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Supplier Selection and Order Allocation Considering Discount Using Meta-Heuristics

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KEYWORDS

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ABSTRACT

In this paper, a multi-objective mathematical model is presented to select suppliers and allocate orders to suppliers under uncertainty conditions. The proposed model is multi-source, multi-customer, multi-product, and multi-period at two levels of supply chain. Objective functions considered in this study are total cost including purchasing, transportation and ordering costs, timely delivering, shipment quality, partial and general coverage of suppliers, and finally, suppliers' weights. The major limitations are price discount on products provided by suppliers which are calculated using signal function. In addition, suppliers' weights in the fifth objective function are calculated using fuzzy TOPSIS technique. Lateness and wastes parameters in this model are considered as uncertain and random triangular fuzzy numbers. Finally, the multi-objective model is solved using two well-known multi-objective algorithms that are Non-dominated Sorting Genetic Algorithm (NSGA-II) and Particle Swarm Optimization (PSO); the results are analyzed using quantitative criteria. Furthermore, Taguchi technique is used to regulate the parameters of two algorithms.

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1. Introduction

Supply chain management (SCM) involves suppliers, manufacturers, distribution centers, and retailers to ensure the efficient flow of raw materials, work-in-process inventory, and finished products among facilities. Simchi-Levi defines SCM as a set of approaches used to efficiently integrate suppliers, manufacturers, warehouses, and stores so that merchandise is produced and distributed at the right quantities, to the right locations, and in the right time in order to minimize sys-tem-wide costs while satisfying service-level requirements [1]. Ghiani expresses that supply chain is a complex logistics system in which raw materials are converted into finished products and then are distributed to the final users [2].

Supplier selection is one of the most critical activities of purchasing management in a supply chain due to the key role of supplier's performance on cost, quality, delivery, and service in achieving the objectives of a supply chain.

Supplier selection is a multiple criteria decision-making (MCDM) problem, which is affected by several conflicting factors. Consequently, a purchasing manager must analyze the trade-off among the several criteria. MCDM techniques

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support the decision-makers' DMs in evaluating a set of alternatives. Depending upon the purchasing situations, criteria have varying importance, and there is a need to weight them [3].

Supply chain management (SCM) plays an important role in competing firms of today's market. Such an integrated supply chain (SC) can reduce total cost compared to the cost when each part decides independently. The SCM is the coordination between location, inventory, transportation, and production for a set, which consists of a network of facilities and distribution options in order to reach the best mix of efficiency and responsiveness for the market being served [4].

Supplier evaluation and selection is a subject of interest for many researchers. Most of these researches can be classified into two categories. The first is using multiple criteria decision-making (MCDM) techniques for evaluating a set of potential or active suppliers in order to rank them. The second is using multi-objective decision making (MODM) techniques in order to allocate a quota of an entire order to selected suppliers.

Supplier selection has attracted many studies in recent years. Fatih et al. developed a multi-objective model to select suppliers using fuzzy TOPSIS technique in a group decision making context [5].

Desheng planned the supplier selection problem as a multi-objective problem which, in addition to taking quantitative factors, they considered qualitative factors using fuzzy logic [6]. Liang developed a fuzzy multi-objective model in a multi-product, multi-period case in two levels. In his model, he considered delivery cost and time as two objective functions and solved his model in a dynamic environment [7]. Torabi and Hassini developed a three-dimension model in a multi-objective fuzzy case as multi-product with fixed demand. Their objective functions are minimizing the deviation variables for store constraint, deviation variable for future coverage constraint and deviation variables cost [8].

Atakhan and Ali Fuat defined a multi-objective model with fuzzy parameters. Then, they solved their model through weighed max – min technique. They obtained the weight of suppliers in their model through TOPSIS technique; utilizing weighing method, they converted objective functions to a single objective function [9].

Hale and Hamidi developed a fuzzy multi – objective model of allocating the order to suppliers. In this model, hierarchical technique is used to obtain the suppliers weights. In addition, they set this weight as an objective function to select the suppliers. Finally, they solved this model using max – min technique of membership function [10].

Onot ranked the suppliers utilizing fuzzy TOPSIS techniques and fuzzy net analysis process. They implemented their technique practically for communications system [11].

Lin developed a model for supplier selection under fuzzy conditions [12]. He considered in his multi-objective model the maximization of suppliers weights as single objective function and the solving of the model alongside the functions of delivery cost and rate. His considered objective functions are cost, delay, and quality, which are considered indefinite and fuzzy.

In addition, suppliers' weight is considered as the objective function obtained by fuzzy ANP (Analytic Network Process) technique. Solving technique in this article is in a two-phase max – min method where objective functions in the second phase are of the same importance.

Amid developed a linear multi-objective model where objective functions and demand are indefinite and fuzzy, then they solved their model using weighted sum technique [13]. Nazari Shirkouhi et al. presented a provider selection problem for several cost levels and products using fuzzy two dimensional and linear multi-objective mutual programming model, where they considered cost, delay, and wastes as objective functions. They converted the proposed two fuzzy techniques of objective functions to a single objective function; eventually, they solved their model using a numerical example [14].

Shaw et al. developed an integer multi-objective model where their objective functions are: purchase cost, delay, wastes or returned products, and environmental effect or greenhouse gases. They converted objective functions to a single objective function using weighed technique where they obtained the suppliers weights through fuzzy hierarchical technique. Finally, they applied their model to an Indian company as a case study [15]. Esfandiari and Seyfbarghy developed a multi-objective model. Their objective techniques consist of minimizing the cost, delay, wastes and maximizing the suppliers' weights. Their model is randomized and demand is achieved through passion probability function. Product cost from the provider side has a linear

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discount. In this model, L–P metric technique is converted to a single objective model and cooling and genetics algorithms are utilized to solve the model [16].

Arikan developed an integer multi-objective model to select the suppliers where his model's objective functions are cost, on time delivery, and delivered units percentage. He converted the objective functions to a single objective using max – min technique and solved the model [17].

Karasakal and Karasakal suggested the partial coverage problem as a branch of maximum coverage problem. In their model, customer's demand coverage rate by every distribution center depended on the inverse of customer distance from that center [18]. Liao et al. suggested the maximum distance constraint on covering the customers demand by distribution centers in the inventory location problem. In this model, if customers are located in the critical coverage distance, all their demands will be supplied; otherwise, the whole demands will remain unresponded [19].

Kokangol and Susuz, by taking capacity, budget and discount conditions into consideration, modeled and solved the supplier selection problem by developing a mixed model through mixing hierarchical analysis techniques, nonlinear mathematical programming model, and multi – objective programming model [20]. Tsai and Wang applied a mixed integer programming procedure to solve the problem and allocate the order to a multi-source and multi-product case in the supply Chain [21]. Their objective functions are cost, minimizing the delay and wastes from supplier side. Two plans for discount of all particles and exponential are applied to the problem, and three objectives, including the cost, number of returned product, and number of particles delivered with delay, are considered.

Meena and Sarmah developed a nonlinear single objective model to select the supplier. This model is a mixed integer-programming model. Customer is confronted with the cost discount and risk from supplier side to select the supplier. Eventually, this model is solved by genetic algorithm due to nonlinearity and complexity [22].

Parkouhi and Ghadikolaei proposed a resilience approach to supplier selection using fuzzy analytical network process and grey VIKOR technique [23].

Firouz et al. proposed an integrated supplier selection and inventory problem with multi-sourcing and lateral transshipment. They proposed a mathematical model to solve the mentioned problem [24].

