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A Complex Design of the Integrated Forward-Reverse Logistics Network under Uncertainty

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KEYWORDS

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Closed-loop logistics,
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ABSTRACT

Design of a logistics network in proper way provides a proper platform for efficient and effective supply chain management. This paper studies a multi-period, multi echelon and multi-product integrated forward-reverse logistics network under uncertainty. First, an efficient complex mixed-integer linear programming (MILP) model by considering some real-world assumptions is developed for the integrated logistics network design to avoid the sub-optimality caused by the separate design of the forward and reverse networks. Then, the stochastic counterpart of the proposed MILP model is used to measure the conditional value at risk (CVaR) criterion, as a risk measure, that can control the risk level of the proposed model. The computational results show the power of the proposed stochastic model with CVaR criteria in handling data uncertainty and controlling risk levels.

1. Introduction

In the last decade, attention to the integrated closed-loop supply chain network because of their economic benefits and environmental legislation increasingly has attracted. Thus, an efficient and robust logistics network leads to a sustainable competitive advantage for firms and helps them to cope with increasing environmental turbulence and uncertainties. Configuration of the logistics network (i.e., determining the number, location, capacity and technology of the facilities) is one of the most important and strategic issues in supply chain management that has a long lasting effect on the total performance of the supply chain [1].

In general, a integrated forward-reverse supply chain network consists of supply raw materials from suppliers, convert these raw materials to end products, shipping them to proper distribution centers and delivering to customer zones, then collection used products and finally recovering or remanufacturing and disposal in suitable way [2].

In most of the past studies the design of forward and reverse logistics networks is considered separately, while the configuration of the forward logistics network is affected by the reverse logistics network and vice versa. Separating the design of forward and reverse logistics may result in sub-optimality, therefore the design of the forward and reverse logistics network should be integrated [3].

A large part of the literature in the logistics network design is related to forward logistics network and smaller part of the literature is associated with the reverse logistics network design. In addition, in recent years a few papers have attended to integrated logistics network design.

The integrated logistics network is designed aimed to integrate the forward and reverse network design decisions to avoid the sub-optimality resulting from separated design. Table 1 shows the relevant literature, in which most of the studies in the logistics network design including forward, reverse and integrated

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models ignore the uncertain and dynamic nature of these problems. Kilibi et al. [4] and Pishvaee et al. [5] mentioned that static and deterministic models are not able to handle the parameters tainted by uncertainty and therefore the decisions resulted from these models may impose high costs to the firms. Risk reduction in environment with imprecise information is one of the most important issues for companies in order to enhance customer service and improve their business processes, thus resulting in increased competitiveness and profitability [6].

Regarding the uncertainty issue, it is crucial to measure and control the negative impact of uncertainty called risk [4]. Risk measurement has an important role in controlling the uncertainty in optimization problems, especially when the losses might be incurred in finance, insurance industry or other investments [7]. The literature of the logistics network design also suffers from the lack of models able to measure and control the risk resulted from the dynamic and uncertain nature of this problem.

Klibi et al. [4] presented a comprehensive critical review on the design of robust value-creating supply chain networks under uncertainty. Also, their review covers optimization models, uncertainty sources, and risk exposures, evaluation criteria of the supply chain design and assessment of supply chain network robustness as a necessary condition to ensure robust value creation.

Tab. 1. Classification of the relevant literature

Reference articles	Problem definition							Modeling		Objective function		
	Deterministic	Stochastic	Dynamic	static	Capacity	Transportation modes	Capacity expansion	MILP	MINLP	CVaR	Min Cost/Max Profit	Others
Forward logistics												
Azaron et al. [8]		●		●	●		●		●		●	●
Thanh et al. [9]	●		●		●		●	●			●	
Bachlaus et al. [10]	●			●	●			●			●	●
Wang and Watada [11]		●		●	●				●		●	
Amiri [12]	●			●	●			●			●	
Ko and Evans [13]	●		●		●				●		●	
Pan and Nagi [14]		●	●		●			●			●	
Dal-Mas et al. [15]		●	●		●			●			●	
Reverse logistics												
Aras et al. [16]	●			●					●		●	
Jayaraman et al. [17]	●			●	●			●			●	
Listes and Dekker [18]		●		●	●			●			●	
Min et al. [19]	●		●					●			●	
Du and Evans [20]	●			●	●			●			●	●
Integrated logistics												
Pishvaee et al. [2] and [5]		●		●	●			●			●	
El-Sayed [21]		●	●		●			●			●	
Fleischmann et al. [22]	●			●				●			●	
Salema et al. [23]		●		●	●			●			●	
Min and Ko [24]	●		●		●				●		●	
Our work		●	●		●	●	●	●		●	●	

Melo et al. [25] presented a general review on the supply chain network design to identify basic features that such models should capture to support decision-making involved in strategic supply chain planning and support a variety of future research directions. Another interesting reviews in this field can be found in Dullaert et al. [27] and Snyder and Lawrence [28]. Some various relevant studies in the logistics network design area have systematically classified and are presented in Table 1.

