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# A Multi Objective Optimization Model for Redundancy Allocation Problems in Series-Parallel Systems with Repairable Components

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## KEYWORDS

Redundancy allocation problem;
Series-parallel system;
Repairable components;
Multi-objective optimization;
Imperialist competitive algorithm

## ABSTRACT

*The main goal in this paper is to propose an optimization model for determining the structure of a series-parallel system. Regarding the previous studies in series-parallel systems, the main contribution of this study is to expand the redundancy allocation parallel problems to systems that have repairable components. Therefore, the considered systems in this paper are the systems that have repairable components in their configurations and subsystems. The suggested optimization model has two objectives: maximizing the system mean time to first failure and minimizing the total cost of the system. The main constraints of the model are: maximum number of the components in the system, maximum and minimum number of components in each subsystem and total weight of the system. After establishing the optimization model, a multi-objective approach of Imperialist Competitive Algorithm is proposed to solve the model.*



## 1. Introduction

Development of a reliable system involves various complex and interrelated factors. The translation of overall system reliability into requirements at various subsystem levels, is an important task for those responsible for the design and development of reliable systems.[1] Enhancing the system reliability or other system performance indicators is one of the most attractive areas for system developers and using redundant components is one of the most frequent approaches in system design process. The series-parallel configuration is a kind of system design that resulted by using redundant components for each subsystem in a simple series system.[2] Fig. **1** shows a simple series system and a series-parallel system design.

Selecting the series-parallel configuration for using redundant component in order to enhance the system reliability, result in a challenge for the designers: decision making about the subsystem components by considering the available resources. This challenge was the main reason to start an interesting stream in the reliability optimization literature, called "redundancy allocation problems in series-parallel systems". Since the first paper on redundancy allocation problem in series-parallel system by Fyffe et.al [1] many researchers have tried to develop this knowledge. Two main approaches in the development of redundancy allocation problem literature could be seen: 1- Proposing new method to solve the previous optimization models on redundancy allocation

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problems and 2- developing the new optimization models for redundancy allocation problems. Coelho [3], Ramirez-Marquez et.al [4], Coit and Liu [5], Ouzineb et.al [6], Ramirez-Marquez and Coit [7], Yun and Kim [8], Nahas et al. [9], Kulturel-Konak et.al [10], Liang and Chen [11], Coit [12], Ha and Kuo [13], Yalaoui et.al [14], You and Chen [15], Liang et.al [16], Juang et.al [17], Coit and Konak [18], Kumar et.al [19], Yeh [2], Wang and Watada [20], Kumar et.al [21], Liu [22] and Afonso et al [23], are some of the studies on redundancy allocation problem in series-parallel system. Conducting a survey of redundancy allocation problems literature reveals that the systems with repairable components have not been involved in previous works. The lack of studies for redundancy allocation problem in series-parallel system with repairable components and subsystems has been concluded in Kuo and Wan [24] after a complete survey on redundancy allocation problem literature. In this study the main goal is to propose an optimization model for the series-parallel systems with repairable components.

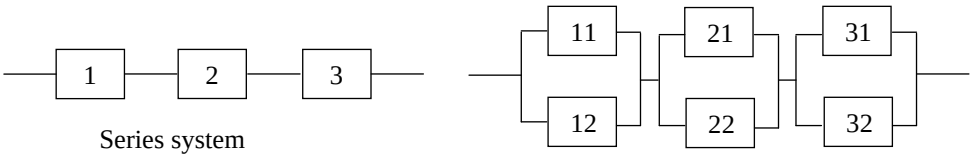


**Fig. 1. The difference between series systems and series-parallel systems**

The main goal in this paper is to optimize the redundancy allocation in a repairable series-parallel system; the main contribution of the paper is to consider the reparability of the system components in the optimization process. To attain the main goal of this study, firstly the optimization model should be established and after that an appropriate algorithm or method for solving the optimization model should be designed and run.

Structure of the paper is organized as follow: the main indicator and measurements for the performance of a repairable system are defined in part 2. In this study one of the system performance indicators (mean time to first failure) is applied to establish the first objective function of the optimization model for redundancy allocation problem, therefore the analytical and mathematical methods for calculating these indicators are introduce in part 3. The assumption, variables and mathematical formulation of the optimization model are explained in part 4. In part 5 the solving approach for the designed optimization model is suggested. A numerical example, which has been solved by the concepts clarified through part 2 to 5, is presented in part 6. Part 7 include the conclusion and some recommendation for future works.