Abdollahi et al. presented a framework for supplier selection based on product-related and organization-related characteristics of the suppliers to be more competitive in market and flexible to overcome probable changes in demands, suppliers, etc [25].

Ruiz-Torres et al. considered the optimal allocation of demand across a set of suppliers given the risk of supplier failures [26].

Gupta & Barua considered supplier selection among SMEs (Small and Medium Enterprises) on the basis of their green innovation ability. They used multiple criteria decision-making models for selecting the best supplier. They used BMW and fuzzy TOPSIS as an integrated model of decision making. A three-phase methodology is used for presenting a framework for supplier selection by large organizations. The first phase involves the selection of criteria of green innovation through literature review and interviews with decision makers. The second phase involves ranking of selection criteria using a novel best-worst method. The third phase involves ranking of suppliers with respect to selection criteria weights obtained in phase two using fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [27].

Hlioui et al. considered joint supplier selection, production and replenishment of an unreliable manufacturing-oriented supply chain [28].

Hamdan and Cheaitou proposed an integrated model of multiple criteria decision making and multi-objective optimization for supplier selection and order allocation with green criteria [29].

Chen and Zou proposed an integrated method for supplier selection from the perspective of risk aversion. In this paper, generalized intuitionistic fuzzy soft set (GIFSS) combined with extending gray relational analysis (GRA) method is proposed to select an appropriate supplier from the perspective of risk aversion in group decision-making environment. The proposed approach consists of two phases [30].

The present study tries to make the model more realistic and wide-spread through utilizing all the indicators as well as taking the model into consideration in all aspects with respect to the number of suppliers, number of customers, being multi periodic and multi product, and using uncertain parameters.

In this study, unlike previous ones, a nonlinear multi-objective programming model is developed

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in which objective functions are total cost, delay, wastes, coverage from suppliers' side and weights. In this model, delay and wastes from suppliers are considered as fuzzy parameters. Finding the suppliers' weights through fuzzy TOPSIS and using triangular fuzzy numbers for measures weights and evaluation of decision-makers for choices are the novelties of objective function in this model. Consideration of coverage by suppliers for selecting and allocating the order to suppliers also are among the innovations of the present study. Supplier selection is performed according to the distance of customer from suppliers and considering the partial and complete coverage.

In the model of the present study, discount constraint is considered to simplify and find the product cost according to the order rate and discounts rates from sign function type and is calculated simply.

2. Fuzzy Sets

Zadeh introduced fuzzy set theory, which is an extension of ordinary set theory, for dealing with uncertainty and imprecision associated with information [31]. As shown in Kaufmann, the preliminary of fuzzy set theory used in this research work is as follows:

Definition 1. A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with ‘‘ordinary’’ (single-valued) numbers. Any fuzzy number can be thought of as a function whose domain is a specified set usually the size of real numbers, and whose range is the span of non-negative real numbers between, and including, 0 and 1. Each numerical value in the domain is assigned a specific ‘‘grade of membership’’ where 0 represents the smallest possible grade, and 1 is the largest possible grade. In this research work, triangular fuzzy numbers are used. A triangular fuzzy number ã (l, m, u) is defined by the following membership function:

$$\mu_{\tilde{a}}(x)=\begin{cases}0 & x\le l\\ \dfrac{x-l}{m-l} & l\le x\le m\\ \dfrac{u-x}{u-m} & m\le x\le u\\ 0 & x\ge u\end{cases} \quad (1)$$

Definition 2. Let Ñ =(n_1,n_2,n_3) and $\tilde{M}$=(m_1,m_2,m_3)be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as follows:

$$d(\tilde{M},\tilde{N})=\sqrt{\frac{1}{3}[(m1-n1)^2+(m2-n2)^2+(m3-n3)^2]} \quad (2)$$

2-1. Fuzzy TOPSIS

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) is one of the most classical methods of solving MCDM problem [32]. This technique is based on the principle that the chosen alternative should have: the longest distance from the negative-ideal solution, i.e., the solution that maximizes the cost criteria and minimizes the benefits criteria, and the shortest distance from the positive-ideal solution, i.e., the solution that maximizes the benefit criteria and minimizes the cost criteria. In classical TOPSIS, the rating and weight of the criteria are known precisely. In fuzzy TOPSIS, all the ratings and weights are defined by means of linguistic variables. A number of fuzzy TOPSIS-based methods and applications have been developed in recent years. The approach to extending the TOPSIS method to fuzzy data used in this study can be outlined as follows:

Step 1: Construct the fuzzy decision matrix

Assume that there are m alternatives A_i (i=1, 2,…., m) to be evaluated against n selection criteria C_j(j=1,2,…,n). The fuzzy MADM can be concisely expressed in matrix format as in formulation (3).

$$\tilde{D}=\begin{bmatrix}\tilde{\mathbf{x}}_{11} & \cdots & \tilde{\mathbf{x}}_{1n}\\ \vdots & \ddots & \vdots\\ \tilde{\mathbf{x}}_{m1} & \cdots & \tilde{\mathbf{x}}_{mn}\end{bmatrix} \quad (3)$$

$$W=[\tilde{\mathbf{w}}_1,\ \tilde{\mathbf{w}}_2,\ \ldots\ldots,\ \tilde{\mathbf{w}}_n] \quad (4)$$

$\tilde{\mathbf{x}}_{ij}$ is the performance rating of the ith alternative $\mathbf{A_i}$with respect to jth criterion; $\mathbf{C_j}$and $\tilde{\mathbf{w}}_j$ represent the weights of the jth criterion $\mathbf{C_j}$. Moreover, $\tilde{\mathbf{x}}_{ij}$ and $\tilde{\mathbf{w}}_j$, i=1,2,…,m and j=1,2,….,n are triangular fuzzy numbers given as $\tilde{w}=(\mathbf{w_{j1}},\mathbf{w_{j2}},\mathbf{w_{j3}})$ and $\tilde{\mathbf{x}}_{ij}=(\mathbf{l_{ij}},\mathbf{m_{ij}},\mathbf{u_{ij}})$.

Step 2: Normalize the fuzzy decision matrix

The raw data are normalized to eliminate anomalies with different measurement units and scales in several MCDM problems. However, the purpose of linear scales transform normalization function used in this study is to preserve the property that the ranges of normalized triangular fuzzy numbers are to be included in [0,1]. If $\tilde{R}$ denotes the normalized fuzzy decision matrix, then

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$\tilde{R}_{ij} = [\tilde{r}_{ij}]_{m*n}$ i = 1,2,...,m and j = 1,2,...,n
where denoted by triangular fuzzy number as $(\mathbf{l_{ij}}, \mathbf{m_{ij}}, \mathbf{u_{ij}})$ for fuzzy data, the normalized values for benefit-related criteria (B) and cost-related criteria (C) are calculated as follows:

$$\tilde{\mathbf{r}}_{\mathbf{ij}} = \left(\frac{\mathbf{l_{ij}}}{\mathbf{u_j^*}}, \frac{\mathbf{m_{ij}}}{\mathbf{u_j^*}}, \frac{\mathbf{u_{ij}}}{\mathbf{u_j^*}}\right) \mathbf{u_j^* = max_i\, u_{ij}} \quad j \in B \qquad (5)$$

$$\tilde{\mathbf{r}}_{\mathbf{ij}} = \left(\frac{\mathbf{l_j^-}}{\mathbf{u_{ij}}}, \frac{\mathbf{l_j^-}}{\mathbf{m_{ij}}}, \frac{\mathbf{l_j^-}}{\mathbf{l_{ij}}}\right) \mathbf{l_j^- = min_i\, l_{ij}} \quad j \in C \qquad (6)$$

