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RESEARCH PAPER

Multi-Product Constrained Economic Production Quantity Models for Imperfect Quality Items with Rework

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ABSTRACT

One of the basic assumptions of classical production-inventory models is that all products are of perfect quality. However, in real manufacturing situations, the production of defective items is inevitable, and a fraction of the items produced may be naturally imperfect. In fact, items may be damaged due to production and/or transportation conditions in the manufacturing process. On the other hand, some reworkable items exist among imperfect items that can be made perfect by additional processing. In addition, the classical production-inventory models assume that there is only one product in the system and that there is an unlimited amount of resources. However, in many practical situations, several products are produced and there are some constraints related to various factors such as machine capacity, storage space, available budget, number of allowable setups, etc. Therefore, we propose new constrained production-inventory models for multiple products where the manufacturing process is defective and produces a fraction of imperfect items. A percentage of defective items can be reworked, and these products go through the rework process to become perfect and return to the consumption cycle. The goal is to determine economic production quantities to minimize the total cost of the system. The analytical solutions are each derived separately by Lagrangian relaxation method, and a numerical example is presented to illustrate and discuss the procedure. A sensitivity analysis is performed to investigate how the variation in the inputs of the models affects the total cost of the inventory system. Finally, some research directions for future works are discussed.

KEYWORDS: *Economic production quantity; Imperfect quality items; Rework; Multi-product multi-constraint; Production-inventory planning.*

1. Introduction

In recent decades, both academics and practitioners have shown that one of the most important problems in manufacturing companies is the decision on production and inventory planning [1]. Until now, various issues in the field of production and inventory control systems have been presented to address these interrelated problems. Manufacturing companies work to meet external demands and carefully adjust their production plans in order to meet the customer demands. Proper inventory planning requires two decisions: (i) how large an item should be produced/ordered, and (ii) when should an item be produced/ordered[2]. The first decision help managers determine production/order lot size, and the second one ensures accurate timing of inventory. The economic order quantity (EOQ) is the first lot-sizing model that has been used extensively to determine the optimal order lot size. The main assumption of this model is that orders are received by the buyer immediately at the moment. It finds the optimal strategy by creating a balance between holding and ordering costs. EOQ has been extended to the economic production quantity (EPQ) model under production conditions, where production occurs at a finite rate, and items are received gradually. The objective is to determine the optimal production lot size to minimize the total cost,

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including production setup cost and inventory cost. In recent years, many studies have been presented in the field of inventory planning and control, in which the classical inventory models have been extended in various directions to deal with more realistic and practical situations in manufacturing companies [1] . One of the main assumptions of classical inventory models is that all items produced are of perfect quality at all times. In real-world situations, production processes are not necessarily perfect, and the production of defective items is inevitable [3-6]. A production process is affected by several undesirable factors and thus, it is rarely possible for a machine/production system to produce items of perfect quality every time. Moreover, in an imperfect production process such as in the textile, glass, and chemical industries, a rework process for imperfect items is typical [7]. The perfect items go into consumption, and the imperfect items with unacceptable quality are discarded as rejects. The remaining imperfect items are reworked to return to the desired quality. The quality-related decisions of a produced item are made by quality control experts in manufacturing companies.

The impact of imperfect items on manufacturing processes has been investigated in the literature [4]. One of the earliest works in this research area was presented by Salameh and Jaber [3], in which they extended the EOQ model with imperfect quality items. The defective items were identified by a 100% inspection policy and sold at a lower price. After that, many authors tried to make the model more realistic by considering real-world situations. Shawky and Abou-El-Ata [8] proposed a constrained production quantity model under a trade policy. Ben-Daya and Rahim [9] studied a multi-stage imperfect production process in which an inspection process was performed to filter and discard the defective items in each stage. Jamal, et al. [10] proposed an inventory model where all defective items can be perfectly reworked. Ben-Daya, et al. [11] considered inventory inspection models with the no inspection, sample inspection, and full inspection policies. Teng and Yang [12] presented and discussed a constrained deterministic inventory model with time-varying demand and cost under generalized holding costs. Ojha, et al. [13] determined the optimal lot size quantity for an imperfect production process under the assumption that all imperfect items are reworkable. Jans and Degraeve [14] presented an overview of dynamic lot-sizing problems with operational and tactical considerations. Biswas and Sarker [15] discussed rework of imperfect items during the production cycle and assumed that production segmentation can be performed along the cycle. Besides, rework of items was considered an imperfect process. Sarker, et al. [16] worked on two models for dealing with reworkable items in a multi-stage production process. The first policy proposed rework of items within the same cycle, while the second proposed accumulation of defective products for rework in a given time period. The superior performance of the second case was confirmed when the holding cost of work-in-process (WIP) items was low. Cárdenas-Barrón [17] extended the EPQ model for reworkable imperfect items by formulating the planned backorder in the problem. Lin [18] integrated the concept of powerful buyer (push system) into the EOQ model for defective items. This model considered multiple delivery police, salvage of defective items, and quantity discount as the options to satisfy the powerful customer. Barzoki, et al. [19] considered an imperfect production process where rework items are fully qualified. Wee and Widyadana [20] presented an EPQ model with rework and preventive maintenance operations for deteriorating items. Two cases of the model were formulated in terms of probability distribution of preventive maintenance time. Yoo, et al. [21] discussed inventory with imperfect production and different inspection options. Wee, et al. [22] relaxed the assumptions about the inspection rate in the EPQ for imperfect items by developing three cases where the production rate can be less than or equal to the inspection rate. Krishnamoorthi and Panayappan [23] studied rework for the production process and evaluated the sales return of poor quality items. Ameli, et al. [24] developed a modified entropic imperfect EPQ model for deterioration items under fuzzy inflation and discount rate. Another assumption in this inventory system was the price dependent demand rate. Sarkar, et al. [25] derived the optimal ordering policy of an imperfect inventory model based on the assumptions of delay in payment and stochastic lead time. Jaber, et al. [26] developed two cases of imperfect EOQ. The proposed policies were repairing defective items and substitution by purchasing from a local supplier. Hauck and Vörös [27] proposed the possibility of investment to speed up the inspection process in the imperfect EOQ model. They also developed a new case where the production quality state can be changed along with the cycles. A fuzzy extension of EPQ for imperfect quality items was presented by Kumar and Goswami [28], who discussed constraints on the allowable shortage and available space of the inventory system. They considered both salvage and rework policies for imperfect items, and a particle swarm optimization approach was used to solve the problem. Al-Salamah [29] studied an imperfect EOQ model under two inspection policies: destructive and non-destructive. Two types of inspection errors were considered to make the model more realistic. Rezaei [30] reviewed the 100% inspection policy for inventory systems with imperfect quality items and concluded that it is more beneficial to have a sampling inspection plan for quality assessment of the received lot. Mahata [31] investigated a new EPQ model with partial order shortage and fuzzy cost parameters. Some other features of this work were random shifting of the machine to the out-of-control state, rework of defective products, and learning during production operation. Aslani, et al. [32] presented an inventory system with partial backordering and random yield production process. Manna, et al. [33] presented a modified imperfect EPQ and assumed that the production rate of the defective products depends on the total production rate of the system. The demand of the system depended on advertising. Mokhtari [7] developed a joint production and lot-sizing decision in an imperfect EPQ system with uncertain demand. To cope with the production of defective items, ordering from an external supplier was possible. Kazemi, et al. [34] incorporated carbon emission concerns into an ordering inventory system and developed a sustainable extension of EOQ. Sebatjane and Adetunji [35] first introduced the concept of imperfect quality items in the EOQ model for growing items. Mokhtari and Asadkhani [5] proposed the batch replacement policy for EOQ with imperfect items under different types of inspection errors. Nobil, et al. [36] considered the lead time for the model of Salameh and Jaber [3] and calculated the optimal reorder point for this inventory system. Alfares and Afzal [37] formulated the order shortage in the inventory model for imperfect quality enhancing items. Fallahi, et al. [1] presented an EPQ model for imperfect quality items where preventive maintenance and multiple shipments delivery policies were considered in the inventory system. Asadkhani, et al. [38] investigated the impact of learning in the inspection process in EOQ models with different policies for defective items.

In the real world, there are usually multiple products that manufacturers should manage, and only a limited number of resources are available to be used [39]. However, traditional production and inventory models ignore such constraints and are therefore impractical. Recently, researchers have discussed the production and inventory problems with various types of constraints, such as limited machine capacity, storage space, budget, number of orders/setups, etc.

