International Journal of Industrial Engineering & Production Research (2017)

September 2017, Volume 28, Number 3
pp. 325- 345

DOI: 10.22068/ijiepr.28.3.325

http://IJIEPR.iust.ac.ir/

The Fuzzy Multi-Depot Vehicle Routing Problem with Simultaneous Pickup and Delivery: Formulation and A Heuristic Algorithm

Ali Nadizadeh

Ali Nadizadeh, *Industrial Engineering Department, Faculty of Engineering, Ardakan University*

KEYWORDS

Vehicle routing problem,
Simultaneous pickup and delivery,
Fuzzy demand,
K-means clustering algorithm.

ABSTRACT

In this paper, the fuzzy multi-depot vehicle routing problem with simultaneous pickup and delivery (FMDVRP-SPD) is investigated. In FMDVRP-SPD, a set of customers with simultaneous pickup and delivery demands should be supplied by a fleet of vehicles that start and end their tours at the same depot. In the problem, both pickup and delivery demands of customers are fuzzy variables. The objective of FMDVRP-SPD is to minimize the total cost of a distribution system, including vehicle traveling cost and vehicle fixed cost. To model the problem, a fuzzy chance-constrained programming model is proposed based on the fuzzy credibility theory. A heuristic algorithm combining K-means clustering algorithm and ant colony optimization is developed for solving the problem. To achieve an appropriate threshold value of parameters of the model, named "vehicle indexes", and to analyze their influences on the final solution, numerical experiments are carried out. Moreover, the efficiency of the heuristic algorithm is demonstrated by using a standard benchmark set of test problems.

1. Introduction

The transportation management includes all stages of the production and distribution systems and represents a related component (generally from 10% to 20%) of the final cost of the goods [1]. Thus, any improvement in transportation management provides reducing the cost of the goods [2]. Vehicle routing problem (VRP), located in the distribution systems, is the classic problem initially introduced by Dantzig and Ramser in 1959 [3]. The problem plays a pivotal role in logistics and is derived from the traveling salesman problem (TSP) [4]. In the VRP, there is a set of customers who have to be visited by a vehicle, and this vehicle has to start and finish its trip at the same depot. This is basically a reflection of real-life distribution problems such as delivering and picking up passengers, mail, packages, and different kind of goods [5]. The VRP is regarded as one of the most challenging integer programming problems. Lenstra and Rinnooy [6] showed that the VRP is an NP-hard combinational optimization problem; therefore, it is difficult to find its optimal solution [7]. While exact algorithms solve small-sized problems [8, 9], issues exist for the large-sized problems or special types of the VRP.

Many variants of the VRP have been developed so far to model real-world problems. The vehicle

* Corresponding author: *Ali Nadizadeh*

Email: nadizadeh@ardakan.ac.ir
Received 20 June 2017; revised 27 September 2017; accepted 8 October 2017

routing problem with simultaneous pickup and delivery (VRP-SPD) is one of the variants of the VRP and belongs to the reverse logistics [10]. There are various real cases for the problems, such as distribution of bottled drinks, chemicals, LPG (liquid petroleum gas) tanks, laundry service of hotels and schools, etc., where customers are typically visited for double service [11]. In the case of the bottled drinks, for instance, full bottles are delivered to customers, and empty ones are brought back either for reuse or for recycle [12]. The VRP-SPD is an Np-hard problem, and even it is more complicated than the VRP because of the fluctuating loads on the vehicle along a route [13]. In the VRP, the total load of each route must not exceed the capacity of the vehicle. However, in VRP-SPD, the net change (decrease or increase) in the vehicle load for each customer of the route must be monitored by the vehicle capacity [14, 15]. For example, the sequence of $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0$ is feasible in Fig. 1, but the routes or sequences of $0 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 0$ or $0 \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow 0$ are not. This event indicates the complexity of the VRP-SPD and suggests the use of extended equations in the modelling of the VRP-SPD against VRP.

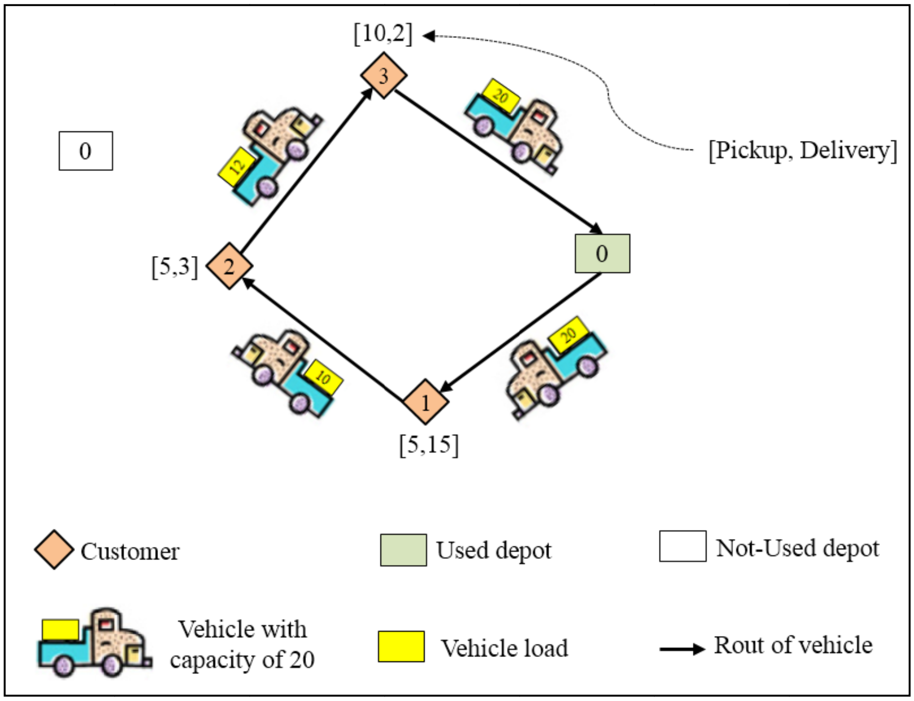

Fig. 1. Illustrative example of the VRP-SPD

Nowadays, due to the development of communication, information technology and the increasing pressure of transportation cost, logistics-based companies often use more than one depot instead of traditional fixed zone service of single depot. The problem is known as the multi-depot vehicle routing problem (MDVRP) [16]. On the other hand, uncertainty in vehicle routing problem arises in modeling a number of business situations that emerge in the area of distribution [17]. Fuzzy logic has been used to solve many applied problems so far. The need to use fuzzy logic in problems arises whenever there are some vague or uncertain parameters. For example, the information about demand of each customer is often not precise enough and customer demand is assumed as a fuzzy number [18]. As an example, based on numerical experiment, it can be concluded that the demand of a customer is “around 40 units”, usually “between 30 and 70 units”, etc. In most cases, there are no sufficient data for fitting a probability distribution to demands of customers, and fuzzy logic can be used to deal with uncertainty of these cases [19].

In this paper, the fuzzy multi-depot vehicle routing problem with simultaneous pickup and delivery (FMDVRP-SPD) is observed. The FMDVRP-SPD is a variant of the MDVRP and belongs to the class of the Np-hard problems. Although the MDVRP has been studied extensively in the literature, the FMDVRP-SPD has received scant attention from researchers so far. Table 1 shows the publications of MDVRP and its variants. As seen in this table and to the best of our knowledge, there are no works on the FMDVRP-SPD in the literature, and this paper is the first attempt in the field of MDVRP that considers the observations of fuzzy and simultaneous pickup and delivery demands, together. More precisely, this paper contributes to the FMDVRP-SPD in the following directions: (a) a fuzzy-chance constrained programming (FCCP) is proposed based on the credibility theory to model the problem; (b) a heuristic algorithm based on K-means clustering algorithm (K-MCA) and ant colony optimization (ACO) are developed to solve the problem; (c) the sensitivity analysis on the main parameters of the model is analyzed.

Tab. 1. summary of the related work on MDVRP

Author(s)	Year	Method used (Contribution or Case study)
Tillman [20]	1969	Clarke & Wright saving-based algorithm
Wren and Holiday [21]	1972	Saving based & refinements
Cassidy and Bennet	1972	Saving based & refinements (School meal delivery problem)
Gillett and Johnson [22]	1976	Clustering & sweep heuristic
Golden and Magnanti [23]	1977	Borderline customer & saving-based
Ball et al. [24]	1983	Saving based & route first, cluster second (distribution of chemical product in the USA and Canada)
Perl and Daskin [25]	1985	Incorporate (P) in location routing & formulation
Benton [26]	1986	Saving method & branch and bound (delivery to retail outlets from a bakery in Indiana)
Perl [27]	1987	(T–C) modified distance formula & a saving variant
Laporte et al. [28]	1988	Branch and bound
Min et al. [29]	1992	Exact methods & heuristic (distribution problem of the hardware products in the USA)
Chao et al. [30]	1993	Record to record
Renaud et al. [31]	1996	Tabu search (TS)
Salhi and Sari [32]	1997	Multi-level heuristic
Cordeau et al. [33]	1997	TS
Thangiah and Salhi [34]	2001	Genetic algorithm & clustering
Tarantilis and Kiranoudis [35]	2002	List-based threshold accepting (Open MDVRP, the distribution of meat in Greece)
Giosa et al. [36]	2002	One & two stage methods (MDVRP with time windows)
Polack et al. [37]	2004	VNS (MDVRP with time windows)
Lim and Wang [38]	2005	Several heuristics (MDVRP with fixed vehicle fleet)
Nagy and Salhi [39]	2005	Combination of a number of heuristics (MDVRP with pickups and deliveries)
Pisinger and Ropke [40]	2007	Adaptive large neighborhood search
Ho et al. [41]	2008	Genetic algorithm
Pepin et al. [42]	2009	Five heuristics & formulations (MDVSP with vehicle scheduling)
Zhang et al. [43]	2011	Formulation & scatter search (MDVRP with weight related cost)
Yu et al. [44]	2011	Ant Systems
Kuo and Wang [45]	2012	VNS (MDVRP with loading cost)
Rahimi-Vahed et al. [46]	2013	A path relinking algorithm (MDVRP with periodic)
Venkata Narasimha et al. [47]	2013	Ant colony optimization (Min-max MDVRP)
Contardo and Martinelli [48]	2014	Formulation and exact algorithm
Escobar et al. [49]	2014	Hybrid granular tabu search procedure
Luo and Chen [50]	2014	Shuffled frog leaping algorithm
Salhi et al. [16]	2014	VNS (MDVRP with heterogeneous vehicles)
Juan et al. [51]	2015	A hybrid approach with ILS
Kachitvichyanukul et al. [52]	2015	A variant of PSO (MDVRP with multiple pickup and delivery)

Wang et al. [53]	2016	Heuristic algorithm (Min-max MDVRP)
Calvet et al. [54]	2016	Hybrid approach combining statistical learning techniques with a metaheuristic (MDVRP with market-segmentation)
Bae and Moon [55]	2016	Heuristic with a genetic algorithm (MDVRP with time windows)
Wang et al. [56]	2016	Heuristic algorithm (MDVRP with minimum service time)

The remainder of this paper is organized as follows. In the next section, the literature review of the works related to the fields of VRP-SPD is summarized. In section 3, some basic concepts of fuzzy theory needed for modeling the problem are given. Section 4 defines the FMDVRP-SPD in more details and presents a fuzzy chance-constrained programming model using the fuzzy credibility theory. Details of the heuristic algorithm to solve the FMDVRP-SPD are presented in section 5. In section 6, numerical experiments are given to reveal the performance of the proposed method. In the final section, the conclusion remarks of the paper are presented.

