



# An Efficient Hybrid Metaheuristic for Capacitated $p$ -Median Problem

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## KEYWORDS

Facility location problem,  
 $p$ -median problem,  
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## ABSTRACT

*Capacitated  $p$ -median problem (CPMP) is a well-known facility-location problem, in which  $p$  capacitated facility points are selected to satisfy  $n$  demand points in such a way that the total assigned demand to each facility does not exceed its capacity. Minimizing the total sum of distances between each demand point and its nearest facility point is the objective of the problem. Developing an efficient solution method for the problem has been a challenge during last decades in literature. In this paper, a hybrid metaheuristic called GACO is developed to find high quality and fast solutions for the CPMP. The GACO combines elements of genetic algorithm and ant colony optimization metaheuristics. Computational results on standard test problems show the robustness and efficiency of the algorithm and confirm that the proposed method is a good choice for solving the CPMP.*

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## 1. Introduction

The capacitated  $p$ -Median problem is a location-allocation problem which intends to specify the location of  $p$  facilities to serve a set of  $n$  candidate points where total allocated demand to each facility not to be more than its capacity limit. The objective of this problem is to minimize the total distances between the demand points and their nearest facilities. The problem is a special case of the plant location problem with cardinality constraints [1]. The CPMP is an NP-hard combinatorial optimization problem [2] with various applications in many practical situations such as clustering and network design problem. The mathematical model of CPMP can be stated as follows: suppose that graph  $G = (V, A)$  is a non-directional complete graph, in which  $V = \{1, 2, \dots, n\}$  and  $A = \{n \times n\}$  are the vertices and edges of the graph, respectively. According to explanations the CPMP problem can be modeled as follows.

$$(CPMP) \quad \min \sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij} \quad (1)$$

$$s.t. \sum_{j \in V} x_{ij} = 1 \quad \forall i \in V \quad (2)$$

$$x_{ij} \leq y_{jj} \quad \forall i, j \in V \quad (3)$$

$$\sum_{j \in V} y_{jj} = p \quad (4)$$

$$\sum_{i \in N} q_i x_{ij} \leq Q_j y_{jj} \quad \forall j \in V \quad (5)$$

$$x_{ij}, y_{jj} \in \{0, 1\} \quad \forall i \in V, j \in V \quad (6)$$

Where  $V = \{1, 2, \dots, n\}$  is a set of vertices and possible medians in the graph,  $d_{ij}$  are distance values,  $x_{ij}$  is allocation decision variable with  $x_{ij} = 1$ , if demand point  $i$  is allocated to median  $j$ , and  $x_{ij} = 0$  otherwise.  $y_{jj}$  is the median decision variable with  $y_{jj} = 1$ , if a facility is located at point  $j$ , and  $y_{jj} = 0$ , otherwise. Finally  $q_i$  and  $Q_j$  are the demand and the capacity of points  $i$  and  $j$ , respectively.

The objective function (1) is to minimize the sum of distances between demand points and median points. Constraints (2) impose that each customer is assigned to a median. Disaggregated inequalities (3) impose that each demand point must be allocated to a point that has

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been selected as median. Constraint (4) ensured that always  $p$  medians are selected. Constraints (5) impose that the sum of demands assigned to a median does not exceed its capacity.

The next section reviews related works. In section 3, a brief introduction to hybrid metaheuristic and the component of the GACO is presented. Section 4 provides the computational results and finally, section 5 deals with some concluding remarks.

## 2. Literature Review

Several algorithms including exact, heuristic and metaheuristic methods have been proposed for solving this problem. In the class of exact solution methods for the CPMP one can find column generation [3], branch and bound and branch-and-price algorithms [4] and [5], cutting plane algorithm [6]. A set partitioning solution method introduced in [7]. In the class of heuristics and metaheuristics there has been an extensive literature using genetic algorithm [8], ant colony optimization [9], scatter search [10] and [11], variable neighborhood search [12], clustering search [13] and [14], which are few works recently studied.