## 2. The Main Indicators of System Performance in Repairable Systems

Differences between the system performance indicators for repairable systems and non-repairable system could be seen in literature.

For repairable systems, *Availability* is the term which replaces *reliability*. Reliability of a non-repairable system at time *t*, is defined as the probability that the system could performed its required function under a given condition for a stated time interval [0,t] [25]. A failure in a non-repairable system before time *t* makes the system unreliable at that time. But for the repairable systems the *availability* is defined as the probability that the system could performed its required function under a given condition at time *t* regardless of its previous fails and repairs during [0,t]. For a non-repairable system *Mean Time to Failure (MTTF)* is defined as the average of system lifetime but in repairable systems *Mean Time to First Failure (MTTFF)*, the term that replaces MTTF, is equal to average time before the system fails for the first time; the repairable system could be repaired after the fail and restarts its function.

The calculation methods for system availability and MTTFF are different from the reliability and MTTF. The analytical and mathematical methods suggested by researchers to calculate the main performance indicators in repairable systems are introduced in the next part.

## 3. Availability

Calculating the availability of a repairable system is one of the interesting areas in system performance indicator literature. In this paper, the considered repair and failure rate for the components of series-parallel system are constant; in other words, failure and repair of the components have exponential distribution. Therefore, a brief review of calculating the availability and MTTFF of these systems is presented in this part. The basic concept of Markove process is used to determine the availability of system. For more details about the availability calculation in repairable systems Birolini [25] and Billinton and Allan [26] could be studied. Suppose that there is a system with an only one repairable component, its failure rate is $\lambda$ and its repair rate is $\mu$; the state space diagram of the system could be established as.

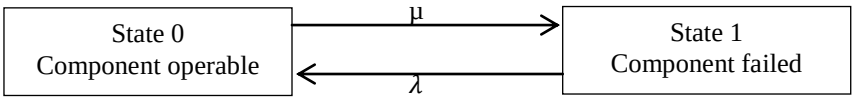


**Fig. 2. The states of a repairable system with one component**

When the system state is 0, it is capable of operation and it is able to perform its function; while in state 1 it is not able to do its function and should be repaired. Hence the availability of the system at time *t* is the probability of residing in the operable state (state 0) at time *t*, and the unavailability is the probability of residing in the failed state (state 1). Billinton and Allan [26] have driven for these probabilities:

$$[P'_0(t) \quad P'_1(t)] = [P_0(t) \quad P_1(t)] \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix} \quad (1)$$

$$P_0(t) = \frac{\mu}{\mu+\lambda}[P_0(0) + P_1(0)] + \frac{e^{-(\lambda+\mu)t}}{\lambda+\mu}[\lambda P_0(0) + \mu P_1(0)] \quad \text{(a)}$$

$$P_1(t) = \frac{\lambda}{\mu+\lambda}[P_0(0) + P_1(0)] + \frac{e^{-(\lambda+\mu)t}}{\lambda+\mu}[\mu P_1(0) - \lambda P_0(0)] \quad \text{(b)} \qquad (2)$$

$P_0(0)$ and $P_1(0)$ are the probability of residing system in each state in t=0 and $P_0(0) + P_1(0)$ is equal to 1. If the system starts its function correctly at t=0, then $P_0(0)=1$ and $P_0(1)= 0$ and could be obtained from:

$$P_0(t) = \frac{\mu}{\mu+\lambda} + \frac{\lambda e^{-(\lambda+\mu)t}}{\lambda+\mu} \quad \text{(a)}$$

$$P_1(t) = \frac{\lambda}{\mu+\lambda} - \frac{\lambda e^{-(\lambda+\mu)t}}{\lambda+\mu} \quad \text{(b)} \qquad (3)$$

Therefore, to calculate the performance indicators of a repairable system, firstly all the possible states of the system should be distinguished and the coefficient matrix in, called transaction matrix, should be established; then the differential equations should be solved. The transaction matrix is a square matrix and its dimensions are equal to number of states. After calculating the $P_i(t)$s, the availability of the system will be equal to the summation of $P_i(t)$s of the states in which the system performs its function correctly. Main challenge in calculating differential equations for a system is the calculation volume required to solve the equations.