To deal with the above-mentioned drawbacks, this paper addresses a stochastic complex mixed-integer linear programming (MILP) model for the integrated logistics network design by considering conditional value-at-risk (CVaR) criterion as a risk measure to assure the robustness of the concerned logistics network. The MILP model is developed for the multi-period, multi echelon and multi-product integrated forward-reverse logistics network considering possibility of handling products in special facilities. In addition, the proposed model is able to support different transportation modes for delivering finished goods from plants to customer zones as well as considering capacity expansion option for plants

besides the features of traditional literature models. We assume that the demand of customers, rate of return of used products and their quality are uncertain input parameters. The CVaR metric which has been proposed by Rockafellar and Uryasev [7] and [28]. CVaR is a measure of risk with significant advantages over value-at-risk (VaR), is utilized in this paper and incorporated to the objective function to measure the investment risk.
In the next section, the model is described and formulated. After introducing CVaR in Section 3, the two-stage stochastic MILP model by considering CVaR criterion is developed in this section. The experimental results are reported in Section 4. Finally, Section 5 concludes this paper and offers some directions for further research.

2. Problem Description and Formulation

The model is formulated for the integrated forward–reverse logistics network design problem. It considers a multi-period, multi-echelon, multi-product network that consists of production, distributors, and customer zones in the forward direction and the collection, recovery and disposal centers in the reverse direction. Because of the economic benefits, the production and recovery and also distribution and collection centers are commonly considered in forward and reverse direction. The presented model covers flexibility in transporting of products between various nodes by introducing transportation modes. This flexibility is especially efficient in cases that the logistics network deal with disruptions or failures in transportation equipment because of risks that are common in shipping products.

2.1. Assumption

The following assumptions are considered in the presented model.

1. The MILP model is a multi-period and any facility can be opened or closed at each period.
2. Shortage is permissible in forward direct, and some demands cannot be satisfied.
3. The returned quantities depend on the customer demand in forward direct.
4. The quantity of disposal used products depends on returns and their quality.
5. The potential locations of production/recovery, distribution/collection and disposal are known and also some location of facilities is pre-determined.
6. The capacity of each location is known for each time period; however, production/recovery centres capacity can be increased.
7. Production/recovery centres have the limited maximal installable capacity.
8. Customers' zones are known and fixed with deterministic demands.
9. Products can be handled in special plants and disposal centres, not in all facilities.
10. If a production/recovery centre performs capacity expansion, it will be active for the end of time horizon.

The following notation is used in the formulation of the proposed model.

Set

P	Set of type product ($p \in P$)
I	Set of potential production/recovery center locations ($i \in I$)
J	Set of potential hybrid distribution-collection center locations ($j \in J$)
K	Fixed locations of customer zones ($k \in K$)
M	Set of potential disposal center locations ($m \in M$)
N	Set of transportation's modes ($n \in N$)
F	Set of capacity option for production/recovery center ($f \in F$)
T	Set of time periods ($t \in T$)

Parameters

d_{pkt}	Demand of customer zone k for product p in period t
r_{pkt}	Rate of return of used product p from customer zone k in period t
S_{pt}	Average disposal fraction of product p in period t
f_{it}	Fixed cost of opening production/recovery center i in period t
g_{jt}	Fixed cost of opening hybrid distribution-collection center j in period t
h_{mt}	Fixed cost of opening disposal center m in period t
Hwa_{fpit}	Fixed cost of adding capacity option f of product p to production/recovery center i at period t
c_{ijpnt}	Shipping cost per unit of product p from production/recovery center i to hybrid distribution-collection center j with mode n in period t
a_{jkpnt}	Shipping cost per unit of product p from hybrid distribution-collection center j to customer zone k with mode n in period t
b_{kjpnt}	Shipping cost per unit of returned product p from customer zone k to hybrid distribution-collection center j with mode n in period t
e_{jipnt}	Shipping cost per unit of recoverable product p from hybrid distribution-collection center j to production/recovery center i with mode n in period t
p_{jmpnt}	Shipping cost per unit of scrapped product p from hybrid distribution-collection center j to disposal center m with mode n in period t
QD_{ipt}	Required quantity of product p for receiving discount from production/recovery center i at period t
ρ_{ipt}	Manufacturing/recovery cost per unit of product p at production/recovery center i in period t
φ_{jpt}	Processing cost per unit of product p at hybrid distribution-collection center j in period t
η_{mpt}	Disposal cost per unit of product p at disposal center m in period t
τ_{ip}	penalty cost per unit of non utilized capacity for product p at production/recovery center i
β_{jp}	penalty cost per unit of non utilized capacity for product p at hybrid distribution-collection center j

α_{mp}	penalty cost per unit of non utilized capacity for product *p* at disposal center *m*
λ_{pkt}	Penalty cost per unit of non-satisfied demand of customer k for product p in period t
cw_{ipt}	Initial capacity of production of product *p* for production/recovery center *i* in period *t*
KA_i	Maximal installable production/recovery capacity at production/recovery center *i*
cy_{jp}	Capacity of handling product *p* in forward flow at hybrid distribution-collection center *j*
cz_{mp}	Capacity of handling scrapped product *p* at disposal center *m*
cyr_{jp}	Capacity of handling returned product *p* in reverse flow at hybrid distribution-collection center *j*
cwr_{ipt}	Capacity of recovery returned product *p* for production/recovery center *i* in period *t*
cwa_{fpi}	Capacity of option *f* for product *p* adding to production/recovery center *i*