Step 3: Construct weighted normalized fuzzy decision matrix
Considering the weight of each criterion, the weighted normalized decision matrix can be computed by multiplying the importance weights of evaluated criteria and the values in the normalized fuzzy decision matrix. Weighted normalized decision matrix $\tilde{V}$ is defined as follows:

$$\tilde{V} = \begin{bmatrix} \tilde{v}_{11} & \cdots & \tilde{v}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{v}_{m1} & \cdots & \tilde{v}_{mn} \end{bmatrix} \quad i = 1,2,...,m;\ j = 1,2,...,n$$

$$\tilde{v}_{ij} = \tilde{r}_{ij} \,.\, \tilde{w}_j \qquad (7)$$

where $\tilde{w}_j$ is fuzzy weight of criterion C_j.
Step 4: Determine the positive ideal solution and the negative ideal solution
Because the positive triangular fuzzy numbers are included in the interval [0,1], the fuzzy positive ideal reference point (FPIRP)denoted by A* and fuzzy negative ideal reference point (FNIRP) denoted by A^- can be defined as follows:

$$A^* = \{\tilde{v}1^*, \tilde{v}2^*, \ldots\ldots \tilde{v}n^*\} \qquad (8)$$

$$\mathbf{A^- = \{\tilde{v}1^-, \tilde{v}2^- \ldots\ldots, \tilde{v}n^-\}} \qquad (9)$$

$$\tilde{v}_j^* = \text{Max}\{\tilde{v}_{ij}\};\ i=1,2,..m\ ;\ j=1,2\ .,n \qquad (10)$$

$$\mathbf{\tilde{v}j^-} = \text{Min}\{\tilde{v}_{ij}\};\ i=1,2,..m\ ;\ j=1,2,\ldots n \qquad (1)$$

where $\tilde{v}_j^* = (1,1,1)$ and $\mathbf{\tilde{v}j^-} = (0,0,0)$, j=1,2,…,n
Step 5: Calculate the distances of each initial alternative to FPIRP and FNIRP.
The distance of each alternative from fuzzy positive ideal reference point and fuzzy negative ideal reference point can be derived respectively as:

$$\boldsymbol{s_i^* = \sum_{j=1}^{n} d\,(\tilde{v}_{ij}, \tilde{v}_j^*)}\ ;\ i=1,\ldots\ldots,m \qquad (2)$$

$$\boldsymbol{s_i^- = \sum_{j=1}^{n} d\,(\tilde{v}_{ij}, \tilde{v}_j^-)}\ ;\ i=1,\ldots\ldots,m \qquad (3)$$

where $d(\tilde{\mathbf{v}}_{\mathbf{ij}}, \tilde{\mathbf{v}}_{\mathbf{j}}^*)$ denotes the distance between two fuzzy numbers and calculated using (2). $\mathbf{s_i^*}$ represents the distance of alternative $\mathbf{A_i}$ from FPIRP and $\mathbf{s_i^-}$ is the distance of alternative $\mathbf{A_i}$from FNIRP.
Step 6: Obtain the closeness coefficient and rank the alternatives
Calculate the closeness coefficient $(\mathbf{CC_i})$ of each alternative as:

$$\boldsymbol{CC_i = \frac{s_i^-}{s_i^- + s_i^*}}\ ;\ i=1,2,\ldots..m \qquad (4)$$

An alternative with CCi approaching 1 indicates that the alter-native is close to the fuzzy positive ideal reference point and far from the fuzzy negative ideal reference point. The alternative with the highest CC value will be the best choice.
Step 7: Calculate the supplier weight

$$W_i = \frac{\mathbf{cci}}{\sum_{i=1}^{m} \mathbf{cci}}\ ;\ i=1,2,\ldots..m \qquad (5)$$

2-2. Covering
As one of the problems in location problem theory, the maximal covering location problem (MCLP) maximizes the number of demand points covered within a specified critical distance or time by a fixed number of facilities. It does not require that all demand points be covered [18]. Our approach can be applied to location problems where the service is at the top level (i.e., fully covered) within a minimum critical distance, which decays with distance (i.e., partially covered) beyond the minimum critical distance until the maximum critical distance and drops to no-service level beyond this range. We believe that modeling such problems by allowing partial coverage (partial service level) is more reasonable than the classical MCLP approach. For instance, it may be important to model the service facility location problems, military logistics problems, and military targeting problems in the presence of partial coverage [18].

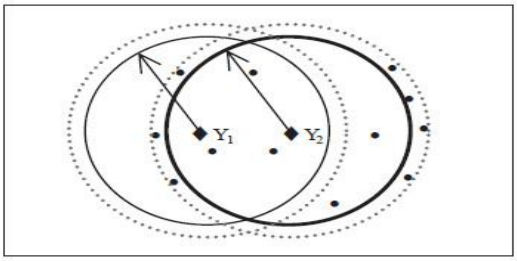

Fig. 1. The effect of partial coverage on the optimal solution to the problem.

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Suppose that there are two potential facility locations, and we would like to choose one with the maximal cover. The solid line shows the minimum critical distance and dotted line shows the maximum critical distance. Location $\boldsymbol{Y_1}$ can cover 6 demand points and location $\boldsymbol{Y_2}$ can cover 5 demand points within the full coverage range. Thus, a classical MCLP solution would choose location $\boldsymbol{Y_1}$ as the location of maximal coverage.

If we employ the partial coverage idea, we may choose location $\boldsymbol{Y_2}$ instead of location $\boldsymbol{Y_1}$, because location $\boldsymbol{Y_2}$ covers 5 demand points fully and an additional 7 demand points partially, while location $\boldsymbol{Y_1}$ covers only 6 demand points fully [18].

Coverings calculated from the following equation:

$$\mathbf{b_i} = \begin{cases} 1 & w_{ij} \le S_j \\ L(\mathbf{w_{ij}}) & \mathbf{S_j} < \mathbf{w_{ij}} < \mathbf{R_j} \\ 0 & w_{ij} \ge R_j \end{cases} \quad (6)$$

$$L(\mathbf{w_{ij}}) = \frac{\mathbf{R_j} - \mathbf{w_{ij}}}{\mathbf{R_j} - \mathbf{S_j}} \qquad 0 < L < 1 \quad (7)$$

3. The Proposed Mathematical Model

Supplier selection problem is always a multiple measure problem, and each measure has a specific importance. This means that each measure must be separately suggested as an objective function.

Now, taking this fact into consideration, there are various measures to select and allocate the order to suppliers. In the present study based on the literature review, five measures are offered to select and allocate the order to the suppliers by independent objective functions. In the following, we will address the modeling of these objective functions and the explanations related to the variables.

Model parameters

Parameters and indices utilized in the model are defined as follows:

Indices

- i = Customer indices (i = 1, 2, …, I)
- j = Supplier indices (j = 1, 2, …, J)
- k = Product indices (k = 1, 2, …, K)
- t = Period indices (t = 1, 2, …, T)
- r = Discount level indices (r = 1, 2, …, R).

Parameters

$\mathbf{P_{ijkt}}$ = cost of each k*th* unit product purchased by customer m in period t from supplier j.

$\tilde{\mathbf{t}}_{\mathbf{jkt}}$: The price of product k in period t by supplier j.

$\widetilde{\mathbf{B}}_{\mathbf{jkt}}$: The price of defective good of product k from supplier j in period t.

$\mathbf{b_{ij}}$: Coverage rate of center j to ther customer i.