Calculating the $P_i(t)$s of a repairable system is an attractive area in repairable systems literature for researchers: Bailey [27], Bhat [28], Grassmann [29], Sumita and Shanthikumar [30], Ross [31], Shaked and Shanthikumar [32], Yoon and Shanthikumar [33], Hoyland and Rausand [34], Moustaf [35], Moustafa [36], Zhang et.al [37], Azaron et.al [38], Li et.al [39], and Amiri and Ghassemi-Tari [40], are some of the studies which have proposes mathematical methods for calculating the $P_i(t)$s in repairable systems. In this paper the recommended mathematical method proposed by Amiri and Ghassemi-Tari [40] is used to determine the availability and MTTFF of the system, therefore this method is clarified in more detail.

$P_0(t)$ and $P_1(t)$ are the probability of residing in state 0 and state 1 respectively; $P'_0(t)$ and $P'_1(t)$ are the derivative of $P_0(t)$ and $P_1(t)$ with respect to *t*. is comprised of a couple of differential equations. There are various methods, by which, these differential equations could be solved. Using Laplace transforms by Birolini [25] and Billinton and Allan [26] could be concluded from:

In the mathematical method suggested by Amiri and Ghassemi-Tari [40] Establishing the transaction matrix of the system is the prerequisite of the performance indexes calculation for a repairable system. To establish the transaction matrix, firstly all the possible states of the system should be determined (like).Then all the possible transactions between states should be detected. Note that the possible transactions are those with only one repair or fail between states transitions during their occurrences.

The dimensions of the transaction matrix are equal to the number of states and each row or column is related to one of the states. The elements of the matrix are determined regarding the repair or failure rate of the transition between corresponding row and column. The summation of the elements in each row should be zero and regarding this consideration, the elements on the diagonal of the matrix could be calculated. Suppose that the transaction matrix is shown by *Q*, a n×n square matrix, *Q* has n non-repeating eigenvalues; *V* is a matrix of eigenvectors of *Q* and $V^{-1}$ is the inverse of *V* and *d* is a diagonal matrix established by using the eigenvalues of *Q* as follows:

$$d = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix} \qquad (4)$$

Now *p(t)*, the state transient probability in the exponential Markov chain with the continuous time could be calculated by.

$$P(t) = V.e^{d.t}.V^{-1} \quad (5)$$

The probability of residing in state *i* (*i=1,2,…n)*, which shown by vector $P_n(t)$, could be derived by:

$$P_n(t) = P_n(0).P(t) \quad (6)$$

Where $P_n(0)$ is the vector of probability of residing the system in state *i* (*i*=1,2,…n) at *t=0.* Using the elements in $P_n(t)$, the availability of the system is the sum of the $P_i(t)$s (*i*=1,2,…n) of the states in which, system does its work correctly. If *B* is the set of functioning states then:

$$A_s(t) = \sum_{j \in B} P_j(t) \quad (7)$$

The availability of the system is defined as the probability of being in the operational mode (to be up) at time *t*, regardless of its historical failures and/or repairs. Another measure applicable for repairable system is *survivability function*. Survivability at time *t* determines the probability that a system does not leave the functioning states (does not fail) during the time interval (0,t] [40]. To calculate the system survivability, the rows and columns related to down states (non-operational states) in the transaction matrix should be deleted; the resulted matrix is called *Q´*. The mathematical operations of to should be performed on matrix *Q´*. The survivability of the system at time *t*, which is shown by $R_s(t)$, is equal to the summation of the $P_j(t)$s resulted by running to on *Q´*. The mean time to first failure (MTTFF) of the system is the expected time to first failure of the system, which could be calculated by:

$$MTTFF = \int_0^\infty R_s(t) \quad (8)$$

Some examples have been solved by Amiri and Ghassemi-Tari [40] to depict the details of their suggested mathematical method.

