda)}\lambda(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}\right)\end{array}\right)$$

Conjecture 4: for a given p, Equation (20) has a unique solution, which satisfies the condition of second degree to become global maximum.
Argumentation: Please refer to Conjecture 1.

Therefore, the optimum answer of T^* for a given p, could be obtained. Furthermore, for each optimum quantity of T^*, the profit function should explored to obtain the optimal price. The first-order condition is:

$$\frac{\partial TP_{4(p,T^*)}}{\partial p}=0$$

$$\frac{\partial TP_{4(p,T^*)}}{\partial p}=\frac{1}{T}\left(\begin{array}{l}-\frac{b\left(-1+e^{T\lambda}\right)p}{\lambda}+\frac{\left(-1+e^{T\lambda}\right)(a-bp)}{\lambda}-C\left(\frac{b\left(1-e^{t_d\lambda}\right)}{\lambda}-\frac{be^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)}{\theta+\lambda}\right)\\ +\frac{bcI_P\left(e^{M\lambda}\theta+e^{-M\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}\\ -\frac{bI_e p\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}+\frac{I_e(a-bp)\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}-\\ h\left(-\frac{b\left(e^{t_d\lambda}\theta+e^{-t_d\theta+T(\theta+\lambda)}\lambda-e^{T\lambda}(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}-\frac{be^{-t_d\theta}\left(e^{t_d(\theta+\lambda)}t_d\theta+e^{T(\theta+\lambda)}t_d\lambda-\frac{e^{t_d\theta}\left(-1+e^{t_d\lambda}\right)(\theta+\lambda)}{\lambda}\right)}{\lambda(\theta+\lambda)}\right)\end{array}\right) \quad (22)$$

Assuming $\lambda<0, |\lambda|>\theta, \beta p<\alpha<2\beta p$, if $\beta c-\beta p+(\alpha-\beta p)<0$ Equation (22), it would have an answer. Moreover, the second derivative of $TP_4(p,T^*)$ with respect to p is always less than or equal to zero:

$$\frac{\partial^2 TP_{4(p,T^*)}}{\partial p^2}=\frac{2\left(\frac{\beta\left(1-e^{t_d\lambda}\right)}{\lambda}-\frac{\beta e^{-t_d\theta}\left(e^{T(\theta+\lambda)}-e^{t_d(\theta+\lambda)}\right)}{\theta+\lambda}\right)-\frac{2\beta Ie\left(1+e^{M\lambda}(-1+M\lambda)\right)}{\lambda^2}}{T}\leq 0$$

Consequently, $TP_4(p,T^*)$ for each T^* is a concave function relative to p. Hence, the obtained quantity of p from Equation (22) will be unique.

5. Solution Algorithm

Based on the presented formulation, an iterative algorithm proposed to solve the model. First, the retailer's qualification to use the trade credit should be identified. In case of presenting the trade credit, based on its amount, it should be identified that which case could be chosen for calculating the optimal amount of the order, price, and total profit. To do so, first the initial amount Q should be obtained to identify whether it is equal to or greater than the amount considered by the supplier. Regarding the equation (7), for calculating Q, quantities of P_j and T are calculated. Then, the following formula used to find the initial amounts of P_j and T by equating its derivative to zero, once relative to T and again relative to p. This is the equation used for calculating the total profit without using earned and payable interest.

$$\frac{1}{T}\left(\begin{array}{l}p\left(\frac{(\alpha-\beta p)e^{-\theta t_d}}{\lambda+\theta}\left[e^{(\lambda+\theta)\mathrm{T}}-e^{(\lambda+\theta)t_d}\right]-\frac{(\alpha-\beta p)}{\lambda}\left[1-e^{\lambda t_d}\right]\right)-A\\ -h\left[\frac{e^{-\theta t_d}(e^{(\theta+\lambda)t_d}\theta+e^{(\theta+\lambda)\mathrm{T}}\lambda-e^{\lambda T+\theta t_d}(\theta+\lambda))(\alpha-\beta p)}{\theta\lambda(\theta+\lambda)}+\right.\\ \left.\frac{e^{-\theta t_d}(\alpha-\beta p)\left(-\frac{e^{\theta t_d}(-1+e^{\lambda t_d})(\lambda+\theta)}{\lambda}+e^{(\theta+\lambda)t_d}\theta t_d+e^{(\theta+\lambda)\mathrm{T}}\lambda t_d\right)}{\lambda(\theta+\lambda)}\right]\\ -c\left[\frac{(\alpha-\beta p)e^{-\theta t_d}}{\lambda+\theta}\left[e^{(\lambda+\theta)\mathrm{T}}-e^{(\lambda+\theta)t_d}\right]-\frac{(\alpha-\beta p)}{\lambda}\left[1-e^{\lambda t_d}\right]\right]\end{array}\right)=0 \quad (23)$$

The proposed algorithm is as follows.
Algorithm 1: find optimal solution
Step 1: by applying Equation (23), the initial amount of p_j and T is identified.
Step2: after obtaining p and T, an initial amount for Q is calculated.

