A Mathematical Method for Managing Inventories in a Dual Channel Supply Chain

E. Teimoury[*], H. Mirzahosseinian & A. Kaboli

E. Teimoury, is assistant professor at the Department of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran
H. Mirzahosseinian, is an MSc student at the same Department and a member of Young Researcher Club, Tehran, Iran
A. Kaboli, .is a P.hD. student and Research assistant in Swiss Federal Institute of Technology, Lausanne, Switzerland

Keywords

ABSTRACT

The advent of e-commerce has prompted many manufacturers to redesign their traditional channel structure by engaging in direct sales. In this paper, we present a dual channel inventory model based on queuing theory in a manufacturer-retailer supply chain, consisting of a traditional retail channel and a direct channel which stocks are kept in both upper and lower echelon. The system receives stochastic demand from the both channel which each channel has an independent demand arrival rate. A lost-sales model which no backorder is allowed is supposed. The replenishment lead times are assumed independent exponential random variables for both warehouse and the retail store. Under the replenishment inventory policy, the inventory position is kept constant at a base-stock level. To analyze the chain performance, an objective function included holding and lost sales costs is defined. At the end, a proposed algorithm named, Best Neighborhood (BN) is used to find a good solution for inventory and the results are compared with Simulated Annealing (SA) solutions.

1. Introduction

The advent of information technology plays a remarkable role in reshaping supply chain behavior in the recent years. Started in the mid 1990's, internet has become an important retail channel. The commercial blossoming of the internet has introduced tremendous opportunities and has underscored the importance of effective supply management. [1]. Recognizing the great potential of the Internet to reach customers, many brand name manufacturers, including Hewlett-Packard, IBM, Eastman Kodak, Nike, and Apple, have added direct channel operations (see [2,3]).

Direct distribution enables companies to bring products to the market faster. Now, companies are hugely benefiting from the early-to-market advantage and making a significant profit margin by eliminating the retailer and distributors margins. It seems doubtful at first sight that selling goods by both retailer and manufacturer makes a profitable frame. Indeed, the competition between manufacturer and retailer could lead to channel conflict [4], pricing policy for different channels [5] and distribution strategies. However, considering the challenges posing in dual channels, examples of today's companies starting a direct channel when they already had a well-structured retail channel, such as IBM, Compaq, HP, and Sony, are great evidence to support the issue that supply chain must react in order to meet consumer expectation instead of insisting on the traditional methods. Many retailers and manufacturers have already learnt that meeting consumer expectation is a valuable area for sharing margins. Frazier [6] stated that such a mixed channel would increase the product's penetration level and revenue on one hand, but would lead to decreased support from the channel partners.

Dual-channel distribution may take many forms, one of which is when a manufacturer both sells through intermediaries and directly to consumers [7]. Chiang et al. [8] pointed out the most important economical reason for use of Multi-channel is reaching potential buyer segments that could not be reached by a single

[*]**Corresponding author:** *E. Teimoury*
E-mail: Teimoury@iust.ac.ir,
E-mail: ha_mirzahosseinian@ind.iust.ac.ir E-mail: amin.kaboli@epfl.ch

channel so that these channels can help to increase the market coverage. Also, they stated that the fundamental task in connection with the two-echelon inventory problem is to find the balance between the stock levels at the top and the bottom echelon.

Although several studies have examined dual channel supply chains, the study of multi-channel supply chain in the internet-enabled versus retail channel has appeared recently in the literature. Rhee and Park [9] study a hybrid channel design problem, assuming that there are two consumer segments: a price sensitive segment and a service sensitive segment. Chiang et al. [10] examine a price-competition game in a dual channel supply chain. Tsay and Agrawal [3] provide an excellent review of recent work in the area and examine different ways to adjust the manufacturer–reseller relationship. Although their results show that a direct channel strategy makes the manufacturer more profitable by posing a viable threat to draw customers away from the retailer, their focus is on channel competition and coordination issues in the setting where the upstream echelon is at once a supplier to and a competitor of the downstream echelon Chiang et al. [8]. Also, their results depend on the assumption that a customer's acceptance of online channel is homogeneous. Teimoury et al. [11] consider more cases than the Chiang et al. [8] case. In another excellent work, Boyaci [12] studies stocking decisions for both the manufacturer and retailer and assumes that all the prices are exogenous and demand is stochastic. In a similar setting, Cattani et al. [13] study pricing strategies of both the manufacturer and the retailer.