## 4. System Assumptions and the Optimization Model

### 4-1. System Assumptions

As mentioned, the main idea in this paper is to expand the redundancy allocation problem into the systems which have repairable subsystems and components. The main assumptions of the considered system in this study are:

- Series-parallel system depicted in Fig. **1** has *m* subsystems.
- There are $O_i$ alternative components could be selected for *i*th subsystem; the decision maker should select one of the alternative components for each subsystem and determine its redundancy level.
- The failure and repair of each alternative component available for each subsystem has exponential distribution with fail rate $\lambda_{ij}$ and repair rate $\mu_{ij}$.
- The system conducts its function perfectly when each subsystem has at least one operable component. Therefore for each subsystem at least one component should be selected.
- The first goal in this study is to maximize the average time before the first failure of the system; the first failure of the system occurs when one of the subsystems and its redundancies fails completely. As Ramirez-Marquez et al. [4] implied the average time before the first failure of the system is equal to minimum of the subsystems MTTFFs. Hence the first goal in optimization process is to maximize the minimum of the subsystems MTTFFs.
- Each alternative component for subsystems has a specific cost. The second goal of the optimization is to minimize the system's cost.
- Total weight of the system, the total number of system components and the components number of each subsystem has some limitations.

### 4-2. Model Variables and Parameters

Considering the above assumptions the parameters and variables of the optimization model are:

- $x_{ij}$: the number of selected component for *i*th subsystem from *j*th alternative (i= 1,2,…m),(j= 1,2,…$O_i$)
- $\lambda_{ij}$: the failure rate of *j*th component for *i*th subsystem (i= 1,2,…m),(j= 1,2,…$O_i$)
- $\mu_{ij}$: the repair rate of *j*th component for *i*th subsystem (i= 1,2,…m),(j= 1,2,…$O_i$)
- m: the number of subsystems
- i: the index of subsystems
- j: the index of existing component types for each subsystem
- $O_i$: the number of existing component types for each subsystem
- $C_{ij}$: the cost of *j*th component type for *i*th subsystem
- $w_{ij}$: the weight of *j*th component type for *i*th subsystem
- $W_s$: the total weight of system
- $C_s$: the total cost of system
- $nmax_i$: the maximum number of components for *i*th subsystem
- $nmin_i$: the minimum number of components for *i*th subsystem
- $N_s$: the maximum total number components of system

### 4-3. Model Construction

According to the model assumptions and model variables and parameters, the details of the optimization model could be defined as below:

***Objective Functions***

As mentioned in the assumption section**,** the first objective function of the optimization model is to maximize the minimum of subsystems MTTFFs; therefore the first objective function could be considered as:

$$max\ z = min\{MTTFF_i\} \tag{9}$$

When the $MTTFF_i$ is:

$$MTTFF_i = f(x_{ij}, \lambda_{ij}, \mu_{ij}) \tag{10}$$

in this study the mathematical method, proposed by Amiri and Ghassemi-Tari [40] is applied to calculate the value of first objective function for each solution of the optimization model.

The second objective function is to minimize the total cost of the system; therefore the second objective function of optimization model is considered as:

$$min\ C_s = \sum_{i=1}^{m}\sum_{j=1}^{o_i} c_{ij} \times x_{ij} \tag{11}$$

***Model Constraints***

The constraint indicates the weight of system in model:

$$\sum_{i=1}^{m}\sum_{j=1}^{o_i} w_{ij} \times x_{ij} \le W_s \tag{12}$$

The constraints that make it possible to select only one type of components for each subsystem:

$$\sum_{j=1}^{o_i} \frac{x_{ij}}{x_{ij}} = 1 \qquad (i = 1,2,\dots m) \tag{13}$$

The constraint of system total components:

$$\sum_{i=1}^{m}\sum_{j=1}^{o_i} x_{ij} \le N_s \tag{14}$$

The constraints of minimum and maximum number of components selected for each subsystem:

$$nmin_i \le \sum_{j=1}^{o_i} x_{ij} \le nmax_i \qquad (i = 1,2,\dots m) \tag{15}$$

The constraints that determines the type and limits of variables:

$$x_{ij} \ge 0\ and\ integer \qquad (i = 1,2,\dots m)\ and (j = 1,2,\dots O_i) \tag{16}$$

## 5. Solving Approach: Multi-Objective Imperialist Competition Algorithm

The main characteristic of the designed model in this paper, unlike the traditional optimization models, is the lack of a polynomial function between the first objective function ($\max z = \min\{MTTFF_i\}$) and the model variables ($x_{ij}$). For calculating the MTTFFs of subsystems, when the components are repairable, the mathematical method proposed by Amiri and Ghassemi-Tari [40] could be applied. On the other hand, one of the advantages of meta-heuristic algorithms is their applicability for solving the optimization models in which there are no polynomial function between objectives or constraints and the variables. Therefore in this paper one of the meta-heuristic algorithms is applied to solve the defined optimization model.