$\mathbf{D_{ikt}}$: Demand of customer i for product k in period t.

$\widetilde{\mathbf{W}}_{\mathbf{j}}$: The weight of supplier j.

$\mathbf{f_{jkt}}$: Fixed cost of ordering for supplier j in period t the for product k.

$\mathbf{P_{ktjr}}$: The price of each unit product k offered by supplier j in period t in discount level r.

$\mathbf{C_{jkt}}$: The capacity of supplier j for product k in period t.

$\mathbf{n_{ikt}}$: Maximum number of supplier for customer i and product k th in period t.

$\mathbf{Q_{ij}}$: Maximum price of recepting the defective good purchased by buyer i from supplier j.

$\mathbf{T_{ij}}$: Maximum price of reception with the delay for purchased goods by buyer i from supplier j.

$\mathbf{S_j}$: Maximum distance for complete coverage by supplier j.

$\mathbf{R_i}$: Maximum distance for partial coverage by supplier j.

$\mathbf{V_{ijk}}$: The cost of shipment per each unit product k from supplier j to customer i in the distance unit.

$\mathbf{w_{ij}}$: Distance of supplier j to customer i.

$\mathbf{H_j}$: Minimum ordering to each supplier.

$\mathbf{O_{it}}$: Maximum capital of customer i in period t.

Decision variables

$\mathbf{X_{ijkt}}$: Amount of goods purchased by buyer i from supplier j in period t for product k .

$\mathbf{Y_{ijkt}}$: If customer i buys product k in period t from supplier j, it is 1; otherwise, it is 0.

Model assumptions

- Objective functions have the same weights from the viewpoint of decision maker.
- Demand is fixed and definitive.
- Shortage is not permissible.
- Discount is universal and a function of the sign.

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- Supply chain is two-dimensional, multi product, multi-period, and multiple-source with several buyers or customers.

3-1. Formulation of objective functions

The cost objective function is composed of 3 parts, including purchase cost, shipment cost, and fixed cost of ordering. In the first part, the suppliers offer the price of each product where this price has the sign function discount. Customers in each period order their products to the suppliers based on the offered price. The second part of objective function is the shipment cost which is calculated based on the customer distance from the supplier and, based on it, ordering amount to the supplier and selection of nearer supplier is performed. In the third part, the fixed cost of ordering for each product of suppliers is offered based on which selection of a supplier with lower cost is considered.

$$\text{Min } z_1 = \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijkt}.\ P_{ijkt} + \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijkt}\ .\ w_{ij}\ .\ v_{ijk}\ .\ y_{ijkt} + \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} f_{jkt}\ .\ y_{ijkt} \quad (8)$$

The second objective function is minimizing the lateness. In this objective function, according to the delay for each product, amount of ordering for each product to the suppliers is defined. On the other hand, since the delays by suppliers have uncertainty, to reach the reality, parameter of delays is considered as random triangular fuzzy numbers. The way to produce such numbers is described.

$$\text{Min } z_2 = \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} \tilde{t}_{jkt}\ .x_{ijkt} \quad (9)$$

The third objective function is minimizing the wastes from suppliers. In this objective function, also, the ordering amount of each product to suppliers is defined according to the percentage of wastes produced for each product by suppliers in each period. This parameter also, due to uncertainty, is represented as random triangular fuzzy figures.

$$\text{Min } z_3 = \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} \tilde{B}_{jkt}.x_{ijkt} \quad (20)$$

The fourth objective function is maximizing the coverage of customers by suppliers. In this objective function, according to the distance of customers from suppliers and partial and complete coverage of suppliers, percentage of coverage of each supplier for each customer is calculated where supplier selection is performed according to the demand coverage rate of each customer by each supplier :

$$\text{Max } z_4 = \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} b_{ij}.\ D_{ikt}\ .y_{ijkt} \quad (10)$$

In the fifth objective function, product-ordering rate is defined according to the suppliers' weights. For making suppliers evaluation more realistic and to select the best suppliers, suppliers' weight is obtained through fuzzy Topsis technique:

$$\text{Max } z_5 = \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} \tilde{W}_{j}.\ x_{ijkt} \quad (11)$$

3-2. Formulation of constraints

The first constraint (demand)

The constraint represents the fact that the ordering rate of each customer for each product in each period from suppliers must be equal to or more than the customer demand for that product in the desired period to not being confronted with any shortage.

$$\sum_{j=1}^{J} x_{ijkt} \geq D_{ikt} \qquad \forall i, \forall k, \forall t \quad (12)$$

The second constraint (observing the order based on the coverage rate)

This constraint indicates that the ordering rate must be equal to or less than the coverage rate of supplier for the desired customer. This constraint is provided for the objective function of coverage, and the supplier with more coverage is selected. On the other hand, this constraint defines the ordering rate after selecting the supplier, which must be observed:

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$$\sum_{j=1}^{J} x_{ijkt} \le \sum_{j=1}^{J} b_{ij}.\, D_{ikt} \quad \forall i, \forall k, \forall t \qquad (13)$$

The third constraint (suppliers' capacity)

Suppliers' capacity constraint explains that ordering rate for each product by the customers in each period must be according to the capacity of each supplier:

$$\sum_{i=1}^{I} x_{ijkt} \le c_{jkt} \quad \forall j, \forall k, \forall t \qquad (14)$$

The fourth constraint (the number of suppliers)

This constraint explains the fact that amount of applying the suppliers to each product in each period by the customer must be according to the amount defined by the managers. In addition, each customer in each period must purchase the product at least from one supplier:

$$1 \le \sum_{j=1}^{J} y_{ijkt} \le n_{ikt} \quad \forall i, \forall k, \forall t \qquad (15)$$

The fifth constraint (the wastes)

Amount of accepting the wastes for each product by each customer in each period from each supplier must correspond to the rate defined by decision-makers.

$$\sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{k=1}^{K} \tilde{B}_{jkt}.\, x_{ijkt} \le \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{k=1}^{K} Q_{ij}.b_{ij}.\, D_{ikt} \ \forall i, \forall j, \forall k, \forall t \qquad (16)$$

The sixth constraint (Delays)

Delays' reception rate for each product by each customer must be as defined by decision makers in each period for each supplier:

$$\sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{k=1}^{K} \tilde{t}_{jkt}.\, x_{ijkt} \le \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{k=1}^{K} T_{ij}.b_{ij}.\, D_{ikt} \ \forall i, \forall j, \forall k, \forall t \qquad (17)$$

The seventh constraint (Discount)

Price of each product offered by the suppliers has a discount of sign function type.

$$a(i,j,k,t,r) = \text{sign}[\text{sign}(x_{ijkt} - q_{jkt(r-1)}) + \text{sign}(q_{jktr} - x_{ijkt})] \quad \forall i, \forall j, \forall k, \forall t, \forall r \qquad (18)$$

$$P_{ijkt} = P_{jkt1} * a(i,j,k,t,1) + P_{jkt2} * a(i,j,k,t,2) + \ldots + P_{jktr} * a(i,j,k,t,r) \quad \forall i, \forall j, \forall k, \forall t \qquad (30)$$

where a(i,j,k,t,r) are positive variables, and their summation is 1. When, in sign function, x is positive, 1 is returned; if x is zero, 0 is returned; when x is negative, -1 is returned. So, a(i, j, k, t, r) corresponding to each discount rate is activated according to the order rate; x and other ranges are zero and become inactivated. This way, price of each product is found.

