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Closed Loop Supply Chain Planning With Vehicle Routing

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KEYWORDS

Closed loop supply chain, Binary programming, vehicle routing, simultaneous pick-up and delivery

ABSTRACT

In the recent decade, special attention is paid to reverse logistic due to economic benefits of recovery and recycling of used products as well as environmental legislation and social concerns. On the other hand, many studies claim that separate, sequential planning of forward and reverse logistic causes sub-optimality. Effective transport activities are also one of the most important components of a logistic system, and it needs an accurate planning. In this study, a mixed integer linear programming model is proposed for integrated forward / reverse supply chain as well as vehicles routing. Logistic network used in this paper is a multi-echelon integrated forward /reverse logistic network, which is comprised of capacitated facility, common facilities of production/recovery and distribution/collection, disposal facilities and customers. The proposed model is multi-period and multi-product with the ability to consider several facilities in each level. Various types of vehicle routing models are also included such as multi-period routing, multi-depot, multi-products, routing with simultaneous delivery and pick-up, flexible depot assignment, and split delivery. The model results present the product flow between the various facilities in forward and reverse directions throughout the planning horizon with the objective minimization of total cost. Numerical example for solving the model using GAMS shows that the proposed model could reach the optimal solution in reasonable time for small and medium real-world problems.

1. Introduction

In today's world, the changes in the economic and industrial spheres are occurring more quickly than in the past. Competition among companies has increased, and costumers want high quality goods at a lower price and low delivery time. To respond to this request and maintaining competitive advantage, companies have increasingly established supply chain networks and integrated logistics. On the other hand and parallel to these developments in the economic

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sphere, because of environmental impact of used products, increasing environmental consciousness of customers and environmental legislations, attention to the reverse logistics (RL) networks has increased. In some countries, environmental legislations force companies to take responsibility for their products over the entire life cycle of the products. From an economic point of view, recovery and recycling of used products is profitable for the companies. In some products, establishment of after-sales repair service results in customer satisfaction; in this way, it indirectly leads to more profits for company. In recent years, many studies have been done on the economic benefits of recycling used products [1]. In the past three decades, many important companies, such as General Motors, Kodak, Xerox, have paid special attention to refurbishing and repairing of the return products.

In the literature, different classifications of reverse logistics' processes are presented. reverse logistics, in general forms, start from end users (i.e., consumers) where used products (returned products) are collected and then attempts to manage end of life products through different decisions such as remanufacturing (to resale them to second markets or if possible to first customers), recycling (to convert them to raw materials), repairing (to sell in the second markets through repairing), and finally, disposing of some hazardous materials.

Many studies in the literature showed that separate or sequential planning of forward and reverse logistics result in sub-optimality. According to these studies, integrated design of forward and reverse logistics courses lead to considerable savings in costs [2]. In the literature, the integrated system includes forward and reverses logistics, which is called closed-loop supply chain (CLSC) that has recently gained significant importance [3]. Typically, a closed-loop supply chain in forward flow includes Supply of raw materials from suppliers, convert raw materials into finished products in plants, and distribution of finished products to the final costumers through the distribution centers. The CLSC in reverse flow includes collecting the returned products from final consumers and remanufacturing, recycling or disposing them in a way that is appropriate. Fig. 1 shows the generic form of CLSC.

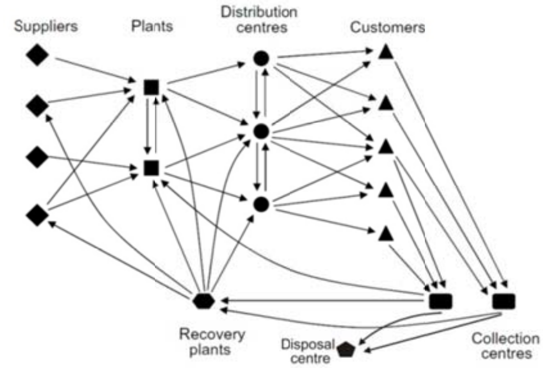


Fig. 1. The generic form of CLSC

One of the most important activities of each supply chain is logistic activities that includes inventory and transportation activities. Wajszczuk [4] showed that the logistic costs are 20%-30% of the total costs of manufacturing companies. Therefore, any optimization in these activities can create sustainable competitive advantage for companies. As mentioned earlier, effective transportation activities are one of the most important components of each logistic system, and it needs an accurate planning. In reverse logistics, non-effective transportation activities limit the economic success of the products recovery. Collection of returned products should be done at the right time and in a route with the lowest cost. Also, transportation and distribution of products in the forward direction is one of the most important problems in the supply chain management.

Transportation activities in CLSC are divided into two types:

Collection of returned products from final costumers and move in the reverse flow.

Distribution of refurbished and new products in forward flow.

An important issue in CLSC is creating synergies between two different flows in different directions. In this case, integrating flow is a suitable approach to cost savings. For example, integration of distribution and collection activities results in the vehicles with full of returned products on the way back to the depot. Hence, an important operational decision concerns finding optimal vehicle routes to transfer the products through the network efficiently by implementation of mathematical models such as vehicle routing models in CLSC distribution planning.

Inventory management is another important operational decision that is known as one of the main tools to improve responsiveness at lower cost, reducing inventory costs, and improving customer service.

This paper is organized as follows: in Section 2, a brief discussion about previous related works is presented; the problem description and its mathematical formulation are comprised in Section 3. The generation of data instances and computational results are presented in Section 4. In Section 5, some concluding remarks are offered.

2. Literature review

2-1. Closed-loop supply chain management

According to the American Reverse Logistics Executive Council, reverse logistics is defined as ‘‘The process of planning, implementing, and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal’’([5],[6]). A review of the supply chain management literature shows that a large part of the studies is in the field of forward logistics network design from supplier to final costumers; a relatively smaller part of the literature is in the field of reverse logistics [3]. A review of the reverse logistics literature shows that studies in this field, especially after 2005, are increasing [7]. Main objective in RL network design is to define the number and location of facilities such as collection centers, remanufacturing centers and disposal centers, as well as determine their capacity and the optimal flow among them. Pishvaee et al. [8] proposed a MILP formulation to design a single product and multiple-echelon reverse logistics network consisting of costumers, collection centers, recovery centers, and disposal centers with limited capacities. As mentioned earlier, separate or sequential planning of forward and reverse logistics results in sub-optimality. The configuration of reverse logistics network, including location of facilities and material flow between them, influences all network components. Some studies in this field show that integrated planning and design of forward and reverse logistics reduces the total supply chain costs ([2],[9],[10],[11]). In general, the purpose of many of these papers is to determine the number, location, and capacity of facilities, such as plants, distribution centers also collection, recovery, disposal centers, as well as determining the optimal products flow among them in forward and reverse direction. Jayarman et al. [12] proposed a MILP model to design an integrated forward/reverse logistics system based on customer demand for remanufactured products. Selma et al. [13] considered a multi-product CLSC with capacitated facilities and uncertainty on demand and returned products. They developed a MILP formulation to design this network. Beamon et al. [14] developed a mixed integer linear programing formulation for a four-echelon CLSC, in which the quality of new and remanufactured products is the same.