Many researchers extensively studied and discussed multi-echelon inventory control policies, but the inventory modeling literature offers little guidance in theoretical basis for multi-echelon dual-channel inventory problem. Our prime object in this paper is to focus on this issue. Also, in the inventory control models, the lost sale cost was considered the same in both channels while this assumption is not adequate with the real world problems in virtue of the customer's behavior complexity. In fact, the costumer's sensitivity of the internet-enabled channel is much more than the retail channel ones and the loyalty to a brand in virtual environments are lesser than the physical environment. So, it seems that the lost sale cost in direct channel is significant. In this paper, we consider the separate lost sale cost for each channel while in the previous models the lost sale cost is considered integrated. Furthermore, parametric analysis based on the model is conducted by varying the key parameters in the cost structure to obtain generalized results.

The rest of the paper is organized as follows: Section 2 sets up the dual channel supply chain model. We analyze the performance of the two-echelon dual-channel system in Section 3. Section 4 and 5 is devoted to SA and the proposed BN algorithm. Section 6 presents the numerical study. The computational results obtained by applying proposed algorithm (BN) and SA are discussed in Section 7. The conclusion is given in Section 8.

2. Dual Channel Model

Consider a two-echelon dual-channel supply system that consists of a manufacturer with a single warehouse at the top echelon and a retail store at the bottom echelon. The topology and product flows of the two-echelon dual-channel supply system are illustrated in Fig. 1.
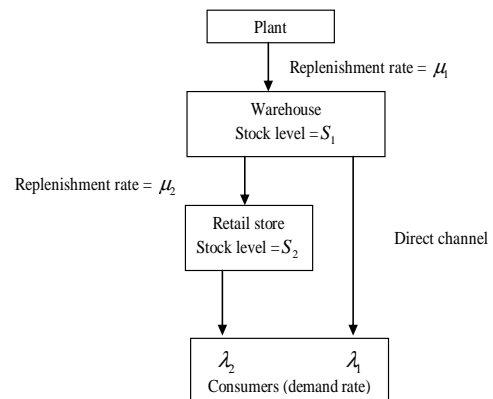

Fig. 1. Two-echelon dual channel inventory system.

Furthermore, we have the following set of assumptions for the inventory model:

The product is available for customers at both retail store and the internet-based direct channel. The system receives stochastic demand from two customer segments: those who prefer the traditional retail store and those who prefer the direct channel in which each segment has an independent demand arrival rate. In fact, the customers arrive at the retail store according to a Poisson process with constant intensity λ_2, orders placed through the direct channel are in accordance with a Poisson process with rate λ_1, and the total demand is elaborated as $\lambda = \lambda_1 + \lambda_2$. The demand of retail customers is met with the on-hand inventory from the bottom echelon while the demand in the internet-enabled channel is fulfilled through direct delivery with the on-hand inventory from the upper echelon. Also, customers are lost when both retail store and the manufacturer warehouse are out of stock simultaneously. The replenishment lead times are assumed independent exponential random variables for both the warehouse and the retail store with mean $(\mu_1)^{-1}$ and $(\mu_2)^{-1}$ respectively. A one-for-one replenishment inventory policy is applied. A customer served from back stock on-hand will trigger a replenishment order immediately by EDI (Electronic Data Interchange) therefore, the information lead time is assumed to be zero. Under this replenishment inventory policy, the inventory position is kept constant at a base-stock level. The base-stock levels at the warehouse and the

retail store are denoted by S_1 and S_2 respectively. We use the following notations in the state of equations and relations:

λ: Total demand.