2. Literature Review

Table 2 summarizes the related works on VRP-SPD, describing their main contributions and/or approaches. Min [57] was the first researcher to tackle the VRP-SPD, considering a real case faced by a public library, with one depot, two vehicles, and 22 customers. To solve the problem, the customers were first clustered into groups and then, in each group, the travelling salesman problems (TSPs) were solved. The infeasible arcs were penalized (their lengths set to infinity), and the TSPs were solved again. Halse [58] solved the VRP-SPD using a cluster-first routing-second approach. In the first stage, the assignment of customers to vehicles was performed, then a routing procedure based on 3-opt was used. Solutions to problems with up to 100 customers were reported to exist in this work.

Nagy and Salhi [39] proposed a method that firstly found a solution to the corresponding VRP problem and then modified the solution to make it feasible for MDVRP-SPD. They both adopted the idea of borderline customers, that is, customers were divided into two subsets, namely borderline and non-borderline customers. The non-borderline customers were assigned to their nearest depots, then the borderline customers were inserted into the single depot vehicle routing one at a time. Dell'Amico et al. [59] found the optimum solution for instances up to 40 customers of VRP-SPD by a proposed exact algorithm based on branch-and-price approach. Gajpal and Abad [60] presented the saving heuristic and the parallel saving heuristic for VRP-SPD. They used a cumulative net-pickup approach to checking the feasibility when two existing routes were merged.

Tab. 2. Related works of the VRP-SPD

Author(s)	Year	contributions and/or approaches.
Min [57]	1989	First work, case study in a public library
Halse [58]	1992	Cluster-first, route-second strategy, 3-opt procedure
Salhi and Nagy [61]	1999	Insertion based heuristics
Dethloff [62]	2001	Constructive heuristic based on the cheapest insertion, radial surcharge and residual capacity
Angelelli and Mansini [63]	2001	Branch-and-price for the VRP-SPD with time windows
Vural [64]	2002	Genetic algorithm
Gokce [65]	2003	Ant colony
Ropke and Pisinger [66]	2004	Large neighborhood search
Nagy and Salhi [39]	2005	Heuristics with different levels of feasibility
Crispim and Brandao [67]	2005	TS + VND
Dell'Amico et al. [59]	2005	Branch-and-price based on dynamic programming and state space relaxation
Chen and Wu [68]	2006	Record-to-record travel + Tabu lists

Montane and Galvao [69]	2006	TS
Gribkovskaia et al. [70]	2007	TS for the VRP-SPD with a single vehicle
Bianchessi and Righini [71]	2007	Constructive and local search heuristics TS+VNS
Subramanian and Cabral [72]	2008	ILS heuristic for the VRP-SPD with time limit
Subramanian et al. [73]	2008	ILS + VND heuristic
Zachariadis et al. [13]	2009	TS + Guided local search
Gajpal and Abad [60]	2009	Ant colony system
Subramanian et al. [74]	2010	RVND + ILS
Zachariadis et al. [75]	2010	Adaptive memory algorithmic
Çatay [14]	2010	Ant colony optimization
Zachariadis and Kiranoudos [76]	2011	Local search approach
Tasan and Gen [77]	2012	Genetic algorithm
Goksal et al. [78]	2013	PSO + VND
Subramanian et al. [79]	2013	ILS + RVND
Rieck,and Zimmermann [80]	2013	Two mixed-integer linear model formulations for the VRP-SPD
Liu et al. [81]	2013	VRP-SPD with time windows for the home health care logistics
Luo and Chen [50]	2014	Shuffled frog leaping algorithm
Juan et al. [51]	2015	A hybrid approach with ILS
Li et al. [82]	2015	ILS heuristic for the VRP-SPD with multi depots

Recently, many metaheuristics have been successfully applied to solve VRP-SPD. For instance, Çatay [14] extended an ant colony algorithm, employing a new saving-based visibility function and a pheromone reinforcement procedure. Zachariadis and Kiranoudos [76] proposed a local search approach which efficiently explored rich solution neighborhoods by statically encoding tentative moves into special data structures. Tasan and Gen [77] developed a genetic algorithm for solving VRP-SPD, which uses permutation-based representation and ensures feasibility. Goksal et al. [78] presented a heuristic solution approach based on particle swarm optimization (PSO) in which a local search was performed by a variable neighborhood descent algorithm (VND). Moreover, it implemented an annealing-like strategy to preserve the swarm diversity. Subramanian et al. [79] proposed a hybrid algorithm for a class of vehicle routing problems with the homogeneous fleet, including VRP-SPD. The hybrid algorithm hybridized an iterated local search-based heuristic approach and a set partitioning formulation, called ILS-RVND-SP. Li et al. [82] developed a metaheuristic based on iterated local search for MDVRP-SPD. In order to strengthen the search, they applied an adaptive neighborhood selection mechanism embedded into the improvement steps and the perturbation steps of iterated local search, respectively. To diversify the search, new perturbation operators are proposed. Their results indicated that the proposed approach outperforms the previous methods for MDVRP-SDP.

3. Fuzzy Credibility Theory

The concept of the fuzzy set was introduced by Zadeh [83] via the membership function and applied to the wide varieties of real problems, thereafter. To measure a fuzzy event, the term fuzzy variable was proposed by Kaufmann [84], and later Zadeh [85] proposed the possibility measure theory of fuzzy variable. Although the possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely necessary in both theory and practice on various problems. In order to define a self-dual measure, a modified form of the possibility theory, called credibility theory, was introduced by Liu [86] and studied very recently by many scholars all around the world. Since a fuzzy version of MDVRP-SPD with credibility theory will be modeled in this paper, a brief introduction to the basic concepts and definitions used is presented as follows [18]:

Let Θ be a nonempty set, and P be the power set of Θ. Each element in P is called an event, and ϕ is an empty set. In order to present an axiomatic definition of possibility, it is necessary to assign a number $\mathrm{Pos}\{A\}$ to each event A, which indicates the possibility that A will occur. To ensure that the number $\mathrm{Pos}\{A\}$ has certain mathematical properties, the following four axioms are approved [86]:

Axiom 3.1 $\mathrm{Pos}\{\Theta\} = 1$;

Axiom 3.2 $\mathrm{Pos}\{\phi\} = 0$;

Axiom 3.3 For each $A_i \in P(\Theta)$, $\mathrm{Pos}\left\{\bigcup_{i=1}^{n} A_i\right\} = \mathrm{supremum}_i \mathrm{Pos}\{A_i\}$;

Axiom 3.4 If Θ_i is a non-empty set, and the set function $\mathrm{Pos}_i\{\}$; $i = 1, 2, ..., n$, satisfies the above three axioms and $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, then, for each $A \in P(\Theta)$, $\mathrm{Pos}\{A\} = \mathrm{supremum}_{(\theta_1,\theta_2,...,\theta_n)\in A}$ $\mathrm{Pos}_1\{\theta_1\} \wedge \mathrm{Pos}_2\{\theta_2\} \wedge \cdots \wedge \mathrm{Pos}_n\{\theta_n\}$.

The above four axioms form the basis of credibility measure theory; all concepts of credibility theory can be obtained by them [86].

Definition 3.5 Let (Θ, $P(\Theta)$, Pos) be a possibility space, and A be a set in $P(\Theta)$, then the necessity measure of A is defined by $\mathrm{Nec}\{A\} = 1 - \mathrm{Pos}\{A^c\}$, such that A^c is the complement of event A.

Definition 3.6 Let (Θ, $P(\Theta)$, Pos) be a possibility space, and A be a set in $P(\Theta)$, and then the credibility measure of A is defined by $Cr\{A\} = \frac{1}{2}$ $(\mathrm{Pos}\{A\} + \mathrm{Nec}\{A\})$.

Considering definition 3.6, the credibility of a fuzzy event is defined as the average of its possibility and necessity. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0 [86]. As mentioned before, the credibility measure is self-dual, and in theory of fuzzy subsets, the law of credibility plays a role similar to that played by the law of probability in measurement theory for ordinary sets [87].