This study proposes a new hybrid metaheuristic which combines elements of genetic algorithm (GA) and ant colony optimization (ACO) metaheuristics that called GACO. In essence, GACO algorithm is an iterative method that in any iteration a population of solutions is randomly constructed from its last generation using crossover, mutation and pheromone trail operators. GACO utilizes population generation, and reproduction and mutation mechanisms from genetic algorithm. Then, as a reinforcement mechanism, it incorporates pheromone trail and selection of elite solutions which are the specific characteristics of ant colony optimization. The proposed algorithm is simple, effective and efficient.

## 3. GACO Algorithm for CPMP

Over the last years, interest in hybrid metaheuristics has risen considerably in the field of optimization. According to [15], combination of one metaheuristic with components from other metaheuristics or with techniques from artificial intelligence and operation research techniques is called a hybrid metaheuristic. The hybridization mechanism allows obtaining robust and better solutions [16]. We have combined the basic elements of GA including crossover and mutation, and pheromone trails from ACO to provide a fast and effective new hybrid algorithm.

Genetic algorithm, developed by Holland (1975), is based on the natural selection recombination of individuals. The individuals which correspond to solutions, composing a population of candidate solutions, are called chromosomes. The quality of candidate solution is measured by fitness function. This

is a measurement of how well the chromosome optimizes the objective function. Two genetic operators called crossover and mutation are used to create new individuals. Due to the selective pressure through a number of generations, the overall trend is towards higher-fitness individuals [17].

Ant colony optimization was proposed by Dorigo (1991) who states that ants can determine the shortest path from nest to food by pheromone traces. The pheromone accumulation and use of the statistical information (pheromone trails) are the basic elements of any ant colony optimization [18].

A solution of the CPMP is composed of two dependent components. The first one is  $p$  number of points that selected as median point and the second component is the remaining  $(n - p)$  points which should be allocated to selected median points. In GACO, the genetic algorithm is used for identifying sets of  $p$  median points and the ant colony optimization carry out the allocation mechanism. Computational results shows that it makes to reach high quality solutions with less computational effort. Figure 1 shows the pseudocode of the proposed algorithm.

### 3.1. Median Selection

In the applied genetic algorithm, each chromosome composed of the  $p$  index of selected median points. It should be noticed that repetition of an index is not allowed in each chromosome and order and succession is not important among genes in each chromosome. To evaluate the fitness of a chromosome, each demand point is first assigned to a median according to allocation mechanism considered to the capacity of median, and then the fitness of a chromosome is computed by calculating the summation of Euclidean distances between each vertex and its assigned median. Therefore, the best solution is the chromosome which has the minimum sum of distances.

**Set parameters**

**Initialize** a random population of individuals

**Repeat**

**Evaluate** fitness of all individuals

**Update** pheromones

**Repeat**

/\* median selection based on genetic algorithm \*/

**Select** parents

**Recombine** parents

**Mutate** offspring

/\* demand allocation based on pheromone trails \*/

**Allocate** demands

**If** (offspring can satisfy acceptance condition)

**Then**

**Add** offspring in new population

**End If**

**Until** (generated offspring less than population size)

**Until** (termination condition satisfied)

**Fig. 1. Pseudocode of GACO**

The initial population has a collection of chromosomes. Each chromosome is constructed by  $p$  randomly selected medians.

Then the remaining  $(n - p)$  demand points is assigned to their nearest median point based on the capacity of nearest median. When the initial population is constructed, an initial amount of pheromone ( $\tau_0$ ) is assigned to each edge of median point and its allocated demand point.

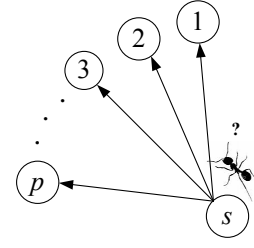


Fig. 2. Ant at a demand point  $s$  moves to a median  $u$ .

### 3.1.1. Generating new Candidate Locations

After the generation of initial population, next populations are constructed by the combination of crossover and mutation operators. Two mechanisms have been tested for the selection of parent in order to create new population. These mechanisms are the well-known roulette wheel method and a ranking-based selection method [8].

### 3.1.2. Crossover

In this paper the crossover operator first calculates a substitution vector for each parent as follows. For each gene of parent 1, operator checks whether the median index of gene is present at the genome of parent 2. If not, that median index is copied to the substitution vector of parent 1.