λ_1: Customer arrival rate at the direct channel.

λ_2: Customer arrival rate at the retail store.

l_1: Opportunity cost of losing a customer at the direct channel.

l_2: Opportunity cost of losing a customer at the retail store.

μ_1: Manufacturer warehouse replenishment rate.

μ_2: Retail store replenishment rate.

S_1: Base-stock level at the manufacturer warehouse.

S_2: Base-stock level at the retail store.

x: On hand inventory at the manufacturer warehouse, $0 \leq x \leq S_1$.

y: On hand inventory at the retail store, $0 \leq y \leq S_2$.

π_{xy}: Steady-state probability.

h_1: Inventory holding cost per item per time unit at the manufacturer warehouse.

h_2: Inventory holding cost per item per time unit at the retail store.

C_H: The average inventory holding cost.

C_L: The average lost sales cost.

L_{S1}: The probability of out of stock at the manufacturer warehouse.

L_{S2}: The probability of out of stock at the retail store.

$L_{S1, S2}$: The probability of out of stock at both channels.

TC: Total cost.

2-1. The Markov Model

Based on the assumptions, the corresponding Markov model can be constructed with the state space (x, y). There are four events which lead to change at state: a customer arrives at the retail store, an order is placed through the direct channel, a replenishment order arrives at the manufacturer warehouse and a replenishment order arrives at the retail store. The transition diagram of the proposed model is as shown in Fig. 2 to verify the balance equations and a better understanding of the system.

Let π_{xy} be the steady-state probability that x items is on-hand at the manufacturer warehouse and y items are on-hand at the retail store. The flow balance equations that require for all states input and output rates to each state are equal are given by:

$$\pi_{xy}\left[\lambda_2(A)+\lambda_1(B)+(S_1-x)\mu_1+\left[(S_2-y)\mu_2\right](B)\right]=\pi_{(x,y+1)}\left[\lambda_2(C)\right]$$
$$+\pi_{(x+1,y)}\left[\lambda_1(D)\right]+\pi_{(x-1,y)}\left[\mu_1(S_1-x+1)(B)\right]$$
$$+\pi_{(x+1,y-1)}\left[\mu_2(S_2-y+1)(E)\right] \text{ for } x=0,1,...,S_1 \text{ and } y=0,1,...,S_2 \tag{1}$$

$$A=\begin{cases}1 & if \quad y>0 \\ 0 & otherwise,\end{cases} \tag{2}$$

$$B=\begin{cases}1 & if \quad x>0 \\ 0 & otherwise,\end{cases} \tag{3}$$

$$C=\begin{cases}1 & if \quad y<S_2 \\ 0 & otherwise,\end{cases} \tag{4}$$

$$D=\begin{cases}1 & if \quad x<S_1 \\ 0 & otherwise,\end{cases} \tag{5}$$

$$E=\begin{cases}1 & if \quad 0<y\leq S_2 \text{ and } 0\leq x<S_1 \\ 0 & otherwise,\end{cases} \tag{6}$$

The left hand side of Eq. (1) represents the average transitions from state (x, y). The first two terms in the bracket specifies the transitions due to receiving demand. Specifically, A and B state whether the retail store and the manufacturer warehouse are out of stock or not, respectively. The last two terms in the bracket are the rates at which in-transit replenishment order arrive at the manufacturer warehouse and the retail store respectively.

Also the right hand side of Eq. (1) represents the average transitions into state (x, y). The first two terms denote the transitions due to satisfying demand by inventory on–hand at the retail store and the manufacturer warehouse, correspondingly. The last two terms indicate the transitions due to receiving in–transit replenishment orders from the manufacturer and the retail store, respectively.