The eighth constraint (Capital)

This constraint represents the amount of capital belonging to each customer in each period, where expenditure rate in supply chain must be equal to this capital :

$$\sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{k=1}^{K} y_{ijkt}.(P_{ijkt} + (w_{ij}.\, v_{ijk}) + (f_{jkt})) \le o_{it} \quad \forall i, \forall j, \forall k, \forall t \qquad (19)$$

The ninth constraint (minimum ordering rate)

Order rate for each customer must be at least equal to the amount defined by the supplier. Otherwise, if the order rate to the supplier is lower than permitted amount, it will not be performed and purchasing from that supplier will not be applicable.

$$(x_{ijkt}).\, y_{ijkt} \ge H_j \quad \forall i, \forall j, \forall k, \forall t \qquad (20)$$

Decision variables constraint

$$X_{ijkt} \ge 0 \qquad (21)$$

$$Y_{ijkt} : 0,1 \qquad (22)$$

Procedure to produce random triangular fuzzy numbers in the model

In this study, some parameters are considered as random triangular fuzzy numbers where the way of constructing these numbers is as follows:

Firstly, using uniform random numbers function in Matlab called Unifrnd, 100 numbers for each parameter matrix solution are produced based on the desired parameter's range. In fact, in each matrix, for each of its solution, which is a fuzzy number, 100 numbers are produced randomly, and then through minimizing the numbers of the first triangular fuzzy number, from mean numbers of middle number and through the maximizing of the numbers, the final fuzzy number is found. Finally, utilizing mean distribution β, triangular fuzzy numbers are converted to crisp. This is done for all the results of the desired parameter matrix.

Betta mean distribution formulation is used for defuzzification of random triangular fuzzy numbers in the objective functions of delays, wastes, and weight [33].

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$$\tilde{B}=(\mathbf{B^p}, \mathbf{Bm}, \mathbf{B^0}) \ ; \ B=\frac{\mathbf{B^p}+4\mathbf{Bm}+\mathbf{B^0}}{6} \quad (23)$$

4. Solution Methodology

Evolutionary algorithms are stochastic search methods that are designed to emulate the language of natural biological evolution. The evolutionary algorithms apply the principle of survival of the fittest individuals to a population of alternative solutions in order to produce better solutions to a problem. Genetic algorithm (GA) and particle swarm optimization (PSO) algorithm are evolutionary algorithms that use some operators in an evolutionary process to obtain the optimal/near optimal solution. It is proven that GA and PSO are very adaptable to a great variety of different complex optimization problems [34-42]. The most important advantage of evolutionary algorithms in multi-objective optimization problems is their capacity to achieve a set of non-dominated solutions without assigning weights to the target functions, which are a function of the decision-makers' views.

4-1. Non-dominating sorting genetic algorithm II (NSGA-II)

The non-dominated sorting genetic algorithm (NSGA) is presented by Deb et al. in which a fitness procedure is implemented in order to choose non-dominated single solutions (chromosomes) from a population [43]. However, this algorithm has had some drawbacks such as computational complexity, non-elitist operation, and the necessity of a sharing parameter that can be quite preventable. Therefore, the non-dominated sorting genetic algorithm II (NSGA-II) is proposed by Deb et al. as a class of multi-objective evolutionary algorithms in which a fast and capable sorting procedure is accompanied by an elitism operation[43]. The pseudo-code of the NSGA-II is illustrated in Fig. 2.

As shown in Fig. 2, the main idea of this algorithm is to reproduce a new population from an initial population and distribute these two populations over the entire Pareto optimal set(s). Meanwhile, in order to find the best possible solutions and acquire the Pareto set(s), we need to prioritize among solutions by assigning a rank to each solution. Therefore, a process, called non-domination sorting, is being applied based on Fig. 3. Note that there are two main parameters in this process: the number of solutions dominating a specific solution (Np) and a set of solutions prevailed by the specific solution (Sp) [43].

Regarding Fig. 3, three points need to be taken into account: (1) this sorting process is an iterative procedure which labels each solution with an unnecessarily unique level/rank. In other words, by this process, it might be possible to have several solutions having the same level/rank; (2) for a minimization problem, the same as our problem, the best level has rank 1, the second level has rank 2, and so on. Now, after applying this approach, each solution recognizes its rank as a fitness evaluation [43].

Step 1: Randomly create an initial population Ipop of P solutions (chromosomes)
Step 2: Calculate all objective functions for each solution in Ipop
Step 3: Specify rank for each solution in Ipop (by non-domination sorting process)
Step 4: Apply the roulette wheel selection based on obtained ranks
Step 5: Apply the crossover scheme on Ipop based on Pc (crossover probability)
Step 6: Apply the mutation scheme on Ipop based on Pm (mutation probability)
Step 7: Acquire new offspring population Opop
Step 8: Combine Ipop and Opop to create a new population Npop
Step 9: Calculate all objective functions for each solution in Npop
Step 10: Specify rank for each solution in Npop (by non-domination sorting process
Step 11: Estimate density for each solution in Npop (by crowding distance calculation
Step 12: Is the stopping criterion met? Yes (go to step 14) / No (go to step 13)
Step 13: Create new Ipop based on obtained ranks (first priority) and crowding distances (second priority) and go to step 2
Step 14: Identify solutions in Npop with rank ≤ 1 as the final non-dominated Pareto set and go to step 15: Terminate the algorithm

Fig 2: The pseudo-code of the NSGA-II

```
for each p∈P     // number/set of solutions in population P
Sp = Ø           // a set of solutions dominated by solution p
Np = 0           // number of solutions dominating solution p
for each q∈P
if (p < q), then     // if p dominates q
Sp=Sp∩{q}        // add q to the set of solutions dominated by p
else if (p > q), then
np=np+1          // increment the domination counter of p
end
if np=0, then
prank=1
```

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F1=F1∩{p} // p belongs to the first front
End
i = 1 // initialize the front counter
while Fi=Ø
Q=Ø // used to store the members of the next front
for each p∈Fi
for each q∈Sp
nq=nq-1
if nq=0 then // q belongs to the next front
qrank=i+1
Q=Q∩{q}
End
i = i +1
Fi=Q
End

Fig 3:Non-domination sorting process for the NSGA-II

Chromosome structure

The way to display the problem solutions is the most important part of designing the genetic algorithm as better as possible. Since each chromosome represents a solution to the problem, the chromosome must be able to display the problem characteristics well. In the present study, the form of solution or problem chromosome consists of 2 parts. The first part of chromosome indicates the order rate for each product by the customer in each period. The second part of chromosome also is considered as a binary variable to select the supplier. Then, in this algorithm, we have a chromosome in the form of a four-dimensional matrix, where the first part is order rate and the second part is selection of suppliers.

A chromosome is defined for each product and each period, where the genes of this chromosome represent the matrix inputs or the number of suppliers and customers. Thus, K * T chromosome or solution is found in the algorithm. Solutions matrices are shown as follows. In MATLAB, order rate is indicated by order count and supplier selection is indicated by order in the function chromosome.