In this line of research, Mondal and Maiti [40] studied a fuzzy inventory model with the budget, setup, and storage space constraints. The proposed problem was a nonlinear programming model and they solved it using a genetic algorithm. In another extension, Baykasoğlu and Göçken [41] used different ranking methods to deal with the resource constraints of the fuzzy multi-product EOQ system. Pasandideh and Niaki [42] first discussed the discrete pallet delivery constraint and the storage space constraint for the multi-product inventory system. A genetic algorithm was designed to solve the problem. Mohan, et al. [43] investigated the concept of payment delay in multiproduct EOQ. To represent a more realistic system, a limit on the total available space of the warehouse was considered. The second case of this model under partial payment delay was also formulated. Pasandideh, et al. [44] incorporated the order shortage in the work of Pasandideh and Niaki [42]. Similarly, genetic algorithm was the proposed solution approach. Pasandideh, et al. [45] derived the optimal replenishment policy for EOQ with multiple products in a two-level supply chain network. The system was operated under vendor managed inventory (VMI) policy and constraints on the setup number and available space. Pal, et al. [46] developed a multi-product EOQ for an inventory system with price-sensitive demand. The objective function of the model was subjected to a price break level. Khalilpourazari, et al. [47] proposed a multi-item EPQ model for an inventory system with constraints on the imposed cost and total available space. Khalilpourazari and Pasandideh [48] discussed nonlinear holding costs in a multi-product EOQ with order shortage. In addition to the total budget and space available, the model constrained the total holding cost and the shortage cost of the system. A metaheuristic moth-flame optimization algorithm was developed to solve the model. Tiwari, et al. [49] proposed an environmentally responsible production system that considered trade credits for customers. They also discussed the rework of imperfect quality items. Khalilpourazari and Pasandideh [50] introduced a bi-objective multi-item partial backorder EOQ to simultaneously optimize the total system cost and the occupied space of the warehouse. The problem also dealt with imperfect quality items and constraints on the holding cost and shortage cost of the system. Khalilpourazari, et al. [39] developed multi-product EPQ under inspection errors, where the order shortage of items was also allowed. In this work, various operational constraints such as allowable holding and shortage cost, occupied space, and available budget were considered. Khalilpourazari, et al. [51] incorporated various sampling inspection plans into the imperfect EOQ model for multiple products under uncertainty. They subjected the mathematical model to real-world constraints such as warehouse capacity and total available budget. Khalilpourazari, et al. [52] presented the multi-item EPQ with imperfect items and rework process which works under the budget constraint. The uncertainty of parameters are discussed by two version of chance constraint programming . Mokhtari and Rezvan [53] extended the multi-product case of EPQ under VMI setting, imposing an upper limit on the total greenhouse gas emission of the system. The work of Nobil, et al. [2] focused on multi-product EPQ in a two-level supply chain where the inventory system is operated under the constrains of single machine capacity, available budget, and maximum shipment. In the work of Mokhtari and Fallahi [54], the available budget for investment was constrained in the production capacity of the EPQ model. This model was formulated under the assumptions of inflation and time value of money. Table 1 presents the key features of the related inventory problems.

Tab. 1. Review of related inventory problems

Paper	Year	Model structure	Number of products	Dealing with imperfect quality items				Real-world constraints				
				Rework (Repair)	Disposal	Salvage	Return	Machine capacity	Storage space	Budget	Number of setups (orders)	Other types
Salameh and Jaber [3]	2000	EOQ	Single			✓						
Ben-Daya and Rahim [9]	2003	EPQ	Single		✓							
Mondal and Maiti [40]	2003	EOQ	Multiple						✓	✓	✓	
Jamal, et al. [10]	2004	EPQ	Single	✓								
Ben-Daya, et al. [11]	2006	EOQ	Single	✓	✓							
Baykasoğlu and Göçken [41]	2007	EOQ	Multiple						✓		✓	
Ojha, et al. [13]	2007	EOQ/EPQ	Multiple	✓								
Biswas and Sarker [15]	2008	EPQ	Single	✓	✓							
Pasandideh and Niaki [42]	2008	EPQ	Multiple						✓			✓
Sarker, et al. [16]	2008	EPQ	Single	✓								
Mohan, et al. [43]	2008	EOQ	Multiple							✓		
Cárdenas-Barrón [17]	2009	EPQ	Single	✓								
Pasandideh, et al. [44]	2010	EPQ	Multiple						✓			✓
Lin [18]	2010	EOQ	Single			✓						
Barzoki, et al. [19]	2011	EPQ	Single	✓		✓						
Pasandideh, et al. [45]	2011	EOQ	Multiple						✓		✓	
Wee and Widyadana [20]	2012	EPQ	Single	✓	✓							
Krishnamoorthi and Panayappan [23]	2012	EPQ	Single	✓			✓					
Yoo, et al. [21]	2012	EPQ	Single	✓	✓	✓	✓					
Pal, et al. [46]	2012	EOQ	Multiple									✓
Wee, et al. [22]	2013	EPQ	Single	✓		✓						✓
Ameli, et al. [24]	2013	EPQ	Single			✓						
Sarkar, et al. [25]	2014	EOQ	Single	✓								
Jaber, et al. [26]	2014	EOQ	Single	✓	✓							
Hauck and Vörös [27]	2015	EOQ	Single		✓							
Kumar and	2015	EPQ	Single	✓		✓				✓		✓

Paper	Year	Model structure	Number of products	Dealing with imperfect quality items				Real-world constraints				
				Rework (Repair)	Disposal	Salvage	Return	Machine capacity	Storage space	Budget	Number of setups (orders)	Other types
Goswami [28]												
Al-Salamah [29]	2016	EPQ	Single	✓	✓	✓	✓					
Khalilpourazari, et al. [47]	2016	EPQ	Multiple						✓			✓
Rezaei [30]	2016	EOQ	Single			✓	✓					
Mahata [31]	2017	EPQ	Single	✓								
Khalilpourazari and Pasandideh [48]	2017	EOQ	Multiple						✓	✓		✓
Aslani, et al. [32]	2017	EOQ	Single				✓					
Manna, et al. [33]	2017	EPQ	Single			✓						
Tiwari, et al. [49]	2018	EPQ	Multiple	✓	✓							✓
Mokhtari [7]	2018	EPQ	Single	✓	✓							
Kazemi, et al. [34]	2018	EOQ	Single		✓	✓						
Khalilpourazari and Pasandideh [50]	2018	EOQ	Multiple				✓					✓
Khalilpourazari, et al. [39]	2019	EOQ	Multiple				✓		✓	✓		
Sebatjane and Adetunji [35]	2019	EOQ	Single			✓						✓
Khalilpourazari, et al. [51]	2019	EOQ	Multiple				✓		✓	✓		
Mokhtari and Asadkhani [5]	2019	EOQ	Single			✓	✓					
Khalilpourazari, et al. [52]	2019	EPQ	Multiple	✓						✓		
Mokhtari and Rezvan [53]	2020	EPQ	Multiple									✓
Nobil, et al. [2]	2020	EPQ	Multiple					✓		✓		✓
Mokhtari and Fallahi [54]	2020	EPQ	Single									✓
Nobil, et al. [36]	2020	EOQ	Single		✓							
Alfares and Afzal [37]	2021	EOQ	Single		✓							
Fallahi, et al. [1]	2021	EPQ	Single		✓							
Asadkhani, et al. [38]	2021	EOQ	Single	✓	✓	✓	✓					
Present work	2021	EPQ	Multiple	✓	✓			✓	✓	✓	✓	

To the best of our knowledge, there is no work in the literature that addresses a multi-product imperfect EPQ model with rework and disposal policies while simultaneously considering four well-known operational constraints on machine capacity, storage space, available budget, and number of setups. In this paper, we propose new production-inventory models for multiple products under EPQ settings where a manufacturer produces multiple products over a finite production rate. It is assumed that the production process is defective and produces a fraction of imperfect items. The imperfect products are also in the rework process to become perfect and return to the consumption cycle. The objective is to determine optimal/economic production quantities so that the total cost of the system is minimized.

The rest of this paper is structured as follows. Section 2 defines the basic single product problem and introduces assumptions. Then, Section 3 extends the basic model to a multi-product problem with machine capacity constraints. Sections 4-6 consider such an extension for the constraints of storage space, budget, and number of setups. Section 7 describes a numerical example and Section 8 performs a sensitivity analysis. Finally, Section 9 concludes the paper.

2. Problem Statement and Assumptions

In the traditional EOQ model, the demand is deterministic and constant over the planning horizon, and the order is received immediately. The model aims to describe the optimal order quantity for items so that total costs, including holding and ordering costs, are minimize. In each inventory cycle T of this model, the order is received immediately, and the stored inventory is then consumed at the demand rate D until it reaches zero during the inventory cycle T. The ordering process incurs a fixed cost, denoted by A, and the inventory is stored at a holding cost per unit time h. The objective is to find the economic order quantity Q so that the total cost of inventory is minimized with ordering and holding costs. Considering the model characteristics, the parameters of the model are defined as $T = Q/D$ and $I_{max} = Q$. The holding cost per cycle is $HC = hQ^2/(2D)$ and the fixed ordering cost per order is $OC = A$. The holding and setup costs per unit time are calculated by dividing HC and OC by the inventory cycle T, as $HCU = HC/T = hQ/2$ and $OC = AD/Q$. Therefore, the total cost is calculated as $TCU = hQ/2 + AD/Q$. Then, the optimal ordering quantity and the total cost are derived by setting the derivative of TCU to zero, as $Q^* = \sqrt{2AD/h}$ and $TCU(Q^*) = \sqrt{2ADh}$. So far, several versions of the EOQ have been proposed by relaxing some basic assumptions or adding new assumptions to the traditional ordering model. As a useful extension of the basic economic order quantity model, the EPQ model determines the optimal production quantity per cycle to minimize the total cost by balancing the production setup cost and the inventory holding cost. The difference between these two models is that the production problem assumes that the manufacturer produces its own quantity. Therefore, orders are available incrementally over a finite rate, while EOQ assumes that the order quantity is received immediately after the total order is placed. In the basic EPQ model, production setup costs are fixed and independent of the quantity produced. Production is done over finite rate incrementally. In each inventory cycle T of the model, production is processed until inventory reaches the maximum level I_{max} in production cycle t^p, and the stored inventory is then consumed at demand rate D until it reaches zero in depletion cycle t^d. The production setup process incurs a fixed cost, denoted by A, and the inventory produced can be stored at a holding cost per unit time, denoted by h. The objective is to find the economic production quantity Q so that the total cost of the inventory system, including setup and holding costs, is minimized. Considering the characteristics of the basic EPQ model, the model parameters are defined as $T = Q/D$, $t^p = Q/P$, $t^d = Q/D - Q/P$, and $I_{max} = Q(1 - D/P)$. The holding cost per cycle is $HC = hQ^2(1 - D/P)/(2D)$ and the setup cost per order is $OC = A$. The holding and setup costs per unit time can be calculated by dividing HC and OC by the inventory cycle T as $HCU = HC/T = hQ(1 - D/P)/2$ and $OC = AD/Q$. Therefore, the total cost involving holding and setup costs is calculated as $TCU = hQ(1 - D/P)/2 + AD/Q$. Therefore, the optimal production quantity and the total cost can be derived by setting the derivative of TCU to zero as $Q^* = \sqrt{2AD/(h(1 - D/P))}$ and $TCU(Q^*) = \sqrt{2ADh(1 - D/P)}$.