Another issue in design of integrated forward/reverse logistics networks that received considerable attention is the use of common facilities such as distribution/ collection centers or manufacturing/ recovery centers. Considering common facilities not only reduces the planning complexity, but also construction and maintenance costs are reduced from economic point of view, duo to using common facilities and infrastructure. Lee and Dong [15] developed a MILP model to design the CLSC network of electronic products with distribution/collection common facilities and solved this model with tabu search meta-heuristic algorithm. But, this article has some weaknesses. For example, simple assumptions such as the specified number and location of common facilities and the use of only one plant. Also, in reverse flow, only remanufacturing process is considered. Zhou and Wang [16] presented a model on design of integrated forward/reverse logistics network with manufacturing/recovery common facilities, which is one of the most popular models for CLSC network design. It also assumes that the quality of remanufactured products and new products is the same. Thus, remanufactured products, such as new products, come to market to satisfy demands. However, their model does not consider some real-world important assumptions (e.g., capacity constraints, multi-product production, and the uncertainty in demand and returned products). Pishvaee et al. [10] proposed a mathematical model to design integrated forward/ reverse logistics network with manufacturing/ recovery and distribution/ collection common facilities and disposal centers in two modes with stochastic and deterministic parameters. Outputs of their model were the product flow among various facilities in forward and reverse direction throughout the planning horizon with the objective of minimizing total cost.

2.2. Vehicle routing problems

As mentioned earlier, finding optimal vehicle routes is one of the most important operational decisions in CLSC management that can significantly reduce costs and improve customer service. Many of the studies in the area of CLSC management did not consider this important operational decision. The Vehicle Routing Problem (VRP), introduced by Dantzig and Ramser [17], is a well-known problem in operations research applied to transportation sciences. Basic formulation of this problem is the so-called Capacitated Vehicle Routing Problem (CVRP), in which geographically dispersed costumers around a central depot have demands for a homogeneous product. They have to be served by homogeneous vehicles with a limited capacity based at the depot. The CVRP aims to determine a set of delivery routes of minimum total cost over a single period, such that: (i) each route starts and ends at the depot; (ii) each customer is served by only one vehicle; (iii) the total demand on each route does not exceed the vehicle capacity. This problem and all of its extensions are NP-hard combinatorial optimization problems [18]. For this reason, the majority of articles focused on the development of effective solution procedures. There are some review papers in this filed in the literature. A comprehensive review can be found in Eksioglu et al. [19].

Since the first article of VRP by Dantzig and Ramser in 1959, this problem has been extended in many ways by introducing additional real-world aspects, resulting in a large number of variants of the VRP. One of the variants of VRP is Multi-depot VRP (MDVRP), which was firstly studied by Sumichrast et al. [20]. MDVRP assumes that multiple depots are geographically scattered among the customers and each customer is visited by a vehicle that is assigned to one of these depots (i.e., each route must start and end at a same depot). Wu et al. [21] developed the MDVRP to multiple-depot location-routing problem. Sepehri et al. [22] proposed a MILP model for multi-product MDVRP in a multi-echelon supply chain. They used Lagrange method to solve the problem by decomposing the model to a single product model. Mahdavi et al. [22] proposed a mathematical model and a heuristic approach for multiple products MDVRP that minimizes total traveled distance.

In another kind of VRP, planning is made over a time horizon, and deliveries to the customer can be made in different days. The objective of this problem is to find a feasible routing solution in each period, such that the total cost of the routes over the planning horizon is minimized. This problem is known as the Periodic Vehicle Routing Problem (PVRP) which is introduced by Beltrami et al. [24] and the first mathematical formulation of this problem is proposed by Christofides et al. [26]. Foster and Ryan [25] presented an extension of PVRP to a problem with pre-defined days to visit each costumer. Francis et al. [27] developed PVRP for a problem, in which the number of visiting each customer was a decision variable of the problem. An extension of PVRP is Multi-depot PVRP (MDPVRP). The MDVRP and the PVRP have received a great deal of attention in the literature, but the MDPVRP has been rarely studied and relatively few studies have been done in this area. such as Kang et al. [28], that used an exact algorithm to solve this problem.

In the basic MDPVRP, every vehicle route has to start and end in the same depot. In other words, the number of vehicles of each depot is fixed and each vehicle starts its travel from its depot and ends in the same depot. A new type of MDPVRP is developed in which no obligation is made to vehicles to return to the initial depot and each vehicle starts from an initial depot and after servicing to sub-set of costumers, ends to a depot that servicing of the customers in the next period be performed at lower cost. The idea of the flexibility in determining the last depot of each route as "flexible assignment" was introduced by Kek et al. [29]. Eydi and Abdorahimi [30] considered a MILP model for the MDVRP with flexible assignment and solved this model with genetic algorithm.

One of the variants of VRP that is widely used to deal with the issues in reverse logistics is the vehicle routing problem with simultaneous pickup and delivery (VRPSPD), in which each costumer requires both delivery and pickup. This problem has been introduced first by Min et al. [31]. A general assumption in this problem is that all deliveries must be originated from the depot, all pickups must be transported back to the depot. Deliveries and pickups must be met simultaneously when each customer is visited by a vehicle and unloading is carried out before loading at each customer. The application of VRPSPD is frequently encountered in the distribution system of grocery store chains. Each grocery store may have both a delivery (e.g., fresh food or soft drink) and pickup (e.g., outdated items or empty bottles) demand [32]. Reverse logistics and CLSC are also another application area for VRPSPD. In this case, integration of forward and reverse flows (for instance, integration of distribution and collection) is a suitable approach to cost savings. Dethloff [33] developed a MILP model for VRPSPD in a reverse logistics network with one depot. This model has been developed in many subsequent studies. Salhi and Nagi [34] proposed multiple depots VRP with simultaneous pickup and delivery.

In the majority of vehicle routing problems, each customer is served by only one vehicle exactly only once. However, this assumption is not always realistic because sometimes the customer demand exceeds the vehicle capacity. In this case, the possibility of multiple visits to the same customer characterizes the VRP with Split Deliveries (SDVRP) Archetti et al. [35] showed that allowing split servicing can reduce routing costs up to 50%.

As said earlier, VRP is an NP-hard problem, thus the majority of studies on this problem have been mainly focused on heuristic and meta-heuristic approaches. Despite the increasing interest in heuristic approaches for VRP, developing efficient mathematical formulations for this problem has received much less attention from researchers. However, efficient mathematical formulations may allow us to solve small or moderate-sized problem instance by using any commercial package or to develop new exact and/or mathematical model-based heuristic algorithms for the problem. This is the main motivation of our paper. On the other hand, according to the literature, there is a few studies on vehicle routing in closed-loop supply chain. Therefore, in this paper, a comprehensive model is proposed to combine different types of vehicle routing problems and implement them in a multi-echelon closed-loop supply chain.

3. Problem Description and Formulation

In this paper, we consider a multi-echelon, multi-period, and multi-product closed-loop supply chain network considering common facilities. This chain consists of production/ recovery centers, distribution/ collection facilities, disposal centers and customers' nodes (see Fig. 2). As we said in the literature review section, with the forward and reverse features of network simultaneously and proposing an integrated model, we aim to prevent sub-optimal solutions. Moreover, employing common facilities for common activities (e.g., production and recovery) saves more money, time and energy and decrease the operational cost. Each common distribution/collection (production/ recovery) center plays a distributor (producer) role in the forward flow on the network, and a collector (recovery) role in the reverse one (e.g., considered at Pishvaee et al. [10]).