C and D state whether the stock in retail store is lower than S_2 and the stock in the manufacturer warehouse is lower than S_1, respectively.
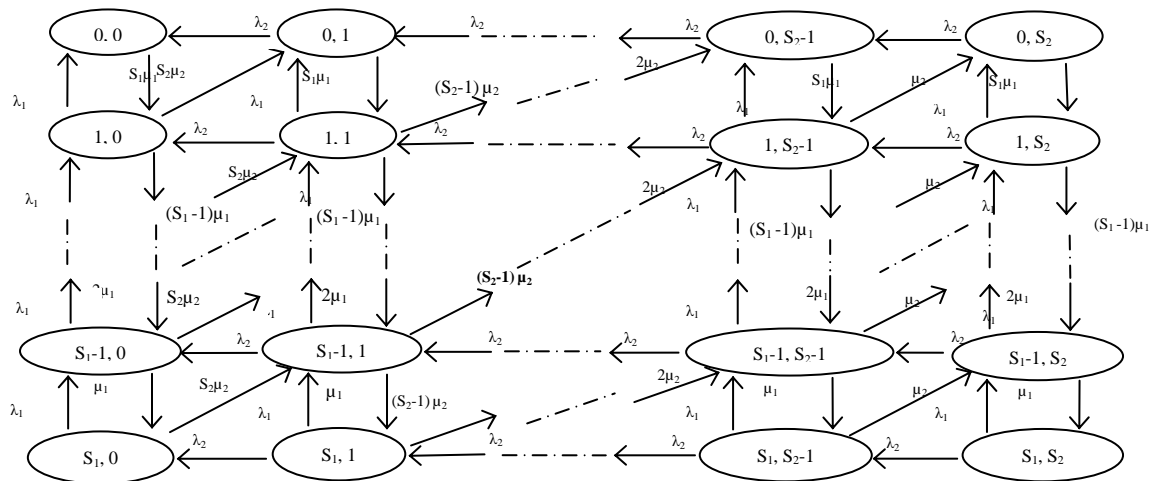

Fig. 2. The transition diagram of the proposed model.

Besides, E states whether the stock in retail store is bigger than zero and smaller than or equal to S_2 and stock in the manufacturer warehouse is bigger than or equal to zero and smaller than S_1, respectively. To find the steady-state probabilities, we can solve the corresponding system of linear equations which contain the balance equations given in Eq. (1) and the normalizing constraint:

$$\sum_{x=0}^{S_1}\sum_{y=0}^{S_2} \pi_{xy} = 1 \tag{7}$$

3. Analysis of Dual Channel model

In this section, we analyze the performance of the two-echelon dual-channel system. Two different operational cost factors are considered: the average inventory holding cost and average lost sales cost. We begin with describing the holding cost.

3.1. Average Inventory Holding Cost

Given the steady-state probabilities, the average inventories for the manufacturer warehouse and the retail store can be modeled in finite horizon, respectively as

$$I_1 = \sum_{x=1}^{S_1}\sum_{y=0}^{S_2} x\pi_{xy} \tag{8}$$

$$I_2 = \sum_{x=0}^{S_1}\sum_{y=1}^{S_2} y\pi_{xy} \tag{9}$$

Let h_1 and h_2 be the inventory holding cost incurred by the firm per item unit at the manufacturer warehouse and the retail store, respectively. Then, the average inventory holding cost, C_H, is represented by

$$C_H = h_1 I_1 + h_2 I_2 \tag{10}$$

The first item of C_H states the inventory holding cost from the manufacturer warehouse and the second item specifies the inventory holding cost from the retail store.

