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RESEARCH PAPER

A Multi-Storehouse Supply Chain Considering Systems' Reliability and Free Shipping

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ABSTRACT

This paper aims to frame a two-player supply chain model with a production system's reliability influenced products' defection rate. Upon generating and inspecting the products, the producer reworks the defectives and sells the perfect and reworked items to a retailer providing him free products' delivery. The retailer stores both types of commodities in the respective showrooms of finite capacities and keeps the excess conforming products in a leased warehouse. Eventually, the formulation of these two partners' profit functions performed, and a numerical illustration demonstrates this model's applicability. Results shows, hiring a storehouse is profitable for the retailer and the deterioration of the production system's reliability impacts adversely on the manufacturer's profit.

KEYWORDS: S*upply chain; Inventory; Two-echelon; Imperfect production; Stochastic demand; Reliability; Warehouse; Pricing.*

1. Introduction

Today's competitive marketing environment compelled businesses or manufacturing organizations to find out strategies for the better management of issues related to adequate production rate, warehouse limitations, smooth customer service, and so forth. In this regard, supply chain management (SCM) provides an integrated way to handle all such issues, becoming an area of interest of many researchers, as well as practitioners.[1]–[9].

Practically, most production systems yield a fraction of defective products in the production period, harming the profitability of the manufacturer. Considering this issue, a recent study [10] regarded an investment for the improvement of the generating system's condition, aiming to lessen the number of defectives. Again, Manna et. al. [11] assumed the production system can shifts to a non-controllable state at any production point, generating a proportion of faulty outputs. An SCM [12] considered the regeneration of the returned and the defective products, intending to make them perfect. Another study [13] assumed that when the system transfers to an uncontrolled state, the production system's reliability can be enhanced using some standby elements the production system mostly depends on.

To reduce the imperfect production induced loss, manufacturers are noticed to rework the defective products to make these usable. In a study, Taleizadeh et al. [14] showed a decrease in the supply chain system's cost when the faulty outputs are reworked than scraping the defectives or selling these at a reduced price. A closed-loop SCM [15] considered rework of used products, including those with manufacturing defects, aiming to reduce environmental pollution. Again, another study [16] assumed two different strategies, the first of which uses the main production system for reworking the defectives and the second one employs a new reworking unit for the same purpose. Moreover, Khanna et al. [17] found the rework of most faulty items

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beneficial to the producer, which affects the consumers' demand positively.

Besides the imperfect production system and the rework of faulty products produced by it, another important factor in the inventory management system is the storage space limitations. In some practical situations, retailers show interest in bulk purchasing for different purposes, such as a rebate to buy the products in a mass, reduction in replenishment and transportation cost, high demand rate of an item, and so on. Therefore, due to the limited capacity of the showroom, another warehouse is used to store the excess items. Researchers considered two warehouse facility while developing their model, among which [18]–[23] are worth mentioning.

From the above discussion, although we found some authors to consider the production of defective items by the faulty machinery system, best known to the authors of this paper, no study studied the influence of the production system's reliability on the proportion of defective items produced. But, in the reality, we see a connection between these two factors. Furthermore, none explored the production and the pricing scheme in a two-member SCM considering a leased storage unit of the retailer, although, in today's busy marketplace, space limitation bounds the retailers to rent a warehouse to store the excess products. Therefore, this paper aims to study these underexplored areas.

machinery system's reliability. The manufacturer sells conforming items to the retailer in bulk after inspection of the products. He sells the outputs with minor defects as the perfect products, after rework. But the items with major faultiness which cannot be restored to perfect products are sold as defective goods to the same retailer after making these usable. These types of products have no guaranty that they will operate successfully in their useful life, therefore, he sells these at a reduced price. The manufacturer provides free delivery of the products to the retailer to encourage him to purchase more. The retailer sells both types of items from the respective showrooms. Due to the space limitation of the showroom of perfect products, he hires extra storage to keep the excess commodities. Eventually, the profit functions of each member have been constructed and this model is solved by the Leader-follower relationship strategy.