Now, consider a triangular fuzzy variable $\tilde{d}$ = (d_1, d_2, d_3) for demand of a customer where $\tilde{d}$ is described by its left boundary d_1 and its right boundary d_3. Thus, the analyst studying that problem can subjectively estimate that, based on his experience or available data, the demand of the customer will not be less than d_1 or greater than d_3. The value of d_2 related to the grade of membership of 1 can also be determined by a subjective estimate. If the certain demand of a customer is considered by the value of r, the possibility, necessity, and credibility are easily obtained as follows [88]:

$$\mathrm{Pos}\ \{\tilde{d} \ge r\} = \begin{cases} 1, & \text{if } r \le d_2 \\ \dfrac{d_3 - r}{d_3 - d_2}, & \text{if } d_2 \le r \le d_3 \\ 0, & \text{if } r \ge d_3 \end{cases} \tag{1}$$

$$\mathrm{Nec}\ \{\tilde{d} \ge r\} = \begin{cases} 1, & \text{if } r \le d_1 \\ \dfrac{d_2 - r}{d_2 - d_1}, & \text{if } d_1 \le r \le d_2 \\ 0, & \text{if } r \ge d_2 \end{cases} \tag{2}$$

$$\mathrm{Cr}\ \{\tilde{d} \ge r\} = \begin{cases} 1, & \text{if } r \le d_1 \\ \dfrac{2d_2 - d_1 - r}{2(d_2 - d_1)}, & \text{if } d_1 \le r \le d_2 \\ \dfrac{d_3 - r}{2(d_3 - d_2)}, & \text{if } d_2 \le r \le d_3 \\ 0, & \text{if } r \ge d_3 \end{cases} \tag{3}$$

4. Problem Definition and Formulation

FMDVRP-SPD extends the basic VRP in such a way that there are multiple depots in different locations and there are some customers who have fuzzy demands, which consist of two parts: the receiving as well as shipping goods. Each vehicle in FMDVRP-SPD is used only in one route and starts and finishes its route at the same depot. Moreover, a fleet of homogeneous vehicles is available at each depot [82]. The objective of the problem is to determine the optimal routes by minimizing the total cost related to the number of vehicles and traveling of vehicles. In short, the following constraints must be met in FMDVRP-SPD:

(1) Each vehicle starts and ends at the same depot.

(2) Each customer is only visited once by a vehicle.

(3) Each customer has a fuzzy demand composed of two parts of pickup and delivery.

(4) The maximum load of each route must not exceed the vehicle capacity at each point of the route.

(5) The total duration of each route (including travel and service time) should not exceed the preset limit.

(6) Number of vehicles located at each depot is predefined, and all the vehicles are homogeneous.

In FMDVRP-SPD, in addition to the above assumptions, the pickup and delivery demands of costumers are considered as triangular fuzzy numbers such as $\tilde{p}_j = (p_{1j}, p_{2j}, p_{3j})$ and $\tilde{d}_j = (d_{1j}, d_{2j}, d_{3j})$ for the j^{th} customer, respectively. Let the vehicles have equal capacity that is denoted by Q, and it can be changed to the triangular fuzzy number as $\tilde{Q} = (Q, Q, Q)$. To model the problem with credibility theory, after serving the first k customers, the available capacity of the vehicle will equal $\tilde{Q}_k = \tilde{Q} + \left(\sum_{j=1}^{k} \tilde{p}_j - \sum_{j=1}^{k} \tilde{d}_j\right) = (q_{1,k},\, q_{2,k},\, q_{3,k})$ in which $\tilde{Q}_k$ is also a triangular fuzzy number by using the rules of fuzzy arithmetic. Thus, after serving k customers, if the capacity of the vehicle in its route is enough and if the relation $\tilde{Q}_k + \left(\tilde{p}_{k+1} - \tilde{d}_{k+1}\right) \geq 0$ is fulfilled, the vehicle certainly can serve the $k+1^{th}$ customer. On the other hand, the credibility that the next customer demand does not exceed the remaining capacity of the vehicle can be obtained as follows:

$$\begin{aligned} \mathrm{Cr} &= Cr\left\{\tilde{Q}_k + \left(\tilde{p}_{k+1} - \tilde{d}_{k+1}\right) \geq 0\right\} = Cr\left\{\left(\tilde{d}_{k+1} - \tilde{p}_{k+1}\right) \leq \tilde{Q}_k\right\} \\ &= Cr\left\{\left(dp_{1,k+1} - q_{3,k},\, dp_{2,k+1} - q_{2,k},\, dp_{3,k+1} - q_{1,k}\right) \leq 0\right\} \end{aligned} \tag{4}$$

$$\mathrm{Cr}\left\{\tilde{d}_{k+1} - \tilde{p}_{k+1} \leq \tilde{Q}_k\right\} = \begin{cases} 0, & \text{if } dp_{1,k+1} \geq q_{3,k} \\ \dfrac{q_{3,k} - dp_{1,k+1}}{2 \times (q_{3,k} - dp_{1,k+1} + dp_{2,k+1} - q_{2,k})}, & \text{if } dp_{1,k+1} \leq q_{3,k},\ dp_{2,k+1} \geq q_{2,k} \\ \dfrac{dp_{3,k+1} - q_{1,k} - 2 \times (dp_{2,k+1} - q_{2,k})}{2 \times (q_{2,k} - dp_{2,k+1} + dp_{3,k+1} - q_{1,k})}, & \text{if } dp_{2,k+1} \leq q_{2,k},\ dp_{3,k+1} \geq q_{1,k} \\ 1, & \text{if } dp_{3,k+1} \leq q_{1,k} \end{cases} \tag{5}$$

Note that the equation of $\tilde{d}_{k+1} - \tilde{p}_{k+1} = \left(dp_{1,k+1},\, dp_{2,k+1},\, dp_{3,k+1}\right)$ is replaced in formulations of (4) and (5). To describe the meaning of the credibility theory for modeling of the FMVRP-SPD, the following statements are considerable; There is no doubt that if the remaining goods in the vehicle are high and the net demand (i.e., different between pickup and delivery demands) at the next customer is low, then the vehicle's chance of being able to finish the next customer's service becomes greater. This means that the greater the difference between available goods and net demand for the next customer, the greater preference to send the vehicle to serve the next customer. According to formulation (5), the preference index is designated by *Cr* which denotes the magnitude of the preference for sending the vehicle to the next customer after it served the current customer. It is obvious that $Cr \in [0, 1]$. When $Cr = 0$, driver is completely sure that he should return the vehicle to the depot. When $Cr = 1$, the driver is absolutely certain that he can serve the next customer by the remaining goods having in his vehicle. Let the dispatcher preference index be designated by Cr^*, where $Cr^* \in [0,1]$. Therefore, according to Cr^* value and the credibility that the next customer demand does not exceed the remaining capacity of the vehicle, a decision must be made as to whether to send the vehicle to the next customer or return that to the depot. Thus, we can say that if relation $Cr \geq Cr^*$ is fulfilled, then the vehicle should be sent to the next customer; otherwise, the vehicle should be returned to the depot and sent back again to the next customer after loading sufficient goods [89]. Moreover, the vehicle routes (or planned routes) are designed in advance by applying the proposed heuristic algorithm. But, the real value of demand of a customer is only characterized when the vehicle reaches the customer. Due to uncertain demand of the customers, a vehicle might not be able to serve a customer once it arrives there due to insufficient capacity. It is assumed that, in such situations, the vehicle returns to the depot to load itself and then returns to the customer where it had a "failure" and continue its service along the rest of the planned route. This arises an additional distance due to route "failure". Hence, an additional distance should be considered for the vehicle due to the route "failure" [88].

Parameter Cr^* has an extremely great impact on both the total length of the planned routes and on the additional distance. For example, lower values of parameter Cr^* express the dispatcher's desire to use all the capacity of the vehicle. These values result in shorter planned routes. But, lower values of parameter Cr^* increase the number of circumstances where a vehicle meets a customer, but it is unable to serve that, thereby increasing the total distance it covers due to the "failure". In this situation, stochastic simulation can be used to evaluate the additional distance due to route "failure". On the other hand, higher values of parameter Cr^* are characterized by less utilization of vehicle capacity along the planned routes and fewer additional distance to cover due to "failures". As a result, the problem logically arises for determining the value of parameter Cr^*, and also other similar parameter of the model will be described in the following, which will result in the least total sum of planned route lengths and additional distance.

The node-based fuzzy chance-constrained programming (FCCP) formulation of the FMDVRP-SPD is expanded as follows. Let $G = (N, A)$ be a complete directed network where $N = N_0 \bigcup N_C$ is a set of nodes in which N_0 and N_C represent the depot and customers nodes, respectively, and $A = \{(i, j): i, j \in N\}$ is the set of arcs. Each arc (i, j) has a nonnegative cost (distance) c_{ij} and travel time t_{ij} that is based on Euclidian distance and triangular inequality holds (i.e., $c_{ij} + c_{jk} \geq c_{ik}$) and $c_{ij} = t_{ij}$. K is the vehicle set and a fleet of m_v identical vehicles with capacity $\tilde{Q}$ with maximum travel time T and fixed operating cost of f available at each depot. Each customer $i \in N_C$ has pickup ($\tilde{p}_i$) and delivery ($\tilde{d}_i$) demands, so that they are fuzzy and $0 \leq \tilde{p}_i, \tilde{d}_i \leq \tilde{Q}$. Moreover, each customer $i \in N_C$ has a service time s_i. The decision variables used for the formulation of the FMDVRP-SPD are given as follows:

$$x_{kij} = \begin{cases} 1 & \text{If vehicle } k \text{ travels directly from node } i \text{ to node } j \quad (\forall\ i, j \in N) \\ 0 & \text{otherwise and if } max\left\{(\tilde{d}_i + \tilde{d}_j),\ (\tilde{p}_i + \tilde{p}_j),\ (\tilde{p}_i + \tilde{d}_j)\right\} > \tilde{Q} \quad (\forall\ i, j \in N_C) \end{cases}$$

$\tilde{l}_{kij}$ = the load of vehicle (or route) k on arc (i, j) $(\forall\ i, j \in N)$