3.2. Average Lost Sale Cost

Recall that when a stock-out occurs in either channel it result in lost sales. Moreover, customers are lost when both retail store and the manufacturer warehouse are out of stock simultaneously. The probabilities that a stock-out occurs only at the manufacturer warehouse and only at the retail store, respectively, depends upon the steady-state probabilities in the following way:

$$L_{S_1} = \sum_{y=1}^{S_2} \pi_{0y} \tag{11}$$

$$L_{S_2} = \sum_{x=1}^{S_1} \pi_{x0} \tag{12}$$

$$L_{S_{1,2}} = \pi_{00} \tag{13}$$

Which $L_{S1,\,S2}$ denote the probability that both channel are simultaneously out of stock. Now, assume that the opportunity cost of losing a customer in the direct channel and the retail store channel are l_1 and l_2 per customer, respectively. Then, we can specify the total average lost sales cost (C_L) as:

$$C_L = L_1\lambda_1\sum_{y=1}^{S_2}\pi_{0y} + L_2\lambda_2\sum_{x=1}^{S_1}\pi_{x0} + (l_1\lambda_1 + l_2\lambda_2)\pi_{00} \tag{14}$$

3.3. Total Cost

To evaluate the performance of the two-echelon dual-channel inventory system, the sum of the inventory holding cost and the lost sales cost are used. Therefore, the total cost is elaborated as $C_H + C_L$. Note that the only decision variables are the base-stock level, S_1 and S_2. So, we have total cost as a function of S_1 and S_2, which is:

$$Tc(S_1,S_2) = \left[h_1\sum_{x=1}^{S_1}\sum_{y=0}^{S_2} x\pi_{xy} + h_2\sum_{y=1}^{S_2}\sum_{x=0}^{S_1} y\pi_{xy} + l_1\lambda_1\sum_{y=1}^{S_2}\pi_{0y} + l_2\lambda_2\sum_{x=1}^{S_1}\pi_{x0} + (l_1\lambda_1 + l_2\lambda_2)\pi_{00} \right] \tag{15}$$

We intend to find the base-stock levels that minimize $Tc=(S_1, S_2)$.

4. Simulated Annealing

Metropolis et al. [14] proposed an algorithm to simulate the evolution of a solid in a heat bath until it reached its thermal equilibrium. The Monte Carlo method was used to simulate the process, which started from a certain thermodynamic state of the system, defined by a certain energy ever and temperature. Then, the state was slightly perturbed. If the change in energy produced by this perturbation was negative, the new configuration was accepted.

If it was positive, it was accepted with a probability given by $e^{-\Delta T / kT}$, where k is the so-called Boltzmann constant, which is a constant of nature that relates temperature to energy [15]. This process is repeated until a frozen state is achieved [16,17]. Thirty years after the publication of Metropolis' approach, Kirkpatrick et al. [18] and Cerny [19] independently pointed out the analogy between this "annealing" process and combinatorial optimization.

These researchers indicated several important analogies: a system state is analogous to a solution of the optimization problem; the free energy of the system (to be minimized) corresponds to the cost of the objective function to be optimized; the slight perturbation1 imposed on the system to change it to another state corresponds to a movement into a neighboring position (with respect to the local search state); the cooling schedule corresponds to the control mechanism adopted by the search algorithm; and the frozen state of the system corresponds to the final solution generated by the search algorithm (using a population size of one). These important analogies led

to the development of an algorithm called "Simulated Annealing".

The SA has two inside and outside loops. The inside loop controls the achievement to equilibrium in the current temperature and outside loop controls the rate of temperature decrease Fig 3. Its parameters are as follows:

EL (Epoch Length), number of accepted solutions in each temperature

T_0, initial temperature

α, rate of the current temperature decrease

X_0, a feasible solution

$F(X)$, the value of objective function for X

---

1. Select an initial (feasible) solution (X_0), $X_b=X_0$, $X=X_0$

2. Select an initial temperature $T_0 > 0$, $T=T_0$

3. Repeat
    Repeat
        Randomly select a neighborhood of X (X_n)
    $\delta = F(X_b) - F(X_n)$
    If $\delta < 0$ then $X_b = X_n$, $X=X_n$
    Else
        Generate random R (uniform distribution in the range $(0, 1)$)
        If $R < exp(-\delta/T)$ then $X= X_n$
    Until max. number of iterations EL reached
    $T = (\alpha \times T)$

4. Until stopping condition is met

5. Print X_b

Fig. 3. Pseudo-code of SA.