Step3: compare Q with Q_L. If Q is less than Q_L, then the first case is chosen to obtain the optimum amounts of p, Q, T and TP, otherwise go to step 4.
Step 4: in this step, the quantity M is examined to choose the cases.
If $M>T$, the second case is chosen to calculate p^*, T^*, Q^* and TP^*.
If $0<M<t_d$, the third case is chosen to calculate these quantities.
If $t_d<M<T$, the fourth case is chosen to calculate these quantities.
Step 5: after choosing appropriate case, we do following sub-steps (here, we explain following sub-steps for first case):
Sub-step 5-1: by using p_j obtained in the first step and by using Equation (15) the amount of T^* will be acquired.
Sub-step 5-2: by using the amount of T^* Obtained in sub-step 5-1 and from Equation (16) the amount p_{j+1} will be obtained.
Sub-step 5-3: p_{j+1} and p_j are compared. If their difference is smaller than a specific amount, then set $p^*=p_{j+1}$ and the amounts of p^* and T^* are the optimum ones. Thus, the algorithm is stopped and we go to sub-step 5-4. Otherwise, set $p=p_{j+1}$ and return back to sub-step 5-1. (In this study, 0.001 is considered to examine the difference between p_j and p_{j+1}; this amount can vary).
Sub-step 5-4: obtain the amounts of TP_1* and $Q*$ using Equations (11) and (7), respectively. Present best-found solutions and stop the algorithm.
End of Algorithm 1.

6. Numerical Analysis and Sensitive Analysis

This section presents a numerical example in order to prove the solution method as well as results. We use the discussed algorithm to solving this numerical example. Moreover, we perform sensitive analysis for values of parameters to showing results. We obtain the results by using Matlab.
A=\$250/per order, c=\$20/per unit, θ=0.08, $t_d=1/12$, $\beta(x)=e^{0.1x}$ *,M=0.06, Ip=0.15,* $f(t,p)=(200-4p)e^{-0.98t}$ *, Ie=0.12.* $Q_{L1}=60$, $Q_{L2}=20$, *h=\$1 per unit time*
First, the model is solved by $Q_{L1}=60$. The following results are reached:
p = 36.0719, T = 0.93384 , Q = 34.972 , TP = 240.6484
Now, the model is solved by $Q_{L2}=20$. Now, the model is solved by $M=1.75$ and the following amounts are acquired.
p = 33.8672 T = 0.67175 Q = 32.3316 TP = 517.058
By comparing the obtained results in two above conditions, we will realize that the retailer would reduce the price to increase selling and the acquired total profit in the second case, (TP = 517.058), is higher than the first (TP = 240.6484); this is due to trade credit. Also, the retailer prefer to order lower quantity with shorter replenishment cycle.

6.1. Sensitive analysis

Doing the sensitive analysis is a suitable way for model performance' comprehension. We can present sensitive analysis for some parameters. In this research, the sensitivity analysis on acquired total profit, replenishment cycle and order amount, will be presented based on different trade credit quantities.

Tab. 1. Sensitivity analysis on total acquired profit, replenishment cycle and order amount based on different quantities of trade credit

M	0	0.03	0.06	0.08	0.40	0.80	1.10	1.40	1.80	2.10
TP	240.65	243.81	247.03	250.32	290.196	339.38	394.41	450.10	526.21	584.55
T	0.93	0.93	0.93	0.92	0.88	0.83	0.722	0.697	0.668	0.6648
Q	34.97	35.00	35.02	35.04	35.07	34.94	32.61	32.49	32.30	32.14