2. Symbol's and Hypotheses

The following symbols are adopted in this model

$I_{1m}(t)$	Manufacturer's conforming items' Inventory level at any time t
$I_{2m}(t)$	Inventory level for less perfect quality items for the manufacturer at any time t
$I_{1r}(t)$	Retailer's conforming items inventory level at any time t
$I_{2r}(t)$	Retailer's non-conforming items inventory level at any time t
t_1	Production period
z	Proportion of faulty outputs production per unit time
D_r	Retailer's conforming products' demand rate
$D_r{}'$	Retailer's non-conforming products' demand rate
D_c	Retailer's selling rate of good products, where, $D_c = x - yS_r, S_r$ is the products retail price
$D_c{}'$	Retailer's selling rate of faulty products
A_m	Manufacturer's set up cost
λ	Reliability parameter
C_p	Products generation cost per unit time
C_m	Manufacturer's screening cost per item.
f_c	Producer's reworking cost per unit
h_m	Manufacturer's conforming outputs' holding cost per unit per unit time
$h_m{}'$	Manufacturer's non-conforming outputs' holding cost per unit per unit time
S_m	Manufacturer's per unit conforming item's selling price
$S_m{}'$	Manufacturer's per unit non-conforming item's selling price

S_r' Retailer's per unit non-conforming item's selling price
A_r Retailer's ordering cost
h_r Retailer's per unt perfect commodity's holding cost per unit time
h_r' Retailer's per unt faulty commodity's holding cost per unit time
h_{rs} Retailer's per unt perfect commodity's holding cost per unit time in the leased warehouse
d_m Manufacturer's per unit conforming item's transportation cost to deliver these to the retailer
d_m' Manufacturer's per unit non-conforming item's transportation cost to deliver these to the retailer
W_1 Capacity of retailer's conforming commodities' showroom
S Capacity of the retailer's secondary (leased) storehouse.

Decision Variables

p_m Manufacturer's production rate per unit time
S_r Retailer's selling price of per unit perfect item

Model's hypotheses:

The following hypotheses are employed to develop this model

i) The manufacturer's faulty generating system yields a mixture of conforming and defective outputs. But, instead of reworking these defectives to make them usable, these items are less reliable and may fail at any time during their serviceable life, hurting the company's reputation. Therefore, he sells these to the retailer at a reduced price.
ii) The retailer's buying rate depends on his selling rate i.e. $D_r = kD_c$.
iii) The generating system's reliability is termed as the probability that the system will operate successfully under the conditions of the intended application, which is generally decreases with time. So, the reliability is $R(t) = e^{-\lambda t}$, where λ is the reliability parameter. We have considered z to be dependent on the generating system's reliability. Therefore, we have taken $z = a - bR(t) = a - be^{-\lambda t}$.

3. Model Formulation

This section aims to formulate mathematical model for both the partners of this SCM. Figure 1 depicts this model's pictorial representation.