We propose a new production-inventory model for imperfect quality items. The following notations are used to formulate the considered single product imperfect EPQ model:

Notations	
P_1	Production rate (unit/time)
D	Demand rate (unit/time)
P_2	Rework rate (unit/time)
A	Setup cost ($/setup)
h	Holding cost ($/unit/time)
t_p	Production period (time)
t_R	Rework period (time)
t_D	Depletion period (time)
CT	Inventory cycle time (time)
β	Percentage of defective items
α	Percentage of reworkable defective items
I_1	Inventory level at start of rework period (unit)
I_2	Inventory level at end of rework period (unit)
I_{max}	Maximum inventory level (unit)
Q	Economic production quantity (unit)

Consider a manufacturing environment in which a manufacturer produces a product to satisfy an external demand D. The demand is assumed to be constant over the time horizon. The manufacturer produces the product at a finite production rate P_1 under EPQ settings. The model assumes an infinite time horizon and zero lead time. In addition, shortage is not allowed and the purchase cost is fixed. Unlike the standard models, the production process is assumed to be defective and to produce a fraction of imperfect items. The reworkable imperfect items go through the rework process to become perfect and return to the consumption cycle. In general, an inventory cycle is composed of production, reworking, and depletion periods. After the production period, a fraction of defective items $(\alpha\beta Q)$ that can be reworked go through a rework process with a rework rate P_2. At the end of the rework period, the stored inventory is consumed until it reaches zero in the depletion period. The objective is to determine the economic production quantity Q so that the total cost is minimized. The total cost consists of setup and holding costs. The holding cost is per item per unit time, denoted by h, while the setup cost is fixed per production cycle, denoted by A. Figure 1 shows a cycle of the proposed production-inventory system. As can be seen, production is done at production rate P_1 within production period t_p. Once the production is finished, the non-reworkable defective items $(1-\alpha)\beta Q$ are discarded from the system, and the reworkable items $\alpha\beta Q$ go through the rework process. During the rework time period t_R, the imperfect items become perfect at a rate of P_2 and return to the system. At the end of the rework period, the stored inventory is consumed during the depletion period t_D until it reaches zero. The next cycles repeat this process continuously. To ensure feasibility and prevent shortage, we consider an initial condition as $I_{max} - \beta Q \geq 0$. Since $I_{max} = Q(1 - D/P_1)$, this condition is simplified as $(1 - D/P_1) - \beta \geq 0$. We first formulate this problem for a single product. Then, we extend this basic model to multiple products with various constraints, namely production machine constraint, shortage space constraint, budget constraint, and number of setups constraint.

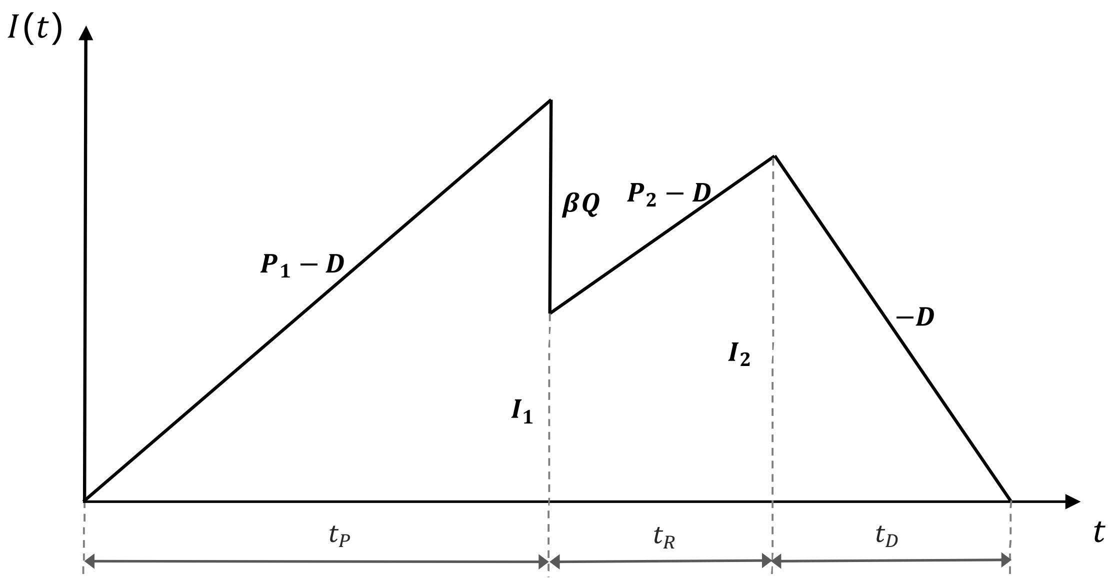

Fig. 1. Inventory level for joint lot-sizing model

Total cost includes setup and holding costs. In the case of a single product, the setup cost SC is incurred per production cycle as follows.

$$SC = A \tag{1}$$

Moreover, to formulate the holding cost, we first calculate the area under the inventory level in three periods, namely production period A_I, rework period A_{II}, and depletion period A_{III}, as follows.

$$A_I = \frac{I_{max} \cdot t_p}{2} \tag{2}$$

Since $I_{max} = Q\left(1 - \frac{D}{P_1}\right)$ and $t_p = \frac{Q}{P_1}$, A_I is rewritten as:

$$A_I = \frac{Q^2}{2P_1}\left(1-\frac{D}{P_1}\right) \tag{3}$$

To calculate A_{II}, we should first formulate the inventory level at the beginning of the rework period I_1 and the inventory level at the end of the rework period I_2.

$$I_1 = Q\left(1-\frac{D}{P_1}\right) - \beta Q \tag{4}$$

$$I_2 = Q\left(1-\frac{D}{P_1}\right) - \beta Q + (P_2 - D)t_R \tag{5}$$

In the rework time period t_R, the reworkable items $\alpha\beta Q$ go through the rework process. Thus, we can calculate t_R in terms of the model parameters as $t_R = \alpha\beta Q/P_2$. Therefore, the inventory level I_2 can be simplified as follows.

$$I_2 = Q\left(1-\frac{D}{P_1}\right) - \beta Q + \alpha\beta Q\left(1-\frac{D}{P_2}\right) \tag{6}$$

Therefore, the area under the inventory level in the rework period is calculated as:

$$A_{II} = \frac{t_R}{2}(I_1 + I_2) \tag{7}$$

which can be rewritten as:

$$A_{II} = \frac{\alpha\beta Q}{2P_2}\left(2Q\left(1-\frac{D}{P_1}\right) - 2\beta Q + \alpha\beta Q\left(1-\frac{D}{P_2}\right)\right) \tag{8}$$

Moreover, the area under the inventory level of the depletion period is formulated as $A_{III} = I_2 t_D/2$ where the depletion period is $t_D = I_2/D$. Hence, we have:

$$A_{III} = \frac{I_2^2}{2D} \tag{9}$$

or:

$$A_{III} = \frac{1}{2D}\left\{Q\left(1-\frac{D}{P_1}\right) - \beta Q + \alpha\beta Q\left(1-\frac{D}{P_2}\right)\right\}^2 \tag{10}$$

Using A_I, A_{II} and A_{III}, the holding cost is formulated by $h\{A_I + A_{II} + A_{III}\}$ as follows:

$$HC(Q) = hQ^2\left\{\frac{G^2}{2D} + \frac{1}{2P_1}\left(1-\frac{D}{P_1}\right) + \frac{\alpha\beta}{2P_2}\left\{\left(1-\frac{D}{P_1}\right) - \beta + G\right\}\right\} \tag{11}$$

where $G = \left(1-\frac{D}{P_1}\right) - \beta + \alpha\beta\left(1-\frac{D}{P_2}\right)$.