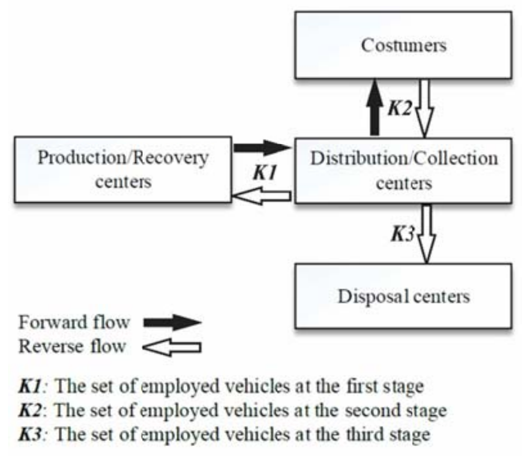


Fig. 2. The considered network

In this network and in the reverse flow, returned products from customers are transported to the distribution/collection centers. Some of these products, which are recoverable, are transported to the production/recovery centers and the others that are not recoverable are transported to disposal centers. According to Zhou and Wang [16], demand of customers can be satisfied through the recovered products and new products. They assumed that the quality of new and recovered products is the same. In the forward flow, the new and recovered products by the production/recovery centers are transported to the end customers through the distribution/collection centers. There is a capacity limitation for all facilities and the transportation fleet.

Moreover, the model deals with transportation planning at each echelon. The employed transportation vehicles in each echelon are independent of other echelons. Travels in all echelons are indirect (tour; such as the routing problem) and each vehicle starts its travel from an initial center and after servicing a subset of nodes, go to a final center.

In each period, each vehicle starts and ends its travel at a center, so that the initial and final centers are not the same necessarily (flexible assignment). In each period, each vehicle starts its travel from the final center in the previous period. In the echelon 1 (2), each vehicle delivers the products to the distribution/ collection centers (costumers) and picks up the returned product from it simultaneously (VRPSPD). In the echelon

1 (2), in each period, the demand of each distribution/ collection center (customer) can be satisfied by multiple visits by multiple vehicles (i.e., split servicing is allowable). But, in the echelon 3 in each period, each distribution/collection center is served by only one vehicle exactly only once.

In the first echelon, produced and recovered products in each production/recovery center in each period can be stocked (regarding to holding capacity), and after that can be distributed in the next period. In each period, each vehicle, regarding its capacity limitation, picks up required products from the stock of a production/ recovery center and starts its travel and delivers products to a subset of distribution/ collection centers (simultaneously, vehicles collect the returned products) and returns to one of the production/ recovery centers with collected returned products. (Delivered products to distribution/collection centers can be distributed to the customers in the next period, and all of the collected returned products in each production/recovery center are remanufactured at the next period). On the other side, the new products are produced regarding the demand of customers. In each of production/recovery center, the products can be stocked to tackle the plausible fluctuation in demand.

In the second echelon, in each period, each vehicle (regarding its capacity limitation) picks up required products from the stock of a distribution/ collection center, delivers them to a subset of customers (simultaneously, vehicles collect the returned products), and returns them to one of the distribution/ collection centers with collected returned products. Shortages in deliveries to customers are allowed (lost sale), but pickups from customers are not allowed. Also, there is no possibility to stock the returned products in the distribution/ collection centers for several periods, so that recoverable proportions of them are transported to the production/ recovery. centers and the rest are transported to the disposal centers in the next period. It is notable that the distribution/collection centers can stock the new and recovered products for distribution to the customers in the next periods.

In the third echelon, in each period, each vehicle starts its travel from a disposal center, picks up the unrecoverable products from a subset of distribution/collection centers, and returns them to one of the disposal centers with collected products. In this echelon, shortage in pickups are not allowed too. The network should be planned with the aim of minimizing total operational cost including of production, remanufacturing, holding, and transportation.

Assumptions:

In the section above, some assumptions were expressed. Additional assumptions which are considered in the modeling are:

- The number of periods during the planning horizon is defined and limited.
- The number, capacity, also location of all facilities are known.
- In each echelon, the vehicles are heterogeneous, with certain capacity and operate independently of other echelon.
- The number and location of all costumers in all periods are known and quantity of delivery demand and returned products to/from each customer and also all parameters are deterministic.

Mixed integer linear programming model (MILP):

Sets:

A	The set of production/recovery centers
B	The set of distribution/collection centers
C	The set of customers
D	The set of disposal centers $i, j \in A/B/C/D$
T	The set of periods $t \in T$
P	The set of products $p \in P$
$K1$	The set of employed vehicles in the first echelon
$K2$	The set of employed vehicles in the second echelon
$K3$	The set of employed vehicles in the third echelon $k \in K1/K2/K3$

Parameters:

$t1_{p,i}$	Time to produce a unit of product p at production/recovery center i (hour)
$t2_{p,i}$	Time to remanufacture a unit of product p at production/recovery center i (hour)

$t3_i$	Time to dispose a unit of product p at production/recovery center i (hour)
$c7_{p,i}$	Production cost of product p per unit at production/recovery center i
$c8_{p,i}$	Remanufacturing cost of product p per unit at production/recovery center i
$c9_i$	Disposal cost of each product per unit at disposal center i
$c_{i,j}$	Travel cost from node i to node j
$t_{i,j}$	Travel time from node i to node j
$hh1$	Holding cost of product per unit (Independent of product type) per period at production/ recovery centers
$hh2$	Holding cost of product per unit (Independent of product type) per period at distribution/ collection centers
$q1$	Shortage cost per unit
$u2_{i,t}$	Production capacity of production/recovery center *i* in period t (hour)
$u4_{i,t}$	Remanufacturing capacity for production/recovery center *i* in period *t* (hour)
$u1_i$	Holding capacity of production/recovery center *i* in period *t* (Independent of product type)
$u3_i$	Holding capacity of distribution/collection center *i* in period *t* (Independent of product type)
$u5_i$	Collection capacity of distribution/collection center *i* in period *t*
$u6_{i,t}$	Disposal capacity of disposal center *i* in period t (hour)
$q1_k$	Capacity of vehicle k which employed in the first echelon $k \in K1$
$q2_k$	Capacity of vehicle k which employed in the second echelon $k \in K2$
$q3_k$	Capacity of vehicle k which employed in the third echelon $k \in K3$
$d_{j,p,t}$	Demand of customer *j* for product *p* in period *t*
$p_{j,p,t}$	Quantity of returned product p of costumer zone j in period t
sp	Proportion of unrecoverable returned products
r_t	Duration of period t (hour)
M	A large positive constant

Variables:

$x1_{i,j,k,t}$	1 if vehicle $k \in K1$ travels from node *i* to node *j* in period t. ($i, j \in B$); otherwise 0
$y1_{i,j,k,t}$	1 if vehicle $k \in K1$ travels from node *i* to node *j* in period t. ($i \in A$, $j \in B$); otherwise 0
$z1_{i,j,k,t}$	1 if vehicle $k \in K1$ travels from node *i* to node *j* in period t. ($i \in B$, $j \in A$); otherwise 0
$x2_{i,j,k,t}$	1 if vehicle $k \in K2$ travels from node *i* to node *j* in period t. ($i, j \in C$); otherwise 0
$y2_{i,j,k,t}$	1 if vehicle $k \in K2$ travels from node *i* to node *j* in period t. ($i \in B$, $j \in C$); otherwise 0
$z2_{i,j,k,t}$	1 if vehicle $k \in K2$ travels from node *i* to node *j* in period t. ($i \in C$, $j \in B$); otherwise 0
$x3_{i,j,k,t}$	1 if vehicle $k \in K3$ travels from node *i* to node *j* in period t. ($i, j \in B$); otherwise 0
$y3_{i,j,k,t}$	1 if vehicle $k \in K3$ travels from node *i* to node *j* in period t. ($i \in D$, $j \in B$); otherwise 0
$z3_{i,j,k,t}$	1 if vehicle $k \in K3$ travels from node *i* to node *j* in period t. ($i \in B$, $j \in D$); otherwise 0
$g1_{p,i,t}$	The amount of product *p* manufactured in production/ recovery center *i* at period *t*
$g2_{p,i,t}$	The amount of product *p* recovered in production/ recovery center *i* at period *t*
$g3_{i,t}$	The amount of product disposed in disposal center *i* at period *t*
$v1_{p,i,t}$	The amount of shortage of product *p* in costumer *i* at period *t*
$h1_{p,i,t}$	Inventory of product *p* in production/recovery center *i* after loading the vehicles at the beginning of period *t*
$s1_{p,i,t}$	Inventory level of product p in production/recovery center *i* after adding new and recovered products at the end of period *t*

$h2_{p,i,t}$	Inventory level of product *p* in distribution/collection center *i* after loading the vehicles at the beginning of period *t*
$s2_{p,i,t}$	Inventory level of product *p* in distribution/collection center *i* after receiving new and recovered products from production/recovery centers at the end of period *t*
$l1_{k,i,p,t}$	Load of product *p* of vehicle $k \in k1$ when leaving the production/recovery center *i* at the beginning of period *t*
$l2_{k,i,p,t}$	Load of product *p* of vehicle $k \in k1$ after servicing the distribution/collection center *i* in period *t*
$a1_{k,i,p,j,t}$	Amount of product *p* delivered from production/recovery center *j* to distribution/collection center *i* by vehicle $k \in k1$ at period *t*
$b1_{k,i,p,j,t}$	Amount of returned product *p* picked up from distribution/collection center *i* and delivered to production/recovery center *j* by vehicle $k \in k1$ at period *t*
$a2_{k,i,p,t}$	Sum of returned product *p* delivered to production/recovery center *i* by vehicle $k \in k1$ at the end of period *t*
$o1_{k,i,t}$	1 if vehicle $k \in k1$ is in the production/recovery center *i* at the end of period *t;* otherwise 0
$n1_{k,i,t}$	1 if vehicle $k \in k1$ is in the production/recovery center *i* at the beginning of period *t;* otherwise 0
$u1_{i,k,t}$	The sub tours elimination variable in the first echelon
$l3_{k,i,p,t}$	Load of product *p* of vehicle $k \in k2$ when leaving the distribution/collection center *i* at the beginning of period *t*
$l4_{k,i,p,t}$	Load of product *p* of vehicle $k \in k2$ after servicing the costumer *i* in period *t*
$a3_{k,i,p,j,t}$	Amount of product *p* delivered from distribution/collection center *j* to costumer *i* by vehicle $k \in k2$ at period *t*
$b3_{k,i,p,j,t}$	Amount of returned product *p* picked up from costumer *i* and delivered to distribution/collection center *j* by vehicle $k \in k2$ at period *t*
$a4_{k,i,p,t}$	Sum of returned product *p* delivered to distribution/collection center *i* by vehicle $k \in k2$ at the end of period *t*
$o2_{k,i,t}$	1 if vehicle $k \in k2$ is in the distribution/collection center *i* at the end of period *t;* otherwise 0
$n2_{k,i,t}$	1 if vehicle $k \in k2$ is in the distribution/collection center *i* at the beginning of period *t;* otherwise 0
$u2_{i,k,t}$	The sub tours elimination variable in the second echelon
$l5_{k,i,p,t}$	Load of product *p* of vehicle $k \in k3$ after servicing the distribution/collection center *i* in period *t*
$a5_{k,i,p,t}$	Sum of returned product *p* delivered to disposal center *i* by vehicle $k \in k3$ at the end of period *t*
$o3_{k,i,t}$	1 if vehicle $k \in k3$ is in the disposal center *i* at the end of period *t*; otherwise 0
$n3_{k,i,t}$	1 if vehicle $k \in k3$ is in the disposal center *i* at the beginning of period *t*; otherwise 0
$b4_{k,i,p,j,t}$	Amount of returned product *p* picked up from distribution/collection center *i* and delivered to disposal center *j* by vehicle $k \in k3$ at period *t*
$u3_{i,k,t}$	The sub tours elimination variable in the third echelon

In terms of the above notations, the proposed MILP model is presented as follows:

Objective function:

Min $$Z = \sum_{t\in T}\sum_{k\in K1}\sum_{i\in A}\sum_{j\in B} y1_{i,j,k,t}c_{i,j} + \sum_{t\in T}\sum_{k\in K1}\sum_{i\in B}\sum_{j\in B, i\neq j} x1_{i,j,k,t}c_{i,j} + \sum_{t\in T}\sum_{k\in K1}\sum_{i\in B}\sum_{j\in A} z1_{i,j,k,t}c_{i,j} +$$
$$\sum_{t\in T}\sum_{k\in K2}\sum_{i\in B}\sum_{j\in C} y2_{i,j,k,t}c_{i,j} + \sum_{t\in T}\sum_{k\in K2}\sum_{i\in C}\sum_{j\in C, i\neq j} x2_{i,j,k,t}c_{i,j} + \sum_{t\in T}\sum_{k\in K2}\sum_{i\in C}\sum_{j\in B} z2_{i,j,k,t}c_{i,j} +$$
$$\sum_{t\in T}\sum_{k\in K3}\sum_{i\in D}\sum_{j\in B} y3_{i,j,k,t}c_{i,j} + \sum_{t\in T}\sum_{k\in K3}\sum_{i\in B}\sum_{j\in B, i\neq j} x3_{i,j,k,t}c_{i,j} + \sum_{t\in T}\sum_{k\in K3}\sum_{i\in B}\sum_{j\in D} z3_{i,j,k,t}c_{i,j} + \quad (1)$$

$$\sum_{t\in T}\sum_{p\in P}\sum_{i\in A} g1_{p,i,t}.c7_{p,i} + \sum_{t\in T}\sum_{p\in P}\sum_{i\in A} g2_{p,i,t}.c8_{p,i} + \sum_{t\in T}\sum_{i\in D} g3_{i,t}.c9_{i} + \sum_{t\in T}\sum_{p\in P}\sum_{i\in A} h1_{p,i,t}.hh1 +$$
$$\sum_{t\in T}\sum_{p\in P}\sum_{i\in B} h2_{p,i,t}.hh2 + \sum_{t\in T}\sum_{p\in P}\sum_{i\in C} v1_{p,i,t}.q1$$