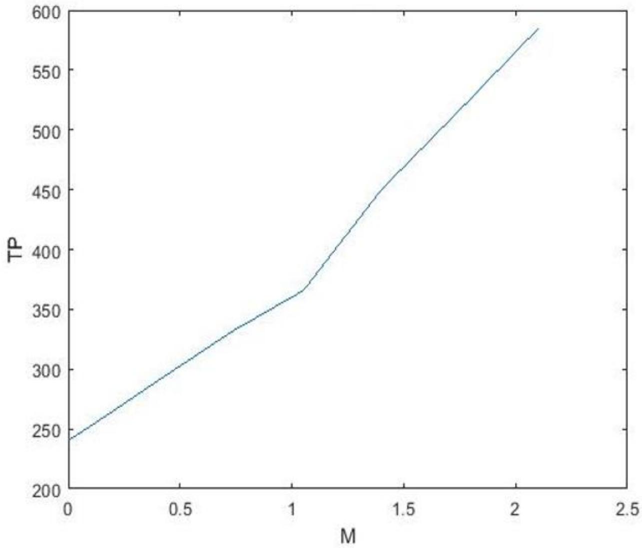

Fig. 2. Sensitivity analysis for total profit based on trade credit amount

Table 1 shows the total profit based on different trade credit rates. As it can be seen from fig. 2, the more trade credit, the more total profit.

Now, sensitivity analysis for replenishment cycle for different trade credit rates will be examined:

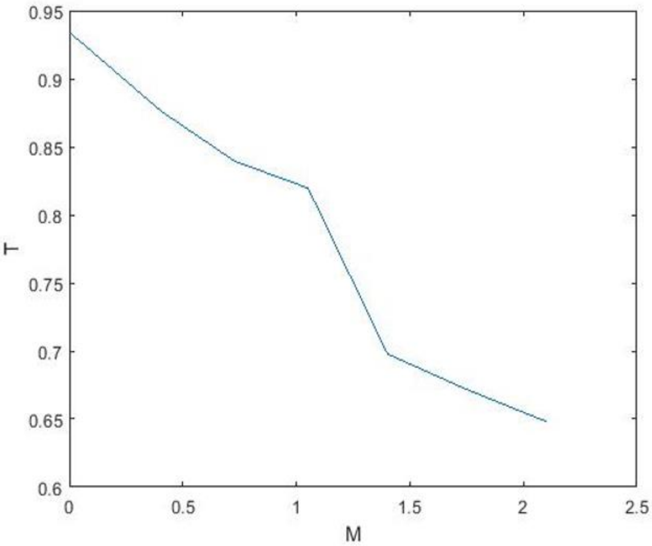

Fig. 3. Sensitivity analysis for replenishment cycle based on trade credit

As it can be seen from Table 1 and fig. 3, along with increasing the trade credit, the replenishment time will be reduced. This reduction means that the retailer refers a shorter replenishment cycle for ordering and earning extra profit.

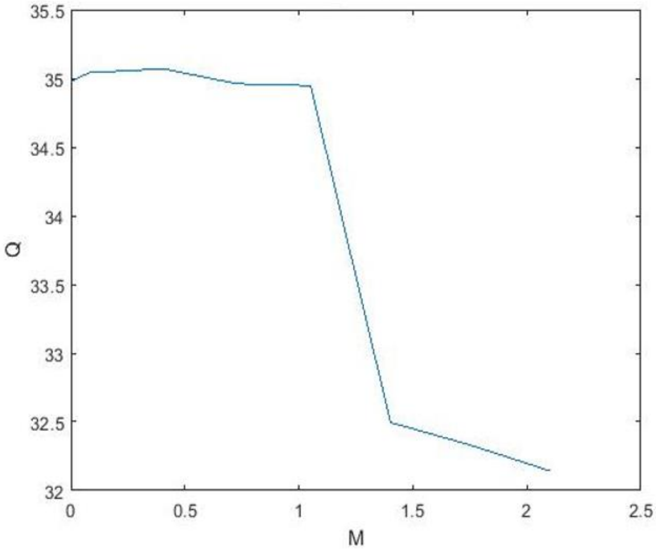

Fig. 4. Sensitivity analysis of order amount based on trade credit

As observed in table 1 and fig. 4, through increasing the trade credit amount, the order amount is reduced. As mentioned in sensitivity analysis of replenishment cycle, the retailer prefers shorter replenishment cycle for ordering with lower amounts.

7. Conclusion

In this research, we studied joint inventory control and pricing for NIDP under QDTC. In this study, the retailer was supposed to order equal to or more than a specific amount identified by the supplier if he/she wants to enjoy the advantages of the trade credit. It was underlined that the optimal answers are present and unique and therefore an appropriate algorithm was proposed for finding the optimal answers. Then by solving a numerical example, the efficiency model was proved. Finally, the results obtained from the example were examined by sensitivity analysis.