Therefore, the total cost is obtained by $SC + HC$ as follows.

$$TC(Q) = A + hQ^2\left\{\frac{G^2}{2D} + \frac{1}{2P_1}\left(1-\frac{D}{P_1}\right) + \frac{\alpha\beta}{2P_2}\left\{\left(1-\frac{D}{P_1}\right) - \beta + G\right\}\right\} \tag{12}$$

Finally, the total cost per unit time is calculated by dividing $TC(Q)$ by the cycle time. The cycle time can be obtained by $CT = t_P + t_R + t_D$ as:

$$CT = \frac{Q}{D}(\beta(\alpha-1)+1) \tag{13}$$

Note that if $0 \le \alpha \le 1$ and $0 \le \beta \le 1$, then we have $\beta(\alpha-1)+1 \ge 0$. The total cost per unit time is obtained as:

$$TCU(Q) = \frac{AD}{Q(\beta(\alpha-1)+1)} + \frac{hDQ}{\beta(\alpha-1)+1}\left\{\frac{G^2}{2D} + \frac{1}{2P_1}\left(1-\frac{D}{P_1}\right) + \frac{\alpha\beta}{2P_2}\left\{\left(1-\frac{D}{P_1}\right) - \beta + G\right\}\right\} \tag{14}$$

The derived total cost per unit time $TCU(Q)$ is a convex function with respect to production quantity Q as:

$$\frac{\partial^2 TCU(Q)}{\partial Q^2} = \frac{2AD}{Q^3(\beta(\alpha-1)+1)} \ge 0 \tag{15}$$

Therefore, to obtain the economic production quantity Q^*, we set the first derivative of $TCU(Q)$ to zero as follows:

$$\frac{\partial TCU(Q)}{\partial Q} = \frac{-AD}{Q^2(\beta(\alpha-1)+1)} + \frac{hD}{(\beta(\alpha-1)+1)}\left\{\frac{G^2}{2D} + \frac{1}{2P_1}\left(1-\frac{D}{P_1}\right) + \frac{\alpha\beta}{2P_2}\left\{\left(1-\frac{D}{P_1}\right) - \beta + G\right\}\right\} = 0 \tag{16}$$

which yields:

$$Q^* = \sqrt{\frac{2AD}{h\left\{G^2+\frac{D}{P_1}\left(1-\frac{D}{P_1}\right)+\frac{\alpha\beta D}{P_2}\left\{\left(1-\frac{D}{P_1}\right)-\beta+G\right\}\right\}}} \quad (17)$$

It is noteworthy that the above formula is reduced to the economic production quantity of the standard EPQ model $\left(\sqrt{2AD/(h(1-D/P_1))}\right)$ when there is no rework period ($\beta = 0$). This verifies the proposed model.

Most real-world inventory systems stock many products, not merely a single product. It is not practical to evaluate each product individually because there may be interactions between products. For example, there may be only one production machine, there may be a limit on storage space capacity, there may be an upper limit on maximum budget, or there may be an upper limit on the number of setups. Products compete for these problems. In the sequel, we consider cases where multiple products compete for constraints on machine usage, storage space, budget in inventory, and number of setups. The following notations are used to formulate the given cases.

Notations	
P_{1j}	Production rate of product j (unit/time)
D_j	Demand rate of product j (unit/time)
P_{2j}	Rework rate of product j (unit/time)
A_j	Setup cost of product j (\$/setup)
h_j	Holding cost of product j (\$/unit/time)
t_{Pj}	Production period of product j (time)
t_{Rj}	Rework period of product j (time)
CT	Inventory cycle time (time)
β_j	Percentage of defective items of product j
α_j	Percentage of reworkable defective items of product j
f_j	Occupied space of product j (m^3/unit)
C_j	Procurement cost of product j (\$/unit)
F	Total available warehouse space (m^3)
M	Total available budget (\$)
N	Allowable production setup number
Q_j	Economic production quantity of product j (unit)

3. Multiple Products With Machine Capacity Constraint

In this section, we consider that a manufacturer produces multiple products $j = 1,2,\dots,N$ over finite production rates P_{1j} each of which faces external demand D_j. It is also considered that the production process is defective and produces some imperfect items. The imperfect items are also subject to a rework process at rework rate P_{2j}. All the characteristics of the products are similar to those of the single product case discussed in Section 2. In addition, there is only one machine, and it is assumed that all products should be produced by this machine. We first derive the possibility condition of production on a single machine. Since the production and rework period of product j per cycle is calculated by $t_{Pj} + t_{Rj}$, the summation of total production and rework times for all products is $\sum_{j=1}^{N}\left(t_{Pj} + t_{Rj}\right)$, which should be smaller or equal to the cycle time CT to ensure feasibility. Therefore, the machine capacity constraint is:

$$\sum_{j=1}^{N}\left(t_{Pj} + t_{Rj}\right) \geq CT \quad (18)$$

By substituting $t_{Pj} = Q_j/P_{1j}$, $t_{Rj} = \alpha_j\beta_jQ_j/P_{2j}$ and $CT = Q_j\left(\beta_j(\alpha_j - 1) + 1\right)/D_j$ into the above inequality and simplifying the result, we have:

$$\sum_{j=1}^{N}\left\{\frac{D_j/P_{1j}+\alpha_j\beta_jD_j/P_{2j}}{\beta_j(\alpha_j-1)+1}\right\} \leq 1 \quad (19)$$

To obtain the economic production quantities, we should derive the total cost per time in terms of CT. For this purpose, we substitute the production quantity Q_j with $CTD_j/\beta_j(\alpha_j - 1) +$

1 in $TCU(Q_j)$, and sum total cost for all products $j = 1, 2, \dots, N$ as follows:

$$TCU(CT) = \sum_{j=1}^{N}\left(\frac{A_j}{CT} + \frac{h_j CT D_j^2}{\left(\beta_j(\alpha_j-1)+1\right)^2}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\}\right) \tag{20}$$

where $G_j = \left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + \alpha_j\beta_j\left(1-\frac{D_j}{P_{2j}}\right)$.

The derived total cost per unit time $TCU(CT)$ is a convex function with respect to CT as:

$$\frac{\partial^2 TCU(CT)}{\partial CT^2} = \sum_{j=1}^{N}\frac{2A_j}{CT^3} \geq 0 \tag{21}$$

Therefore, to obtain the economic cycle time CT^*, we set the first derivative of $TCU(CT)$ to zero as follows:

$$\frac{\partial TCU(CT)}{\partial CT} = \sum_{j=1}^{N}\left(\frac{-A_j}{CT^2} + \frac{h_j D_j^2}{\left(\beta_j(\alpha_j-1)+1\right)^2}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\}\right) = 0 \tag{22}$$

which yields:

$$CT^* = \sqrt{\frac{2\sum_{j=1}^{N}A_j}{\sum_{j=1}^{N}\left(\frac{h_j D_j^2}{\left(\beta_j(\alpha_j-1)+1\right)^2}\left\{\frac{G_j^2}{D_j} + \frac{1}{P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\}\right)}} \tag{23}$$

Then, the economic production quantity for product j is calculated by $Q_j^* = \frac{CT^* D_j}{\left(\beta_j(\alpha_j-1)+1\right)}$

4. Multiple Products With Storage Space Constraint

Consider a manufacturer that produces multiple products, each of which operates under the conditions discussed in Section 2. However, the space available for storing the inventory is limited, which is a real frequently occurred assumption in practice. The space occupied by each unit is represented by f_j and the total available space is denoted by F. To obtain the economic production quantities, we derive the total cost per unit time in terms of $Q_1, Q_2, \dots, Q_N$ as follows.

$$TCU(Q_1, Q_2, \dots, Q_N) = \sum_{j=1}^{N}\left(\frac{A_j D_j}{Q_j\left(\beta_j(\alpha_j-1)+1\right)} + \frac{h_j D_j Q_j}{\beta_j(\alpha_j-1)+1}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\}\right) \tag{24}$$

Therefore, the problem is transformed into a constrained optimization problem, $\min TCU(Q_1, Q_2, \dots, Q_N)$, subject to $\sum_{j=1}^{N} f_j Q_j\left(1-\frac{D_j}{P_{1j}}\right) \leq F$. To solve this problem, the Lagrangian Relaxation (LR) approach can be used. The LR is an analytical approach usually used to solve constrained optimization problems by relaxing the complicating constraint in the original problem. Consider a general function $f(X_1, X_2, \dots, X_n)$ to be minimized with a constraint $g(X_1, X_2, \dots, X_n) \leq B$. For this case, the LR approach constructs the unconstrained Lagrangian function as:

$$LR(X_1, X_2, \dots, X_n, \lambda) = f(X_1, X_2, \dots, X_n) + \lambda[g(X_1, X_2, \dots, X_n) - b] \tag{25}$$

where λ is the Lagrangian multiplier. Then, the Kuhn-Tucker condition is used to optimize the $LR(X_1, X_2, \dots, X_n, \lambda)$ as follows.

$$\frac{\partial LR(X_1, X_2, \dots, X_n, \lambda)}{\partial X_j} = \frac{\partial f(X_1, X_2, \dots, X_n)}{\partial X_j} + \lambda\frac{\partial g(X_1, X_2, \dots, X_n)}{\partial X_j} = 0\,, \tag{26}$$

$$\frac{\partial LR(X_1,X_2,\dots,X_n,\lambda)}{\partial \lambda} = g(X_1,X_2,\dots,X_n) - B = 0 \tag{27}$$

By solving the above system of equations for X_j and λ, the optimal solution is obtained.
For our problem, the Lagrangian function is constructed as follows:

$$LR(Q_1,Q_2,\dots,Q_N) = \sum_{j=1}^{N}\left(\frac{A_jD_j}{Q_j(\beta_j(\alpha_j-1)+1)} + \frac{h_jD_jQ_j}{\beta_j(\alpha_j-1)+1}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\}\right) + \theta\left(\sum_{j=1}^{N} f_jQ_j\left(1-\frac{D_j}{P_{1j}}\right) - F\right) \tag{28}$$

where θ denotes the Lagrangian multiplier. To solve the new unconstrained problem, we set the derivative of $LR(Q_1,Q_2,\dots,Q_N)$ with respect to Q_j and θ to zero as follows:

$$\frac{\partial LR(TCU(Q_1,Q_2,\dots,Q_N))}{\partial Q_j} = \frac{-A_jD_j}{Q_j^2(\beta_j(\alpha_j-1)+1)} + \frac{h_jD_j}{\beta_j(\alpha_j-1)+1}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\} + \theta f_j\left(1-\frac{D_j}{P_{1j}}\right) = 0 \tag{29}$$