Subject to:

$$\sum_{i\in A} y1_{i,j,k,t} + \sum_{i\in B, i\neq j} x1_{i,j,k,t} \le 1 \qquad \forall j\in B, t, k\in K1 \qquad (2)$$

$$\sum_{i\in A}\sum_{j\in B} y1_{i,j,k,t} = \sum_{i\in B}\sum_{j\in A} z1_{i,j,k,t} \qquad \forall k\in K1, t \qquad (3)$$

$$\sum_{i\in A}\sum_{j\in B} y1_{i,j,k,t} \le 1 \qquad \forall k\in K1, t \qquad (4)$$

$$\sum_{i\in A} y1_{i,j,k,t} + \sum_{i\in B, i\neq j} x1_{i,j,k,t} = \sum_{i\in B, i\neq j} x1_{j,i,k,t} + \sum_{i\in A} z1_{j,i,k,t} \qquad \forall j\in B, k\in k1, t \qquad (5)$$

$$\sum_{i\in B}\sum_{j\in B, i\neq j} x1_{i,j,k,t} \le \left(\sum_{i\in A}\sum_{j\in B} y1_{i,j,k,t}\right).M \qquad \forall k\in K1, t \qquad (6)$$

$$u1_{i,k,t} - u1_{j,k,t} + (|B|+1).x1_{i,j,k,t} \le |B| \qquad \forall i,j\in B (i\neq j), k\in k1, t \qquad (7)$$

$$\sum_{i\in A} n1_{k,i,t} = 1 \qquad \forall k\in K1, t \qquad (8)$$

$$\sum_{i\in A} o1_{k,i,t} = 1 \qquad \forall k\in K1, t \qquad (9)$$

$$\sum_{j\in B} y1_{i,j,k,t} \le n1_{k,i,t} \qquad \forall k\in K1, t, i\in A \qquad (10)$$

$$\sum_{j\in B} z1_{j,i,k,t} \le o1_{k,i,t} \qquad \forall k\in K1, t, i\in A \qquad (11)$$

$$o1_{k,i,t-1} = n1_{k,i,t} \qquad \forall k\in K1, t\ge 2, i\in A \qquad (12)$$

$$o1_{k,i,t} \ge n1_{k,i,t} + (1 - \sum_{j\in B} y1_{i,j,k,t}) - 1 \qquad \forall k\in K1, t, i\in A \qquad (13)$$

$$l1_{k,i,p,t} = \sum_{j\in B} a1_{k,j,p,i,t} \qquad \forall k\in K1, t, i\in A, p \qquad (14)$$

$$\sum_{p\in P} l1_{k,i,p,t} \le q1_k \qquad \forall k\in K1, t, i\in A \qquad (15)$$

$$l2_{k,j,p,t} \ge \left[l1_{k,i,p,t} - a1_{k,j,p,i,t} + \sum_{i\in A} b1_{k,j,p,i,t} - M.(1 - y1_{i,j,k,t}) \right] \qquad \forall k\in K1, t, p, i\in A, j\in B \qquad (16)$$

$$l2_{k,j,p,t} \le \left[l1_{k,i,p,t} - a1_{k,j,p,i,t} + \sum_{i\in A} b1_{k,j,p,i,t} + M.(1 - y1_{i,j,k,t}) \right] \qquad \forall k\in K1, t, p, i\in A, j\in B \qquad (17)$$

$$l2_{k,j,p,t} \ge \left[l2_{k,i,p,t} - \sum_{i\in A} a1_{k,j,p,i,t} + \sum_{i\in A} b1_{k,j,p,i,t} - M.(1 - x1_{i,j,k,t}) \right] \qquad \forall k\in K1, t, p, i\in B, j\in B, i\neq j \qquad (18)$$

$$l2_{k,j,p,t} \le \left[l2_{k,i,p,t} - \sum_{i\in A} a1_{k,j,p,i,t} + \sum_{i\in A} b1_{k,j,p,i,t} + M.(1 - x1_{i,j,k,t}) \right] \qquad \forall k\in K1, t, p, i\in B, j\in B, i\neq j \qquad (19)$$

$$\sum_{i\in A}\sum_{k\in K1} b1_{k,j,p,i,t} = \left(\sum_{k\in K2} a4_{k,j,p,t-1}\right).(1-sp) \qquad \forall j\in B, t, p \qquad (20)$$

$$\sum_{p\in P} l2_{k,j,p,t} \le q1_k \qquad \forall k\in K1, t, j\in B \qquad (21)$$

$$\sum_{p\in P} a1_{k,j,p,i,t} \le \left[y1_{i,j,k,t} + \sum_{i\in B, i\neq j} x1_{i,j,k,t} \right].M \qquad \forall k\in K1, t, i\in A, j\in B \qquad (22)$$

$$\sum_{p\in P}\sum_{j\in B} a1_{k,j,p,i,t} \le \left[\sum_{j\in B} y1_{i,j,k,t}\right].M \qquad \forall k\in K1, t, i\in A \qquad (23)$$

$$\sum_{i\in A}\sum_{p\in P} b1_{k,j,p,i,t} \le \left[\sum_{i\in A} y1_{i,j,k,t} + \sum_{i\in B, i\neq j} x1_{i,j,k,t}\right].M \qquad \forall k\in K1, t, j\in B \qquad (24)$$

$$\sum_{j\in B}\sum_{p\in P} b1_{k,j,p,i,t} \le \left[\sum_{j\in B} z1_{j,i,k,t}\right].M \qquad \forall k\in K1, t, i\in A \qquad (25)$$

$$\sum_{i\in B} y2_{i,j,k,t} + \sum_{i\in C, i\neq j} x2_{i,j,k,t} \le 1 \qquad \forall j\in C, t, k\in K2 \qquad (26)$$

$$\sum_{i\in B}\sum_{j\in C} y2_{i,j,k,t} = \sum_{i\in C}\sum_{j\in B} Z2_{i,j,k,t} \qquad \forall k\in K2, t \qquad (27)$$

$$\sum_{i\in B}\sum_{j\in C} y2_{i,j,k,t} \le 1 \qquad \forall k\in K2, t \qquad (28)$$

$$\sum_{j\in B} y2_{j,i,k,t} + \sum_{j\in C, i\neq j} x2_{j,i,k,t} = \sum_{j\in C, i\neq j} x2_{i,j,k,t} + \sum_{j\in B} z2_{i,j,k,t} \qquad \forall i\in C, k\in k2, t \qquad (29)$$

$$\sum_{i\in C}\sum_{j\in C, i\neq j} x2_{i,j,k,t} \le \left(\sum_{i\in B}\sum_{j\in C} y2_{i,j,k,t}\right).M \qquad \forall k\in K2, t \qquad (30)$$

$$u2_{i,k,t} - u2_{j,k,t} + (|C|+1).x2_{i,j,k,t} \le |C| \qquad \forall i, j\in C\,(i\neq j), k\in k2, t \qquad (31)$$

$$\sum_{i\in B} n2_{k,i,t} = 1 \qquad \forall k\in K2, t \qquad (32)$$

$$\sum_{i\in B} o2_{k,i,t} = 1 \qquad \forall k\in K2, t \qquad (33)$$

$$\sum_{j\in C} y2_{i,j,k,t} \le n2_{k,i,t} \qquad \forall i\in B, k\in k2, t \qquad (34)$$

$$\sum_{j\in C} z2_{j,i,k,t} \le o2_{k,i,t} \qquad \forall i\in B, k\in k2, t \qquad (35)$$

$$o2_{k,i,t-1} = n2_{k,i,t} \qquad \forall k\in K2, t\ge 2, i\in B \qquad (36)$$

$$o2_{k,i,t} \ge n2_{k,i,t} + (1 - \sum_{j\in C} y2_{i,j,k,t}) - 1 \qquad \forall k\in K2, t, i\in B \qquad (37)$$