Variables

X_{ijpnt}	Quantity of product *p* shipped from production/ recovery center *i* to hybrid distribution-collection center *j* with mode *n* in period *t*
U_{jkpnt}	Quantity of product *p* shipped from hybrid distribution-collection center *j* to customer zone *k* with mode *n* in period *t*
δ_{pkt}	Quantity of non-satisfied demand of customer *k* for product *p* in period *t*
Q_{kjpnt}	Quantity of returned product *p* shipped from customer zone *k* to hybrid distribution-collection center *j* with mode *n* in period *t*
V_{jipnt}	Quantity of recoverable product *p* shipped from hybrid distribution-collection center *j* to production/recovery center *i* with mode *n* in period *t*
L_{jmpnt}	Quantity of scrapped product *p* shipped from hybrid distribution-collection center *j* to disposal center *m* with mode *n* in period *t*
W_{it}	$=\begin{cases}1 & \text{If a production/recovery center is opened at location } i, \\ 0 & \text{Otherwise}\end{cases}$
Y_{jt}	$=\begin{cases}1 & \text{If a hybrid distribution-collection center is opened at location } j, \\ 0 & \text{Otherwise}\end{cases}$
Z_{mt}	$=\begin{cases}1 & \text{If the disposal center is opened at location } m, \\ 0 & \text{Otherwise}\end{cases}$
RE_{ipt}	$=\begin{cases}1 & \text{If a production/recovery center opened at location } i \text{ can be manufacture product } p \text{ in period } t \\ 0 & \text{Otherwise}\end{cases}$
RS_{mpt}	$=\begin{cases}1 & \text{If a disposal center opened at location } m \text{ can be disposal product } p \text{ in period } t \\ 0 & \text{Otherwise}\end{cases}$
wa_{fpit}	$=\begin{cases}1 & \text{If capacity option } f \text{ is added to production/recovery center } i \text{ at period } t, \\ 0 & \text{Otherwise}\end{cases}$

$$
\begin{aligned}
\text{Min} \quad & \sum_t \Bigg[\sum_i f_{it} W_{it} + \sum_j g_{jt} Y_{jt} + \sum_m h_{mt} Z_{mt} + \sum_i \sum_j \sum_p \sum_n (\rho_{ipt} + c_{ijpnt}) X_{ijpnt} \\
& + \sum_j \sum_k \sum_p \sum_n (\phi_{jpt} + a_{jkpnt}) U_{jkpnt} + \sum_k \sum_j \sum_p \sum_n (\phi_{jpt} + b_{kjpnt}) Q_{kjpnt} + \\
& \sum_j \sum_i \sum_p \sum_n (\rho_{ipt} + e_{jipnt}) V_{jipnt} + \sum_j \sum_m \sum_p \sum_n (\eta_{mpt} + p_{jmpnt}) L_{jmpnt} \\
& + \sum_i \sum_p \tau_{ip} \Bigg[(W_{it} cw_{ipt} + \sum_f cwa_{fpi} wa_{fpit} - \sum_j \sum_n X_{ijpnt}) \\
& \qquad + (W_{it} cwr_{ipt} + \sum_f cwa_{fpi} wa_{fpit} - \sum_j \sum_n V_{jipnt}) \Bigg] \\
& + \sum_j \sum_p \beta_{jp} \Bigg[(Y_{jt} cy_{jp} - \sum_k \sum_n U_{jkpnt}) + (Y_{jt} cyr_{jp} - \sum_k \sum_n Q_{kjpnt}) \Bigg] \\
& + \sum_m \sum_p \alpha_{mp} (Z_{mt} cz_{mp} - \sum_j \sum_n L_{jmpnt}) \Bigg] + \sum_f \sum_p \sum_i Hwa_{fpi1} wa_{fpi1} \\
& + \sum_t \Bigg[\sum_f \sum_p \sum_i Hwa_{fpit} (wa_{fpi(t+1)} - wa_{fpit}) + \sum_p \sum_k \lambda_{pkt} \delta_{pkt} \Bigg]
\end{aligned} \tag{1}
$$

s.t.

$$\sum_j \sum_n U_{jkpnt} + \delta_{pkt} \geq d_{pkt} \quad \forall p,k,t \tag{2}$$

$$\sum_j \sum_n Q_{kjpnt} \geq r_{pkt} d_{pkt} \quad \forall p,k,t \tag{3}$$

$$\sum_i \sum_p \sum_n X_{ijpnt} - \sum_k \sum_p \sum_n U_{jkpnt} = 0 \quad \forall j,t \tag{4}$$

$$\sum_i \sum_p \sum_n V_{jipnt} - \sum_k \sum_p \sum_n (1 - S_{pt}) Q_{kjpnt} = 0 \quad \forall j,t \tag{5}$$

$$\sum_m \sum_p \sum_n L_{jmpnt} - \sum_k \sum_p \sum_n S_{pt} Q_{kjpnt} = 0 \quad \forall j,t \tag{6}$$