And

$$\frac{\partial LR(TCU(Q_1,Q_2,\dots,Q_N))}{\partial \theta} = \sum_{j=1}^{N} f_jQ_j\left(1-\frac{D_j}{P_{1j}}\right) - F = 0 \tag{30}$$

which is simplified as:

$$Q_j^* = \sqrt{\frac{2A_jD_j}{h_j\left\{G_j^2+\frac{D_j}{P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right)+\frac{\alpha_j\beta_jD_j}{P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right)-\beta_j+G_j\right\}\right\}+2\theta f_jD_j(\beta_j(\alpha_j-1)+1)\left(1-\frac{D_j}{P_{1j}}\right)}} \tag{31}$$

$$\sum_{j=1}^{N}\left\{f_j\sqrt{\frac{2A_jD_j}{h_j\left\{G_j^2+\frac{D_j}{P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right)+\frac{\alpha_j\beta_jD_j}{P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right)-\beta_j+G_j\right\}\right\}+2\theta f_jD_j(\beta_j(\alpha_j-1)+1)\left(1-\frac{D_j}{P_{1j}}\right)}}\left(1-\frac{D_j}{P_{1j}}\right)\right\} = F \tag{32}$$

We first obtain the optimal value of θ from Eq. (32), and it is then substituted into Eq. (31) to calculate the economic production quantity Q_j $(j = 1,2,\dots,N)$.

5. Multiple Products With Budget Constraint

In real-world situations, the budget available to invest in the inventory is limited. In this section, we consider a manufacturer that operates with multiple products under EPQ with imperfect quality and rework, which have a limited amount of budget M to invest in inventories. Therefore, the problem is formulated as a constrained optimization problem as $\min\ TCU(Q_1,Q_2,\dots,Q_N)$ subject to $\sum_{j=1}^{N} C_jQ_j\left(1-\frac{D_j}{P_{1j}}\right) \le M$, where C_j is the unit purchase price of product j. The Lagrangian function is constructed for this case as follows:

$$LR(Q_1,Q_2,\dots,Q_N) = \sum_{j=1}^{N}\left(\frac{A_jD_j}{Q_j(\beta_j(\alpha_j-1)+1)} + \frac{h_jD_jQ_j}{\beta_j(\alpha_j-1)+1}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\}\right) + \varphi\left(\sum_{j=1}^{N} C_jQ_j\left(1-\frac{D_j}{P_{1j}}\right) - M\right) \tag{33}$$

where φ denotes the Lagrangian multiplier. To solve this unconstrained problem, we set the derivative of $LR(Q_1,Q_2,\dots,Q_N)$ with respect to Q_j and φ to zero as follows:

$$\frac{\partial LR(TCU(Q_1,Q_2,\dots,Q_N))}{\partial Q_j} = \frac{-A_jD_j}{Q_j^2(\beta_j(\alpha_j-1)+1)} + \frac{h_jD_j}{\beta_j(\alpha_j-1)+1}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\} + \varphi C_j\left(1-\frac{D_j}{P_{1j}}\right) = 0 \quad (34)$$

And

$$\frac{\partial LR(TCU(Q_1,Q_2,\dots,Q_N))}{\partial \varphi} = \sum_{j=1}^{N} C_jQ_j\left(1-\frac{D_j}{P_j}\right) - M = 0 \quad (35)$$

which yields:

$$Q_j^* = \sqrt{\frac{2A_jD_j}{h_j\left\{G_j^2+\frac{D_j}{P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right)+\frac{\alpha_j\beta_jD_j}{P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right)-\beta_j+G_j\right\}\right\}+2\varphi C_jD_j(\beta_j(\alpha_j-1)+1)\left(1-\frac{D_j}{P_{1j}}\right)}} \quad (36)$$

$$\sum_{j=1}^{N}\left\{C_j\sqrt{\frac{2A_jD_j}{h_j\left\{G_j^2+\frac{D_j}{P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right)+\frac{\alpha_j\beta_jD_j}{P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right)-\beta_j+G_j\right\}\right\}+2\varphi C_jD_j(\beta_j(\alpha_j-1)+1)\left(1-\frac{D_j}{P_{1j}}\right)}}\left(1-\frac{D_j}{P_{1j}}\right)\right\} = M \quad (37)$$

The optimal value of φ is obtained from Eq. (23), and it is then substituted into Eq. (22) to calculate the economic production quantity $Q_j (j = 1, 2, \dots, N)$.

6. Multiple Products With Number of Setups Constraint

In this section, we consider an upper bound for the number of setups carried out per time unit L. The problem is formulated as a constrained optimization problem as $\min\ TCU(Q_1, Q_2, \dots, Q_N)$ subject to $\sum_{j=1}^{N}\frac{D_j}{Q_j(\beta_j(\alpha_j-1)+1)} \le N$. The Lagrangian function is constructed for this case as follows:

$$LR(Q_1,Q_2,\dots,Q_N) = \sum_{j=1}^{N}\left(\frac{A_jD_j}{Q_j(\beta_j(\alpha_j-1)+1)} + \frac{h_jD_jQ_j}{\beta_j(\alpha_j-1)+1}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\}\right) + \tau\left(\sum_{j=1}^{N}\frac{D_j}{Q_j(\beta_j(\alpha_j-1)+1)} - L\right) \quad (38)$$

where φ denotes the Lagrangian multiplier. We set the derivative of $LR(Q_1, Q_2, \dots, Q_N)$ with respect to Q_j and τ to zero as follows:

$$\frac{\partial LR(TCU(Q_1,Q_2,\dots,Q_N))}{\partial Q_j} = \frac{-A_jD_j}{Q_j^2(\beta_j(\alpha_j-1)+1)} + \frac{h_jD_j}{\beta_j(\alpha_j-1)+1}\left\{\frac{G_j^2}{2D_j} + \frac{1}{2P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right) + \frac{\alpha_j\beta_j}{2P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right) - \beta_j + G_j\right\}\right\} - \frac{\tau}{Q_j^2}.\frac{D_j}{\beta_j(\alpha_j-1)+1} = 0 \quad (39)$$

And

$$\frac{\partial LR(TCU(Q_1,Q_2,\dots,Q_N))}{\partial \tau} = \sum_{j=1}^{N}\frac{D_j}{Q_j(\beta_j(\alpha_j-1)+1)} - L = 0 \quad (40)$$

which yields:

$$Q_j^* = \sqrt{\frac{2(A_j+\tau)D_j}{h_j\left\{G_j^2+\frac{D_j}{P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right)+\frac{\alpha_j\beta_jD_j}{P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right)-\beta_j+G_j\right\}\right\}}} \quad (41)$$

$$\sum_{j=1}^{N}\left\{\frac{D_j}{\beta_j(\alpha_j-1)+1}\sqrt{\frac{h_j\left\{G_j^2+\frac{D_j}{P_{1j}}\left(1-\frac{D_j}{P_{1j}}\right)+\frac{\alpha_j\beta_j D_j}{P_{2j}}\left\{\left(1-\frac{D_j}{P_{1j}}\right)-\beta_j+G_j\right\}\right\}}{2(A_j+\tau)D_j}}\right\}=\mathrm{L} \tag{42}$$

The optimal value of τ is obtained from Eq. (41), and it is then substituted into Eq. (42) to calculate the economic production quantity $Q_j(j=1,2,\dots,N)$.

7. Illustrative Example

To investigate the performance of the proposed EPQ models, a numerical example is used and discussed in this section with three products ($N=3$). Table 2 shows the demand rates, production rates, rework rates, setup costs, holding costs, rate of imperfect products, rate of reworkable items, occupied space, purchasing cost, total available budget, total available budget, and total number of allowed setups for the three products.

Tab. 2. Specifications of numerical example

		D_j	P_{1j}	P_{2j}	A_j	h_j	β_j	α_j	f_j	C_j	F	M	L
Product	$j=1$	1000	6500	8500	150	10	0.15	0.8	1	50			
	$j=2$	3000	7000	9000	250	8	0.2	0.7	2	25	500	5000	12
	$j=3$	2000	8500	10000	200	12	0.1	0.9	1	75			

Before calculating the solutions, we first investigate the feasibility conditions for the three products. To this end, we evaluated three conditions: (i) $P_{1j}>D_j$, (ii) $P_{2j}>D_j$, and (iii) $\left(1-D_j/P_{1j}\right)-\beta_j\geq 0$ and concluded that the feasibility condition is met. For the case with a single machine constraint, we evaluated the possibility condition for producing three products on one machine, i.e., $\sum_{j=1}^{N}\left\{\frac{D_j/P_{1j}+\alpha_j\beta_j D_j/P_{2j}}{\beta_j(\alpha_j-1)+1}\right\}\leq 1$ and ensured that the model is feasible for this case. Table 3 presents the optimal results of the models for the numerical example considered.