The node-based FCCP formulation of the FMDVRP-SPD is proposed as follows:

$$Minimize \quad \sum_{k \in K} \sum_{i \in N_0} \sum_{j \in N_C} f x_{kij} + \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{kij} + B \quad (6)$$

subject to:

$$\sum_{k \in K} \sum_{j \in N_C} x_{kij} \leq m_v \qquad \forall\ i \in N_0 \qquad (7)$$

$$\sum_{k \in K} \sum_{j \in N} x_{kij} = \sum_{k \in K} \sum_{j \in N} x_{kji} = 1 \qquad \forall\ i \in N_C \qquad (8)$$

$$\sum_{j \in N_C} x_{kij} = \sum_{j \in N_C} x_{kji} \leq 1 \qquad \forall\ k \in K,\ i \in N_0 \qquad (9)$$

$$\sum_{i \in S} \sum_{j \in S} x_{kij} \leq |S| - 1 \qquad \forall\ S \subseteq N_C,\ k \in K \qquad (10)$$

$$\sum_{i \in N_0} \sum_{j \in N_0} x_{kij} = 0 \qquad \forall\ k \in K \qquad (11)$$

$$\sum_{i \in N} \sum_{j \in N} (c_{ij} + s_i) x_{kij} \leq T \qquad \forall\ k \in K \qquad (12)$$

$$Cr\left(\tilde{l}_{kij} \leq \tilde{Q}\right) \geq Cr^1 \qquad \forall\ k \in K,\ \forall\ i, j \in N \qquad (13)$$

$$Cr\left(\sum_{k \in K} \sum_{j \in N} \tilde{l}_{kij} - \sum_{k \in K} \sum_{j \in N} \tilde{l}_{kji} = \tilde{d}_i - \tilde{p}_i\right) \geq Cr^2 \qquad \forall\ i \in N_C \qquad (14)$$

$$x_{kij} \in \{0, 1\} \qquad \forall\ k \in K,\ \forall\ i, j \in N \qquad (15)$$

$$\tilde{l}_{kij} \geq 0 \qquad \forall\ k \in K,\ i, j \in N \qquad (16)$$

In the above formulation, objective function (6) minimizes the sum of the fixed cost related to the number of vehicles, the total travel cost of all the vehicles, and total additional distances. Note that the total additional distances, denoted by B, will be obtained by the stochastic simulation algorithm presented in section 5.3.2. Constraints (7) guarantee that the number of vehicles departing from each depot should not be more than the number of available vehicles. Constraints (8) state that each customer should be visited exactly once and served within one route only. Constraints (9) represent that each vehicle starts and ends at the same depot. Constraints (10) eliminate sub-tours and ensure that the solution is connected. Constraints (11) indicate that the vehicles cannot travel directly between two depots. Constraints (12) express the limitation of travel distance of vehicles. Fuzzy chance constraints (13) indicate that total loads in each point of a route must not exceed the vehicle capacity at a confidence level. Fuzzy chance constraints (14) show the flow conservation equations at a confidence level. Finally, constraints (15) and (16) specify the binary variables and the range of decision variables, respectively. Note that, for convenience of the computational experiments in Section 6, confidence levels of Cr^1 and Cr^2 in the model are called "vehicle indexes".

5. The Proposed Heuristic Algorithm

A heuristic algorithm with three phases is proposed to solve the FMDVRP-SPD. Fig. 2 shows three phases of the heuristic algorithm, schematically. In the first phase, a K-means clustering algorithm is applied to all customers grouped (Fig. 2(a)). In the second phase, the clusters of customers are assigned to the depots based on proximity of the centroid of clusters and depots (Fig. 2(b)). In the final phase, ant colony optimization (ACO) is used to proper routes of vehicles obtained (Fig. 2(c)). In this phase, stochastic simulation is also applied to the additional distances related to the routes "failure".

It is important to note that the heuristic algorithm is repeated for a predefined number of iterations. When the algorithm achieves a better solution, it is replaced by the last best-known solution. Moreover, since in the first phase, clustering centroids are initialized randomly, clusters formed in each iteration of the heuristic algorithm are different together. This can ensure that the proposed algorithm avoids confining sub-optimal solutions. The details of each phase of the heuristic algorithm are summarized in the following sections.

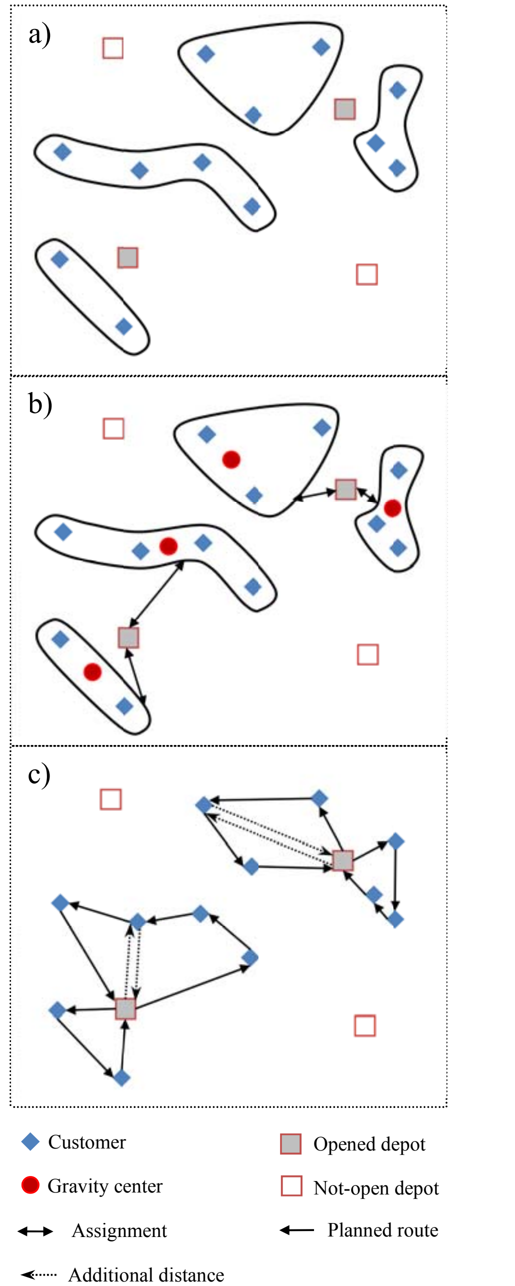

Fig. 2. Illustrative example of the heuristic algorithm.

5-1. K-means clustering algorithm

K-means clustering algorithm (K-MCA) was introduced by Hartigan and Wong [90]. This algorithm is one of the well-known squared error-based clustering algorithms, which is both easy to implement and reasonably effective in solving many practical problems [91-93]. The aim of K-MCA is to divide M points in N dimensions into

K clusters so that the within-cluster sum of squares is minimized [94]. The initial clustering centroids are classified optimally according to the minimum value of evaluating indicator Jc which indicates the sum of error squares. It is defined as in the following equation:

$$J_c = \sum_{k=1}^{K} \sum_{p \in X_k} \left\| p - m_k \right\|^2 \tag{17}$$

where X_k denotes the set of clustering centers, m_k is the average value of the clustering center k, and p is the data included (i.e., coordinates customers) in the clustering center k. The search of objective function is along the sum of error squares decreasing direction. The K-means clustering algorithm can resolve the clustering problems effectively, but it depends on K value. Improper K gives significant influences on the actual effect on the algorithm [95]. In this paper, for the first phase of the heuristic algorithm, the following relation is used to determine K value:

$$K = \left\lceil max \left\{ \frac{\sum_{i=1}^{N_C} p_{i3}}{Q}, \frac{\sum_{i=1}^{N_C} d_{i3}}{Q} \right\} \right\rceil \tag{18}$$

where p_{i3} and d_{i3} are the right boundary of fuzzy pickup and delivery demands of the i^{th} customer, respectively. Q is the vehicle capacity, N_C represents the set of customers, and $\lceil b \rceil$ denotes the smallest integer number of greater than b. It is obvious that the more the number of clusters is, the more vehicles will be used, causing an increase in the transportation cost. Applying equation (20), the minimum number of vehicles with high fuzzy credibility value will be achieved. K-MCA uses different agglomerative techniques which can be mainly classified as follows: (1) nearest neighbor, (2) farthest neighbor, (3) weighted average, (4) moving average, and (5) unweighted centroid [96]. In the proposed heuristic algorithm, the nearest neighbor (also called single linkage) agglomerative technique is utilized. K-MCA with this technique is initialized with randomly assigned K cluster centroids, and these cluster centroids are updated after the assignment of all data points to the closest clusters [94].

Typical convergence criteria in K-MCA are:

(1) no (or minimal) reassignment of data points to new cluster centroid is reached,
(2) minimal decrease in squared error is achieved. In the presented K-MCA, in addition to the above criteria, once a new customer is selected to be included in a cluster, the following capacity conditions must be held:
3) Total fuzzy pickup demands of the current members of the cluster with new customers should be less than the capacity of the vehicle (i.e., $\left(\sum_{i=1}^{r} \tilde{p}_i\right) + \tilde{p}_{r+1} \le \tilde{Q}$).
4) Total fuzzy delivery demands of the current members of the cluster with new customer should be less than the capacity of the vehicle (i.e., $\left(\sum_{i=1}^{r} \tilde{d}_i\right) + \tilde{d}_{r+1} \le \tilde{Q}$).
5) Total fuzzy pickup demands of current members of the cluster with fuzzy delivery demand of new customer should be less than the capacity of the vehicle (i.e., $\left(\sum_{i=1}^{r} \tilde{p}_i\right) + \tilde{d}_{r+1} \le \tilde{Q}$).

The pseudo-code of the K-MCA is shown in Algorithm 1.

Algorithm 1: K-means clustering algorithm.
01: input the number of clustering K and coordinate all customers
02: for i = 1, 2, ..., K do
03: initialize clustering centroids randomly
04: end for
05: while termination criteria not satisfied do
06: calculate distances and classify
07: calculate the average distance value of each cluster
08: make new clustering centroids
09: end while
10: display K clusters

5-2. Allocating clusters to depot(s)

In this phase, the clusters are respectively allocated to the depots. Each depot can serve as many clusters according to the available vehicles located at each depot, denoted by m_v in formulation. To allocate the clusters, the Euclidian distance of the gravity center of a cluster to all depots is calculated. Afterwards, the cluster is allocated to the nearest depot. If the nearest depot is not an available vehicle, the next nearest depot is the candidate depot to allocate the cluster. This procedure (i.e., allocating cluster to the nearest depot with available vehicle(s)) is repeated until all clusters are covered.

5-3. Routing

5-3-1. Ant colony optimization

In the last phase of the heuristic algorithm, the routing problem for each cluster and the relevant depot is solved. The routing problem of FMDVRP-SPD is the same as TSP, which is solved using ACO. ACO is an intelligent algorithm inspired by the foraging behavior of ants [97]. Ants use a special chemical substance on their way called pheromone to communicate and exchange knowledge during individuals so that other ants can pass the same route. The pheromone of the shorter route increases; therefore, more ants move from that way. This behavior has inspired people to create artificial ant systems to resolve combinatorial optimization problems and obtain approximately optimal solutions [95, 98, 99].