The first part of chromosome (order raste to the suppliers)

The second part of chromosome (suppliers selection)

Chromosome 1*1 :

$$\overbrace{\begin{pmatrix} x_{1111} & \cdots & x_{1j11} \\ \vdots & \ddots & \vdots \\ x_{i111} & \cdots & x_{ij11} \end{pmatrix}}\overbrace{\begin{pmatrix} y_{1111} & \cdots & y_{1j11} \\ \vdots & \ddots & \vdots \\ y_{i111} & \cdots & y_{ij11} \end{pmatrix}}$$

Chromosome k*t :

$$\overbrace{\begin{pmatrix} x_{11kt} & \cdots & x_{1jkt} \\ \vdots & \ddots & \vdots \\ x_{i1kt} & \cdots & x_{ijkt} \end{pmatrix}}\overbrace{\begin{pmatrix} y_{11kt} & \cdots & y_{1jkt} \\ \vdots & \ddots & \vdots \\ y_{i1kt} & \cdots & y_{ijkt} \end{pmatrix}}$$

Fig. 4. Chromosome structure in the non-dominated multi objective genetic algorithm

Crossover operator

In the crossover operator, initial population is constructed in a number equal to n crossover, and then selection is performed randomly [44]. In fact, crossover is a function taking the location of two parents and producing two offspring. In other words, each parent produces two springs. For this operation, crossover is an arithmetic crossover, which is for a continuous space. Equations (36) and (37) are parents, and equations (38) and (39) are the offspring of parents that are, in general, defined as equations (40) and (41).

$X_1=(x_{11},x_{12},\ldots\ldots\ldots\ldots,x_{1n})$ (24)
$X_2=(x_{21},x_{22},\ldots\ldots\ldots\ldots,x_{2n})$ (25)

Offspring are generated by crossing the parents:

$Y_1=(y_{11},y_{12},\ldots\ldots\ldots\ldots\ldots,y_{1n})$ (26)
$Y_2=(y_{21},y_{22},\ldots\ldots\ldots\ldots\ldots..,y_{2n})$ (27)

In a general case:

$Y_{1i} = \alpha_i x_{1i} + (1- \alpha_i) x_{2i}$ (40)
$Y_{2i} = \alpha_i x_{2i} + (1- \alpha_i) x_{1i}$ (28)
$0 \leq \alpha \leq 1$

α is equal to the parents' elements which is a approximation of uniform crossover in the discrete space.

Mutation operator

In the crossover operator, the initial population is generated equal to the number of n mutation. Then, selection is performed randomly [44]. For mutation operator, Gaussian technique in the continuous space is used, such that the amount of selected variable x is between x_{min} and x_{max}, where variable x wants to be converted to x'. Δx has a normal distribution with mean 0 and variance$\boldsymbol{\sigma^2}$; here, we have :

$\Delta x \sim N(0,\sigma^2)$ (29)
$x'=x+\Delta x \sim N(x,\sigma^2) \Longrightarrow x+\sigma N(0,1)$ (30)

where Δx is defined by normal random function.

σ is defined as a parameter in the algorithm where we can consider some percent of variables diversity which is p mutation, e.g., 0.1 of diversity of upper and lower limits of variables

$\sigma = 0.1*(varmax - varmin)$ (31)

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To select the number of selected elements or variables, α rate is defined as the mutation rate or effect rate, represented by μ.

μ percentage of the population is selected and the operation mentioned in equation (43) is applied to population [44].

Concerning the crossover and mutation schemes, it should be noted that to diversify the search space more efficiently, the operations of crossover and mutation are classified in a way that cannot be conducted on the same chromosomes.

Now, even though the sorting process can differentiate between solutions by assigning a rank to each of them, there might be some solutions having the same rank; so, how can we choose the best available solutions? Deb et al. [43] answered this question by another approach, called a crowding distance criterion. This criterion measures the density of other solutions distributed around a particular solution. As a matter of fact, MOOPs urge to not only obtain a set of solutions, but also acquire evenly distributed solutions. Hence, the density of the solutions' position is also important.

The foregoing framework is depicted in Fig. 5 as a schematic illustration in which the set of black solutions (L solutions) identifies the non-dominated Pareto set. In addition, the area surrounded by the dotted line clarifies the value of crowding distance criterion of solution i in the Pareto set. Regarding this fact, whenever two different solutions have the same rank, the bigger the criterion value, the more suitable the solution [43].

Additionally, the whole coding process of the crowding distance criterion is according to Fig. 8.

$$\mathbf{d_j(k)} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \frac{\mathbf{f_i(k-1)-f_i(k+1)}}{\mathbf{f_i^{max}-f_i^{min}}} \quad (32)$$

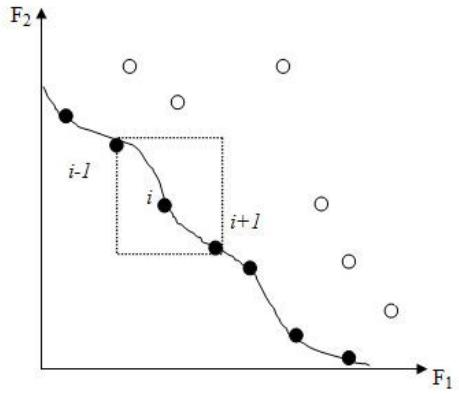

Fig. 5. Schematic illustration of crowding distance criterion

Step 1: l=|Γ| // number of solutions in Γ (non-dominated set)
Step 2: for each i∈Γ
Step 3: Γ[i]distance=0// initialize the distance of solution i
Step 4: for each objective m
Step 5: Γ=sort(Γ,m) // sort the non-dominated set based on the value of each objective function
Step 6: Γ[0]distance=Γ[l]distance=0 // boundary points are always selected
Step 7: for i = 2 to (l – 1) // for all other points
Step 8: Γ[i]distance=Γ[i]distance+(Γ[i+1].m-Γ[i-1].m)/(fmmax-fmmin)

Fig 6. Algorithmic procedure of crowding distance criterion

4-2. Particle swarm optimization algorithm (PSO)

Particle swarm optimization (PSO) is introduced by [45]. The concept is mainly from collective animal behavior. Although its development is a little late compared to GA, it is now applied to solution of optimization. Particle swarm contains two concepts: one is that the proposed individual will refer to their own experience or experience of others in decision making according to the human decision process. The other is to propose simple rules to modularize collective natural behavior. Basically, the complicated collective behavior can be simulated by the three following aspects: follow the individual closest to objects, move towards object, and move toward group center. In the original PSO, particle i is represented as $\boldsymbol{X_i}$ =($\boldsymbol{X_{i1}}$, $\boldsymbol{X_{i2}}$, ..., $\boldsymbol{X_{iD}}$), which represents a potential solution to a problem in D-dimensional space. Each particle keeps a memory of its previous best position Pbest, and a velocity along each dimension, represented as $\boldsymbol{v_i}$=($\boldsymbol{v_{i1}}$, $\boldsymbol{v_{i2}}$, ..., $\boldsymbol{v_{iD}}$). At each iteration, the position of the

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particle with the best fitness value in the search space, designated as g and P vectors of the current particle, is combined to adjust the velocity along each dimension, and that velocity is then used to compute a new position for the particle [45]. The method could be divided into $\boldsymbol{G_{Best}}$ and $\boldsymbol{L_{Best}}$ versions, whose main difference is their definition of the best. In $\boldsymbol{G_{Best}}$ version, the particle swarm optimizer keeps track of the overall best value, and its location, obtaining thus far by any particle in the population, which is called $\boldsymbol{G_{Best}}$ (Gbestid). For $\boldsymbol{L_{Best}}$ version, in addition to gbest, each particle keeps track of the best solution, called $\boldsymbol{L_{Best}}$ (Gbestid), and it is attained within a local topological neighborhood of particles. However, the particle velocities in each dimension are held to a maximum velocity $\boldsymbol{v_{max}}$ and the velocity in that dimension is limited to $\boldsymbol{v_{max}}$. The updated rule is as follows:

$$\mathbf{V}_{i,t}^{new} = W \times \mathbf{V}_{i,t}^{old} - 1 + C_1 \times Rand1 \times (Pbestid - X_{i,t} - 1) + C_2 \times rand2 \times (Gbestid - X_{i,t} - 1) \quad (33)$$

$$\mathbf{X}_{i,t}^{new} = \mathbf{X}_{i,t}^{old} - 1 + \mathbf{V}_{i,t}^{new} \quad (34)$$

where $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ determine the relative influence of the social and cognition components (learning factors), while $\boldsymbol{rand_1}$ and $\boldsymbol{rand_2}$ denote two random numbers uniformly distributed in the interval [0, 1]. After the first version of PSO is proposed, many efforts have been made to improve the performance of PSO.

