

# Optimization Models for a Deteriorating Single Server Queuing Production System

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## KEYWORDS

Single server queue,  
Maintenance,  
stochastic model,  
Gradual deterioration,  
Production planning

## ABSTRACT

*In this paper a single server queuing production system is considered which is subject to gradual deterioration. The system is discussed under two different deteriorating conditions. A planning horizon is considered and server which is a D/M/1 queuing system is gradually deteriorates through time periods. A maintenance policy is taken into account whereby the server is restored to its initial condition before some distinct periods. This system is modeled to obtain optimal values of arrival rates and also optimal maintenance policy which minimizes production, holding and maintenance costs and tries to satisfy demands through time periods. The model is also considered to control customers' sojourn times. For each deteriorating condition a model is developed. Models are solved by GA based algorithms and results for a sample are represented.*

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## 1. Introduction

In a global competitive market, manufacturing looks for ways to reduce costs due to machine failures. In fact, in many companies, production planning is widely linked with machine maintenance. There exists a wide literature addressing the issue of production planning and an equally broad literature tackling maintenance planning questions.

Production planning models typically seek to balance the costs of system setups with the costs of production and materials holding, while maintenance models typically attempt to balance the costs and benefits of sound maintenance plans in order to optimize the performance of the production system.

In both domains, issues of production and maintenance modeling have experienced an evident success both from theoretical and applied viewpoints. Paradoxically the issue of combining production and maintenance plans has received much less attention. The large part

of the production planning models assume that the system will function at its maximum performance during the planning horizon, and the large part of the maintenance planning models disregard the impact of maintenance on the production capacity and do not explicitly consider the production requirements. Actually, apart from the preventive maintenance actions that can be scheduled during down times, any unplanned maintenance action disturbs the production plan.

It is therefore crucial that both production and maintenance aspects of a production system to be concurrently considered during the elaboration of optimal production and maintenance plans.

Although there are some literature about integration of production and maintenance planning, almost majority of them consider production planning at the operational level which consists of scheduling, such as Sortakul (2007) and Yulan et al (2007) and Pan et al (2009). Hence there are poor literature related to integrated production and maintenance planning at the tactical level.

Now a brief literature review of the integrated production and maintenance domain is given.

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Srinivasan and Lee (1996) examined the integrated effects of preventive maintenance and production with inventory policies on the operating costs of the production unit.

Wienstein and Chung (1999) presented an integrated production and maintenance planning model to resolve the contradictory objectives of system reliability and profit maximization. By their work, First an aggregate production plan is generated, and then a master production schedule is developed to minimize the weighted deviations from the indicated aggregate production goals.

Finally, work center loading requirements, determined through rough cut capacity planning, are used to simulate machine failures during the aggregate planning horizon. Rezg et al (2004) proposed optimization problem by considering preventive maintenance and inventory control in a production line made up of  $N$  machines. Cormier et al (2009) developed a mathematical modeling framework for simultaneously generating production plans to minimize maintenance cost which consists of preventive replacements and corrective replacements and production costs such as holding costs and shortage costs. They considered that demand is constant over an infinite planning horizon.

Boukas and Haurie (1990), Gharbi and Kenne (2000) and Kenne and Gharbi (2001) are some of the papers that used Markov models to consider ordering of the production flow and preventive maintenance. Sarker and Haque (2000) executed a simulation of a production system with a random failure. Iravani and Duenyas (2002) presented an integrated maintenance with repair and production with inventory model using a Markov decision process. Kenne and Gharbi (2004) studied the stochastic optimization of a problem of production control with corrective and preventive maintenance. They proposed a method to find the optimal age of preventive maintenance and production rates for a production system composed of identical machines.

Cassady and Kutanoglu (2005) presented a model that integrates maintenance planning and production scheduling of a single machine. Aghezzaf et.al (2007) proposed a model that has integrated production and maintenance planning on a capacitated production system throughout a specified finite planning horizon to find an integrated lot-sizing and preventive maintenance strategy of the system that satisfies the demand for all items over the entire horizon without backlogging, and which minimizes the expected sum of production and maintenance costs.

They considered that the production system is subject to random failures, and that any maintenance action carried out on the system, in a period, reduces the system's available production capacity during that period.

Allen et al (2007) proposed a model that can be applied to schedule maintenance activities in support of the

operation of manufacturing and production plans. It was considered a multi-component system that optimizes both cost and reliability simultaneously.

Aghezzaf (2008) proposed a model to integrate production planning and preventive maintenance as a nonlinear mixed-integer program where each production line implements a cyclic preventive maintenance policy.

Kenne et al (2008) proposed the production planning problem of a stochastic manufacturing system consisting of one machine producing one part type with control on preventive and corrective maintenance rates. They developed the stochastic optimization model of the considered problem with three decision variables: production rate, preventive and corrective maintenance rates and two state variables were included, age of the machine and stock.

Yang et al (2009) considered the preventive maintenance of unreliable single server queues with a multi-state deteriorating server subject to random shocks. They presented the system size distribution and sojourn time distribution. Also they analyzed the proposed maintenance policy based on the cost, considering holding cost and repair cost. Maintenance, deteriorating systems, and shock models have been extensively researched in the literature like Wang (2002). Kaufman and Lewis (2007) considered a single server queue with a multi-state deteriorating server. They modeled the deterioration of a server as a Markov chain and analyzed an optimal maintenance structure, using a semi-Markov decision process.