5-3-2. Stochastic simulation

As mentioned before, the pickup and delivery demands of each customer are triangular fuzzy numbers, so they cannot be directly considered as deterministic numbers such as other algorithms that solve the deterministic MDVRP-SPD. Since the real value of demand is identified as the vehicle reaches the customer, the simulation experiment is utilized to specify the deterministic value of pickup and delivery demands of each customer. For each feasible planned route that the solution of the heuristic algorithm stands for, additional distances due to route "failures" (B) are obtained by a stochastic simulation algorithm. The following stochastic simulation algorithm with four steps is proposed to reveal the real pickup and delivery demands of each customer:

Step 1: For each customer, estimate the additional distances by simulating "real" pickup and delivery demands. The "real" pickup demands were generated by the following steps: (1) randomly generate a real number p in the interval between the left and right bounds of the triangular fuzzy number representing pickup demand of the customer and compute its membership m; (2) generate a random number r; $r \in [0,1]$; (3) compare r and m, if $r \leq m$, then "real" pickup demand of the customer is adopted as p; otherwise, it is not accepted. In this case, random numbers p and r are generated again and again until random numbers p and r are found, such that relation $r \leq m$ is satisfied; (4) check and repeat (1) to (3), and terminate the process when each customer has a simulation "real" value of pickup demand. Note that the above process is the same for "real" delivery demands.

Step 2: Move along the route designed by ACO and calculate the additional distance due to route "failures" in terms of the "real" pickup and delivery demands.

Step 3: Repeat Steps 1 and 2 for M times.

Step 4: Compute the average additional distances that come out of simulation, and return it as the additional distance the route.

6. Computational Results

6-1. Sensitivity analysis on parameters of "vehicle indexes"

In this section, numerical experiments are given to reveal the performance of the FCCP model of the FMDVRP-SPD and the efficiency of the heuristic algorithm. To evaluate the sensitivity of the parameters of the model, different sizes of instances are considered to conduct computational experiments. It is assumed that there are 30 customers and 2 depots with 3 vehicles located at each depot for a small-sized instance, 50 customers and 4 depots with 3 vehicles located at each depot for a medium-sized instance, and finally 100 customers and 10 depots with 4 vehicles located at each depot as a large-sized instance. In each instance, the coordinates of all customers and depots are generated randomly in [50×50]. Moreover, the triangular fuzzy demands of customers, such as $\tilde{d} = (d_1, d_2, d_3)$, are selected randomly. More precisely, d_1, d_2, and d_3 are randomly generated within [10,25], [26,50] and [51,80], respectively. The general specifications of the three test instances are listed in Table 3. In Table 3, the serving time of customers is expressed based on the distance scale, and the name of each instance can be summarized as the number of depots, $|N_0|$ and the number of customers, $|N_C|$ (i.e., $|N_0| \times |N_C|$). It is important to note that the test instances are similar to the real-world cases, and the obtained results can be applied for real applications.

Tab. 3. The relative data of the three instances

ID of Instances	Vehicle capacity	No. of vehicles at each depot	Fixed cost of vehicles	Max. duration of each vehicle
2 × 30	600	2	20	∞

4 × 50	500	3	20	∞
10 × 100	400	4	20	∞

The heuristic algorithm is encoded in MATLAB 7.10.0 on a computer, holding Intel® Core™ Duo CPU T2450 2.00 GHz and 1.00 GB of RAM. The value of "vehicle indexes" (i.e., Cr^1 and Cr^2 in the formulation) varied within the interval of 0.1 to 1 with a step of 0.1. The average computational results of 10 times are presented in Tables 4–6 for three different size test instances, respectively. The columns of all tables are respectively named as the "vehicle indexes", the planned routes, the additional distances, the routing costs that include the planned routes and additional distances, the vehicle costs, the total costs that consist of routing costs as well as vehicle costs, and finally the CPU time of solutions. For convenience, the results of Tables 4–6 are depicted in Figs. 3–5, respectively. As seen in Tables 4–6 and also in Figs. 3–5, when the value of "vehicle indexes" is equal to 0.6, the total cost has a minimum amount.

Tab. 4. Computational results for 2 × 30 instance with different "vehicle indexes"

"Vehicle indexes"	Planned distances	Additional distances	Routing costs	Vehicle costs	Total costs	CPU Time (second)
0.1	250.7	67.2	317.9	40	357.9	6
0.2	255.3	58.7	313.9	40	353.9	6
0.3	261.1	47.0	308.1	40	348.1	7
0.4	266.7	39.1	305.8	40	345.8	6
0.5	270.4	32.3	302.8	40	342.8	6
0.6	275.6	18.6	294.2	40	**334.2***	6
0.7	285.9	9.3	295.2	60	355.2	5
0.8	293.5	4.2	297.7	60	357.7	5
0.9	296.9	0.5	297.4	80	377.4	5
1.0	307.7	0.0	307.7	80	387.7	5

*Bold number indicates the minimum total cost

Tab. 5. Computational results for 4 × 50 instance with different "vehicle indexes"

"Vehicle indexes"	Planned distances	Additional distances	Routing costs	Vehicle costs	Total costs	CPU Time (second)
0.1	317.0	80.3	397.3	60	457.3	15
0.2	320.7	71.2	391.9	60	451.9	14
0.3	323.3	58.5	381.8	80	461.8	15
0.4	328.8	52.1	380.8	80	460.8	17
0.5	343.6	39.3	382.9	80	462.9	14
0.6	350.1	20.9	371.0	80	**451.0***	16
0.7	378.8	6.8	385.7	100	485.7	14
0.8	386.0	2.6	388.6	120	508.6	14
0.9	394.0	0.3	394.3	140	534.3	15
1.0	450.7	0.0	450.7	140	590.7	15

*Bold number indicates the minimum total cost

Tab. 6. Computational results for 10 × 100 instance with different "vehicle indexes"

"Vehicle indexes"	Planned distances	Additional distances	Routing costs	Vehicle costs	Total costs	CPU Time (second)
0.1	463.9	198.9	662.8	140	802.8	38
0.2	489.7	170.5	660.2	140	800.2	35
0.3	505.7	143.4	649.2	160	809.2	41

0.4	519.7	131.3	651.0	180	831.0	32
0.5	533.4	88.5	621.8	200	821.8	39
0.6	538.3	31.4	569.7	220	**789.7***	44
0.7	568.0	7.9	575.9	260	835.9	35
0.8	572.5	1.0	573.5	300	873.5	31
0.9	600.2	0.0	600.2	320	920.2	37
1.0	616.8	0.0	616.8	340	956.8	39

*Bold number indicates the minimum total cost

According to Figs. 3–5, lower values of "vehicle indexes" denote a tendency to use total vehicle capacity. These values are associated with the routes with the shorter planned distances. Furthermore, lower values of "vehicle indexes" increase the number of cases in which a vehicle visits customers, but it is unable to serve them, thereby increasing the total additional distance due to the route "failure". Higher values of "vehicle indexes" are characterized by less utilization of vehicle capacity along with fewer additional distance to cover due to "failures". Consequently, in this analysis, the proper value of "vehicle indexes" is approximately around 0.6, considering the total cost.

Moreover, at high "vehicle indexes" value, to ensure high service to customers, the decision-maker considers fewer customers for each cluster to increase the number of clusters. In addition, because each cluster is supported by one vehicle, the cost of employing the vehicle may be high. As seen in Table 4–6, when the value of "vehicle indexes" has increased, the cost (or the number) of deploying the vehicles is also grown.

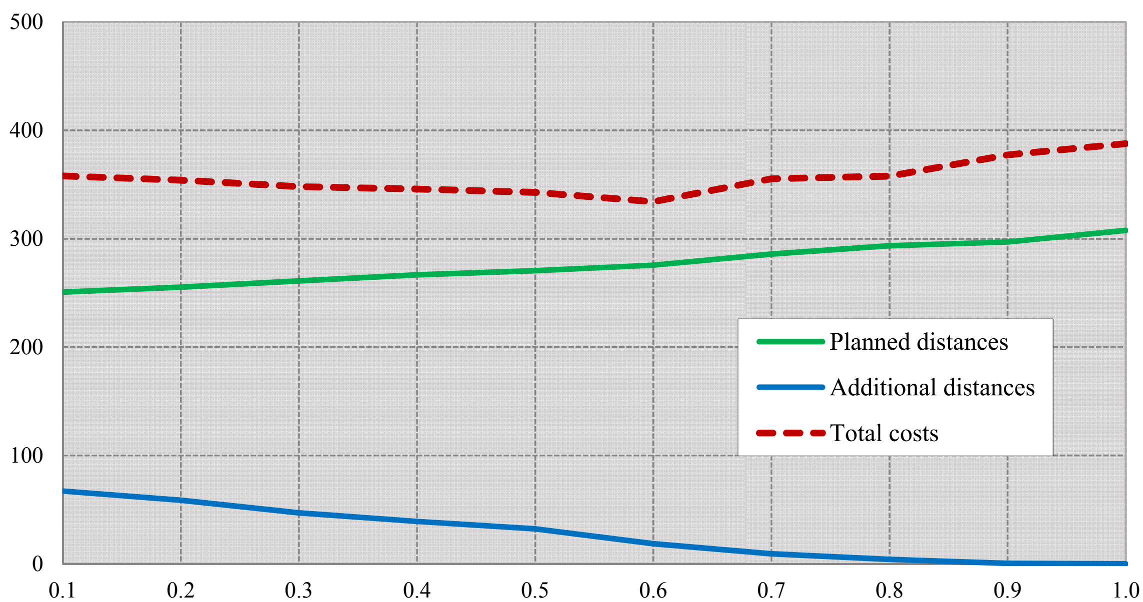

Fig. 3. The cost changes with various "vehicle indexes" for 2 × 30 instance.