Major cycle of MOPSO

Leader selection is the first step in the major cycle of MOPSO, where a probability distribution is defined. Then, using a roulette cycle, sampling is performed from this probability distribution so that it is defined that what cell will be selected. Then, a case is selected among the members of this cell. Members of non-dominated particles are placed in a repository. In the selection, a cell is selected meeting the competency condition; thus, we have:

$$n_i < n_j \Longrightarrow p_i \geq p_j \quad (35)$$

n_i : The population of cell i.

p_i : The population to select cell i.

Boltzman technique is used to define p :

$$P_i \propto \exp(-\beta_{ni}) p_i = \frac{e^{-\beta ni}}{\sum_j e^{-\beta nj}} \quad (36)$$

Mutation

Uniform distribution is used to define the mutated particles rate [44]. Mutation probability is given as follows:

$$P_m = \left(1 - \frac{it-1}{maxit-1}\right)^{5/\mu} \quad (50)$$

μ : Mutation rate which controls the plot slope, it : Iteration.

In the meta-heuristic algorithms, it is not possible to code the constraints directly in the model; thus, in this section, penalty or violation function is used. If the limit is met, penalty is not added to the objective function; but, if this limit is not met, penalty is not added to the objective function; but, if this limit is not met, penalty amount is multiplied to a coefficient called alpha and is added to the objective function [44]. Penalty amount in confronting various limits is explained in the following equations :

$$Violation(g \leq g_0) = \max\left(\frac{g}{g0} - 1, 0\right) \quad (37)$$

$$Violation(g \geq g_0) = \max\left(1 - \frac{g}{g0}, 0\right) \quad (38)$$

$$Violation(g = g_0) = \left|\frac{g}{g0} - 1\right| \quad (39)$$

Violation objective function is converted to the following equation:

$\hat{z} = z + \alpha$

Parameter setup for non-dominated multi-objective genetic algorithm

To set the algorithm parameters, three levels are considered. This test is designed in Mini Tab and Taguchi technique. The suggested test is defined by Taguchi technique L27(3**5).

That is, 27 tests are designed from 5 parameters and 3 levels. Function S/N is also defined for minimization as follows:

S/N= -10*Log10(Sum(Y**2)/n) (54)

Table 1 displays the levels considered for non-dominated multi-objective genetic algorithm. In the following, three problems are defined for each suggested test where, by implementing the algorithm for each test, objective functions are found. Amount of each objective function is obtained by the minimum amount of each objective function in pareto boundary (non-dominated solutions), which is the same as unfeasted solution. In table 2, three test problems are shown.

For each problem of each test, separate objective functions are found. In this part, for each test, the mean of each objective function is obtained from 3 problems.

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Total $Z=w_1*Z_1+w2*Z_2+w_3*Z_3+w_4*Z_4+w_5*Z_5$(40)

Tab. 1. The levels defined for parameters of algorithm NSGA–II

Parameters	Levels		
	Level 1	Level 2	Level 3
Maximum Number of Iterations	10	15	20
Population Size	50	75	100
Crossover Percentage	0.3	0.5	0.7
Mutation Percentage	0.1	0.3	0.5
Mutation Rate	0.01	0.03	0.05

Amount of each objective function obtained for each problem is converted to an objective function through the weighting technique. The following equation indicates the general objective function [46].

Amount of w indicates that the weight or significance functions are of equal significance for decision-maker, considered as 0.2.

As observed, the best ratios are located in the upper part of each section.

The levels considered for the parameters of multi-objective algorithm of particle swarm optimization are shown as in table (4).

Tab. 2. Test problem to find the best parameter amount

Problem number	1	2	3
Customers	2	3	5
Suppliers	3	4	6
Products	2	4	7
Period	2	3	3

Tab. 3. The best parameters of NSGA – II

Parameters	Parameter
Maximum number of Iterations	20
Population size	75
Crossover percentage	0.7
Mutation percentage	0.3
Mutation Rate	0.01

Performance measures

To evaluate the efficiency of two meta-heuristic algorithms, we will define 5 sample examples in various aspects. Then, ratios of each measure to each function of each sample example are obtained; finally, mean amount of each measure is defined among the objective functions in each sample example.

Table 6 indicates the input parameters of five test examples.

Tab. 4. The levels defined for the parameters of MOPSO algorithm

Parameters	Levels		
	Level 1	Level 2	Level 3
Number of Maximum solutions	10	15	20
Population Size	50	75	100
Repository Size	0.3	0.5	0.7

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Mutation Percentage	0.1	0.3	0.5
Mutation Rate	0.01	0.03	0.05

Tab. 5. The best parameters of MOPSO algorithm

Parameters	Ratios
Maximum number of Iterations	15
Population size	75
Crossover percentage	25
Mutation percentage	0.5
Mutation Rate	0.03