Tab. 3. Optimal results of numerical example

		Unconstrained multi-product EPQ	Multi-product EPQ under single machine constraint	Multi-product EPQ under storage space constraint	Multi-product EPQ under budget constraint	Multi-product EPQ under number of setups constraint
Optimal production quantity	Q_1^*	194.31	177.84	138.77	37.94	300.81
	Q_2^*	612.42	550.53	210.12	86.42	830.26
	Q_3^*	298.56	348.48	186.27	37.67	427.21
Optimal total cost	$TCU_1{}^*$	1591.7	1597.9	1682.75	4231.68	1746.13
	$TCU_2{}^*$	2605.6	2620.4	4244.19	9416.03	2727.24
	$TCU_3{}^*$	2706.6	2739.0	3013.41	10896.98	2882.17
Optimal Lagrangian multiplier		–	–	$\theta^*=0.0046$	$\varphi^*=0.0024$	$\tau^*=209.4879$

As we expected, the optimal cost is larger in the single machine model than in the multi-product model without machine constraint. This reveals that machine capacity is an active constraint in this example. The optimal costs in all other constrained cases are larger than those of the multi-product unconstrainted case, because the constraints $F=500$, $M=5000$ and $L=12$ are active. Moreover, as we expected, the economic production quantities in the models with space and budget constraint become smaller than those of the unconstrained case, while they increase in the model with a constraint on the number of setups. This is due to the inherent nature of the constraints.

8. Sensitivity Analysis

In this section, we perform a sensitivity analysis by changing the value of the input parameters to assess the behavior of the optimal total cost of all proposed models. The impact of six parameters on each model is studied, of which the related parameters of the defective items (α, β and P_2) and the demand rate of the system are constant. The parameters vary in the range of -50% to $+50\%$, and the behavior of the objective function is reported in both the percentage change and the output value.

8.1. Sensitivity analysis of unconstrainted multi-product EPQ

The sensitivity analysis of the unconstrained multi-product EPQ is performed by varying six parameters of the models, and the results are presented in Table 4. As can be seen, the model is more sensitive to inventory holding cost, production rate, and demand rate. Figure 2 shows that when the parameters increase, the total cost of the system increases. Figure 3 shows that when the rework rate increases, better performance of the system is achieved and an increase in the percentage of defective items and reworkable items results in less cost to the inventory system.

Tab. 4. Sensitivity of optimal total cost in unconstrained multi-product EPQ model

% Change in parameter	Parameter h_j Total cost	h_j %Change	P_{1j} Total cost	P_{1j} %Change	P_{2j} Total cost	P_{2j} %Change	D Total cost	D %Change	β_j Total cost	β_j %Change	α_j Total cost	α_j %Change
−50%	4881.80	−29.289%	–*	–	6882.20	−0.314%	5400.54	−21.775%	6917.20	+0.192%	6939.80	+0.520%
−40%	5347.75	−22.540%	5741.50	−16.837%	6889.45	−0.209%	5807.82	−15.876%	6915.32	+0.165%	6931.52	+0.400%
−30%	5776.23	−16.334%	6193.21	−10.294%	6894.62	−0.134%	6153.61	−10.867%	6913.06	+0.133%	6923.83	+0.289%
−20%	6175.05	−10.557%	6502.34	−5.817%	6898.50	−0.078%	6447.54	−6.610%	6910.42	+0.094%	6916.69	+0.185%
−10%	6549.63	−5.132%	6729.38	−2.528%	6901.51	−0.034%	6696.12	−3.009%	6907.37	+0.050%	6910.06	+0.089%
0	6903.91	0	6903.91	0	6903.91	0	6903.91	0	6903.91	0	6903.91	0
+10%	7240.88	+4.881%	7042.56	+2.008%	6905.88	+0.028%	7074.07	+2.464%	6900.02	−0.056%	6898.22	−0.082%
+20%	7562.86	+9.545%	7155.50	+3.644%	6907.52	+0.052%	7208.74	+4.415%	6895.69	−0.119%	6892.94	−0.159%
+30%	7871.67	+14.018%	7249.34	+5.003%	6908.91	+0.072%	7309.25	+5.871%	6890.90	−0.188%	6888.06	−0.230%
+40%	8168.82	+18.322%	7328.59	+6.151%	6910.09	+0.089%	7376.19	+6.840%	6885.64	−0.265%	6883.55	−0.295%
+50%	8455.53	+22.475%	7396.43	+7.134%	6911.12	+0.104%	7409.44	+7.322%	6879.89	−0.348%	6879.38	−0.355%

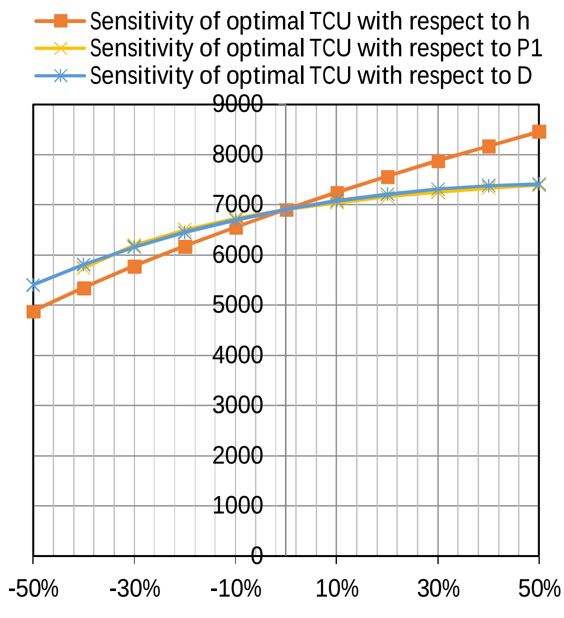

Fig. 2. Sensitivity of unconstrainted multi-product EPQ model to h, P_1 and D

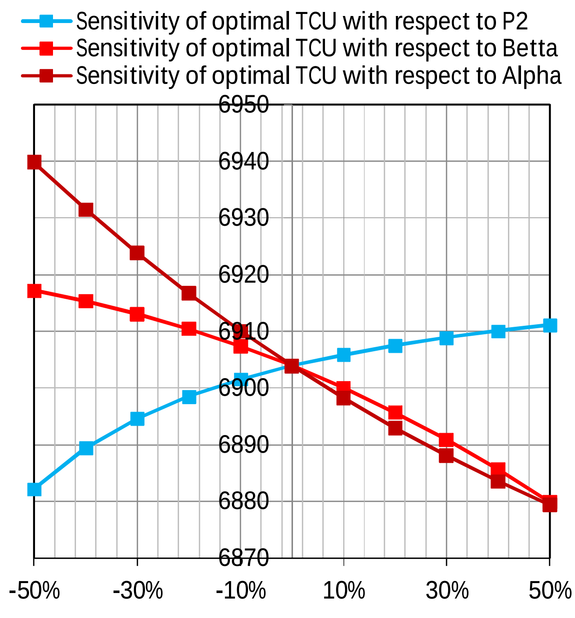

Fig. 3. Sensitivity of unconstrainted multi-product EPQ model to P_2, α and β

* The model is infeasible in this value

8.2. Sensitivity analysis of multi-product EPQ with machine capacity constraint

The model's behavior is assessed by varying some input parameters, and the results are summarized in Table 5. For the feasible values of P_1 and D, as shown in Figure 4, there is an increasing trend in total cost as the parameters increase. Increasing the inventory holding cost also leads to worse solutions. Like the unconstrained model, the inventory system has better performance under the increase in the percentage of defective and reworkable defective items. Increasing the rework rate of defective items has a negative effect on the total cost. The related results are shown in Figure 5.

Tab. 5. Sensitivity of optimal total cost in multi-product EPQ model with machine capacity constraint

% Change in parameter	h_j		P_{1j}		P_{2j}		D		β_j		α_j	
	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change
−50%	4919.61	−29.289%	–	–	–	–	5431.51	−21.932%	6969.93	+0.180%	6990.69	+0.479%
−40%	5389.16	−22.540%	–	–	6943.85	−0.194%	5842.07	−16.031%	6968.15	+0.155%	6983.00	+0.368%
−30%	5820.96	−16.334%	–	–	6948.68	−0.125%	6191.49	−11.008%	6966.02	+0.124%	6975.86	+0.266%
−20%	6222.87	−10.557%	–	–	6952.31	−0.073%	6489.63	−6.723%	6963.52	+0.088%	6969.23	+0.170%
−10%	6600.35	−5.132%	–	–	6955.12	−0.032%	6743.30	−3.077%	6960.64	+0.047%	6963.08	+0.082%
0	6957.38	0	6957.38	0	6957.38	0	6957.38	0	6957.38	0	6957.38	0
+10%	7296.96	+4.881%	7092.69	+1.945%	6959.22	+0.026%	–	–	6953.71	−0.053%	6952.09	−0.076%
+20%	7621.43	+9.545%	7203.51	+3.538%	6960.76	+0.049%	–	–	6949.64	−0.111%	6947.20	−0.146%
+30%	7932.63	+14.017%	7295.97	+4.867%	6962.06	+0.067%	–	–	6945.13	−0.176%	6942.67	−0.211%
+40%	8232.08	+18.322%	7374.29	+5.992%	6963.17	+0.083%	–	–	6940.19	−0.247%	6938.48	−0.272%
+50%	8521.01	+22.474%	7441.51	+6.959%	6964.13	+0.097%	–	–	6934.79	−0.325%	6934.62	−0.327%