$$\sum_j \sum_p \sum_n V_{jipnt} - \sum_j \sum_p \sum_n X_{ijpnt} \leq 0 \quad \forall i,t \tag{7}$$

$$\sum_k \sum_p \sum_n Q_{kjpnt} - \sum_k \sum_p \sum_n U_{jkpnt} \leq 0 \quad \forall j,t \tag{8}$$

$$wa_{fpit} \leq W_{it} \quad \forall f,p,i,t \tag{9}$$

$$wa_{fpit} \leq wa_{fpi(t+1)} \quad \forall f,p,i,t \tag{10}$$

$$\sum_j \sum_n X_{ijpnt} \leq (cw_{ipt} W_{it} + \sum_f cwa_{fpi} wa_{fpit}) RE_{ipt} \quad \forall i,p,t \tag{11}$$

$$\sum_j \sum_n V_{jipnt} \leq (cwr_{ipt} W_{it} + \sum_f cwa_{fpi} wa_{fpit}) RE_{ipt} \quad \forall i,p,t \tag{12}$$

$$\sum_{i}\sum_{n} X_{ijpnt} \le cy_{jp} Y_{jt} \quad \forall j,p,t \tag{13}$$

$$\sum_{k}\sum_{n} Q_{kjpnt} \le cyr_{jp} Y_{jt} \quad \forall j,p,t \tag{14}$$

$$\sum_{j}\sum_{n} L_{jmpnt} \le cz_{mp} Z_{mt} RS_{mpt} \quad \forall m,p,t \tag{15}$$

$$\sum_{p}\sum_{t} cw_{ipt} W_{it} + \sum_{p}\sum_{f}\sum_{t} cwa_{fpi} wa_{fpit} \le KA_i \quad \forall i \tag{16}$$

$$W_{it}, Y_{jt}, Z_{mt}, RE_{ipt}, RS_{mpt}, wa_{fpit} \in \{0,1\} \quad \forall i,j,m,f,p,t \tag{17}$$

$$X_{ijpnt}, U_{jkpnt}, Q_{kjpnt}, V_{jipnt}, L_{jmpnt} \ge 0 \quad \forall i,j,p,k,n,m,t \tag{18}$$

Objective function (1) minimizes the total costs including the fixed opening cost, transportation cost, processing cost, penalty cost for non-utilized capacities, adding capacity option cost and shortage cost. Constraint (2) ensures that all demands of customers are not satisfied. Constraint (3) ensures that the returned products are collected from all customers. Constraints (4) to (6) assure the flow balance at production/recovery and hybrid distribution/collection centers in forward and reverse flows. Constraints (7) to (9) cite the logical constraints. Constraint (10) ensures when a capacity option added to special production/recovery center, it will be active for the end of time horizon.

Equations (11) and (12) express the capacity constraints and also possibility of producing/recovering special product in particular production/recovery centers. Constraints (13) to (15) are capacity constraints on hybrid distribution/ collection and disposal centers. Constraint (16) ensures the limited maximal installable capacity.

Finally, Constraints (17) and (18) enforce the binary and non-negativity restrictions on corresponding decision variables. The resulting model is a MILP model with *2(IJPNT+JKPNT)+(JMPNT)* continuous variables and *(I+J+M)T+(FPIT)* binary variables. The number of constraints is *(6I + 2K +6J+M+9P+2F+14T)*, excluding Constraints (17) and (18).

3. Two-Stage Stochastic Programming with Considering CVaR

Risk measures, such as VaR and CVaR, can be countered by stochastic optimization. CVaR measures the risk of an investment in a conservative way, focusing on the less profitable outcomes. The α-conditional value-at-risk (α-CVaR) is the minimizing of "the expected value of the costs in the (1- α)×100% worst cases" (Schultz and Tiedemann, [29]), where $\alpha \in (0,1)$ is a confidence level and pre-determined probability.

As well as, minimizing the CVaR leads to minimize the VaR and the CVaR, and it is more conservative than the VaR. It means that the CVaR is greater than or equal to the VaR. More detailed concept of the CVaR is discussed by Rockafellar and Uryasev [29] and Rockafellar and Uryasev [7].

In the following, the mathematical model of the CVaR is represented. We assume that positive values of $f(x,\omega)$ represent losses.

Assume that ω has a finite discrete distribution with N realizations and corresponding probabilities given as π_θ for ω_θ, $\theta=1,..,N$, (θ is representative a particular scenario) with $\pi_\theta>0$ and $\sum_\theta \pi_\theta=1$. For $f(x,\omega)$, the α-CVaR can be stated by the following minimization formula:

$$F_\alpha(x,\eta) = \eta + \frac{1}{1-\alpha} \mathrm{E}\left[(f(x,\omega)-\eta)^+\right]$$

Where,

$$(f(x,\omega)-\eta)^+ = \max\{f(x,\omega)-\eta, 0\}$$

Let the α-CVaR for loss random variable $f(x,\omega)$ is denoted by $\psi_\alpha(x)$. So, the α-CVaR equation can be restated as follows:

$$\psi_\alpha(x) = \min\left\{\eta + \frac{1}{1-\alpha}\mathrm{E}\left[\max\{f(x,\omega)-\eta,0\}\right]\right\}$$