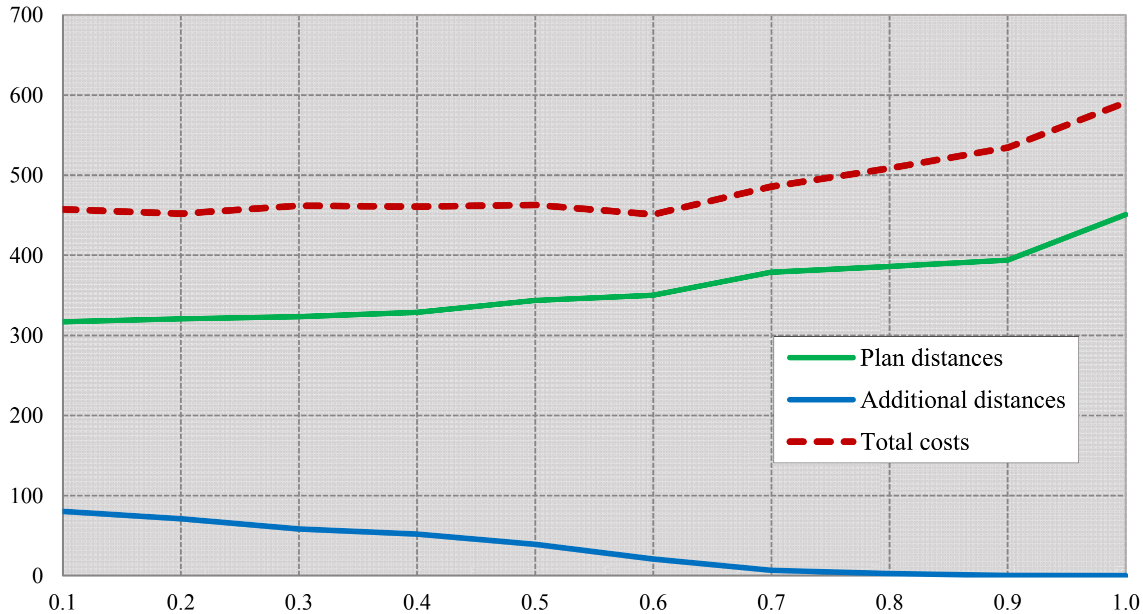

Fig. 4. The cost changes with various "vehicle indexes" for 4 × 50 instance.

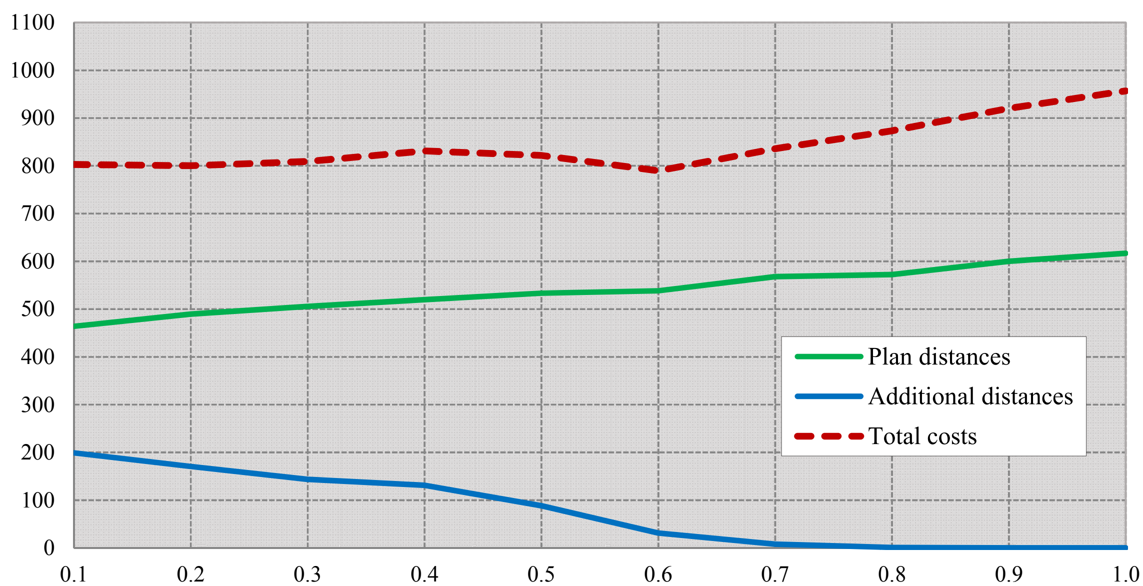

Fig. 5. The cost changes with various "vehicle indexes" for 10 × 100 instance.

6-2. Performance evaluation of the heuristic algorithm

To evaluate the efficiency of the presented heuristic algorithm, a computational experiment is carried out in this section. The efficiency of the proposed method is evaluated using 14 standard benchmark test problems of CVRP presented by [100]. It is noted that each test problem of FMDVRP-SPD can be reduced to a CVRP. Actually, if the number of depots equals 1, the pickup demands equal 0, and the left and right bounds of the triangular fuzzy demands are equal; then, the FMDVRP-SPD is changed to CVRP. The comparative results are summarized in Table 7. The first column of Table 7 represents the ID number of each test problem. The second column reports the best-known solutions (BKS) that are given in the literature [101]. The solutions and CPU times obtained by two approaches: SS-ACO of [102] and PSO of [103], as shown in the next columns. The last column of the table shows the solution and CPU time of the heuristic algorithm.

Results of Table 7 indicate that the heuristic algorithm in comparison with two approaches has been able to obtain 8 best-known solutions out of the 14 test problems (see the last row of Table 7). It is noted that since the computer systems of the researchers are different together, then it seems that the comparison between run time of the approaches with the heuristic algorithm is not accurate. Consequently, computational results express that the heuristic algorithm is competitive with other algorithms in terms of solution quality.

Tab. 7. Computational results of heuristic algorithm on standard test problems of CVRP.

ID of Instance	BKS	SS-ACO		PSO		Heuristic algorithm	
		Best Cost	CPU time (s)	Best Cost	CPU time (s)	Best Cost	CPU time (s)
C1	524.61	524.61	32.39	524.61	–	524.61	14
C2	835.26	835.26	41.23	844.42	–	835.26	23
C3	826.14	830.14	70.67	829.4		826.14	36
C4	1028.42	1038.2	147.83	1048.89	–	1061.39	48
C5	1291.29	1307.18	416.98	1323.89	–	1351.36	52
C6	555.43	559.12	38.28	555.43	–	555.43	17
C7	909.68	912.68	53.01	917.68	–	909.68	28
C8	865.94	869.34	123.68	867.01	–	865.94	37
C9	1162.55	1179.4	306.85	1181.14	–	1173.56	41
C10	1395.85	1410.26	596.03	1428.46	–	1455.36	45
C11	1042.11	1044.12	136.64	1051.87	–	1050.36	39
C12	819.56	824.31	91.88	819.56	–	819.56	38
C13	1541.14	1556.52	275.04	1546.2	–	1560.39	339
C14	866.37	870.26	217.33	866.37	–	866.37	33
No. of BKS		2		4		8	

–: The problem is not solved in the corresponding study.
Bold numbers indicate that best-known solution values are attained by the corresponding approach.

7. Conclusion and Future Research

One of the main requirements of the transportation management is to provide goods or services from a supply center to various dispersed points with significant economic implications. In this study, a relevant problem in the transportation management and reverse logistic was investigated. The fuzzy multi-depot vehicle routing problem with simultaneous pickup and delivery (FMDVRP-SPD) was considered. The problem via fuzzy credibility theory and chance-constrained programming was modeled. Since the problem was Np-hard, a heuristic algorithm with three iterative phases that integrated K-means clustering algorithm (K-MCA) and ant colony optimization (ACO) was proposed to solve the problem. In the third phase of the heuristic algorithm, the additional distances due to fuzzy demands and route "failures" were estimated by stochastic simulation for each planned route. To obtain the best sensitive parameters of the model, named "vehicle indexes", three test instances with different sizes which are compatible with real data were generated. The computational experiments showed that the "vehicle indexes" greatly influence the planned routes, additional distances, and fixed cost of vehicles. Finally, numerical experiments with standard test problems of CVRP were carried out to show the efficiency of the proposed heuristic algorithm. This paper has some capable future research clues: the first phase of the heuristic algorithm (i.e., K-means clustering algorithm) can be strengthened through the improvement process such as swapping or re-assignment methods (i.e., 2-Opt or 3-Opt), considering the FMDVRP-SPD with fuzzy time windows; the other is developing the FMDVRP-SPD by some more realistic assumptions, e.g., heterogeneous vehicles and depots with unequal capacities.

References

[1] Dinc Yalcın, G. and N. Erginel, *Fuzzy multi-objective programming algorithm for vehicle routing problems with backhauls.* Expert Systems with Applications, Vol. 42, No. 13 (2015), pp. 5632-5644.

[2] Nadizadeh, A., et al., *Using greedy clustering method to solve capacitated location-routing problem.* African Journal of Business Management, Vol. 5, No. 17, (2011), pp. 7499-7506.

[3] Dantzig, G.B. and J.H. Ramser, *The Truck Dispatching Problem.* Management Science, Vol. 6, No. 1, (1959), pp. 80-91.

[4] Kourank Beheshti, A. and S.R. Hejazi, *A Quantum Evolutionary Algorithm for the Vehicle Routing Problem with Single-sided Time Window Setting.* International Journal of Industiral Engineering & Producion Research, Vol. 25, No. 4, (2014), pp. 287-295.

[5] Karakatič, S. and V. Podgorelec, *A survey of genetic algorithms for solving multi depot vehicle routing problem.* Applied Soft Computing, Vol. 27, (2015), pp. 519-532.

[6] Lenstra, J.K. and A.H.G. Rinnooy Kan, *Complexity of vehicle routing and scheduling problems.* Networks, Vol. 11, (1981), pp. 221–227.

[7] Zare Mehrjerdi, Y., *Stochastic Approach to Vehicle Routing Problem: Development and Theories.* International Journal of Industiral Engineering & Producion Research, Vol. 24, No. 4, (2013), pp. 285-293.

[8] Hà, M.H., et al., *An exact algorithm and a metaheuristic for the generalized vehicle routing problem with flexible fleet size.* Computers & Operations Research, Vol. 43, (2014), pp. 9-19.

[9] Karimi, H. and A. Seifi, *Acceleration of Lagrangian Method for the Vehicle Routing Problem with Time Windows.* International Journal of Industiral Engineering & Producion Research, Vol. 23, No. 4, (2012), pp. 309-315.