The findings demonstrated that when the QDTC is presented by the supplier, the retailers are encouraged to buy more and consequently to benefit more. Eventually, s/he could enjoy the advantages of this technique through additional profit. Via applying this technique, the retailer prefers to decrease the replenishment cycle and then he/she increases the number of ordering and thus his/her total profit is boosted. In this way, s/he can earn the most amount of profit.

We added the QDTC approach to the price-dependent strategy and inventory control for NIDP. The demand function was detected to be dependent on the time and price. Other researchers are suggested to consider probable demand function for future studies. Moreover, supplementary techniques such as advertisement tasks could be added to the model for further study paths.

References

[1] Wu, K. S., Ouyang, L. Y., and Yang, C. T., "*An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging,"* International Journal of Production Economics, Vol. 101, No. 2, (2006), pp. 369-384.

[2] Yang, C. T., Ouyang, L. Y., and Wu, H. H., "*Retailer's optimal pricing and ordering policies for non-instantaneous deteriorating items with price-dependent demand and partial backlogging,"* Mathematical Problems in Engineering, Vol. 2009, (2009).

[3] Ouyang, L. Y., Wu, K. S., and Yang, C. T., "*A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments,"* Computers & Industrial Engineering, Vol. 51, No. 4, (2006), pp. 637-651.

[4] Geetha, K., and Uthayakumar, R., "*Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments,"* Journal of computational and applied mathematics, Vol. 233, No. 10, (2010), pp. 2492-2505.

[5] Maihami, R., and Kamalabadi, I. N., "*Dynamic Pricing and Ordering Policy for Non-Instantaneous Deteriorating Items*," Advanced Materials Research, Vol. 433, (2012), pp. 6607-6615.

[6] Maihami, R., Ghalehkhondabi, I., Ahmadi, E., "*Pricing and inventory planning for non-instantaneous deteriorating products with greening investment: A case study in beef industry*," Journal of Cleaner Production, Vol. 295, (2021), pp. 126368.

[7] Rezagholifam, M., Sadjadi, S.J., Heydari, M., Karimi,., "*Optimal pricing and ordering strategy for non-instantaneous deteriorating items with price and stock sensitive demand and capacity constraint*," International Journal of Systems Science: Operations & Logistics, Vol. 9, (2022), pp.121-132.

[8] Sundararajan, R., Palanivel, M., Uthayakumar, R., "*An inventory system of non-instantaneous deteriorating items with backlogging and time discounting*," International Journal of Systems Science: Operations & Logistics, Vol. 7, (2020), pp.233-247.

[9] Chakraborty, D., Jana, D.K., Roy, T.K., "*Multi-warehouse partial backlogging inventory system with inflation for non-instantaneous deteriorating multi-item under imprecise environment*," Soft Computing, Vol. 24, (2020), pp. 14471-14490.

[10] Li, G., He, X., Zhou, J., Wu, H., "*Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items*," Omega, Vol. 84, (2019), pp. 114-126.

[11] Barman, A., Das, R., De, P.K., "*An analysis of optimal pricing strategy and inventory scheduling policy for a non-instantaneous deteriorating item in a two-layer supply chain*," Applied Intelligence, Vol. 52, (2021), pp. 4626-4650.

[12] Tashakkor, N., Mirmohammadi, S.H., Iranpoor, M., "*Joint optimization of dynamic pricing and replenishment cycle considering variable non-instantaneous deterioration and stock-dependent demand*," Computers & Industrial Engineering, Vol. 123, (2018), pp. 232-241.

[13] Khan, M.A.A., Halim, M.A., AlArjani, A., Shaikh, A.A., Uddin, M.S., "*Inventory management with hybrid cash-advance payment for time-dependent demand, time-varying holding cost and non-instantaneous deterioration under backordering and non-terminating situations*," Alexandria Engineering Journal, Vol. 61, No. 11, (2022), pp. 8469-8486.

[14] Mahdavisharif, M., Kazemi, M., Jahani, H., Bagheri, F., "*Pricing and inventory policy for non-instantaneous deteriorating items in vendor-managed inventory systems: a Stackelberg game theory approach*," International Journal of Systems Science: Operations and Logistics, Vol. 10, (2022), pp. 1-28.

[15] Almathkour, F., Benkherouf, L., "*Optimal policies for finite horizon model with time-varying demand rate, non-instantaneous deterioration and backlogging*," International Journal of Systems Science: Operations and Logistics, Vol. 10, (2022), pp. 1-18.