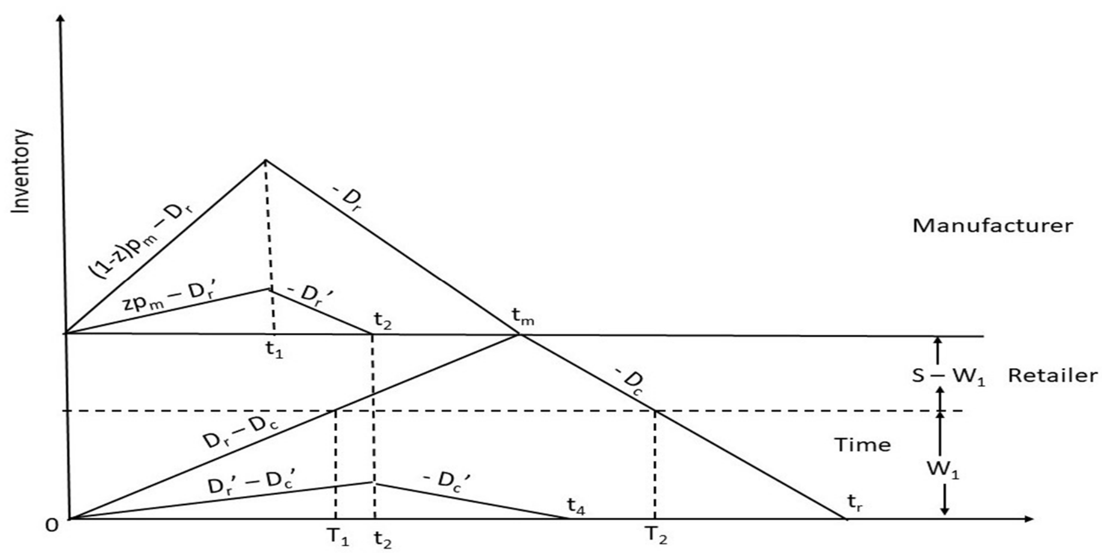

Fig. 1. Pictorial depiction of this SCM

Here, z is the proportion of generating faulty products, therefore, (1-z) is the percentage of perfect items production per unit time. So, the manufacturer's conforming products' inventory level accumulates at a rate $(1\text{-}z)p_m - D_r$, where p_m is the product generation rate and D_r is the retailer's demand. Similarly, his non-conforming items' inventory accumulates at a rate $zp_m - D_r'$. The manufacturing ends at time $t = t_1$, then the producer's stock level for both perfect and defective outputs deplete due to selling and reach the zero level at time t_m and t_2 respectively. Again, the retailer buy's both the perfect and defective items in the period t_m and t_2 respectively and sells these to the customers at a rate D_c and D_c' respectively. He stores the conforming products received during $[0, T_1]$ in his showroom and after that he starts keeping the excess products in the rented warehouse and continues up to time t_m. After that the inventories of these two types of products start decreasing to meet the customers' demand and reach the zero level at time t_r and t_4 respectively.

3.1. Manufacturer's model

For conforming outputs:

The manufacturer generates the items during $[0, t_1]$ at a rate p_m and sells the superior quality outputs to the retailer at a rate D_r, after inspecting these. The stock level of these outputs starts decreasing after the production period is over and exhausted at $t = t_m$. These changes in manufacturer's inventory level is given by

$$\frac{dI_{1m}(t)}{dt} = \begin{cases} (1-z)p_m - D_r, & 0 \le t \le t_1 \\ -D_r, & t_1 \le t \le t_m \end{cases} \quad (1)$$

(where $z = \left(a - be^{-\lambda t}\right)$ and $D_r = kD_c$), with the boundary conditions –

$$I_{1m}(0) = 0, I_{1m}(t_m) = 0$$

Solving the equation (1) we have –

$$I_{1m}(t) = \begin{cases} (p_m - D_r)t - \left\{at + \frac{b}{\lambda}\left(e^{-\lambda t} - 1\right)\right\}p_m, & 0 \le t \le t_1 \\ -D_r(t - t_m), & t_1 \le t \le t_m \end{cases} \quad (2)$$

The manufacturer purchases Q units of raw material to produce the products at a rate D_m up to time t_1. So,

$$t_1 = \frac{Q}{D_m} \quad (3)$$

Now, from the continuity condition of $I_{1m}(t)$ at $t = t_1$ we have –

$$(p_m - D_r)t_1 - \left\{at_1 + \frac{b}{\lambda}\left(e^{-\lambda t_1} - 1\right)\right\}p_m = -D_r(t_1 - t_m)$$

$$\Rightarrow D_r t_m = \left\{(1-a)t_1 - \frac{b}{\lambda}(1 - \lambda t_1 - 1)\right\}p_m$$

$$\Rightarrow t_m = \frac{(1 - a + b)p_m t_1}{D_r}$$

$$\Rightarrow t_m = \frac{(1-a+b)p_m Q}{D_m D_r} \quad \text{(using equation 3)} \quad (4)$$

Now, the set-up cost $= A_m$

Making charge $= C_p p_m t_1 = \frac{C_p p_m Q}{D_m}$

Inspection charge $= C_m p_m t_1 = \frac{C_m p_m Q}{D_m}$

Holding expense –

$$= h_m\left[\int_0^{t_1} I_{1m}(t)dt + \int_{t_1}^{t_m} I_{1m}(t)dt\right]$$

$$= h_m\left[\int_0^{t_1}\left\{(p_m - D_r)t - \left\{at + \frac{b}{\lambda}\left(e^{-\lambda t} - 1\right)\right\}p_m\right\}dt + \int_{t_1}^{t_m}\{-D_r(t - t_m)\}dt\right]$$

$$= h_m\left[\frac{(p_m - D_r)t_1^2}{2} - \left\{\frac{at_1^2}{2} - \frac{b}{\lambda}\left(\frac{e^{-\lambda t_1}}{\lambda} - \frac{1}{\lambda} + t_1\right)\right\}p_m + \frac{D_r(t_m - t_1)^2}{2}\right]$$