By introducing additional variables Z_θ for representing (Max{ $f(x,\omega)$-$\eta,0$}) for all $\theta=1,..,N$ and using a well-known idea in linear programming, this nonlinear programming problem can be transformed into a linear programming problem. Also, by expanding the expected value of (Max{ $f(x,\omega)$-$\eta,0$})for all scenarios, we achieve the following equivalent linear programming problem [30].

$$\psi_\alpha(x) = \min\left\{\eta + \frac{1}{1-\alpha}\sum_{\theta=1}^{N}\pi_\theta z_\theta\right\}$$

s.t.

$$f(x,\omega_\theta) - \eta - z_\theta \le 0 \qquad \forall \theta,$$

$$z_\theta \ge 0 \qquad \forall \theta,$$

It should be noted that in the optimum solution η^* is corresponding to the α-VaR. In the following, we introduce two-stage stochastic linear programming (SLP) by considering the CVaR criterion for the compact model of MILP described in Section 2.

$$\min \quad \sum_t (f(y_t - y_{t-1}) + h(z_t - z_{t-1})) + \sum_\theta \sum_t \pi_{\theta t} c_t x_{\theta t} + \xi(\eta + \frac{1}{1-\alpha}\sum_t \sum_\theta \pi_{\theta t} z_{\theta t})$$

s.t.

$$\begin{aligned}
& Ax_{\theta t} \geq d_{\theta t} \\
& Nx_{\theta t} = 0 \\
& Bx_{\theta t} \leq C(y_t + z_t) \\
& M(y_t + z_t) \leq K \\
& z_t \leq y_t \\
& f(y_t - y_{t-1}) + h_t(z_t - z_{t-1}) + \pi_{\theta t} c_{\theta t} x_{\theta t} - \eta - z_{\theta t} \leq 0 \qquad \forall \theta, t \\
& y_t, z_t \in \{0,1\}, \quad x_{\theta t}, z_{\theta t}, \eta \in R^+
\end{aligned}$$

In the above compact form, *f* and *h* correspond to fixed opening and capacity expansion costs, respectively. C_t corresponds to transportation, processing, penalty and shortage costs. The matrices *A, B, M* and *N* are coefficient matrices of the constraints. *K* is the scalar in the related constraints.

All *y* and *z* are the binary decision variables for the opening and adding capacity, respectively. All the continuous decision variables include into vector ***x***. Let Ω be the set of all possible scenarios, θ a particular scenario and $\pi_{\theta t}$ probability of occurrence scenario θ in period *t*. because θ is a finite number (number of scenarios) the expected value function become a summation on θ. ξ is the weighting factor for the risk measure and we assume that ξ=1 in all computations which is represented in the next Section. Let the assumptions and definitions in Section 2 for different scenarios, for example $d_{pk\theta t}$ is Demand of customer zone k for product *P* in period *t* for scenario θ. As well as, other parameters and decision variables including index θ are similar to counterpart of them which are described in Section 2; however, they have been tainted by uncertainty. Thus, they have got index θ. The counterpart two-stage SLP model by considering the CVaR criterion in α confidence level for the MILP model described in Section 2 can be stated as follows:

$$\begin{aligned}
\text{Min} \quad & \sum_t \Bigg[\sum_i f_{it} W_{it} + \sum_j g_{jt} Y_{jt} + \sum_m h_{mt} Z_{mt} + \sum_i \sum_j \sum_p \sum_n \sum_\theta \pi_{\theta t} (\rho_{ipt} + c_{ijpnt}) X_{ijpn\theta t} \\
& \sum_j \sum_k \sum_p \sum_n \sum_\theta \pi_{\theta t} (\phi_{jpt} + a_{jkpnt}) U_{jkpn\theta t} + \sum_k \sum_j \sum_p \sum_n \sum_\theta \pi_{\theta t} (\phi_{jpt} + b_{kjpnt}) Q_{kjpn\theta t} \\
& + \sum_j \sum_i \sum_p \sum_n \sum_\theta \pi_{\theta t} (\rho_{ipt} + e_{jipnt}) V_{jipn\theta t} + \sum_j \sum_m \sum_p \sum_n \sum_\theta \pi_{\theta t} (\eta_{mpt} + p_{jmpnt}) L_{jmpn\theta t} \\
& + \sum_i \sum_p \sum_\theta \pi_{\theta t} \tau_{ip} \Bigg[(W_{it} cw_{ipt} + \sum_f cwa_{fpi} wa_{fpit} - \sum_j \sum_n X_{ijpn\theta t}) \\
& \qquad + (W_{it} cwr_{ipt} + \sum_f cwa_{fpi} wa_{fpit} - \sum_j \sum_n V_{jipn\theta t}) \Bigg] \\
& + \sum_j \sum_p \sum_\theta \pi_{\theta t} \beta_{jp} \Bigg[(Y_{jt} cy_{jp} - \sum_k \sum_n U_{jkpn\theta t}) + (Y_{jt} cyr_{jp} - \sum_k \sum_n Q_{kjpn\theta t}) \Bigg] \\
& + \sum_m \sum_p \alpha_{mp} (Z_{mt} cz_{mp} - \sum_j \sum_n L_{jmpn\theta t}) \Bigg] + \sum_f \sum_p \sum_i Hwa_{fpi1} wa_{fpi1} \\
& + \sum_t \Bigg[\sum_f \sum_p \sum_i Hwa_{fpit} (wa_{fpi(t+1)} - wa_{fpit}) + \sum_p \sum_k \sum_\theta \pi_{\theta t} \lambda_{pkt} \delta_{pk\theta t} \Bigg] \\
& + \xi(\eta + \frac{1}{1-\alpha} \sum_{t \in T} \sum_{\theta \in \Omega} \pi_{\theta t} z_{\theta t})
\end{aligned} \tag{19}$$