[16] Udayakumar, R., "*An EOQ model for non-instantaneous deteriorating items with time-dependent demand under partial backlogging*," Journal of Management Analytics, Vol. 9, No. 4, (2022), pp. 1-18.

[17] Tavassoli, S., Manavizadeh, N., Rezaei, A., Rabbani, M., "*A Lot-Sizing Model for Non-Instantaneous Deteriorating Products under Advance Payment and Non-Linear Partial Backlogging*," Iranian Journal of Management Studies, Vol. 15, No. 1, (2022), pp. 85-110.

[18] Chandramohan, J., Chakravarthi, R.P.A. and Ramasamy, U. "*A comprehensive*

inventory management system for non-instantaneous deteriorating items in supplier- retailer-customer supply chains," Supply Chain Analytics, Vol. 3, (2023).

[19] Priyamvada, P., Rini, R. and Jaggi, C.K. "*Optimal inventory strategies for deteriorating items with price-sensitive investment in preservation technology,*" RAIRO - Operations Research, Vol. 56, No. 2, (2022), pp. 601-617.

[20] Shinn S.W., Hwang, H., Park, S.S., "*Joint price and lot size determination under conditions of permissible delay in payments and quantity discounts for freight cost,"* European Journal of Operational Research, Vol. 91, (1996), pp. 528-42.

[21] Hwang, H., and Shinn, S.W., "*Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments,"* Computers & Operations Research, Vol. 24, No. 6, (1997), pp. 539-547.

[22] Maihami, R., and Kamalabadi, I. N., "*Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging,"* Mathematical and Computer Modelling, Vol. 55, Nos. 5-6, (2012), pp. 1722-1733.

[23] Maihami, R., and Kamalabadi, .I.N., "*Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand,"* International Journal of Production Economics, Vol. 136, No. 1, (2012), pp. 116-122.

[24] Aliabadi, L., Yazdanparast, R., Nasiri, M.M., "*An inventory model for non-instantaneous deteriorating items with credit period and carbon emission sensitive demand: a signomial geometric programming approach*," International Journal of Management Science and Engineering Management, Vol. 14, (2019), pp. 124-136.

[25] Udayakumar, R., Geetha, K.V., Sana, S.S., "*Economic ordering policy for non-instantaneous deteriorating items with price and advertisement dependent demand and permissible delay in payment under inflation*," Mathematical Methods in Applied Sciences, Vol. 44, No. 9, (2020), pp. 7697-7721.

[26] Das, S.C., Manna, A.K., Rahman, M.S., Shaikh, A.A., "*An inventory model for non-instantaneous deteriorating items with preservation technology and multiple credit periods-based trade credit financing via particle swarm optimization*," Soft Computing, Vol. 25, (2021), pp. 5365-5384.

[27] Rapolu, C.N., Kandpal, D.H., "*Joint pricing, advertisement, preservation technology investment and inventory policies for non-instantaneous deteriorating items under trade credit*," OPSEARCH, Vol. 57, (2020), pp. 274-300.

[28] Tripathy, M., Sharma, G. and Sharma, A.-K. "*An EOQ inventory model for non-instantaneous deteriorating item with constant demand under progressive financial trade credit facility,*" OPSEARCH, Vol. 59, (2022), pp. 1215-1243.

[29] Choudhury, M. and Mahata, G.C. "*Non-instantaneous deteriorating items inventory models with fixed lifetime products under hybrid partial prepayment and trade credit in supply chain,*" Journal of Industrial and Management Optimization, Vol. 20, No. 1, (2024), pp. 221-259.

[30] Viswanath, J. and Thilagavathi, R. "*A study of two warehouse inventory dynamism for non-instantaneous deteriorating item with Weibull demand considering trade credit policy,*" OPSEARCH, (2024).

[31] Singh, A. and Goel, A. "*Warehouse Inventory Model for Perishable Items with Hybrid Demand and Trade Credit Policy,*" Process Integration and Optimization for Sustainability, (2024).

[32] Momena, A.F., Haque, R., Rahaman, M. and Mondal, S.P. "*A Two-Storage Inventory Model with Trade Credit Policy and Time-Varying Holding Cost under Quantity Discounts,*" Logistics, Vol. 7, No. 4, (2023).

[33] Mashud, A.H.M., Hasan, M.R., Wee, H.M. and Daryanto, Y. "*Non-instantaneous deteriorating inventory model under the joined effect of trade-credit, preservation technology and advertisement policy,*" Kybernetes, Vol. 49, No. 6, (2020), pp. 1645-1674.