$$l3_{k,i,p,t} = \sum_{j\in C} a3_{k,j,p,i,t} \qquad \forall k\in K2, t, p, i\in B \qquad (38)$$

$$\sum_{p} l3_{k,i,p,t} \le q2_{k} \qquad \forall k\in K2, t, i\in B \qquad (39)$$

$$l4_{k,j,p,t} \ge \left[l3_{k,i,p,t} - a3_{k,j,p,i,t} + \sum_{i\in B} b3_{k,j,p,i,t} - M.(1 - y2_{i,j,k,t})\right] \qquad \forall k\in K2, t, p, i\in B, j\in C \qquad (40)$$

$$l4_{k,j,p,t} \le \left[l3_{k,i,p,t} - a3_{k,j,p,i,t} + \sum_{i\in B} b3_{k,j,p,i,t} + M.(1 - y2_{i,j,k,t})\right] \qquad \forall k\in K2, t, p, i\in B, j\in C \qquad (41)$$

$$l4_{k,j,p,t} \ge \left[l4_{k,i,p,t} - \sum_{i\in B} a3_{k,j,p,i,t} + \sum_{i\in B} b3_{k,j,p,i,t} - M.(1 - x2_{i,j,k,t})\right] \qquad \forall k\in K2, t, p, i\in C, j\in C, i\neq j \qquad (42)$$

$$l4_{k,j,p,t} \le \left[l4_{k,i,p,t} - \sum_{i\in B} a3_{k,j,p,i,t} + \sum_{i\in B} b3_{k,j,p,i,t} + M.(1 - x2_{i,j,k,t})\right] \qquad \forall k\in K2, t, p, i\in C, j\in C, i\neq j \qquad (43)$$

$$\sum_{p} l4_{k,j,p,t} \le q2_{k} \qquad \forall k\in K2, t, j\in C \qquad (44)$$

$$\sum_{i\in B}\sum_{k\in K2} b3_{k,j,p,i,t} = p_{j,p,t} \qquad \forall t, p, j\in C \qquad (45)$$

$$\sum_{p} a3_{k,j,p,i,t} \leq \left[y2_{i,j,k,t} + \sum_{i\in C, i\neq j} x2_{i,j,k,t} \right].M \qquad \forall k \in K2, t, j\in C, i\in B \qquad (46)$$

$$\sum_{p}\sum_{j\in C} a3_{k,j,p,i,t} \leq \left[\sum_{j\in C} y2_{i,j,k,t} \right].M \qquad \forall k \in K2, t, i\in B \qquad (47)$$

$$\sum_{p}\sum_{i\in B} b3_{k,j,p,i,t} \leq \left[\sum_{i\in B} y2_{i,j,k,t} + \sum_{i\in C, i\neq j} x2_{i,j,k,t} \right].M \qquad \forall k \in K2, t, j\in C \qquad (48)$$

$$\sum_{p}\sum_{j\in C} b3_{k,j,p,i,t} \leq \left[\sum_{j\in C} z2_{j,i,k,t} \right].M \qquad \forall k \in K2, t, i\in B \qquad (49)$$

$$\sum_{k\in K3}\sum_{i\in D} y3_{i,j,k,t} + \sum_{k\in K3}\sum_{i\in B, i\neq j} x3_{i,j,k,t} \leq 1 \qquad \forall j\in B, t \qquad (50)$$

$$\sum_{k\in K3}\sum_{j\in D} z3_{i,j,k,t} + \sum_{k\in K3}\sum_{j\in B, i\neq j} x3_{i,j,k,t} \leq 1 \qquad \forall i\in B, t \qquad (51)$$

$$\sum_{i\in D}\sum_{j\in B} y3_{i,j,k,t} = \sum_{i\in B}\sum_{j\in D} Z3_{i,j,k,t} \qquad \forall k\in K3, t \qquad (52)$$

$$\sum_{i\in D}\sum_{j\in B} y3_{i,j,k,t} \leq 1 \qquad \forall k\in K3, t \qquad (53)$$

$$\sum_{j\in D} y3_{j,i,k,t} + \sum_{j\in B, i\neq j} x3_{j,i,k,t} = \sum_{j\in B, i\neq j} x3_{i,j,k,t} + \sum_{j\in D} z3_{i,j,k,t} \qquad \forall i\in B, k\in k3, t \qquad (54)$$

$$\sum_{i\in B}\sum_{j\in B, i\neq j} x3_{i,j,k,t} \leq \left(\sum_{i\in D}\sum_{j\in B} y3_{i,j,k,t} \right).M \qquad \forall k\in K3, t \qquad (55)$$

$$u3_{i,t} - u3_{j,t} + (|B|+1).\sum_{k\in K3} x3_{i,j,k,t} \leq |B| \qquad \forall i,j\in B (i\neq j), k\in k3, t \qquad (56)$$

$$\sum_{i\in D} n3_{k,i,t} = 1 \qquad \forall k\in K3, t \qquad (57)$$

$$\sum_{i\in D} o3_{k,i,t} = 1 \qquad \forall k\in K3, t \qquad (58)$$

$$\sum_{j\in B} y3_{i,j,k,t} \leq n3_{k,i,t} \qquad \forall i\in D, k\in k3, t \qquad (59)$$

$$\sum_{j\in B} z3_{j,i,k,t} \leq o3_{k,i,t} \qquad \forall i\in D, k\in k3, t \qquad (60)$$

$$o3_{k,i,t-1} = n3_{k,i,t} \qquad \forall k\in K3, t\geq 2, i\in D \qquad (61)$$

$$o3_{k,i,t} \geq n3_{k,i,t} + (1 - \sum_{j\in B} y3_{i,j,k,t}) - 1 \qquad \forall k\in K3, t, i\in D \qquad (62)$$

$$l5_{k,j,p,t} \geq \left[\left(\sum_{i\in D} b4_{k,j,p,i,t} \right) - M.(1 - \sum_{i\in D} y3_{i,j,k,t}) \right] \qquad \forall k\in K3, t, p, j\in B \qquad (63)$$

$$l5_{k,j,p,t} \leq \left[\left(\sum_{i\in D} b4_{k,j,p,i,t} \right) + M.(1 - \sum_{i\in D} y3_{i,j,k,t}) \right] \qquad \forall k\in K3, t, p, j\in B \qquad (64)$$

$$l5_{k,j,p,t} \geq \left[l5_{k,i,p,t} + \left(\sum_{i\in D} b4_{k,j,p,i,t} \right) - M.(1 - x3_{i,j,k,t}) \right] \qquad \forall k\in K3, t, p, i\in B, j\in B, i\neq j \qquad (65)$$

$$l5_{k,j,p,t} \leq \left[l5_{k,i,p,t} + \left(\sum_{i\in D} b4_{k,j,p,i,t} \right) + M.(1 - x3_{i,j,k,t}) \right] \qquad \forall k\in K3, t, p, i\in B, j\in B, i\neq j \qquad (66)$$

$$\sum_{p} l5_{k,j,p,t} \leq q3_{k} \qquad \forall k\in K3, t, j\in B \qquad (67)$$

$$\sum_{k\in K3}\sum_{i\in D} b4_{k,j,p,i,t} = \left[\sum_{k\in K2} a4_{k,j,p,t-1}\right].sp \qquad \forall j\in B,t,p \qquad (68)$$