$$= \frac{h_m}{2}\left[(p_m - D_r)t_1^2 - \left\{at_1^2 - \frac{2b}{\lambda}\left(\frac{\lambda t_1^2}{2} - \frac{\lambda^2 t_1^3}{6}\right)\right\}p_m + D_r(t_m - t_1)^2\right]$$

$$= \frac{h_m}{2}\left[\frac{(p_m - D_r)Q^2}{D_m^2} - \left\{a - b + \frac{b\lambda Q}{3D_m}\right\}\frac{p_m Q^2}{D_m^2} + \frac{D_r\{(p_m - D_r) - (a-b)p_m\}^2 Q^2}{D_m^2 D_r^2}\right]$$

$$= \frac{h_m p_m Q^2}{2D_m^2}\left[\frac{\{(p_m - D_r) - (a-b)p_m\}(1-a+b)}{D_r} - \frac{b\lambda Q}{3D_m}\right]$$

Shipping expense

$$= d_m\left[\int_0^{t_1} I_{1m}(t)dt + \int_{t_1}^{t_m} I_{1m}(t)dt\right]$$

$$= \frac{d_m p_m Q^2}{2D_m^2}\left[\frac{\{(p_m - D_r) - (a-b)p_m\}(1-a+b)}{D_r} - \frac{b\lambda Q}{3D_m}\right]$$

For non-conforming outputs:

The following differential equations describes the changes in the manufacturer's inventory level of imperfect products

$$\frac{dI_{2m}(t)}{dt} = \begin{cases} zp_m - D_r', & 0 \le t \le t_1 \\ -D_r', & t_1 \le t \le t_2 \end{cases} \quad (5)$$

(where $= \left(a - be^{-\lambda t}\right)$), with the boundary conditions $I_{2m}(0) = 0, I_{2m}(t_2) = 0$

Solving the equation (5) we have –

$$I_{2m}(t) = \begin{cases} (ap_m - D_r')t + \frac{bp_m}{\lambda}\left(e^{-\lambda t} - 1\right), & 0 \le t \le t_1 \\ -D_r'(t - t_2), & t_1 \le t \le t_2 \end{cases} \quad (6)$$

From the continuity condition of $I_{2m}(t)$ at $t = t_1$ we have –

$$(ap_m - D_r')t_1 + \frac{bp_m}{\lambda}\left(e^{-\lambda t_1} - 1\right) = -D_r'(t_1 - t_2)$$

$$\Rightarrow D_r' t_2 = ap_m t_1 + \frac{bp_m}{\lambda}(1 - \lambda t_1 - 1)$$

$$\Rightarrow t_2 = \frac{(a-b)p_m t_1}{D_r'} = \frac{(a-b)p_m Q}{D_m D_r'} \quad (7)$$

Holding cost of the manufacturer –

$$= h_m'\left[\int_0^{t_1} I_{2m}(t)dt + \int_{t_1}^{t_2} I_{2m}(t)dt\right]$$

$$= h_m'\left[\int_0^{t_1}\left\{(ap_m - D_r')t + \frac{bp_m}{\lambda}\left(e^{-\lambda t} - 1\right)\right\}dt + \int_{t_1}^{t_2}\{-D_r'(t - t_2)\}dt\right]$$

$$= h_m'\left[\frac{(ap_m - D_r')t_1^2}{2} - \frac{bp_m}{\lambda}\left(\frac{e^{\lambda t_1}}{\lambda} - \frac{1}{\lambda} + t_1\right) + \frac{D_r'}{2}(t_2 - t_1)\right]$$

$$= \frac{h_m'}{2}\left[(ap_m - D_r')t_1^2 - bp_m\left(t_1^2 - \frac{\lambda t_1^3}{3}\right) + D_r'(t_2 - t_1)^2\right]$$

$$= \frac{h_m' p_m Q^2}{2D_m^2}\left[\frac{\{(a-b)p_m - D_r'\}(a-b)}{D_r'} + \frac{b\lambda Q}{3D_m}\right]$$

Shipping charge

$$= d_m'\left[\int_0^{t_1} I_{2m}(t)dt + \int_{t_1}^{t_2} I_{2m}(t)dt\right]$$

$$= \frac{d_m' p_m Q^2}{2D_m^2}\left[\frac{\{(a-b)p_m - D_r'\}(a-b)}{D_r'} + \frac{b\lambda Q}{3D_m}\right]$$