This means that median index may be transferred to parent 2 as a result of crossover, since this transfer would not create duplicate median indices in parent 2's genotype. Once the two substitution vectors are calculated, the median indices which can be exchanged are identified. If there is at least one median index in the substitution vectors of each parent, the crossover operation can be performed by two randomly selected subsets of elements, one for each substitution vector. Then these subsets of indices will be randomly swapped between the two parents. This procedure avoids median index duplication in any of the two produced children.

### 3.1.3. Mutation

Here mutation rate is set to a very low level and the current index of a randomly selected gene is replaced by another randomly generated index considering that the new median index is not present in the current genotype of the individual.

## 3.2. Demand Allocation Mechanism

In GACO demand points allocated to appropriate medians based on ant colony system algorithm. In GACO, when located at a demand point  $s$ , ant moves to a median  $u$  chosen according to *pseudorandom proportional rule* [18], given by equations (7) and (8), see Figure 2.

$$r = \begin{cases} \arg \max_{u \in C_s^k} \left\{ [\tau_{s,u}]^\alpha \cdot [\eta_{s,u}]^\beta \right\}, & \text{if } q \leq q_0; \\ R, & \text{otherwise;} \end{cases} \quad (7)$$

$$p_{s,u}^k = \frac{[\tau_{s,u}]^\alpha [\eta_{s,u}]^\beta}{\sum_{u \in C_s^k} [\tau_{s,u}]^\alpha [\eta_{s,u}]^\beta}, \quad \text{if } u \in C_s^k, \quad (8)$$

Where  $q$  is a random variable uniformly distributed in  $[0, 1]$ ,  $q_0$  ( $0 \leq q_0 \leq 1$ ) is a parameter, and  $R$  is a random variable selected according to the probability distribution given by Equation (8). The parameter  $\eta_{s,u} = 1/d_{s,u}$  is a heuristic value that is available a priori,  $\alpha$  and  $\beta$  are two parameters which determine the relative influence of the pheromone trail and the heuristic information, and  $C_s^k$  is the set of medians that has enough capacity to provide service to demand point  $s$ , when ant  $k$  being at demand point  $s$ . By this probabilistic rule, the probability of choosing a particular median increases with the value of the associated pheromone trail  $\tau_{s,u}$  and of the heuristic information value  $\eta_{s,u}$ .

The role of parameters  $\alpha$  and  $\beta$  is as follows. If  $\alpha = 0$ , the nearest medians are more likely to be selected, this corresponds to a greedy algorithm.

If  $\beta = 0$ , only pheromone amplification is at work, that is, only pheromone is used, without any heuristic bias. In other words, with probability  $q_0$  the ant makes the best possible move as indicated by the learned pheromone trails and the heuristic information (in this case, the ant is exploiting the learned knowledge), while with probability  $(1 - q_0)$  it performs a biased exploration of the arcs. Tuning the parameter  $q_0$  allows modulation of the degree of exploration and the choice of whether to concentrate the search of the system around the best-so-far solution or to explore other medians [18].

The applied updating rule has a similar framework to the one introduced in [19]. To propose a suitable updating rule here, pheromone trail levels are updated at the edges of each demand point ( $s$ ) and their median point ( $u$ ) as follows:

$$\tau_{s,u} = (1 - \rho) \cdot \tau_{s,u} + \sigma \cdot \mu_{s,u}; \quad (9)$$

$$\mu_{s,u} = \frac{\sum_{l \in D(u)} d_{l,u}}{d_{s,u}}, \quad u = \{1, \dots, p\}; \quad (10)$$

where  $\sum_{l \in D(u)} d_{l,u}$  is the sum of distances between each median point  $u$  and its allocated demand points  $D(u)$ . In Equation (10),  $\mu_{s,u}$  is a reasonable measure of allocating demand point  $s$  to median point  $u$ . The value of  $\tau_{s,u}$  is the history information which we want to retain from the last iteration. It could help retain partial profitable information and produce better solutions. The parameters  $\rho$  and  $\sigma$  control the relative weight of pheromone trail values.

After producing new candidate locations and allocating demand points to them, the next stage is evaluating the quality of the generated individuals. A chromosome will enter into next generation if it has fitness at least  $k$  ( $0 < k < 1$ ) percent of the best fitness of the current population and not duplicated in the new generating population. This criterion is imposed as an additional exploitation mechanism for refinement of the current solution.