$$\sum_j \sum_n U_{jkpn\theta t} + \delta_{pk\theta t} \geq d_{pk\theta t} \quad \forall p, k, \theta, t \tag{20}$$

$$\sum_j \sum_n Q_{kjpn\theta t} \geq r_{pk\theta t} d_{pk\theta t} \quad \forall p, k, \theta, t \tag{21}$$

$$\sum_i \sum_p \sum_n X_{ijpn\theta t} - \sum_k \sum_p \sum_n U_{jkpn\theta t} = 0 \quad \forall j, \theta, t \tag{22}$$

$$\sum_i \sum_p \sum_n V_{jipn\theta t} - \sum_k \sum_p \sum_n (1 - S_{p\theta t}) Q_{kjpn\theta t} = 0 \quad \forall j, \theta, t \tag{23}$$

$$\sum_m \sum_p \sum_n L_{jmpn\theta t} - \sum_k \sum_p \sum_n S_{p\theta t} Q_{kjpn\theta t} = 0 \quad \forall j, \theta, t \tag{24}$$

$$\sum_j \sum_p \sum_n V_{jipn\theta t} - \sum_j \sum_p \sum_n X_{ijpn\theta t} \leq 0 \quad \forall i, \theta, t \tag{25}$$

$$\sum_k \sum_p \sum_n Q_{kjpn\theta t} - \sum_k \sum_p \sum_n U_{jkpn\theta t} \leq 0 \quad \forall j, \theta, t \tag{26}$$

$$wa_{fpit} \leq W_{it} \quad \forall f, p, i, t \tag{27}$$

$$wa_{fpit} \le wa_{fpi(t+1)} \quad \forall f, p, i, t \tag{28}$$

$$\sum_j \sum_n X_{ijpn\theta t} \le (cw_{ipt} W_{it} + \sum_f cwa_{fpi} wa_{fpit}) RE_{ipt} \quad \forall i, p, \theta, t \tag{29}$$

$$\sum_j \sum_n V_{ijpn\theta t} \le (cwr_{ipt} W_{it} + \sum_f cwa_{fpi} wa_{fpit}) RE_{ipt} \quad \forall i, p, \theta, t \tag{30}$$

$$\sum_i \sum_n X_{ijpn\theta t} \le cy_{jp} Y_{jt} \quad \forall j, p, \theta, t \tag{31}$$

$$\sum_k \sum_n Q_{kjpn\theta t} \le cyr_{jp} Y_{jt} \quad \forall j, p, \theta, t \tag{32}$$

$$\sum_j \sum_n L_{jmpn\theta t} \le cz_{mp} Z_{mt} RS_{mpt} \quad \forall m, p, \theta, t \tag{33}$$

$$\sum_p \sum_t cw_{ipt} W_{it} + \sum_p \sum_f \sum_t cwa_{fpi} wa_{fpit} \le KA_i \quad \forall i \tag{34}$$

$$\begin{aligned}
&\sum_i f_{it} W_{it} + \sum_j g_{jt} Y_{jt} + \sum_m h_{mt} Z_{mt} + \sum_i \sum_j \sum_p \sum_n \pi_{\theta t} (\rho_{ipt} + c_{ijpnt}) X_{ijpn\theta t} \\
&\sum_j \sum_k \sum_p \sum_n \pi_{\theta t} (\phi_{jpt} + a_{jkpnt}) U_{jkpn\theta t} + \sum_k \sum_j \sum_p \sum_n \pi_{\theta t} (\phi_{jpt} + b_{kjpnt}) Q_{kjpn\theta t} \\
&+ \sum_j \sum_i \sum_p \sum_n \pi_{\theta t} (\rho_{ipt} + e_{jipnt}) V_{jipn\theta t} + \sum_j \sum_m \sum_p \sum_n \pi_{\theta t} (\eta_{mpt} + p_{jmpnt}) L_{jmpn\theta t} \\
&+ \sum_i \sum_p \pi_{\theta t} \tau_{ip} \Bigg[(W_{it}\, cw_{ipt} + \sum_f cwa_{fpi} wa_{fpit} - \sum_j \sum_n X_{ijpn\theta t}) \\
&\qquad + (W_{it}\, cwr_{ipt} + \sum_f cwa_{fpi} wa_{fpit} - \sum_j \sum_n V_{jipn\theta t}) \Bigg] \\
&+ \sum_j \sum_p \pi_{\theta t} \beta_{jp} \Bigg[(Y_{jt} cy_{jp} - \sum_k \sum_n U_{jkpn\theta t}) + (Y_{jt} cyr_{jp} - \sum_k \sum_n Q_{kjpn\theta t}) \Bigg] \\
&+ \sum_m \sum_p \alpha_{mp} (Z_{mt} cz_{mp} - \sum_j \sum_n L_{jmpn\theta t}) \Bigg] + \sum_f \sum_p \sum_i Hwa_{fpi1} wa_{fpi1} \\
&+ \sum_t \Bigg[\sum_f \sum_p \sum_i Hwa_{fpit} (wa_{fpi(t+1)} - wa_{fpit}) + \sum_p \sum_k \pi_{\theta t} \lambda_{pkt} \delta_{pk\theta t} \Bigg] - \eta - z_{\theta t} \le 0 \\
&\forall \theta \in \Omega, t \in T
\end{aligned} \tag{35}$$