Cost of rework $= f_c D_r' t_2 = \frac{f_c(a-b)p_m Q}{D_m}$

or equivalently

$$f_c\int_0^{t_1}\left(a - be^{-\lambda t}\right)p_m dt = f_c p_m\left(at_1 + \frac{b}{\lambda}\left(e^{-\lambda t_1} - 1\right)\right) = f_c p_m\left(at_1 + b(\lambda t_1 - t_1)\right)$$

Total revenue from sales

$= S_m D_r t_m + S'_m D'_r t_2 = \frac{S_m(1-a+b)p_m Q}{D_m} + \frac{S'_m(a-b)p_m Q}{D_m}$

Therefore, the manufacturer's total expense –

$$TC_m = \frac{1}{t_m}\left[A_m + A'_m + \frac{(C_p+C_m)p_m Q}{D_m} + \frac{f_c(a-b)p_m Q}{D_m} + \frac{(h_m+d_m)p_m Q^2}{2D_m^2}\left\{\frac{\{(p_m-D_r)-(a-b)p_m\}(1-a+b)}{D_r} - \frac{b\lambda Q}{3D_m}\right\} + \frac{(h'_m+d'_m)p_m Q^2}{2D_m^2}\left\{\frac{\{(a-b)p_m-D'_r\}(a-b)}{D'_r} + \frac{b\lambda Q}{3D_m}\right\}\right]$$

$$\Rightarrow TC_m = \frac{D_m D_r(A_m+A'_m)}{(1-a+b)p_m Q} + \frac{D_r(C_p+C_m)}{(1-a+b)} + \frac{f_c(a-b)D_r}{(1-a+b)} + \frac{(h_m+d_m)D_r Q}{2(1-a+b)D_m}\left\{\frac{\{(p_m-D_r)-(a-b)p_m\}(1-a+b)}{D_r} - \frac{b\lambda Q}{3D_m}\right\} + \frac{(h'_m+d'_m)D_r Q}{2(1-a+b)D_m}\left\{\frac{\{(a-b)p_m-D'_r\}(a-b)}{D'_r} + \frac{b\lambda Q}{3D_m}\right\} \quad (8)$$

Therefore, total profit earned by the manufacturer –

$$\pi_m = \frac{1}{t_m}\left(\frac{S_m(1-a+b)p_m Q}{D_m} + \frac{S'_m(a-b)p_m Q}{D_m}\right) - TC_m$$

$$\Rightarrow \pi_m = S_m D_r + \frac{S'_m D_r(a-b)}{(1-a+b)} - \left\{\frac{D_m D_r(A_m+A'_m)}{(1-a+b)p_m Q} + \frac{D_r(C_p+C_m)}{(1-a+b)} + \frac{f_c(a-b)D_r}{(1-a+b)} + \frac{(h_m+d_m)D_r Q}{2(1-a+b)D_m}\left\{\frac{\{(p_m-D_r)-(a-b)p_m\}(1-a+b)}{D_r} - \frac{b\lambda Q}{3D_m}\right\} + \frac{(h'_m+d'_m)D_r Q}{2(1-a+b)D_m}\left\{\frac{\{(a-b)p_m-D'_r\}(a-b)}{D'_r} + \frac{b\lambda Q}{3D_m}\right\}\right\} \quad (9)$$

3.2. Retailer's model

For conforming items:

The following differential equations describes the changes in retailer's inventory level of his perfect products

$$\frac{dI_{1r}(t)}{dt} = \begin{cases} D_r - D_c, & 0 \le t \le t_m \\ -D_c & t_m \le t \le t_r \end{cases} \quad (10)$$

with boundary conditions –

$$I_{1r}(t) = \begin{cases} 0, & at\ t = 0 \\ W_1 & at\ t = T_1 \\ S & at\ t = t_m \\ W_1 & at\ t = T_2 \\ 0 & at\ t = t_r \end{cases} \quad (11)$$

The solutions of equation (10) are-

$$I_{1r}(t) = \begin{cases} (D_r - D_c)t, & 0 \le t \le T_1 \\ W_1 + (D_r - D_c)(t - T_1) & T_1 \le t \le t_m \\ S - D_c(t - t_m) & t_m \le t \le T_2 \\ W_1 - D_c(t - T_2) & T_2 \le t \le t_r \end{cases} \quad (12)$$