#### 4. Computational Results

We used three sets of standard test instances which have been introduced in [20]. The first set A, includes 10 small size test problems, that there are 50 points with different demand values, and capacity of each point is 120 units, out of which 5 points should be selected as median among all candidates. In the second sets (B) which includes 10 medium size test problems, there are 100 points and 10 points should be selected as medians. The third set C includes 6 large scale problems. Computational experiments were carried out on a Xeon 3.2 GHz processor and 2 GB RAM, the algorithm was coded and executed in Java environment. Parameter values for the proposed method were tuned through several executions and finally are set as given:

population size = 150;  
 number of iterations = 500;  
 mutation probability = 0.2 ;  
 maximum number of chromosomes that are selected for mutation 60%;  
 $\alpha$ ,  $\beta$ ,  $\rho$  and  $\sigma$  are set to 0.5, 1.5, 0.5 and 0.7, respectively;  
 the ranking based-selection method is selected for parent selection, and  
 modified one point crossover is chosen for crossover.

The results are summarized in Tables 1, 2 and 3. The column named Best-known represents the best known

solution of the instance which was obtained in literature, and  $S_{best}$  refers to the best solution found by the proposed hybrid metaheuristic and  $Dev.$  (in %) is the relative gap between best-known solution and  $S_{opt}$ , according to Equation (11). The last column is referred to the average solution of proposed method in 10 runs.

$$Dev = \frac{S_{Best} - Best\_known}{Best\_known} \times 100. \quad (11)$$

Tab. 1. Results for the set A

Instance		GACO			
#	Best-Known	$S_{best}$	Time (s)	% Dev.	Ave.
1	713	713	6.2	0.0	713
2	740	740	5.4	0.0	740
3	751	751	7.0	0.0	751
4	651	651	9.5	0.0	651
5	664	664	15.0	0.0	664
6	778	778	7.8	0.0	778
7	787	787	11.0	0.0	787
8	820	820	10.1	0.0	820
9	715	715	14.2	0.0	715
10	829	829	17.6	0.0	829
Mean	-	-	10.38	0.0	-
Max.	-	-	17.60	0.0	-

Table 1 shows the proposed method could find optimal solution in all instances in very short computational time.

Tab. 2. Results for the set B

Instance		GACO			
#	Best-Known	$S_{best}$	Time (s)	% Dev.	Ave.
1	1006	1006	25.4	0.0	1006
2	966	968	34.0	0.5	971
3	1026	1026	29.4	0.0	1028
4	982	986	38.3	0.3	989
5	1091	1096	94.7	0.5	1100
6	954	957	74.9	0.3	960
7	1034	1034	98.1	0.0	1036
8	1043	1050	72.9	0.7	1054
9	1031	1037	89.1	0.6	1041
10	1005	1022	140.6	1.6	1028
Mean	-	-	69.7	0.4	-
Max.	-	-	140.6	1.6	-

Tab. 3. Results for the set C

Instances		ANN	GACO			
#	Size ( $p$ $n$ )	Best Solution	$S_{best}$	Time (s)	% Dev.	Ave.
1	25 500	122420	121319	111.8	-0.91	122884
2	30 600	60421	60376	196.4	-1.39	61112
3	50 708	120753	120185	519.2	-0.47	120716
4	57 818	139626	139618	635.2	-0.01	140081
5	60 920	166270	163332	474.9	-1.80	164711
6	70 1030	NA	173903	643.1	*	174541
Mean	-	-	-	430.1	-0.92	-
Max.	-	-	-	635.2	-1.80	-

### 5. Conclusion

This paper proposed a new solution method for the capacitated  $p$ -median problem, named GACO. In the proposed method, genetic algorithm component is used for median selection and demand points are allocated to selected medians based on pheromone trail concept in ant colony optimization considering the capacity limit of each median. The main idea and motivation for this is to guide and improve search process to faster converge toward promising area of search spaces. In order to show the validity of proposed hybrid algorithm, we compared the obtained results over the standard test problems. The results indicate that the proposed algorithm is an effective and efficient method for solving CPMP.

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