$$W_{it}, Y_{jt}, Z_{mt}, RE_{ipt}, RS_{mpt}, wa_{fpit} \in \{0,1\} \quad \forall i, j, m, f, p, t \tag{36}$$

$$X_{ijpn\theta t}, U_{jkpn\theta t}, Q_{kjpn\theta t}, V_{jipn\theta t}, L_{jmpn\theta t}, z_{\theta t}, \eta \ge 0 \quad \forall i, j, p, k, n, m, \theta, t \tag{37}$$

In the above model, the solution is not optimal in general for the individual scenarios in different periods [31]. Future periods and related scenarios are a description of a future situation and the course of events that enables one to progress from the original situation to the future situation.

4. Computational Results

In this Section, at first the comparison results of stochastic model without CVaR criterion respect to deterministic model are reported, and then the outcome results for risk measures are highlighted. In the stochastic models, by increasing the number of scenarios significantly increases the computational time with limited benefit in solution accuracy [21]. Our experiments on the presented stochastic model by considering the CVaR criterion also show the accuracy of this claim.

Here to assess the performance of the presented model, two test problems are selected. Each problem includes three periods and each period consists of four scenarios. First scenario in each period of each problem that has a higher probability is considered as nominal data for deterministic model. The data in test problems are generated randomly and the interested readers can reach the data set and LINGO codes for test problems from authors. Test problems are solved with LINGO 8.0 on a Pentium dual-core 2.66 GHZ computer with 4 GB RAM.

As shown in Table 2, the stochastic model results in a higher objective function value compared with deterministic model. In addition, the number of variables and constraints for the two models shows the higher degree of complexity of the stochastic model. Also, the stochastic model by considering the CVaR criterion has a higher objective function value and higher degree of the complexity respect to the stochastic model without the CVaR criterion. It is should be noted that the stochastic model by considering the CVaR criterion results the same solution compared with the stochastic model without the CVaR criterion in optimum solutions, exclude the objective function value.

It is can be concluded from Table 3 that the stochastic model opens more facilities or facilities with a more capacity compared to the deterministic model to assure robustness of the logistics network in dealing with the uncertainty conditions and other issues, such as shortage possibility. Realization of scenarios in Table 4 confirms this judgment.

Due to the strategic nature of facility location decisions in the logistics network design, changing facility location impossible in the short run; however, the quantity of flow between facilities as a tactical decision can be changed in short run [2]. Therefore, strategic decisions (binary variables) should be determined independently from scenario realizations, whereas the tactical decisions can be updated. To assess the performance of deterministic and stochastic model without the CVaR under each scenario, at the first, the models are solved by Lingo 8.0.

Then, the solutions of the two models are obtained under realization of each scenario in any period by allowing the models to update their continuous decision variables with fixed binary variables that are acquired from the first step solution for all scenarios. Because of this, the solution is not optimal in realization of scenarios. As illustrated in Table 4, the stochastic model results better solutions than the deterministic one in 75% of the cases and also, difference between solutions of stochastic linear programming model and deterministic model is considerable. It is obvious that the increase in demand and shortage costs increases the total costs for both of the models.

However, as depicted in Figures 1 and 2, the total cost is more sensitive to a demand compared with shortage costs. This observation can be explained by the impact of the demand on the costs of both forward and reverse networks while the shortages occur in the forward direction.

Thus, the increase in a demand has more impact on the total costs compared with the increase in shortage costs. Total costs augmented slightly when the shortage costs increases; however, in some instances, total costs increase with a jump. Since the CVaR is the expected loss exceeding the VaR; so, as it is shown in Table 5 in the certain confidence level, the CVaR is larger or equal to the VaR. In addition, this result is obtained under different periods for risk metrics (see Figure 3). Obviously, by increasing in the confidence level, the VaR and CVaR measures increase respect to measures with a less confidence level. The results illustrated in Table 5 confirm this idea. From Table 5 results, it can be concluded the CVaR measure is more conservative than the VaR measure, and it is more suitable for risk-averse organizations.