5. Proposed Algorithm

Our proposed algorithm named, Best Neighborhood (BN) begins its process with the primary random solution; in each process by checking all neighborhood points, it moves toward the point that makes the most decrease on the objective function amount. This procedure is continued while there is not any neighborhood point of the existing point to make a lower amount of the objective function.

6. Numerical Study

In this section we generated ten test problems by randomly choosing a value for each parameter from the given interval in Table 1.

Tab. 1. The based characteristics of the system.

Base Parametric Values for Sport Shoes Company	Quantities
System-related parameters:	
Manufacturer warehouse replenishment rate	$\mu_1 \in$ (10, 30)
Retail store replenishment rate	$\mu_2 \in$ (5, 20)
Customer-related parameters:	
Customer arrival rate at the direct channel	$\lambda_1 \in$ (20, 40)
Customer arrival rate at the retail store	$\lambda_2 \in$ (5, 20)
Cost-related parameters:	
Opportunity cost of losing a customer at the direct channel	$l_1 \in$ (500, 1500)\$
Opportunity cost of losing a customer at the retail store	$l_2 \in$ (300, 1000)\$
Inventory holding cost per item per time unit at the warehouse	h_1 (5, 75)\$/year
Inventory holding cost per item per time unit at the retail store	h_2 (10, 90)\$/year

7. Computational Results

This section presents the results obtained by the proposed algorithm, Best Neighborhood (BN), and Simulated Annealing (SA). All computational results described in this section are produced by a Matlab code on a conventional Pentium V 3 GHz computer with 512Mb. Both algorithms start from a point that average inventory holding cost is equal to average lost sales cost. For solving the examples with Simulated Annealing (SA), we set the SA parameters as follows: EL=50, T0=10, α=0.02. Also, the Algorithm is stopped when the temperature becomes less than 0.1. Table 2 summarizes the comparative results for solving the 10 test problems.

Tab. 2. SA and BN performances for test problems.

Problem	Solution		Objective Function		CPU Time(s)	
	BN	SA	BN	SA	BN	SA
1	(7, 3)	(7, 3)	721.99	721.99	1.07	0.35
2	(6, 5)	(6, 5)	817.11	817.11	1.01	0.52
3	(12, 7)	(12, 7)	113.51	113.51	1.12	0.26
4	(10, 5)	(10, 5)	251.26	251.26	1.12	0.43
5	(9, 6)	(9, 6)	129.07	129.07	1.14	0.69
6	(11, 4)	(11, 4)	158.67	158.67	1.09	0.26
7	(9, 7)	(9, 7)	531.28	531.28	1.07	0.49
8	(8, 6)	(8, 6)	329.54	329.54	1.04	0.15
9	(7, 4)	(7, 4)	706.01	706.01	1.07	0.36
10	(14, 7)	(14, 7)	138.93	138.93	1.21	0.96

Columns 1 and 2 present the solution values found by the tested algorithms, their objective functions are included in columns 3 and 4, and their computational time is shown in columns 5 and 6. As it is obvious both algorithms have the same solution and objective function. But the performance of SA is better than BN, in CPU time.

7.1. Parametric Analysis

In this section, we conduct a parametric analysis to study the effect of lost sales cost on stock levels in both echelons. We use the values shown in Table 3 for parameters.

Tab. 3. The based characteristics of the system.