Tab. 6. Input parameters of 5 random problem

Problem No	5
Customers	0
Suppliers	9
Products	0
Period	4

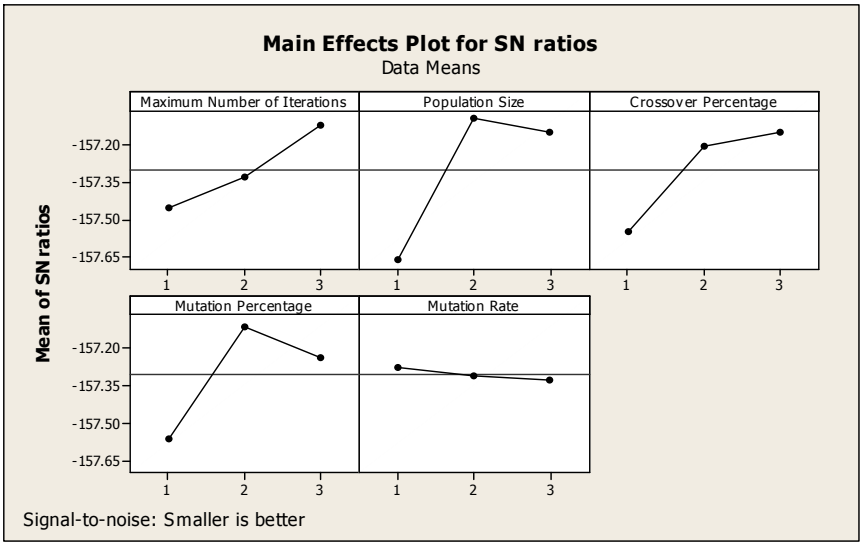

Fig. 7. Ratio of NSGA – II algorithm parameters in 3 levels

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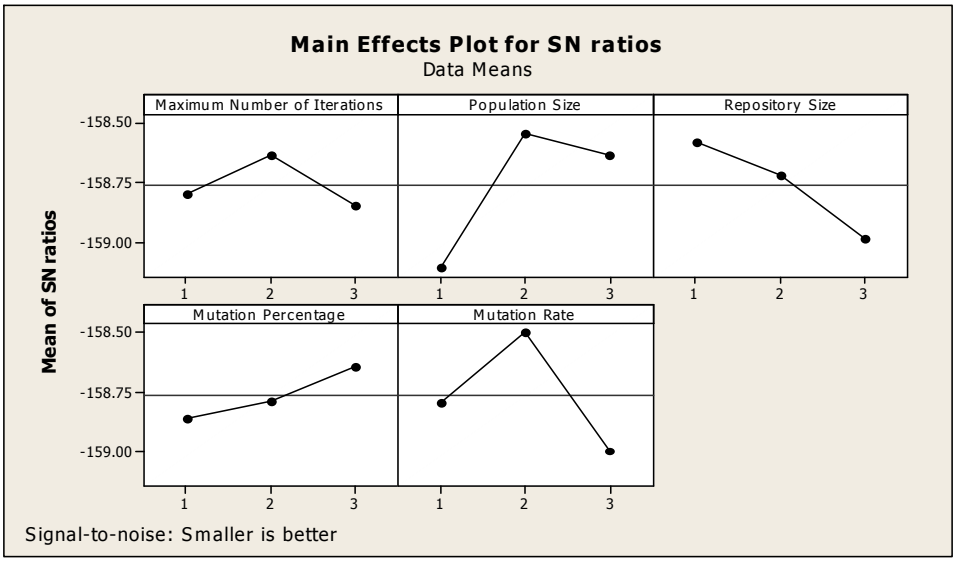

Fig. 8. Ratios of MOPSO algorithm parameters in 3 levels

Distance from ideal point

One of the measures to evaluate the algorithms is the distance from the ideal point. This measure calculates the distance of all points from the best population size. The following equation indicates how to calculate this measure [47]:

$$MID=\frac{\sum_{i=1}^{n} cin}{n} \quad (41)$$

Solutions distribution

Another measure is distribution of algorithm solutions, such that the algorithm would cover all the solution spaces' points. This measure calculates the relative distance of the subsequent solutions. The following solution indicates how to calculate this measure [47]:

$$S=\sqrt{1/n\sum_{i=1}^{n}(di-\bar{d}\ )^2} \quad (42)$$

Where we have :

$$\bar{d}=\sum_{i=1}^{n}\frac{di}{|n|} \quad (43)$$

$$d_i=\min_{\{k\in N\ \&\ k\neq 1\}}\sum_{m=1}^{2}\left|fm^i-fm^k\right| \quad (44)$$

Algorithm solving time

The final measure is algorithm solving or implementation time. Algorithms are programmed using MATLAB 7.14.0.739 (R2012a) and implemented on a PC under windows 7, 2.40 GHz, RAM 4 GB.

Results obtained by implementation of each measure are obtained by Matlab and displayed in the following figures:

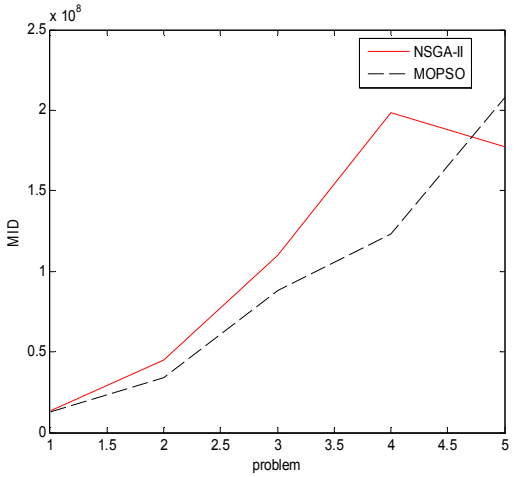

Fig. 9. Schematic of the ratios of the measure distance from algorithms' ideal point

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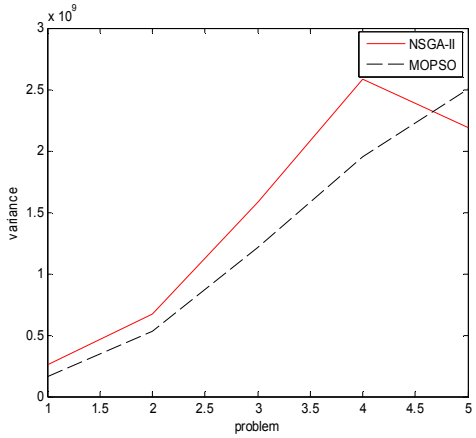

Fig. 10. Schematic of the algorithms variance measure ratios

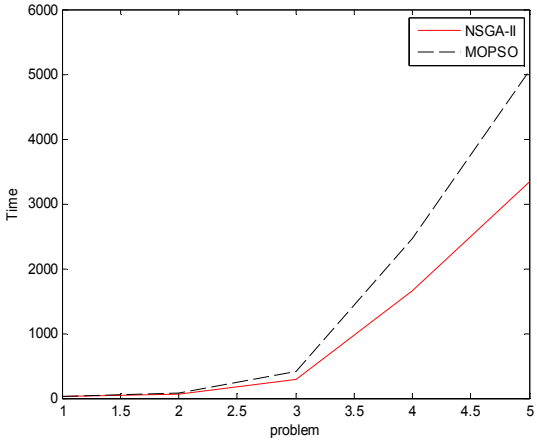

Fig. 11. Schematic of the ratios of algorithms implementation time

5. Conclusion

A multi-objective mathematical model is presented in this paper in order to select suppliers and allocate orders to suppliers under uncertainty conditions. The multi-objective model is solved using two well-known multi-objective algorithms: Non-dominated Sorting Genetic Algorithm (NSGA-II) and Particle Swarm Optimization (PSO); the results are analyzed using quantitative criteria.

After performing the tests and evaluating the solutions of two algorithms by three measures including ideal point, variance, and solving time, it is concluded that NSGA-II is superior in the measures of solving time and variance, and meta-heuristic algorithm of particle crowd has a smaller distance from the ideal point.

Thus, NSGA – II algorithm in the measures of time and variance is superior with respect to sample example; in numerical case, algorithm MOPSO is superior in the measure of distance from the ideal point. However, with respect to the statistical analysis and performing one-way variance analysis and hypothesis testing, we concluded that NSGA-II can be completely comparable, and means of the two algorithms measures do not differ significantly. In addition, implementing the non-parametric test also indicates the accuracy of this claim. Then, it can be concluded that to solve the model for selecting and allocating order to the supplier in the widespread case, that is, in multi-objective, multiple customer, multi product and multi period and in the multi objective cases, non-dominated ordering multi objective meta-heuristic algorithm or particle crowd may be used to solve the model. However, it is worth considering that if the algorithm implementation time and variance is of importance to decision-makers, NSGA – II algorithm may be used, and when distance from the ideal point is important, MOPSO algorithm may be used. However, in a general case, both algorithms have the ability to compete with each other.

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For future studies, the following suggestions are offered:

- Considering other objective functions such as risk, reliability, locating costs, green supply chain, etc.
- Considering parameters that are more uncertain such as fuzzy, probable or gray demand.
- Using other uncertain techniques to find the suppliers weights such as fuzzy hierarchical, fuzzy network analysis, etc.
- Taking the shortage into consideration in the mode.
- Considering and developing the model in other supply chains such as distributor, customer, or retailer.
- Using other techniques for setting parameter for meta-heuristic algorithms.
- Using other measures to estimate the solutions and evaluate the meta-heuristic algorithms.
- Using other meta-heuristic algorithms to solve the algorithm such as cooling, etc.

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