Imperialist Competition Algorithm (ICA) is one of the meta-heuristic algorithms, which has been applied for solving many optimization models since its introduction. ICA is a powerful optimization technique proposed by Atashpaz-Gargari and Lucas in 2007 [41]. ICA has three main steps:

- Establishing the first generation of solutions which are called countries in ICA
- Establishing the empires by classifying the generated countries to imperialists and colonies and assigning each colony to one of the imperialist randomly
- Moving the colonies to their relevant imperialists in a imperialist competitive process to achieve better solutions for the optimization model

Since occurrences of the stopping criterion of the algorithm, the third step is repeated and the best solution for the optimization model would be selected.

In this work ICA is applied to solve the optimization method, clarified in previous sections. ICA, proposed by Atashpaz-Gargary and Lucas [42], is a Meta-heuristic algorithm applicable for solving the optimization models. This algorithm is simulates the social political process of imperialism and imperialistic competition [43]. Like other evolutionary algorithm, ICA starts with an initial population. Population individuals, called country, are classified into two types: colonies and imperialists. An empire consists of an imperialist and some colonies. Imperialistic competition among these empires forms the basis of this evolutionary algorithm. During this competition, weak empires collapse and powerful ones take possession of their colonies. So, to prevent the collapse of an empire, during the progress of algorithm the imperialist countries try to make their colonies more powerful. Imperialistic competition hopefully converges to a state in which only one empire exists

and its colonies are in the same position and have the same cost as the imperialist.

The first version of ICA was proposed for solving the optimization models with only one objective; however this algorithm was also applied in multi-objective optimization models by many researchers. Abdi et.al [44], Abedinia et.al [45], Mohammadi et.al [46], and Ghanavati et.al [47], are some of the applications of ICA in solving the multi-objective optimization. Some adjustments should be made in the original version of ICA to apply it in a multi-objective optimization model. In this study these adjustments are done as follow:

### 5.1. Countries Structure

The structure designed to depict the solutions of a specific model could be unique. In ICA the structure of countries shows the method of defining the decision variables of the model. The role of countries structure is exactly like the role of chromosomes in genetic algorithm (GA). Regarding the model assumptions and its variables in this study, the structure of the countries is defined as Fig. 3.

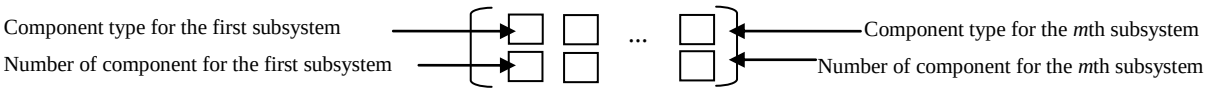


**Fig. 3. The structure of the optimization problem's solutions**

As shown in Fig. 3, the solutions of the optimization models are matrixes with two rows, the number of columns are equal to the number of subsystems. The elements of the first row illustrate the type of component, selected for the related subsystem. The element in second row of each subsystem column, verifies the number of selected components for related subsystem.

### 5.2. Initial Countries Generation

One of the main characteristics of the meta-heuristic algorithms is their random behavior in searching the feasible solution space. The start population of these algorithms is generated by stochastic process. On the other hand, Generating new population by current population is done by using the random numbers and parameters. In this study the first generation of countries is produced randomly; firstly all the matrixes of solutions elements are established by random numbers between 0 and 1.

Then, the first element of each column is multiplied by the number of existing component types for the related subsystem while the result is rounded to the first larger integer number. The second element of each column is multiplied by the maximum number of components could be selected for related subsystem while the result is rounded to the nearest integer number. The results are matrixes with integer elements, which could be considered as countries and the start population of algorithm.

### 5.3. Imperialists Selection

After generating countries, a non-dominance technique and a crowding distance are used to select the imperialists. At first, the countries which are not dominated by other countries are selected; if the number of non-dominated countries is more than required imperialists, the countries with larger crowding distance are selected as imperialist countries between them. Indeed there are two main criteria for comparing the solutions for a multi-objective optimization method: between different solutions for a multi-objective optimization model the non-dominated solutions are preferred and between the non-dominated solutions the ones with larger crowding distance are preferred. The crowding distance, $d_i$ , of a solution *i* for a multi-objective optimization model is a measure of the search space around *I* which is not occupied by any other solution in population [48]. The remaining countries, considered as colonies, distributed among the imperialists with respect to imperialist power.