Base Parametric Values for Sport Shoes Company	Quantities
System-related Parameters:	
Manufacturer warehouse replenishment rate	$\mu_1 = 20$
Retail store replenishment rate	$\mu_2 = 10$
Customer-related Parameters:	
Customer arrival rate at the direct channel	$\lambda_1 = 30$
Customer arrival rate at the retail store	$\lambda_2 = 10$
Cost-related Parameters:	
Opportunity cost of losing a customer at the direct channel	$l_1 = 1000\$$
Opportunity cost of losing a customer at the retail store	$l_2 = 600\$$
Inventory holding cost per item per time unit at the warehouse	$h_1 = 75\$/year$
Inventory holding cost per item per time unit at the retail store	$h_2 = 80\$/year$

In the first diagram, the lost sales cost in retailer channel is assumed to be constant then the effect of lost sales cost in direct channel variation on stock levels is studied. Please see the Fig. 4. In Fig.5, the lost sales cost in direct channel is assumed to be constant and the effect of lost sales cost for retailer channel variation on stock levels is studied. Finally, In Fig.6, the effect of lost sales cost in both channels simultaneous variation on stock levels is studied.
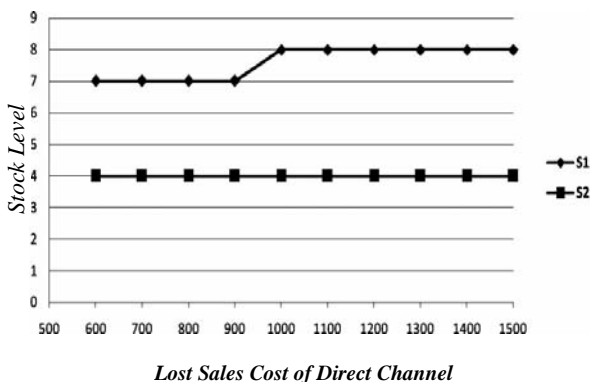
Lost Sales Cost of Direct Channel

Fig. 4. Effect of lost sales cost in direct channel variation on stock levels.

Lost Sales Cost of Retailer Channel

Fig. 5. Effect of lost sales cost in retailer channel variation on stock levels.
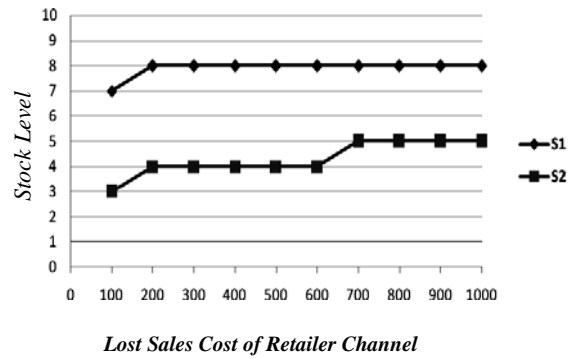
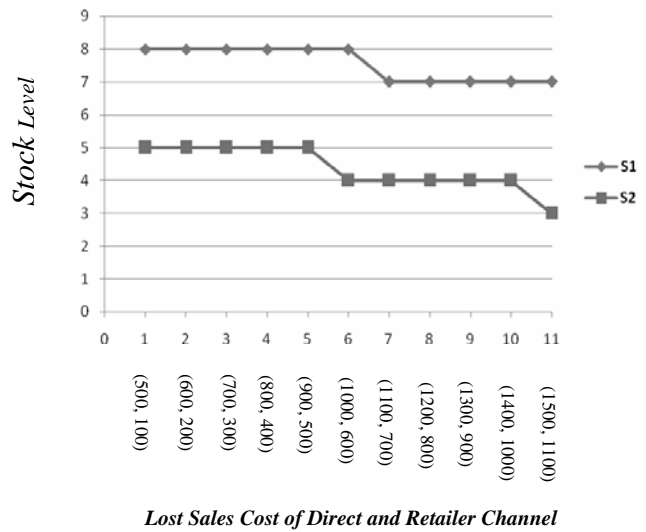*Lost Sales Cost of Direct and Retailer Channel*

Fig. 6. Effect of lost sales cost in both channel simultaneous variations on stock levels.

8. Conclusion

In the inventory control models, the lost sale cost was considered the same in the both channel while this assumption is not adequate with the real world problems in virtue of the customer's behaviors complexity.