$$\sum_{p}\sum_{i\in D} b4_{k,j,p,i,t} \le \left[\sum_{i\in D} y3_{i,j,k,t} + \sum_{i\in B, i\ne j} x3_{i,j,k,t}\right].M \qquad \forall t,k\in K3, j\in B \qquad (69)$$

$$\sum_{p}\sum_{j\in B} b4_{k,j,p,i,t} \le \left[\sum_{j\in B} z3_{j,i,k,t}\right].M \qquad \forall i\in D,t,k\in K3 \qquad (70)$$

$$\sum_{i\in B}\sum_{k\in K2} a3_{k,j,p,i,t} + v1_{p,j,t} \ge d_{j,p,t} \qquad \forall p,t,j\in C \qquad (71)$$

$$\sum_{p\in P}\left(g1_{p,i,t}.t1_{p,i}\right) \le u2_{i,t} \qquad \forall i\in A,t \qquad (72)$$

$$\sum_{p\in P}\left(g2_{p,i,t}.t2_{p,i}\right) \le u4_{i,t} \qquad \forall i\in A,t \qquad (73)$$

$$g3_{i,t}.t3_i \le u6_{i,t} \qquad \forall i\in D,t \qquad (74)$$

$$g2_{p,i,t} = \sum_{k\in K1} a2_{k,i,p,t-1} \qquad \forall i\in A,t,p\in P \qquad (75)$$

$$g3_{i,t} = \sum_{p}\sum_{k\in K3} a5_{k,i,p,t-1} \qquad \forall i\in D,t \qquad (76)$$

$$a2_{k,i,p,t} = \sum_{j\in B} b1_{k,j,p,i,t} \qquad \forall i\in A\ ,t,p,k\in K1 \qquad (77)$$

$$a4_{k,i,p,t} = \sum_{j\in C} b3_{k,j,p,i,t} \qquad \forall i\in B,t,p,k\in K2 \qquad (78)$$

$$a5_{k,i,p,t} = \sum_{j\in B} b4_{k,j,p,i,t} \qquad \forall i\in D,t,p,k\in K3 \qquad (79)$$

$$\sum_{p}\sum_{k\in K2} a4_{k,i,p,t} \le u5_i \qquad \forall i\in B,t \qquad (80)$$

$$s1_{p,i,t} = h1_{p,i,t} + g1_{p,i,t} + g2_{p,i,t} \qquad \forall i\in A,t,p \qquad (81)$$

$$h1_{p,i,t} = s1_{p,i,t-1} - \sum_{k\in K1} l1_{k,i,p,t} \qquad \forall i\in A\ ,t,p \qquad (82)$$

$$\sum_{p\in P} s1_{p,i,t} \le u1_i \qquad \forall i\in A,t \qquad (83)$$

$$s2_{p,j,t} = h2_{p,j,t} + \sum_{i\in A}\sum_{k\in K1} a1_{k,j,p,i,t} \qquad \forall j\in B,t,p \qquad (84)$$

$$h2_{p,j,t} = s2_{p,j,t-1} - \sum_{k\in K2} l3_{k,j,p,t} \qquad \forall j\in B,t,p \qquad (85)$$

$$\sum_{p} s2_{p,j,t} \le u3_j \qquad \forall j\in B,t \qquad (86)$$

$$\sum_{i\in A}\sum_{j\in B} y1_{i,j,k,t}.t_{i,j} + \sum_{i\in B}\sum_{j\in B, i\ne j} x1_{i,j,k,t}.t_{i,j} + \sum_{i\in B}\sum_{j\in A} Z1_{i,j,k,t}.t_{i,j} \le r_t \qquad \forall k\in K1,t \qquad (87)$$

$$\sum_{i\in B}\sum_{j\in C} y2_{i,j,k,t}.t_{i,j} + \sum_{i\in C}\sum_{j\in C, i\ne j} x2_{i,j,k,t}.t_{i,j} + \sum_{i\in C}\sum_{j\in B} Z2_{i,j,k,t}.t_{i,j} \le r_t \qquad \forall k\in K2,t \qquad (88)$$

$$\sum_{i\in D}\sum_{j\in B} y3_{i,j,k,t}.t_{i,j} + \sum_{i\in B}\sum_{j\in B, i\ne j} x3_{i,j,k,t}.t_{i,j} + \sum_{i\in B}\sum_{j\in D} Z3_{i,j,k,t}.t_{i,j} \le r_t \qquad \forall k\in K3,t \qquad (89)$$

binary variable:x1,y1,z1,x2,y2,z2,x3 ,y3 ,z3,n1,o1,n2,o2,n3,o3 ; (90)

positive variable:g1 ,g2 ,g3 ,s1 ,s2 ,h1 ,h2 ,v1,l1,a1,a4, l3 ,a3 ,l4 ,b3 ,l2 ,l5 ,b1 ,b4 ,a2 ,a5 ; (91)

$$1 \le u1_{i,k,t} \le |B| + 1 \qquad \forall i\in B,k\in k1,t \qquad (92)$$

$$1 \le u2_{i,k,t} \le |C| + 1 \qquad \forall i\in C,k\in k2,t \qquad (93)$$

$$1 \le u3_{i,t} \le |B| + 1 \qquad \forall i\in B,k\in k3,t \qquad (94)$$

$$s1_{p,i,0}=C \qquad \forall i \in A\ , p \qquad (95)$$

$$s2_{p,j,0}=C \qquad \forall j \in B, p \qquad (96)$$

The objective function (1) minimizes total costs throughout the planning horizon, which includes total travel costs in all stages, production and recovery costs in production/ recovery centers and disposal costs in disposal centers, total holding costs in production/ recovery centers and distribution/ collection centers, and total shortage costs. Constraints (2)-(25) are related to the first echelon. Constraints (2) consider that each vehicle visits each distribution/ collection center at most once. According to constraint (3), each vehicle route starts and ends at a production / recovery center. The start and end node need not be the same. Constraint (4) ensures that in each period, each vehicle starts their route at most from one production/ recovery center. Constraint (5) defines a flow by which the number of arrivals is equal to the number of departures for all distribution/ collection centers. Constraint (6) ensures that each vehicle can travel among distribution/ collection centers if it leaves a production/recovery center. Constraint (7) is sub-tours elimination constraint. Constraints (8) to (13) are constraints of flexible assignment. According to the Constraints (8) to (11), each vehicle ($k \in k1$) route starts and ends at a production/recovery center and the start and end node need not be the same. Constraint (12) ensures that origin of each vehicle at the beginning of period *t* is same as destination of that vehicle at the end of period *t-1*. Constraint (13) ensures that vehicle $k \in k1$ stays in its place without moving if it is idle in period *t*. Constraint (14) defines the load of vehicle $k \in k1$ when leaving the production/ recovery center *i* at the beginning of each period. Constraint (15) is the capacity constraint for vehicle $k \in k1$. Constraints (16) to (19) define the load of product *p* of vehicle $k \in k1$ after having serviced distribution/collection center *j* in period *t*. Constraint (20) imposes that, in period *t* and for each product, the flow exiting from each distribution/ collection center to all production/recovery centers be equal to the flow entering to each distribution/collection center in period *t-1* from all customers by various vehicles multiplied by the (1-*sp*). Constraint (21) is the capacity constraint for vehicle $k \in k1$. Constraints (22) and (23) cite the logical constraints related to variable $a1_{k,i,p,j,t}$; Constraints (24) and (25) cite the logical constraints related to variable $b1_{k,i,p,j,t}$.