[34] Mondal, R., Das, S., Akhtar, M., Shaikh, A.A. and Bhunia, A.K. "*A two-warehouse inventory model for deteriorating items with partially backlogged demand rate under trade credit policies,*" International Journal of System Assurance Engineering and Management, (2024).

[35] Chang, C. T., Ouyang, L. Y., and Teng, J. T., "*An EOQ model for deteriorating items under supplier credits linked to ordering quantity,"* Applied Mathematical Modelling, Vol. 27, No. 12, (2003), pp. 983-996.

[36] Chung, K. J., Goyal, S.K., and Huang, Y. F., "*The optimal inventory policies under permissible delay in payments depending on the ordering quantity,"* International Journal of production economics, Vol. 95, No. 2, (2005), pp. 203-213.

[37] Chang, H. C., Ho, C.-H., Ouyang, L. Y., Su, C. H., "*The optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity,"* Applied Mathematical Modelling, Vol. 33, No. 7, (2009), pp. 2978-2991.

[38] Chen, S. C., Cárdenas-Barrón, L.E., and Teng, J. T., "*Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity,"* International Journal of Production Economics, Vol. 155, (2014), pp. 284-291.

[39] Chung, K. J., Cárdenas-Barrón, L.E., and Ting, P. S., "*An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit,"* International Journal of Production Economics, Vol. 155, (2014), pp. 310-317.

[40] Ouyang, L. Y., Ho, C. H., and Su, C. H., "*An optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit,"* Computers & Industrial Engineering, Vol. 57, No. 3, (2009), pp. 920-930.

[41] Yang, C. T., Pan, Q., Ouyang, L. Y., and Teng, J. T., "*Retailer's optimal order and credit policies when a supplier offers either a cash discount or a delay payment linked to order quantity,"* European Journal of Industrial Engineering, Vol. 7, No. 3, (2013), pp. 370-392.

[42] Shah, N.H., Gor, A.S., and Jhaveri, C.A., "*Optimal pricing and ordering policy for an integrated inventory model with quadratic demand when trade credit linked to order quantity,"* Journal of modelling in management, Vol. 7, No. 2, (2012), pp. 148-165.

[43] Shah, N.H., and Cárdenas-Barrón, L.E., "*Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount,"* Applied Mathematics and Computation, Vol. 259, (2015), pp. 569-578.

[44] Chang, C.-T., Cheng, M.-C., and Ouyang, L.-Y., "*Optimal pricing and ordering policies for non-instantaneously deteriorating items under order-size-dependent delay in payments,"* Applied

Mathematical Modelling, Vol. 39, No. 2, (2015), pp. 747-763.

[45] Shah, N.H., Keswani, M., Khedlekar, U.K. and Prajapati, N.M. "*Non-instantaneous controlled deteriorating inventory model for stock-price-advertisement dependent probabilistic demand under trade credit financing,*" OPSEARCH, Vol. 61, (2024), pp. 421-459.

[46] Mahato, C., Mahato, F. and Mahata, G.C. "*Bi-level trade credit policy under pricing and preservation technology in inventory models for non-instantaneous deteriorating items under carbon tax policy,*" SADHANA, Vol. 48, No. 103, (2023).

[47] Tiwari, S., Cárdenas-Barrón, L.E., Malik, A.I. and Jaggi, C.K. "*Retailer's credit and inventory decisions for imperfect quality and deteriorating items under two-level trade credit,*" Computers & Operations Research, Vol. 138, (2022).

[48] Soni, H.N. and Chauhan, A.D. "*Joint inventory, promotion and preservation decisions for deteriorating items with maximum lifetime and stochastic demand under two-level partial trade credit,*" International Journal of Operational Research, Vol. 45, No. 2, (2022), pp. 241-281.

[49] Das, S., Choudhury, M. Mahato, C. and Mahata G.C. "*Dual-channel supply chain inventory model under two-warehouse setting with order volume-linked trade credit, all-units discount and partial backordering,*" Soft Computing, Vol. 27, (2023), pp. 15817-15.

Follow this article at the following site:

Hamed Salehi Mourkani, Anwar Mahmoodi & Isa Nakhai Kamalabadi. Joint Pricing and Inventory Control for Non-instantaneous Deteriorating Items When Trade Credit is Linked to Order Quantity. IJIEPR 2024; 35 (3) :1-20
URL: http://ijiepr.iust.ac.ir/article-1-1706-en.html