Separating the lost sale cost in both channels would make a significant effect on the inventory rate in each level of the chain. Also, it could be helped to recognize that each level with the more lost sale cost would try to hold more inventories in comparison to another level. In this article, the authors tried to analyze and illustrate these remarkable notes; the analytic results and numerical examples show up the efficacy of the proposed approach.

References

[1] Yue, X., Liu, J., *"Demand Forecast Sharing in a Dual-Channel Supply Chain*," European Journal of Operational Research, Vol. 174, 2006, pp. 646–667.

[2] Wilder, C., *HP's Online Push,* Information Week, 1999.

[3] Tsay, A., Agrawal, N., *"Channel Conflict and Coordination in the E-Commerce Age*," Production and Operations Management, Vol. 13, 2004, pp. 93–110.

[4] Webb, K.L., *"Managing Channels of Distribution in the Age of Electronic Commerce*," Industrial Marketing Management, Vol, 31, 2002, pp. 95–102.

[5] Tang, F.F., Xing, X., *"Will the Growth of Multi-Channel Retailing Diminish the Pricing Efficiency of the Web*," Journal of Retailing , Vol. 77, 2001, pp. 319–333.

[6] Frazier, G.L., "*Organizing and Managing Channels of Distribution,"* J Acad Mark Sci, Vol. 27, 1999, pp. 226–241.

[7] Preston, L.E., Schramm, A.E., "*Dual Distribution and its Impact on Marketing Organization*," California Management Review , Vol. 8 , 1965, pp. 59–69.

[8] Chiang, W.K., Monahan, G.E., "*Managing Inventories in a Two-Echelon Dual-Channel Supply Chain*," European Journal of Operational Research, Vol.162, 2005, pp. 325-341.

[9] Rhee, B., Park, S.,"*Online Store as a New Direct Channel and Emerging Hybrid Channel System*", Working Paper, The Hong Kong University of Science & Technology,1999.

[10] Chiang, W.K., Chhajed, D., Hess, J.D., *"Direct Marketing, Indirect Profits: A Strategic Analysis of Dual-Channel Supply Chain Design,"* Management Science, Vol.49, 2003, pp.1-20.

[11] Teimoury, E., Mirzahosseinian, H., Kaboli, A., *"Managing Inventories in a Dual Channel Supply Chain*," Proceedings of the International Conference of Iranian Operations Research Society, 2008.

[12] Boyaci, T., *"Competitive Stocking and Coordination in a Multiple-Channel Distribution System*," Working paper, McGill University, 2004.

[13] Cattani, K., Gilland, W., Heese, S., Swaminathan, J., "*Boiling Frogs: Pricing Strategies for a Manufacturer Adding an Internet Channel*," Production and Operations Management, Vol. 15, 2006, pp. 40–56.

[14] Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A., Teller, E., *"Equation of State Calculation by Fast Computing Machines,"* Journal of Chemical Physics, Vol. 21, 1953, pp. 1087-1092.

[15] Flannery, B.P., Teukolsky, S.A., Vetterling, W.T., *Numerical Recipes in Pascal,* Cambridge University Press, Cambridge, UK, 1989.

[16] Dowsland, K.A., *Simulated Annealing. In C. R. Reevees,* editor, Modern Heuristic Techniques for Combinatorial Problems, John Wiley & Sons, 1993.

[17] Sait, S.M., Youseff, H., Ali, H., *"Fuzzy Simulated Evolution Algorithm for Multi-objective Optimization of VLSI Placement*," Congress on Evolutionary Computation, Washington D.C., USA.1999. pp. 91-97.

[18] Kirkpatrick, S., Gellatt, C., Vecchi, M., *"Optimization by Simulated Annealing,"* Science, Vol. 220, 1983, pp. 671-680.

[19] Cerny, V., *"A Thermo dynamical Approach to the Traveling Salesman Problem: An Efficient Simulation Algorithm,"* Journal of Optimization Theory and Applications, Vol.45, 1985, pp. 41-51.