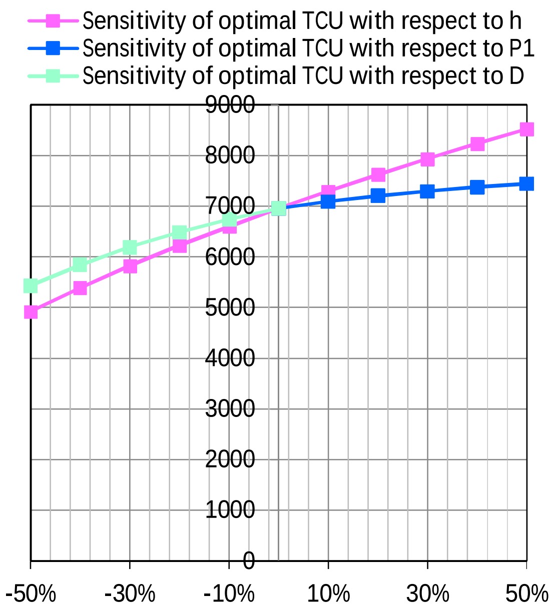

Fig. 4. Sensitivity of multi-product EPQ model with machine capacity constraint to h, P_1 and D

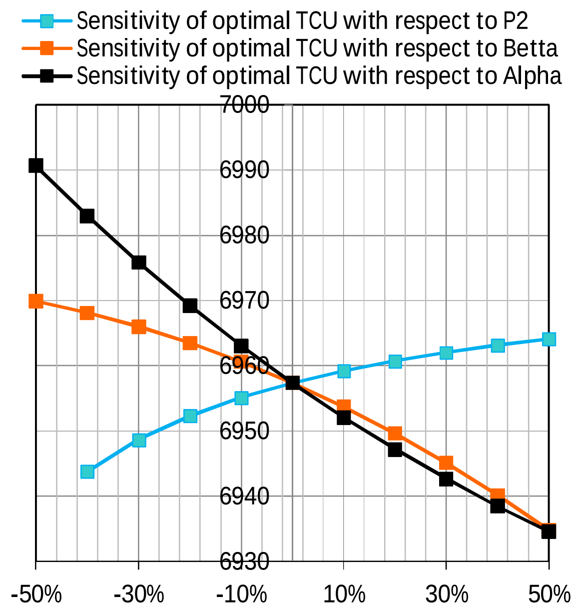

Fig. 5. Sensitivity of multi-product EPQ model with machine capacity constraint to P_2, α and β

8.3. Sensitivity analysis of multi-product EPQ with storage space constraint

The occupied space of the products and the total available storage space are two unique parameters of this model. We investigated the impact of f_j, F and D along with the parameters related to imperfect items, and the results are given in Table 6. It can be concluded that the increase in the warehouse's total available space reduces the total cost imposed on the inventory system. Figures 6 and 7 illustrate the analyses schematically. On the other hand, the total cost generally increases when the demand rate and the occupied space of each product increase. Interestingly, the total cost in this model shows an unfavorable behavior when the percentage of defective items and reworkable items increases. While producing more defective items causes an increase in the total cost of the system, lower costs are expected when a larger percentage of defective items can be reworked.

Tab. 6. Sensitivity of optimal total cost in multi-product EPQ model with storage space constraint

% Change in parameter	Parameter											
	f_j		F		P_{2j}		D		β_j		α_j	
	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change
−50%	6926.17	−22.529%	15438.95	+72.688%	8931.52	−0.099%	6170.85	−30.978%	8828.80	−1.248%	9349.01	+4.571%
−40%	7121.47	−20.345%	13161.57	+47.215%	8934.46	−0.066%	6870.73	−23.149%	8850.83	−1.001%	9261.50	+3.592%
−30%	7459.57	−16.563%	11582.13	+29.549%	8936.52	−0.043%	7498.23	−16.130%	8872.96	−0.754%	9176.98	+2.647%
−20%	7893.46	−11.710%	10439.28	+16.766%	8938.13	−0.025%	8052.76	−9.928%	8895.27	−0.504%	9095.36	+1.734%
−10%	8393.45	−6.117%	9587.75	+7.241%	8939.33	−0.011%	8533.76	−4.548%	8917.79	−0.252%	9016.58	+0.853%
0	8940.35	0	8940.35	0	8940.35	0	8940.35	0	8940.35	0	8940.35	0
+10%	9521.47	+6.500%	8441.46	−5.580%	8941.18	+0.009%	9271.73	+3.707%	8963.09	+0.254%	8866.70	−0.824%
+20%	10128.23	+13.287%	8053.90	−9.915%	8941.81	+0.016%	9526.85	+6.560%	8985.98	+0.510%	8795.49	−1.620%
+30%	10754.63	+20.293%	7751.79	−13.294%	8942.35	+0.022%	9704.33	+8.545%	9008.99	+0.768%	8726.61	−2.391%
+40%	11396.41	+27.472%	7516.53	−15.926%	8942.86	+0.028%	9802.56	+9.644%	9032.19	+1.027%	8660.03	−3.135%
+50%	12050.43	+34.787%	7334.35	−17.964%	8943.31	+0.033%	9819.70	+9.836%	9055.55	+1.289%	8595.63	−3.856%

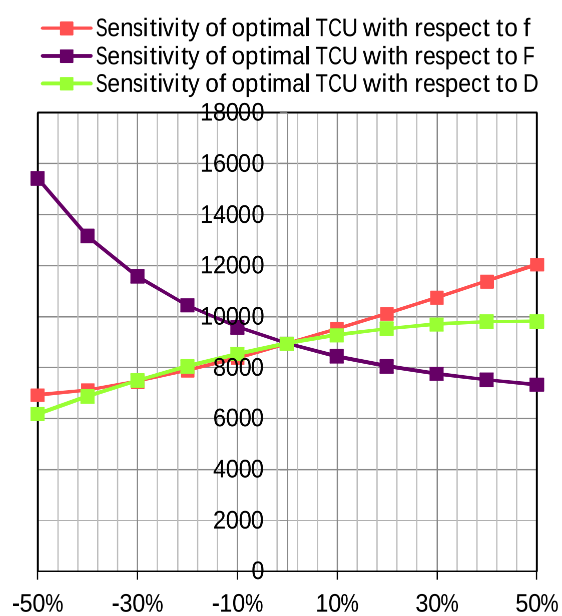

Fig. 6. Sensitivity of multi-product EPQ model with storage space constraint to f_j, F and D

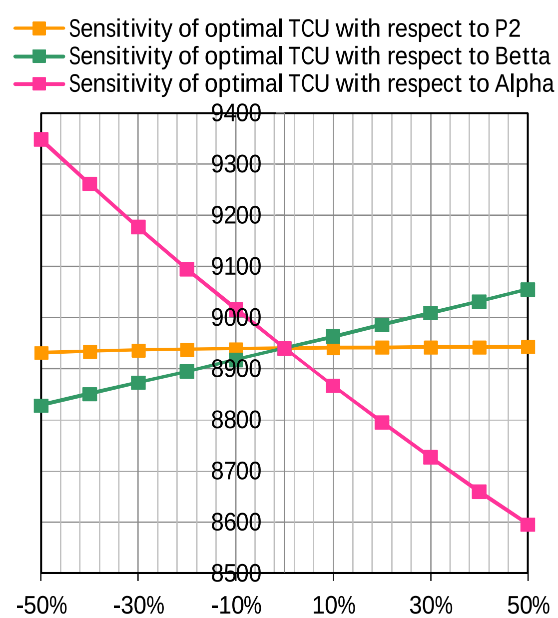

Fig. 7. Sensitivity of multi-product EPQ model with storage space constraint to P_2, α and β

8.4. Sensitivity analysis of multi-product EPQ with budget constraint

As shown in Figure 8, the violation of the total available budget indicates that the model's objective function yields lower costs when management has access to more financial resources, and the multi-product imperfect EPQ performs better when the purchase price of products and the demand rate are low. Figure 9 shows the behavior of the related parameters of the defective items. Although producing a batch with more defective items leads to higher costs, the presence of more reworkable products reduces the total inventory cost. All the results are presented in Table 7.

Tab. 7. Sensitivity of optimal total cost in multi-product EPQ model with budget constraint

% Change in parameter	Parameter: C_j Total cost	C_j %Change	M Total cost	M %Change	P_{2j} Total cost	P_{2j} %Change	D Total cost	D %Change	β_j Total cost	β_j %Change	α_j Total cost	α_j %Change
−50%	12988.26	−47.083%	48377.16	+97.098%	24541.31	−0.014%	14989.20	−38.931%	24176.47	−1.500%	26024.01	+6.027%
−40%	15235.07	−37.929%	40401.27	+64.603%	24542.43	−0.009%	17301.29	−29.511%	24248.92	−1.205%	25712.65	+4.759%
−30%	17528.09	−28.587%	34717.79	+41.447%	24543.22	−0.006%	19414.66	−20.901%	24321.94	−0.908%	25409.33	+3.523%
−20%	19849.81	−19.128%	30467.08	+24.129%	24543.84	−0.003%	21327.67	−13.107%	24395.58	−0.608%	25113.77	+2.319%
−10%	22190.60	−9.591%	27171.56	+10.702%	24544.30	−0.002%	23038.45	−6.137%	24469.85	−0.305%	24825.67	+1.145%
0	24544.69	0	24544.69	0	24544.69	0	24544.69	0	24544.69	0	24544.69	0
+10%	26908.44	+9.630%	22404.12	−8.721%	24545.00	+0.001%	25843.67	+5.292%	24620.17	+0.308%	24270.62	−1.117%
+20%	29279.42	+19.290%	20628.30	−15.956%	24545.25	+0.002%	26932.03	+9.727%	24696.28	+0.618%	24003.19	−2.206%
+30%	31655.96	+28.973%	19133.06	−22.048%	24545.46	+0.003%	27805.57	+13.285%	24773.03	+0.930%	23742.15	−3.270%
+40%	34036.86	+38.673%	17858.29	−27.242%	24545.65	+0.004%	28459.02	+15.948%	24850.44	+1.246%	23487.30	−4.308%
+50%	36421.25	+48.387%	16759.92	−31.717%	24545.83	+0.005%	28885.61	+17.686%	24928.52	+1.564%	23238.41	−5.322%