### 5.4. Calculation of Imperialists Cost and Assigning the Colonies to Imperialist

To calculate the cost of each country (Imperialist or colony), is applied as follow:

$$C_{i,n} = \frac{\left| f_{i,n}^{p} - f_{i}^{p,best} \right|}{f_{i,total}^{p,max} - f_{i,total}^{p,min}} \qquad (17)$$

Where $C_{i,n}$ is the normalized value of objective function *i* for imperialist *n*, and $f_{i,n}^{p}$ is the value of the objective function *i* for imperialist *n*. Also $f_{i}^{p,best}$ is the best value for objective function *i* in all empires; $f_{i,total}^{p,max}$ and $f_{i,total}^{p,min}$ are the maximum and minimum values of *i*th objective function in each iteration in related empire, respectively. Finally, the normalized cost value of each imperialist ($C_n$) is obtained as below:

$$C_n = \sum_{i=1}^{r} C_{i,n} \qquad (18)$$

Where *r* is the number of objective functions. The power of each imperialist is calculated by. The colonies are distributed among the imperialists according to the normalize power of each imperialist country calculated by:

$$P_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right| \quad (19)$$

The initial number of colonies of an empire will be calculated as follow:

$$NC_n = round\{P_n \times N_{col}\} \quad (20)$$

Where $NC_n$ is the initial number of colonies of the *n*th imperialist, and $N_{col}$ is the number of all colonies. $NC_n$ of colonies are randomly selected and assign to the *n*th imperialist. Therefore, more powerful imperialist takes a greater number of colonies while imperialist with weaker power has less number.

### 5.5. Total Power for an Empire

The total power of an empire is mainly affected by the power of the imperialist country; however, the power of the colonies of an empire has an eligible effect on the total power of that empire. Therefore, the total power of an empire is calculated as below:

$$TP\ of\ Emp_n = Total\ Cost\ (Imperialist_n) + \xi\ mean\ \{Total\ Cost\ (Colonies\ of\ Empire_n)\} \quad (21)$$

Where *TP of Emp*$_n$ is the total power of the *n*th empire and $\xi$ is a positive number which is considered to be less than 1. The total cost of imperialists and colonies are calculated by and. It is necessary to mention that the effect of colonies on an empire's cost is related the value of $\xi$; therefore greater value of $\xi$ resulted in greater contribution of colonies in the total cost of an empire and vice versa.

### 5.6. Searching Solution Space

In this study the assimilation technique is applied for searching the solution space. After dividing colonies between imperialists, colonies are moved towards their related imperialist.

This movement is shown in Fig. **4**, in which *d* is the distance between colony and imperialist; if *C* and *I* are the matrix of a colony and its related imperialist, respectively, then d= C-I; *x* is the amount of moving of colony towards the imperialist which has a uniform distribution between 0 and $\beta \times d$; $\beta$ is a number greater than 1.

A $\beta$ > 1, causes the colonies to get closer to the imperialist state from both sides [49].

For moving colony towards imperialist in other direction, an angle, $\theta$, is applied with a uniform distribution between $-\gamma$and $\gamma$. It has been estimated that $\pi/4$ is the most proper value of $\gamma$.

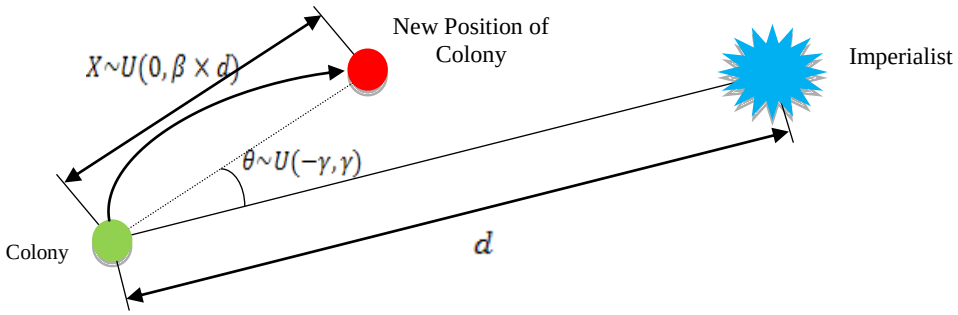


**Fig. 4. Moving colonies toward the imperialists with a random angle θ**

More details and examples about the assimilation method are presented in Abdi et.al [44] and Abedinia et.al [45].