Now, from the continuity condition of $I_{1r}(t)$ at $t = T_1$ we have –

$$(D_r - D_c)T_1 = W_1$$
$$\Rightarrow T_1 = \frac{W_1}{D_r - D_c} \quad (13)$$

Again, from the second equation of equation (12), $I_{1r}(t_m) = S$ gives –

$$S = W_1 + (D_r - D_c)(t_m - T_1)$$
$$\Rightarrow (t_m - T_1) = \frac{S - W_1}{(D_r - D_c)} \quad (14)$$

From the third equations of the equation (12), $I_{1r}(T_2) = W_1$ gives,

$$W_1 = S - D_c(T_2 - t_m)$$
$$\Rightarrow T_2 = \frac{S - W_1}{D_c} + \frac{S}{D_r - D_c} \quad (15)$$

Again, from the equation (12), $I_{1r}(t_r) = 0$ gives,

$$0 = W_1 - D_c(t_r - T_2)$$
$$\Rightarrow t_r = \frac{W_1}{D_c} + T_2 = \frac{SD_r}{D_c(D_r - D_c)} \quad (16)$$

Now, purchasing cost of the retailer = $S_m D_r t_m = \frac{S_m(1-a+b)p_m Q}{D_m}$

Holding cost of the retailer in the showroom –

$$= h_r\left[\int_0^{T_1} I_{1r}(t)dt + \int_{T_1}^{T_2} W_1 dt + \int_{T_2}^{t_r} I_{1r}(t)dt\right]$$

$$= h_r\left[\int_0^{T_1}(D_r - D_c)t dt + \int_{T_1}^{T_2} W_1 dt + \int_{T_2}^{t_r}\{W_1 - D_c(t-T_2)\}dt\right]$$

$$= h_r\left[\frac{(D_r - D_c)T_1^2}{2} + W_1(T_2 - T_1) + W_1(t_r - T_2) - \frac{D_c(t_r - T_2)^2}{2}\right]$$

$$= h_r\left[\frac{(D_r - D_c)T_1^2}{2} + W_1(t_r - T_2 + T_2 - t_m + t_m - T_1) - \frac{D_c(t_r - T_2)^2}{2}\right]$$

$$= \frac{h_r D_r W_1(2S - W_1)}{2D_c(D_r - D_c)}$$

Holding cost of the retailer in the secondary warehouse (hired) –

$$= h_{rs}\left[\int_{T_1}^{t_m}\{I_{1r}(t) - W_1\}dt + \int_{t_m}^{T_2}\{I_{1r}(t) - W_1\}dt\right]$$

$$= h_{rs}\left[\int_{T_1}^{t_m}\{(D_r - D_c)(t - T_1)\}dt + \int_{t_m}^{T_2}\{(S - W_1) - D_c(t - t_m)\}dt\right]$$

$$= h_{rs}\left[\frac{(D_r - D_c)(t_m - T_1)^2}{2} + (S - W_1)(T_2 - t_m) - \frac{D_c(T_2 - t_m)^2}{2}\right]$$

$$= \frac{h_{rs}(S - W_1)^2 D_r}{2D_c(D_r - D_c)}$$

For non-conforming items:

The changes of retailer's imperfect products' inventory are given by

$$\frac{dI_{2r}(t)}{dt} = \begin{cases} D_r' - D_c' & 0 \le t \le t_2 \\ -D_c' & t_2 \le t \le t_4 \end{cases} \quad (17)$$

with boundary conditions $I_{2r}(0) = 0 \; and \; I_{2r}(t_4) = 0$

The solutions of the system of equations (17) are -

$$I_{2r}(t) = \begin{cases} (D_r' - D_c')t & 0 \le t \le t_2 \\ -D_c'(t - t_4) & t_2 \le t \le t_4 \end{cases} \quad (18)$$

Employing continuity condition of $I_{2r}(t)$ at t = t_2 we obtain –

$$(D_r' - D_c')t_2 = -D_c'(t_2 - t_4)$$

$$\Longrightarrow t_4 = \frac{D_r' t_2}{D_c'} = \frac{(a-b)p_m Q}{D_m D_c'} \quad (19)$$