Tab. 2. Computational results under the described data

Problem size $I \times J \times K \times M \times P \times N \times F \times T$	Optimal value of objective function			Number of variables			Number of constraints		
	Deterministic	Stochastic Without CVaR	Stochastic with CVaR	Deterministic	Stochastic Without CVaR	Stochastic with CVaR	Deterministic	Stochastic Without CVaR	Stochastic with CVaR
5×8×10×3×2×2×3×3	75210170	91475730	159193000	3234	12930	12937	615	1824	1858
10×15×20×4×2×2×3×3	217519000	218790500	375104500	11646	46578	46585	1187	3504	3538

Tab. 3. Share of different type of costs in the objective function

Problem size $I \times J \times K \times M \times P \times N \times F \times T$	value of fixed opening costs		value of transportation and processing costs		value of non utilized capacity penalty costs		value of capacity expansion costs	
	Deter.	Stochastic	Deter.	Stochastic	Deter.	Stochastic	Deter.	Stochastic
5×8×10×3×2×2×3×3	37554800	44271100	15142870	18027230	21240580	2516150	1271929	2554179
10×15×20×4×2×2×3×3	103055100	115082100	24495800	27428870	27940420	35292480	3730134	4653857

Tab. 4. Objective function under realization of scenarios

Problem size $I \times J \times K \times M \times P \times N \times F \times T$	Period	Scenario Ω	Scenario probability $(\pi(\theta,t))$	Value of objective function	
				deterministic	stochastic
5×8×10×3×2×2×3×3	1	1	0.4	26285610	28729290
		2	0.2	52071680	29462170
		3	0.3	75873650	30296140
		4	0.1	89470790	28869340
	2	1	0.5	28940460	31583490
		2	0.2	46961820	31438770
		3	0.15	63755480	30747460
		4	0.15	84434490	30713650
	3	1	0.55	20005170	29274550
		2	0.15	50183690	29243320
		3	0.2	74824870	30869210
		4	0.1	87290960	41519840
10×15×20×4×2×2×3×3	1	1	0.5	88992890	63192490
		2	0.2	137132600	65324060
		3	0.2	159691200	92114770
		4	0.1	174074000	100415900
	2	1	0.45	61601150	62041830
		2	0.25	72691570	66314160
		3	0.2	92114770	85732670
		4	0.1	111102900	104721400
	3	1	0.5	54914530	86304980
		2	0.25	84752250	85732670
		3	0.15	109383000	93672080
		4	0.1	120890300	105152200

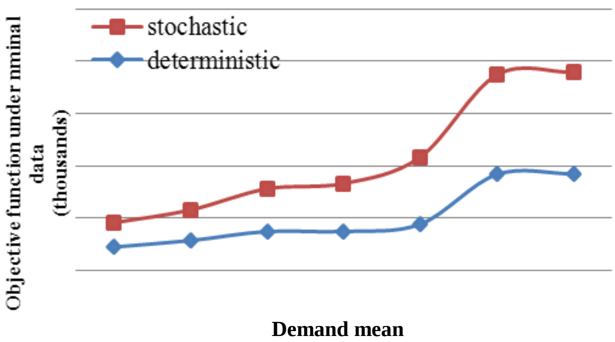

Fig. 1. Total costs vs. Demand

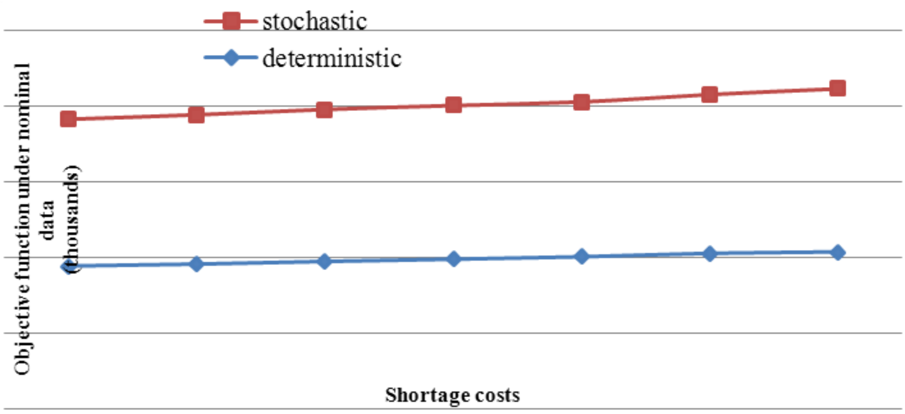

Fig. 2. Total costs vs. shortage cost

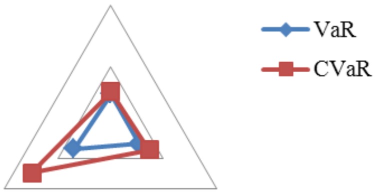

Fig. 3. Risk metrics in different periods

5. Conclusion

In this paper, we presented a scenario-based stochastic optimization model by considering the CVaR criterion. Motivated from shortcomings in the literature, our model considered an integrated dynamic MILP model for facility location with capacity expansion and different transportation modes in the supply chain network design. In the proposed model, the demand of customers, return ratio and quality of returns assumed to be uncertain. Finally, the performance and behaviour of the proposed models were investigated through numerical experiments. Computational results showed the strength of stochastic model in handling data uncertainty and controlling risk level under uncertainty. Considering other risk measures, such as minimum variance (Markowitz, [32]) and comparing the performance with the CVaR and VaR measures can be considered for future research.

Since the computational time increases significantly when the size of problem and the number of scenarios increased; introducing an efficient exact or heuristic solution methods is another need that can be covered in further studies.

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