### 5.7. Penalty Function

In the application of meta-heuristic algorithms, using penalty function is one of the most accepted solutions to face the solutions which violate the constraints. In this study the value of objective for these kinds of solutions is calculated by:

$$Z_{new} = Z_{old} + \sqrt{dv^c} \quad (22)$$

Where $Z_{new}$ is the value of objective function for the solutions which violate the constraints. $z_{old}$ is value of objective function for solution regardless of violation of constraints; *dv* is the number of constraints which have been violated by related solution and *c* is the amount of violence.

## 6. Numerical Example

In this part, the result of applying the proposed optimization model and its solving method to optimize the structure of a system with 4 subsystems is presented. The system has 4 subsystems with series-parallel configuration and the parameters of alternative components for each subsystem is depicted in Tab. 1:

**Tab. 1. Subsystems alternative components**

| Subsystem | Maximum number of components in subsystem | Minimum number of components in subsystem | Component types | Failure rate | Repair rate | Weight | Cost |
|---|---|---|---|---|---|---|---|
| 1 | 5 | 1 | 1 | 2 | 10 | 100 | 120 |
| | | | 2 | 5 | 18 | 80 | 100 |
| | | | 3 | 4 | 25 | 85 | 140 |
| | | | 4 | 2 | 8 | 90 | 110 |
| 2 | 6 | 1 | 1 | 5 | 40 | 250 | 400 |
| | | | 2 | 6 | 42 | 200 | 380 |
| | | | 3 | 10 | 100 | 200 | 500 |
| 3 | 4 | 1 | 1 | 4 | 22 | 450 | 800 |
| | | | 2 | 4 | 28 | 550 | 800 |
| | | | 3 | 7 | 20 | 250 | 800 |
| | | | 4 | 7 | 18 | 200 | 800 |
| 4 | 4 | 1 | 1 | 5 | 30 | 500 | 1200 |
| | | | 2 | 7 | 35 | 500 | 1500 |
| | | | 3 | 3 | 25 | 500 | 1500 |

maximum number of total component in system = 20
maximum weight of the system = 4500

After establishment of the optimization model with two objective functions, maximizing the minimum of subsystems MTTFFs and minimizing the total cost of system, and the related constraints of system the model was solved by the ICA approach as clarified in previous part by using MATLAB software. For solving the optimization model firstly, the parameters of ICA approach should be defined and tuned.

The definition of the ICA approach parameters was done by using the RSM method: The independent parameters which can affect the quality of the solutions resulted by running ICA are number of population (*n-pop*), number of imperialist (*n-imp*), Probability of using assimilation method for moving the colonies toward their related imperialist or percentage of assimilation ($P_A$), the contribution of the colonies on total power of an empire ($\xi$) and the amount of colony movement toward its related empire ($\beta$). Each factor was measured at two levels, which could be coded as value -1 when the factor was at its low level and +1 when the factor was at its high level. Coded variable were defined as follows:

$$x_i = \frac{r_i - (\frac{h+l}{2})}{(\frac{h-l}{2})} \qquad (23)$$

Where $x_i$ and $r_i$ are coded variable and natural variable, respectively. *h* and *l* represent high level and low level of factor. ICA was run in a limited time by using different combination of independent parameters and the resulted Pareto solutions for each run were recorded in a database.

Then, all the Pareto solutions in databases were combined and a non-dominance technique was applied to determine the non-dominated solutions. Each combination of independent parameters which had more contribution in the final non-dominated solutions is selected for the final run of ICA. The result of the parameters tuning by Expert Design software (software could be applied for running RSM) could be seen in Tab. 2.

**Tab. 2. the parameters for ICA**

| Parameters | Optimal value |
|---|---|
| *n-Pop* | 193 |
| *N-imp* | 5 |
| $P_A$ | 0.54 |
| $\xi$ | 0.195 |
| $\beta$ | 1.8 |

Like solving the other multi-objective optimization models, the application of ICA for solving the model, resulted in Pareto optimal solutions. The Pareto optimal solutions contain the solutions that were not dominated by the other solutions. The considered stopping criterion for ICA was number of function calls (NFCs) which was equal to 40,000**.** The Pareto optimal solutions for the optimization model of the considered system are depicted in Tab. 3.