Constraints (26) to (49) are corresponding with the constraints (2) to (25) but for the second echelon.

Constraints (50) to (70) are related to the third echelon. Constraints (50) to (56) guarantee that the established tours in third echelon are feasible according to the problem assumptions. Constraints (57) to (62) are flexible assignment constraints and have similar function with constraints (8) to (13) but in third echelon. Constraints (63) to (66) define the load of product *p* of vehicle $k \in k3$ after having serviced distribution/collection center *j* in period *t*, and Constraint (67) is the capacity constraint for vehicle $k \in k3$. Constraint (68) imposes that, in period *t* and for each product, the flow exiting from each distribution/ collection center to all disposal centers is equal to the flow entering to each distribution/collection center in period *t-1* from all customers by various vehicles multiplied by the *sp*. Constraints (69) and (70) cite the logical constraints related to variable $b4_{k,i,p,j,t}$.

Constraint (71) defines that a portion of customer demand is satisfied and the rest will be lost. Constraints (72)-(73) are production and recovery capacity constraints of production/recovery centers, and constraint (74) is disposal capacity constraint of disposal centers. According to constraint (75), The amount of product *p* recovered in production/ recovery center *i* at period *t* is equal to quantity of recoverable product *p* shipped from various distribution/collection centers by various vehicles ($k \in k1$) in period *t-1,* and according to constraint (76), all of unrecoverable products shipped from various distribution/collection centers to each disposal center in each period are disposed in the next period. Constraints (77) to (79) are logical constraints. Constraint (80) is the collection capacity constraint of each distribution/ collection center. Constraints (81) and (82) define the variables $s1_{p,i,t}$ and $h1_{p,i,t}$; constraint (83) is the holding capacity constraint of each production/ recovery center. Constraints (84) and (85) define the variables $s2_{p,i,t}$ and $h2_{p,i,t}$, and constraint (86) is the holding capacity constraint of each distribution/ collection center. Constraints (87) to (89) ensure that total travel time of all vehicles in all echelons and in each period is less than duration of that period. Constraints (90) and (91) enforce the binary and non-negativity restrictions on corresponding decision variables; constraint (92) to (94) are the upper and lower bounds for sub-tours elimination variable. Finally, constraints (95) and (96) define inventory of various products in the production/ recovery centers and distribution/ collection centers at the beginning of planning horizon.

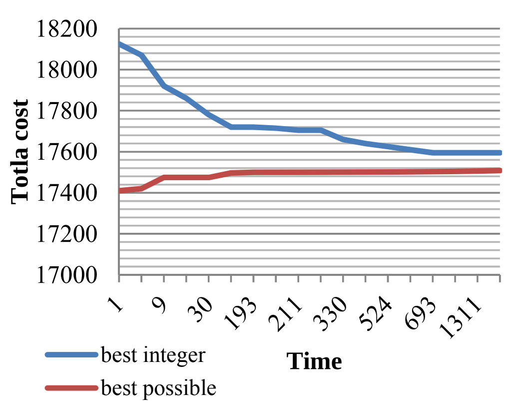
4. Computational Experiments

To evaluate the performance of the model, some random examples with different dimensions are solved by Branch and Cuts algorithm in GAMS 23.6. We have used a system Intel core i7 1.73 GHz, 4GB RAM. The results are presented in table 1.

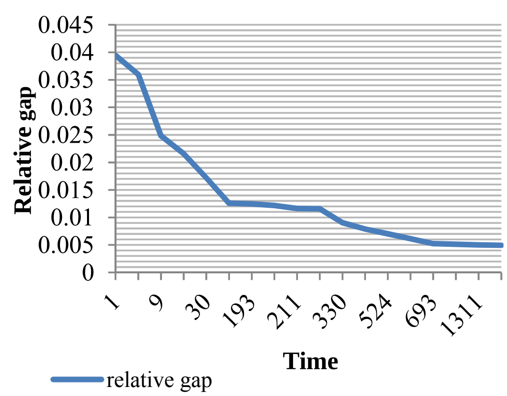
Tab. 1. Numerical Examples

No.	Problem dimension A,B,C,D,P,K1,K2,K3,T	optimal solution time (seconds)	optimal objective function values (seconds)	near to optimal solution time (seconds)	best integer (objective function)	Best node (objective function)	Absolute gap	Relative gap
1	1,2,3,1,1,2,4,2,3	25	1130	-	-	-	-	-
2	2,2,3,1,2,4,4,2,5	650	6666	-	-	-	-	-
3	2,3,5,1,2,4,4,3,5	-	-	4000	12260	11982.8832	277.1168	0.0226
4	3,3,5,1,3,4,5,3,7	-	-	5000	17595	17570.2	24.8	0.00141

Solving different problems with different dimensions represents the model ability to achieve optimal solution in proper time for small problems such as examples 1 and 2. It also represents its ability to achieve near-optimal solution in reasonable time for larger problems such as examples 3 and 4. Due to limited memory of computer and the large computation time, optimal solution cannot be achieved for large-sized problems. Hence, we investigated these problems regarding the gap between the best integer solution and the best possible solution (lower bound) and their convergence by Branch and Cut algorithm. Solving the problem by Branch and Cut algorithm in GAMS, the best integer solution and the best possible solution (i.e., relaxed LP) are, respectively, returned as output in columns best integer and best node (best possible). In addition, we should consider the convergence of solution to the optimal solution. As an example, in Problem 4, this is studied by diagrams of Fig. 3 in 2000 seconds.In Fig. 3a, the convergence between the best integer solution and the best feasible solution is studied. As we can see, any significant improvement cannot be made in the solutions after 600 seconds. Also, in Fig. 3b, the relative gap between the solutions is less than 0.005 after 600 seconds. This means that the integer solution reached in that time is the optimal, or the one so near to it.



(a)



(b)

Fig. 3. The convergence (a) and the relative gap (b) between the best integer solution and the best possible solution.

5. Conclusion and Future Researches

This paper contributes to vehicle routing and production planning in a closed-loop supply chain. In this paper, a comprehensive MILP model was presented to combine different types of VRP assumptions and applying them to a multi echelon closed loop supply chain. Indeed, the model contributes to the literature by considering more real-world complexities. The logistic network, which is considered in this paper, is a multi-echelon closed-loop supply chain network with facilities and limited capacity which is comprised: common facilities of production/recovery and distribution/collection, disposal centers, and customers. The proposed model is a multi-period and multi-product one with the ability to regard several facilities at each level of the chain. Transportation in all echelons of the network is indirect (tours). In this model, various types of routing assumptions are included such as multi-depot, multi-products, periodic routing, flexible assignment, routing with simultaneous delivery and pick-up, and split servicing. Obtained numerical results by Branch and Cuts algorithm shows that the proposed model capable to reach the optimal solution in reasonable time for small- and medium-sized problems. The current paper can be extended in several ways; first, combination the model by stochastic assumptions such as demand uncertainty. Second, development of the model to a time-dependent model by considering traffic congestions. Third, developing a meta-heuristic method to solve the large-sized problems. Forth, extending the model to a multi-objective one by considering different and conflict objectives such as tardiness minimization and balancing the shipped load by various vehicles.

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