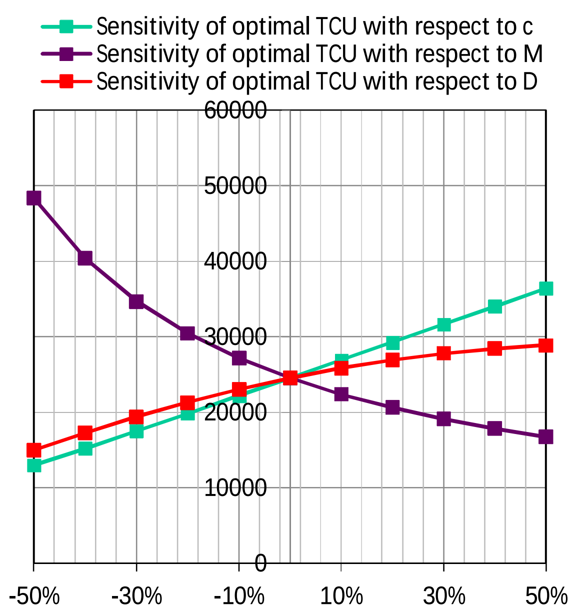

Fig. 8. Sensitivity of multi-product EPQ model with budget constraint to c_j, M and D

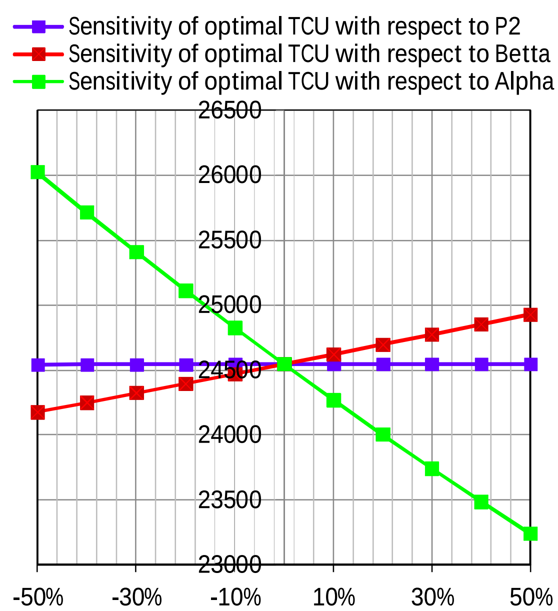

Fig. 9. Sensitivity of multi-product EPQ model with budget constraint to P_2, α and β

8.5. Sensitivity analysis of multi-product EPQ with number of setups constraint

Finally, Table 8 presents the results of the sensitivity analysis of the multi-product EPQ with a constraint on the number of setups. From Figure 10, we can conclude that there is a negative correlation between the total cost and the upper bound on the number of setups. The model's total cost increases when there is a higher demand for products, or holding cost of items has an increasing trend. If the rework rate decreases or the percentage of defective items and reworkable items increases (see Figure 11), the total cost decreases, and the production-inventory system performs better.

Tab. 8. Sensitivity of optimal total cost in multi-product EPQ model with number of setups constraint

% Change in parameter	Parameter											
	h_j		L		P_{2j}		D		β_j		α_j	
	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change	Total cost	%Change
−50%	4882.59	−33.620%	11067.31	+50.462%	7325.58	−0.407%	5432.21	−26.148%	7373.96	+0.251%	7405.04	+0.673%
−40%	5378.67	−26.876%	9863.34	+34.094%	7335.60	−0.271%	5903.63	−19.739%	7371.33	+0.215%	7393.58	+0.517%
−30%	5873.69	−20.146%	9010.79	+22.504%	7342.76	−0.174%	6331.85	−13.917%	7368.26	+0.173%	7383.03	+0.374%
−20%	6368.04	−13.425%	7937.75	+7.915%	7348.05	−0.102%	6716.84	−8.683%	7364.58	+0.123%	7373.16	+0.240%
−10%	6861.94	−6.710%	7602.10	+3.352%	7352.25	−0.045%	7058.17	−4.043%	7360.27	+0.064%	7363.94	+0.114%
0	7355.53	0	7355.53	0	7355.53	0	7355.53	0	7355.53	0	7355.53	0
+10%	7848.90	+6.707%	7177.37	−2.422%	7358.20	+0.036%	7608.16	+3.435%	7350.14	−0.073%	7347.68	−0.107%
+20%	8342.10	+13.413%	7052.80	−4.116%	7360.49	+0.067%	7815.29	+6.251%	7344.13	−0.155%	7340.40	−0.206%
+30%	8835.17	+20.116%	6923.73	−5.870%	7362.46	+0.094%	7976.08	+8.437%	7337.57	−0.244%	7333.75	−0.296%
+40%	9328.14	+26.818%	6904.82	−6.127%	7364.06	+0.116%	8089.20	+9.974%	7330.29	−0.343%	7327.53	−0.381%
+50%	9821.03	+33.519%	6909.38	−6.066%	7365.45	+0.135%	8152.68	+10.837%	7322.34	−0.451%	7321.78	−0.459%

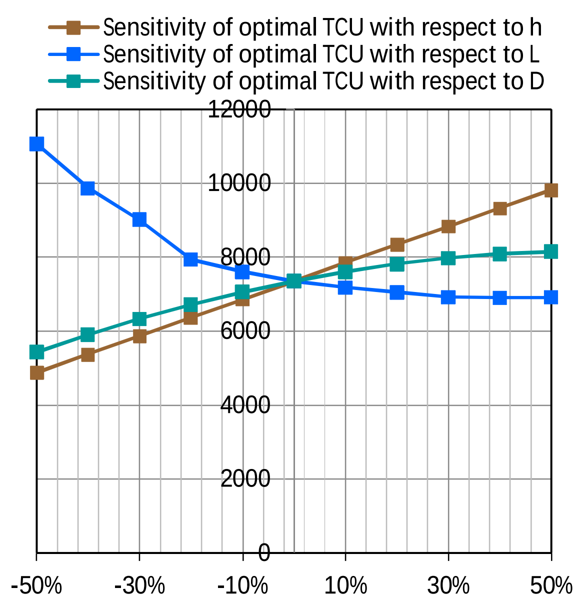

Fig. 10. Sensitivity of multi-product EPQ model with number of setups constraint to h, L and D

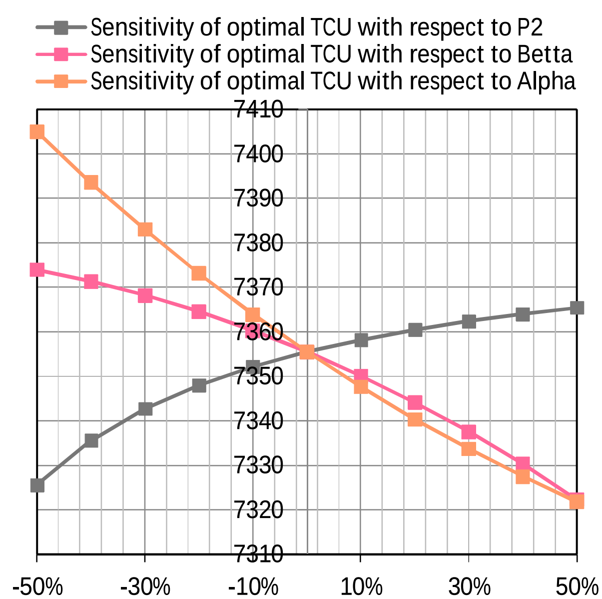

Fig. 11. Sensitivity of multi-product EPQ model with number of setups constraint to P_2, α and β

9. Conclusion

This paper proposes new constrained production-inventory models for multiple products where the manufacturing process is defective and produces a fraction of imperfect items. The imperfect products go through the rework process to become perfect and return to the consumption cycle. The objective is to determine economic production quantities so that the total cost of the system is minimized. Initially, a basic model with a single product is developed. The feasibility condition is also derived for this case. Then, multi-product constrained models are presented as extensions of the basic model to multi-product cases. The closed-form analytical solutions are derived separately for each case, and a numerical example is presented to illustrate and discuss the procedure. The results reveal that the holding and setup costs are the same in the optimal solution of the basic model. It is also observed that the presence of the active constraint causes a decrease in the optimal production quantities for the cases with the space and budget constraints and leads to an increase in the optimal quantities for the case with the constraint on the number of setups. A sensitivity analysis is performed for all the proposed models to study the behavior of the total cost in case of violating some parameters of the production-inventory system.

This work can be extended in several directions. A reasonable suggestion for future research is to adapt the models for a more general case where shortage (backorder or lost sale) is allowed. Moreover, it is interesting to include uncertain parameters, such as demand and defective rates, in the proposed models. As future extensions of this work, one can consider the learning effect of inspecting imperfect items or the quantity discount in purchasing.

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