Therefore, total cost of the retailer –

Expense for holding the items in the showroom is -

$$= h_r'[\int_0^{t_2} I_{2r}(t)\, dt + \int_{t_2}^{t_4} I_{2r}(t)dt]$$

$$= h_r'\left[\int_0^{t_2}(D_r' - D_c')t\, dt + \int_{t_2}^{t_4}\{-D_c'(t - t_4)\}dt\right]$$

$$= \frac{h_r'}{2}[(D_r' - D_c')t_2{}^2 + D_c'(t_2 - t_4)^2]$$

$$= \frac{h_r'(a-b)^2(D_r' - D_c')p_m^2 Q^2}{2D_m^2 D_r' D_c'}$$

Now, purchasing cost of the defective items $= S_m' D_r' t_2 = \frac{S_m'(a-b)p_m Q}{D_m}$

$$TC_r = \frac{1}{t_r}\left[A_r + A'_r + \frac{S_m(1-a+b)p_mQ}{D_m} + \frac{S'_m(a-b)p_mQ}{D_m} + \frac{h_rD_rW_1(2S-W_1)}{2D_c(D_r-D_c)} + \frac{h_{rs}(S-W_1)^2D_r}{2D_c(D_r-D_c)} + \frac{h_r'(a-b)^2(D_r'-D_c')p_m^2Q^2}{2D_m^2D'_rD'_c}\right]$$

where $D_r = kD_c$

$$\Rightarrow TC_r = \frac{(k-1)D_c(A_r + A'_r)}{Sk} + \frac{S_m(k-1)D_c(1-a+b)p_mQ}{SD_mk} + \frac{S'_m(k-1)D_c(a-b)p_mQ}{SD_mk} + \frac{h_rW_1(2S-W_1)}{2S} + \frac{h_{rs}(S-W_1)^2}{2S} + \frac{h_r'(a-b)^2(D_r'-D_c')(k-1)D_cp_m^2Q^2}{2SD_m^2D'_rD'_ck} \quad (20)$$

where, $D_c = (x - yS_r)$

Therefore, total profit of the retailer –

$$\pi_r = S_rD_c + \frac{S'_r(a-b)(k-1)D_c}{SD_mk} - \left\{\frac{(k-1)D_c(A_r + A'_r)}{Sk} + \frac{S_m(k-1)D_c(1-a+b)p_mQ}{SD_mk} + \frac{S'_m(k-1)D_c(a-b)p_mQ}{SD_mk} + \frac{h_rW_1(2S-W_1)}{2S} + \frac{h_{rs}(S-W_1)^2}{2S} + \frac{h_r'(a-b)^2(D_r'-D_c')(k-1)D_cp_m^2Q^2}{2SD_m^2D'_rD'_ck}\right\} \quad (21)$$

4. Solution Methodology

In the leader-follower relationship approach, the players' actions are sequential, where the leading personality makes his decisions first and then the others decide subsequently. Here, the members keep a decent relationship relying upon what each of them is willing to exchange [24], [25]. In this model, the manufacturer provides free delivery of the products to the retailer to encourage him to buy more products. Therefore, the manufacturer's dominating role regards him as the leader and considers the retailer as his follower. Here, first, the manufacturer decides on his decision variable by optimizing his total cost, and then, the retailer decides on his decision variable depending on the manufacturer's decision. Here, the production rate and the selling price are the decision variables of the manufacturer and the retailer, respectively.

Differentiating manufacturer's profit function π_m (from equation (9)) twice with respect to p_m we have –

$$\frac{\partial(\pi_m)}{\partial p_m} = \frac{D_mD_r(A_m + A'_m)}{(1-a+b)p_m^2Q} - \frac{(h_m+d_m)Q(1-a+b)}{2D_m} - \frac{(h'_m+d'_m)D_rQ(a-b)^2}{2(1-a+b)D_mD'_c} \quad (22)$$

And,

$$\frac{\partial(\pi_m)}{\partial p_m} = -\frac{2D_mD_r(A_m + A'_m)}{(1-a+b)p_m^3Q} < 0$$

This proves the concavity of the manufacturer's profit function and his optimum production rate is determined by equating the right side of equation (22) to zero.