[10] Mirzabaghi, M., A. Rashidi Komijan, and A.H. Sarfaraz, *Closed loop supply chain planning with vehicle routing.* International Journal of Industiral Engineering & Producion Research, Vol. 27, No. 3, (2016), pp. 285-301.

[11] Yao, B., et al., *A two-stage heuristic algorithm for the school bus routing problem with mixed load plan.* Transportation Letters, (2016), pp. 1-15.

[12] Nadizadeh, A. and B. Kafash, *Fuzzy capacitated location-routing problem with simultaneous pickup and delivery demands.* Transportation Letters, (2017), pp. 1-19.

[13] Zachariadis, E.E., C.D. Tarantilis, and C.T. Kiranoudis, *A hybrid metaheuristic algorithm for the vehicle routing problem with simultaneous delivery and pick-up service.* Expert Systems with Applications, Vol. 36, No. 2, Part 1, (2009), pp. 1070-1081.

[14] Catay, B., *A new saving-based ant algorithm for the Vehicle Routing Problem with Simultaneous Pickup and Delivery.* Expert Systems with Applications, (2010), Vol. 37, pp. 6809–6817.

[15] Polat, O., et al., *A perturbation based variable neighborhood search heuristic for solving the Vehicle Routing Problem with Simultaneous Pickup and Delivery with Time Limit.* European Journal of Operational Research, Vol. 242, No. 2, (2015), pp. 369-382.

[16] Salhi, S., A. Imran, and N.A. Wassan, *The multi-depot vehicle routing problem with heterogeneous vehicle fleet: Formulation and a variable neighborhood search implementation.* Computers & Operations Research, Vol. 52, Part B, (2014), pp. 315-325.

[17] Ghaffari-Nasab, N., S. Ghazanfar Ahari, and M. Ghazanfari, *A hybrid simulated annealing based heuristic for solving the location-routing problem with fuzzy demands.* Scientia Iranica Transactions E: Industrial Engineering, (2013), Vol. 20, No. 3, pp. 919–930.

[18] Zare Mehrjerdi, Y. and A. Nadizadeh, *Using Greedy Clustering Method to Solve Capacitated Location-Routing Problem with Fuzzy Demands.* International Journal of Industiral Engineering & Producion Research, Vol. 27, No. 1, (2016), pp. 1-19.

[19] Nadizadeh, A. and H.H. Nasab, *Solving the dynamic capacitated location-routing problem with fuzzy demands by hybrid heuristic algorithm.* European Journal of Operational Research, Vol. 238, No. 2, (2014), pp. 458-470.

[20] F.A., T., *The multiple terminal delivery problem with probabilistic demands.* Transportation Science, Vol. 3, (1969), pp. 192–204.

[21] Wren, A. and A. Holiday, *Computer scheduling of vehicles from one or more depots to a number of delivery points.* Operational Research Quarterly, Vol. 23, (1972), pp. 333–344.

[22] Gillett, B.E. and J.W. Johnson, *Multi-terminal vehicle dispatching algorithm.* Omega, Vol. 4, (1976), pp. 711–718.

[23] Golden, B. and T. Magnanti, *Implementing vehicle routing algorithms.* Vol. 7, (1977), pp. 113–148.

[24] Ball, M., et al., *Planning for truck fleet size in the presence of a common carrier option.* Decision Science Vol. 14, (1983), pp. 103–120.

[25] Perl, J. and M.S. Daskin, *A warehouse location-routing.* Transportation Research, Vol. 19B, (1985), pp. 381–396.

[26] Benton, W.C., *Evaluating a modified heuristic for the multiple vehicle scheduling problem.* Working paper RS86-14, College Administration Science, The Ohio State University, Columbus, OH, (1986).

[27] Perl, J., *The multi-depot routing allocation problem.* American Journal of Mathematical & Management Sciences, Vol. 7, (1987), pp. 8–34.

[28] Laporte, G., Y. Nobert, and S. Taillefer, *Solving a family of multi-depot vehicle routing and location-routing problems.* Transportation Science, Vol. 22, (1988), pp. 161-172.

[29] Min, H., J. Current, and D. Schilling, *The delivery depot vehicle routing problem with backhauling.* Journal of Business Logistics, Vol. 13, (1992), pp. 259–288.

[30] Chao, I.M., B. Golden, and E. Wasil, *A new heuristic for the multi-depot vehicle routing problem that improves upon best-known solutions.* American Journal Mathematics Management Science Vol. 13, (1993), pp. 371–406.

[31] Renaud, J., G. Laporte, and F.F. Boctor, *A tabu search heuristic for the multi-depot vehicle routeing problem.* Computers & Operations Research, Vol. 23, (1996), pp. 229–235.

[32] Salhi, S. and M. Sari, *A multi-level composite heuristic for the multi-depot vehicle fleet mix problem.* European Journal of Operational Research, Vol. 103, (1997), pp. 95–112.

[33] Cordeau, J.F., M. Gendreau, and G. Laporte, *A tabu search heuristic for periodic and multi-depot vehicle routing problem.* Networks Vol. 30, (1997), pp. 105–119.

[34] Thangiah, S.R. and S. Salhi, *Genetic clustering: an adaptive heuristic for the multidepot vehicle routing problem.* Applied Artificial Intelligence, Vol. 15, (2001), pp. 361–383.

[35] Tarantilis, C.D. and C.T. Kiranoudis, *Distribution of fresh meat.* Journal of Food Engineering, Vol. 51, (2002), pp. 85–91.

[36] Giosa, I., I. Tansini, and I. Viera, *New assignment algorithms for the multi-depot vehicle routing problem.* European Journal of Operational Research, Vol. 53, (2002), pp. 997–984.

[37] Polack, M., R.F. Hartl, and K. Doerner, *A variable neigbourhood search for the multi depot vehicle routing problem with time windows.* Journal of Heuristics, Vol. 10, (2004), pp. 627–627.

[38] Lim, A. and F. Wang, *Multi-depot vehicle routing problem: a one-stage approach.* IEEE Transactions on Automation Science and Engineering, Vol. 4, (2005), pp. 397–402.

[39] Nagy, G. and S. Salhi, *Heuristic algorithms for single and multiple depot vehicle routing problems with pickups and deliveries. .* European Journal of Operational Research, Vol. 162, (2005), pp. 126–141.

[40] Pisinger, D. and S. Ropke, *A general heuristic for vehicle routing problem.* Computers & Operations Research, Vol. 34, (2007), pp. 2403–2435.

[41] Ho, W., et al., *A hybrid genetic algorithm for the multi-depot vehicle routing problem.* Engineering Applications of Artificial Intelligence, Vol. 21, (2008), pp. 548–557.

[42] Pepin, A.-S., et al., *A comparison of five heuristics for the multiple depot vehicle scheduling problem.* Journal of Scheduling, Vol. 12, (2009), pp. 17–30.

[43] Zhang, H., J. Tang, and R.Y.K. Fung, *A scatter search for multi-depot vehicle routing problem with weight-related cost.* Asian Pacific Journal of Operations Research, Vol. 28, (2011), pp. 323–348.

[44] Yu, B., Z.-Z. Yang, and J.-X. Xie, *A parallel improved ant colony optimization for multi-depot vehicle routing problem.* Journal of the Operational Research Society, Vol. 62, No. 1, (2011), pp. 183-188.

[45] Kuo, Y., . and C.C. Wang, *A variable neighborhood search for the multi-depot vehicle routing problem with loading.* Expert Systems with Applications, Vol. 39, (2012), pp. 6949–6954.

[46] Rahimi-Vahed, A., et al., *A path relinking algorithm for a multi-depot periodic vehicle routing problem.* Journal of Heuristics, Vol. 19, No. 3, (2013), pp. 497-524.

[47] Venkata Narasimha, K., et al., *An ant colony optimization technique for solving min–max Multi-Depot Vehicle Routing Problem.* Swarm and Evolutionary Computation, Vol. 13, (2013), pp. 63-73.

[48] Contardo, C. and R. Martinelli, *A new exact algorithm for the multi-depot vehicle routing problem under capacity and route length constraints.* Discrete Optimization, Vol. 12, (2014), pp. 129-146.

[49] Escobar, J.W., et al., *A hybrid Granular Tabu Search algorithm for the Multi-Depot Vehicle Routing Problem.* Journal of Heuristics, Vol. 20, No. 5, (2014), pp. 483-509.

[50] Luo, J. and M.-R. Chen, *Improved Shuffled Frog Leaping Algorithm and its multi-phase model for multi-depot vehicle routing problem.* Expert Systems with Applications, Vol. 41, No. 5, (2014), pp. 2535-2545.

[51] Juan, A.A., et al., *Combining biased randomization with iterated local search for solving the multidepot vehicle routing problem.* International Transactions in Operational Research, Vol. 22, (2015), pp. 647-667.

[52] Kachitvichyanukul, V., P. Sombuntham, and S. Kunnapapdeelert, *Two solution representations for solving multi-depot vehicle routing problem with multiple pickup and delivery requests via PSO.* Computers & Industrial Engineering, Vol. 89, (2015), pp. 125-136.

[53] Wang, X., B. Golden, and E. Wasil, *The min-max multi-depot vehicle routing problem: heuristics and computational results.* Journal of the Operational Research Society, Vol. 66, No. 9, (2015), pp. 1430-1441.

[54] Calvet, L., et al., *Combining statistical learning with metaheuristics for the Multi-Depot Vehicle Routing Problem with market segmentation.* Computers & Industrial Engineering, Vol. 94, (2016), pp. 93-104.

[55] Bae, H. and I. Moon, *Multi-depot vehicle routing problem with time windows considering delivery and installation vehicles.* Applied Mathematical Modelling, Vol. 40, Nos. 13–14, (2016), pp. 6536-6549.

[56] Wang, X., et al., *The min–max split delivery multi-depot vehicle routing problem with minimum service time requirement.*

Computers & Operations Research, Vol. 71, (2016), pp. 110-126.

[57] Min, H., *The multiple vehicle routing problem with simultaneous delivery and pick-up points.* Transportation Research Part A: General, Vol. 23, No. 5, (1989), pp. 377-386.