**Tab. 3. the Pareto optimal solutions for optimization model resulted by using the ICA**

| No. | System structure | | | | | MTTFF | Cost | weight |
|---|---|---|---|---|---|---|---|---|
| 1 | Component type | 1 | 2 | 1 | 3 | 0.1667 | 2480 | 1250 |
| | numbers | 1 | 1 | 1 | 1 | | | |
| 3 | Component type | 4 | 1 | 1 | 1 | 0.25 | 3170 | 1540 |
| | numbers | 1 | 2 | 1 | 1 | | | |
| 5 | Component type | 4 | 2 | 4 | 3 | 0.5 | 4870 | 1890 |
| | numbers | 1 | 2 | 2 | 2 | | | |
| 7 | Component type | 2 | 2 | 4 | 1 | 0.8163 | 4980 | 1960 |
| | numbers | 2 | 2 | 2 | 2 | | | |
| 9 | Component type | 4 | 2 | 2 | 3 | 1.0815 | 5620 | 2680 |
| | numbers | 2 | 2 | 2 | 2 | | | |
| 11 | Component type | 4 | 2 | 2 | 3 | 1,6754 | 6760 | 3430 |
| | numbers | 2 | 3 | 3 | 2 | | | |
| 13 | Component type | 4 | 2 | 1 | 3 | 1.9354 | 7380 | 3630 |
| | numbers | 2 | 3 | 3 | 3 | | | |
| 15 | Component type | 1 | 2 | 2 | 3 | 4.3630 | 8370 | 4050 |
| | numbers | 3 | 3 | 3 | 3 | | | |
| 17 | Component type | 4 | 2 | 2 | 3 | 6.4691 | 8460 | 4020 |
| | numbers | 3 | 3 | 3 | 3 | | | |
| 19 | Component type | 4 | 2 | 1 | 3 | 7.8894 | 9580 | 4370 |
| | numbers | 3 | 4 | 4 | 3 | | | |
| 2 | Component type | 4 | 2 | 1 | 3 | 0.2 | 2500 | 1240 |
| | numbers | 1 | 1 | 1 | 1 | | | |
| 4 | Component type | 4 | 1 | 1 | 1 | 0.333 | 3970 | 1990 |
| | numbers | 1 | 2 | 2 | 1 | | | |
| 6 | Component type | 4 | 2 | 1 | 3 | 0.6282 | 4960 | 2480 |
| | numbers | 2 | 2 | 2 | 2 | | | |
| 8 | Component type | 2 | 1 | 2 | 3 | 0.8772 | 5020 | 2760 |
| | numbers | 2 | 2 | 2 | 2 | | | |
| 10 | Component type | 4 | 2 | 4 | 3 | 1.2245 | 5960 | 2180 |
| | numbers | 2 | 3 | 2 | 2 | | | |
| 12 | Component type | 4 | 2 | 1 | 3 | 1.8590 | 6760 | 3130 |
| | numbers | 2 | 3 | 3 | 2 | | | |
| 14 | Component type | 4 | 2 | 4 | 1 | 4.128 | 7470 | 2970 |
| | numbers | 3 | 3 | 3 | 3 | | | |
| 16 | Component type | 4 | 2 | 2 | 3 | 5.7877 | 8430 | 4020 |
| | numbers | 3 | 3 | 3 | 3 | | | |
| 18 | Component type | 4 | 2 | 2 | 3 | 6.5440 | 8780 | 4220 |
| | numbers | 3 | 4 | 3 | 3 | | | |
| 20 | Component type | 4 | 2 | 1 | 3 | 11.5181 | 9700 | 4460 |
| | numbers | 4 | 4 | 4 | 3 | | | |

## 7. Conclusion

The previous studies in redundancy allocation problems in series-parallel systems have not been considered the systems with repairable components. In this paper the main idea is to propose a multi-objective approach for redundancy allocation problems in series-parallel systems with repairable components. The considered system in this study has components with constant failure and repair rate, therefore considering systems comprising of components without exponential distribution for their repair and failure times could be a good challenge for future studies.

In this study the designed optimization model is solved by a meta-heuristic algorithm, called Imperialist Competitive Algorithm (ICA); the main goal of the paper was to propose an optimization model and a solving algorithm to attain the optimize structure of a repairable series-parallel system. Using other algorithms to solve the proposed optimization model and comparing the results by the solutions resulted in this paper could be the goal for feature works.

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