Again, differentiating the retailer's profit function π_r in respect of S_r we get –

$$\frac{\partial(\pi_r)}{\partial S_r} = (x - 2yS_r) - \frac{S_r'(a-b)(k-1)y}{SD_mk} + y\left\{\frac{(k-1)(A_r + A_r')}{Sk} + \frac{S_m(k-1)(1-a+b)p_mQ}{SD_mk} + \frac{S_m'(k-1)(a-b)p_mQ}{SD_mk} + \frac{{h_r}'(a-b)^2({D_r}' - {D_c}')(k-1)p_m^2Q^2}{2SD_m^2D_r'D_c'k}\right\} \quad (23)$$

And,

$$\frac{\partial^2(\pi_r)}{\partial S_r^2} = -2y < 0$$

This proves the concavity of the retailer's profit function and his optimum selling price is obtained by equating the right side of the equation (23) to zero.

5. Numerical Example

Example-1: A manufacturer produces plastic water tanks. During the production period, a proportion of defective items produce due to faulty machinery system. He sorts all the defective products, reworks all these and sells these at a reduced price. A retailer procures both types of products from him and sells these from the respective showrooms. The reworked products' showroom is large enough to store the products, but, as he purchases the perfect products in a bulk, a rented storage facility is used to keep the extra items. We considered the following hypothetical values of the variables with appropriate units.

$A_m = 300, A_m' = 150, Q = 400, D_m = 100, C_m = 1, f_c = 4, a = 0.4, b = 0.05, h_m = 0.5, h_m' = 0.3, d_m = 0.3, d_m' = 0.2, \lambda = 0.04, D_r' = 3, D_r = 36, c_p = 20, A_r = 200, A_r' = 150, h_r = 2, h_r' = 0.5, D_c' = 2, x = 75, y = 0.5, S_m = 100, S_m' = 50, S = 90, k = 1.8, S_r' = 70$

Table 1 depicts the optimum results

Tab. 1. Optimum results.

p_m	S_r	π_m	π_r
43.44	107.39	1694.66	832.91

6. Sensitivity Analysis

Here, the sensitivity of the chain's partners' profit, distinctive associated cost, and products' selling price to different parameters is studied and the results are presented in tables 2 -4.

Tab. 2. Results when λ changes.

λ	Reworking cost of manufacturer	Total profit of manufacturer
0.03	77.87	2011.24
0.04	77.98	1694.66
0.05	78.09	1272.89
0.06	78.20	903.72

Tab. 3. Results when S changes.

S	W_1	Selling price of retailer (S_r)	Holding cost of retailer (for perfect items)	Total profit of retailer
70	40	116.65	62.93	493.20

75	40	113.87	66.01	586.56
80	40	111.44	69.00	674.31
85	40	109.30	71.90	756.36
90	40	107.39	74.72	832.91

Tab. 4. Results when W_1 changes.

S	W_1	Selling price of retailer (S_r)	Holding cost of retailer (for perfect items)	Total profit of retailer
90	40	107.39	74.72	832.91
90	45	107.39	77.62	830.00
90	50	107.39	80.22	827.41
90	55	107.39	82.51	825.12
90	60	107.39	84.50	823.13

6.1. Observations

The following observations witnessed from the above tables –

i) The producer's reworking cost escalates with the increase in reliability parameter λ, but his profit shrinks rapidly (from table 2). These findings are quite practical as the equation $z = a - be^{-\lambda t}$ shows the number of defective outputs increases proportionately with λ, resulting in a rise in the reworking cost, and the manufacturer's profit declines as a consequence.

ii) The holding cost increases proportionately with the capacity of the secondary warehouse (leased), but the retail price decreases (table 3). These findings are quite practical because, as the retailer procures the products in a bulk reducing the replenishment cost of the items, he can sell these products at a comparatively lower price than before. Again, as the retailer hired this storage facility on a rental basis, the carrying cost increases with the increase in its size. Moreover, the retailer may gain more profit in this case, which fulfils the main aim of the strategy of hiring such a storehouse.

iii) Table 4 depicts the holding charges for conforming items to vary proportionately with its specified showroom (W_1), but the retailer's profit varies inversely, and the retail price remains fixes with this parameter.

7. Conclusions

This manufacturer-retailer SCM with retailer's three storehouses considers generating system's reliability dependent rate of producing defectives. The producer sorts and reworks all the defectives and sells the perfect and reworked items to the same retailer. The retailer owns two different showrooms for both types of products to market these, but the capacity limitations bound him to recruit a leased storage facility to store the excess conforming commodities. This model proves this strategy profitable for the retailer and shows the production system's reliability deterioration impacts adversely the manufacturer's profit.

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