[58] Halse, K., *Modeling and solving complex vehicle routing problems*, in *Institute of Mathematical Statistics and Operations Research*. (1992), Technical University of Denmark: Denmark.

[59] Dell'Amico, M., G. Righini, and M. Salanim, *A branch-and-price approach to the vehicle routing problem with simultaneous distribution and collection.* Transportation Science Vol. 40, No. 2, (2006), pp. 235–247.

[60] Gajpal, Y. and P. Abad, *An ant colony system (ACS) for vehicle routing problem with simultaneous delivery and pickup.* Computers & Operations Research, Vol. 36, No. 12, (2009), pp. 3215-3223.

[61] Salhi, S. and G. Nagy, *A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling.* Journal of the Operational Research Society Vol. 10, (1999), pp. 1034–1042.

[62] Dethloff, J., *Vehicle routing and reverse logistics: the vehicle routing problem with simultaneous delivery and pick-up.* OR Spektrum, Vol. 23, No. 1, (2001), pp. 79–96.

[63] Angelelli, E. and R. Mansini, *A branch-and-price algorithm for a simultaneous pickup and delivery problem. Quantitative approaches to distribution logistics and supply chain management*. (2002), Berlin, Heidelberg: Springer.

[64] Vural, A.V., *A GA based meta-heuristic for capacitated vehicle routing problem with simultaneous pick-up and deliveries*, in *Master's thesis, Graduate School of Engineering and Natural Sciences*. (2003), Sabanci University.

[65] Gokce, E.I., *A revised ant colony system approach to vehicle routing problems*, in *Master's thesis, Graduate School of Engineering and Natural Sciences*. (2004), Sabanci University.

[66] Ropke, S. and D.A. Pisinger, *A unified heuristic for a large class of vehicle routing problems with backhauls*. 2004, University of Copenhagen: Technical Report (2004)/14.

[67] Crispim, J. and J. Brandao, *Metaheuristics applied to mixed and simultaneous extensions of vehicle routing problems with backhauls.* Journal of the Operational Research Society, Vol. 56, No. 7, (2005), pp. 1296–1302.

[68] Chen, J.F. and T.H. Wu, *Vehicle routing problem with simultaneous deliveries and pickups.* Journal of the Operational Research Society Vol. 57, No. 5, (2006), pp. 579–587.

[69] Alfredo Tang Montané, F. and R.D. Galvão, *A tabu search algorithm for the vehicle routing problem with simultaneous pick-up and delivery service.* Computers & Operations Research, Vol. 33, No. 3, (2006), pp. 595-619.

[70] Gribkovskaia, I., et al., *General solutions to the single vehicle routing problem with pickups and deliveries.* European Journal of Operational Research, Vol. 180, No. 2, (2007), pp. 568-584.

[71] Bianchessi, N. and G. Righini, *Heuristic algorithms for the vehicle routing problem with simultaneous pick-up and delivery.* Computers & Operations Research, Vol. 34, No. 2, (2007), pp. 578-594.

[72] Subramanian, A. and L. dos Anjos Formiga Cabral, *An ILS Based Heuristic for the Vehicle Routing Problem with Simultaneous Pickup and Delivery and Time Limit*, in *Evolutionary Computation in Combinatorial Optimization: 8th European Conference, EvoCOP 2008, Naples, Italy, March 26-28, 2008. Proceedings*, J. van Hemert and C. Cotta, Editors. Springer Berlin Heidelberg: Berlin, Heidelberg. (2008), pp. 135-146.

[73] bramanian, A., L.A.F. Cabral, and L. Ochi, *An efficient ILS heuristic for the vehicle routing problem with simultaneous pickup and delivery*. (2008), Universidade Federal

Fluminense, available at /http://www.ic.uff.br/PosGraduacao/RelTecnicos/401.pdfS.: Technical Report.

[74] Subramanian, A., et al., *A parallel heuristic for the Vehicle Routing Problem with Simultaneous Pickup and Delivery.* Computers & Operations Research, Vol. 37, No. 11, (2010), pp. 1899-1911.

[75] Zachariadis, E.E., C.D. Tarantilis, and T. Kiranoudos, *An adaptive methodology for the vehicle routing problem with simultaneous pick-ups and deliveries.* European Journal of Operational Research, Vol. 202, (2010), pp. 401–411.

[76] Zachariadis, E.E. and T. Kiranoudos, *A local search metaheuristic algorithm for the vehicle routing problem with simultaneous pick-ups and deliveries.* Expert Systems with Applications, Vol. 38, (2011), pp. 2717–2726.

[77] Tasan, A.S. and M. Gen, *A genetic algorithmbased approach to vehicle routing problem with simultaneous pick-up and deliveries.* Computers & Industrial Engineering, Vol. 62, (2012), pp. 755–761.

[78] Goksal, F.P., I. Karaoglan, and F. Altiparmak, *A hybrid discrete particle swarm optimization for vehicle routing problem with simultaneous pickup and delivery.* Computers & Industrial Engineering, Vol. 65, (2013), pp. 39–53.

[79] Subramanian, A., E. Uchoa, and L.S. Ochi, *A hybrid algorithm for a class of vehicle routing problems.* Computers & Operations Research, Vol. 40, (2013), pp. 2519–2531.

[80] Rieck, J. and J. Zimmermann, *Exact Solutions to the Symmetric and Asymmetric Vehicle Routing Problem with Simultaneous Delivery and Pick-Up.* Business Research, Vol. 6, No. 1, (2013), pp. 77-92.

[81] Liu, R., et al., *Heuristic algorithms for a vehicle routing problem with simultaneous delivery and pickup and time windows in home health care.* European Journal of Operational Research, Vol. 230, No. 3, (2013), pp. 475-486.

[82] Li, J., et al., *Iterated local search embedded adaptive neighborhood selection approach for the multi-depot vehicle routing problem with simultaneous deliveries and pickups.* Expert Systems with Applications, Vol. 42, No. 7, (2015), pp. 3551-3561.

[83] Zadeh, L.A., *Fuzzy Sets.* Information and Control, Vol. 8, (1965), pp. 338-353.

[84] Kaufmann, A., *Introduction to the theory of fuzzy subsets*. (1975), New York: Academic Press.

[85] Zadeh, L.A., *Fuzzy sets as a basis for a theory of possibility.* Fuzzy Sets and Systems, Vol. 1, No. 1, (1978), pp. 3-28.

[86] Liu, B., *Uncertain Theory: An introduce to its axiomatic foundations*. (2004), Springer, Berlin.

[87] Erbao, C. and L. Mingyong, *The open vehicle routing problem with fuzzy demands.* Expert Systems with Applications, Vol. 37, No. 3, (2010), pp. 2405–2411.

[88] Erbao, C. and L. Mingyong, *A hybrid differential evolution algorithm to vehicle routing problem with fuzzy demands.* Journal of Computational and Applied Mathematics, Vol. 231, No. 1, (2009), pp. 302–310.

[89] Zare Mehrjerdi, Y. and A. Nadizadeh, *Using greedy clustering method to solve capacitated location-routing problem with fuzzy demands.* European Journal of Operational Research, Vol. 229, No. 1, (2013), pp. 75-84.

[90] Hartigan, J.A. and M.A. Wong, *A k-means clustering algorithm.* Applied Statistics, Vol. 28, No. 1, (1979), pp. 100–108.

[91] Cinar, D., K. Gakis, and P.M. Pardalos, *A 2-phase constructive algorithm for cumulative vehicle routing problems with limited duration.* Expert Systems with Applications, Vol. 56, (2016), pp. 48-58.

[92] Cao, B. and F. Glover, *Creating balanced and connected clusters to improve service delivery routes in logistics planning.* Journal of Systems Science and Systems, Vol. 19, (2010), pp. 453–480.

[93] Xu, R. and D. Wunsch, *Survey of clustering algorithms.* IEEE Transactions on Neural Networks, Vol. 16, (2005), pp. 645–678.

[94] Ekici, A. and A. Retharekar, *Multiple agents maximum collection problem with time dependent rewards.* Computers & Industrial Engineering, Vol. 64, (2013), pp. 1009–1018.

[95] Gao, S., et al., *Ant colony optimization with clustering for solving the dynamic location routing problem.* Applied Mathematics and Computation, Vol. 285, (2016), pp. 149-173.

[96] Day, W.H.E. and H. Edelsbrunner, *Efficient algorithms for agglomerative hierarchical clustering methods.* Journal of Classification, Vol. 1, No. 1, (1984), pp. 7–24.

[97] Dorigo, M., V. Maniezzo, and A. Colorni, *The Ant System: Optimization by a colony of cooperating agents.* IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, Vol. 26, No. 1, (1996), pp. 29-41.

[98] Bonabeau, E., M. Dorigo, and G. Theraulaz, *Inspiration for optimization from social insect behaviour.* Nature Vol. 406, No. 6791, pp. 39–42.

[99] Dorigo, M. and T. Stützle, *Ant Colony Optimization*. (2004), MIT Press.

[100] Christofides, N., A. Mingozzi, and P. Toth, *Exact algorithms for the vehicle routing problem, based on spanning tree and shortest path relaxations.* Mathematical Programming, Vol. 20, No. 1, (1981), pp. 255-282.

[101] Hosseinabadi, A.A.R., et al., *A new efficient approach for solving the capacitated Vehicle Routing Problem using the Gravitational Emulation Local Search Algorithm.* Applied Mathematical Modelling, Vol. 49, Supplement C (2017), pp. 663-679.

[102] Zhang, X. and L. Tang, *A new hybrid ant colony optimization algorithm for the vehicle routing problem.* Pattern Recognition Letters, Vol. 30, No. 9, (2009), pp. 848-855.

[103] Ai, T.J. and V. Kachitvichyanukul, *A particle swarm optimization for the vehicle routing problem with simultaneous pickup and delivery.* Comput. Oper. Res., Vol. 36, No. 5, (2009), pp. 1693-1702.

Follow This Article at The Following Site

Nadizadeh A. The fuzzy multi-depot vehicle routing problem with simultaneous pickup and delivery: Formulation and a heuristic algorithm. IJIEPR. 2017; 28 (3) :325-345 . DOI: 10.22068/ijiepr.28.3.325
URL: http://ijiepr.iust.ac.ir/article-1-772-en.html