Njike et al (2009) proposed the interaction between defective products and optimal control of production rate, lead time and inventory to minimize the expected overall cost due to maintenance activities, inventory holding and backlogs. They considered two maintenance conditions of a machine controlled by two decision variables: production and maintenance rates.

A set of papers introduced dynamic models of manufacturing systems with operational failure and partially observable defective quality states, Kim and Gershwin (2005), Gershwin and Kim (2005) and Gershwin and Kim (2008). Hajej et al. (2009) proposed combinatorial model between production and maintenance plan for a manufacturing system satisfying a random demand in order to establish an optimal production planning and scheduling maintenance strategy.

They considered the influence of the production rate on the degradation degree with a constrained stochastic production maintenance planning problem under hypotheses of inventory and failure rate variables to minimize the average total holding, production and maintenance costs. Noureifath et al. (2010) combined preventive maintenance with tactical production planning in multi-state systems. In order to satisfy the demand for all products over the entire horizon they determined an integrated lot-sizing and preventive maintenance strategy of the system in a manner that the

sum of preventive and corrective maintenance costs, setup costs, holding costs, backorder costs, and production costs are minimized. Recently, Nodem et al. (2011) proposed a model based on semi-markov decision process which is to find the optimal production, repair/replacement and preventive maintenance policies for a deteriorating manufacturing system. They used stochastic dynamic programming method to obtain the optimality conditions. Zied et al. (2011) acquired combined plans of production and maintenance, for a manufacturing system which is to satisfy a random demand.

They first minimized the average total inventory and production cost and acquired an optimal production plan. Using this production plan, they established an optimal maintenance schedule which minimized maintenance cost. Najid et al. (2011) considered integration of production and maintenance in a manner that production lot sizes of various items are determined and preventive maintenances are planned.

Their model took into account reliability of the production line and demand shortages. To tackle with this problem they proposed a linear mixed integer model. In this paper some new models are introduced in which production, inventory and maintenance decisions are taken. The models are also set to control queuing variables of the system.

This model can be applied to assembling lines and synchronized manufacturing based systems. The general purpose of this paper is to develop an integrated production and maintenance model which considers queuing theory. The proposed models are to determine arrival rate of customers to production line and maintenance policy which minimize the expected total production and maintenance costs over a finite planning horizon. The model takes into account the fact that the production system deteriorates consecutively.

Two different deteriorating conditions, that is, deterioration with constant rate and stochastic deterioration, are discussed. Determining arrival rates of customers and combining production planning and maintenance issues in addition to considering a control on customers' sojourn times are contributions of current paper. In the next section, we present assumptions and notation that are employed for the development of the models.

Then the model is defined and derived. In Sections 3 solution methodologies are proposed to solve the models. An illustrative numerical example and final concluding remarks are given in the subsequent sections.

## 2. Problem Formulation

We consider a single server queuing production system being deteriorated to inferior states in progress of time periods. The discipline of the system is First-Come-First-Served (FCFS). It is assumed that planning

horizon consists of  $T$  time periods and demand in each period is constant and known upfront. Customers or parts arrive at the system with definite arriving times and with rate  $\lambda$ .

The server operates in several maintenance states; the initial state of the server is one which is the best condition of the server and server fails at state  $\beta$ . The maintenance states are arranged according to the extent of deterioration of the system. We consider this production unit under two different deteriorating conditions: i) deterioration due to aging and, ii) deterioration due to aging and random shocks. In the first deteriorating condition it's assumed that at the beginning of the production process, the machine is in its best operating condition, i.e. the state of the server is 1, after a period of time and in response to deterioration of the server the state of the system shifts to the next state that is state 2.

Generally, if operating condition of the system at current period is in state  $s$ , in the next period the system will be in state  $s+1$  (Fig. 1). The gradual deterioration of the server deals with a set of maintenance states as  $S = \{1, 2, \dots, \beta\}$ .

In the second deteriorating condition both gradual deterioration and deterioration imposed by random shocks are taken into account: i) from the current operating state  $s$  to the next inferior operating state ( $s + 1$ ) with transition probability  $p_s$ , due to aging, or ii) from the current operating state  $s$  to the failure state  $F$  with transition probability  $q_s = 1 - p_s$ , due to a random shock (Fig. 2).

Since state  $\beta - 1$  is followed by break down,  $q_{\beta-1}$  is always 1. We assume that the deterioration of the production unit is continuously monitored so that the current operating state of the system can be detected.

The service times are represented by independent exponential random variables with rate  $\mu_s$ , where  $s$  denotes the operating state of the server. According to the conditions of the server the service times are arranged in an increasing order as  $\mu_1 > \dots > \mu_{\beta-1}$ . Since at  $\beta$  server fails,  $\mu_\beta$  is irrational and in order to model the system,  $\mu_\beta$  is replaced with zero. It's clear that in each period we deal with a D/M/1 model.

To preserve the system in an acceptable condition, a preventive maintenance policy is performed. During the planning horizon and before some distinct periods a maintenance operation is executed. After each maintenance operation the operating state of the server is restored to its initial state. Maintenance operations are executed out of working shifts.

For each deteriorating condition a model is proposed. The system under the first deteriorating condition is formulated as a non-linear model and under the second deteriorating condition as a non-linear and stochastic model. Models are solved to obtain optimal constant arrival rates of customers and optimal maintenance policy among all time periods.

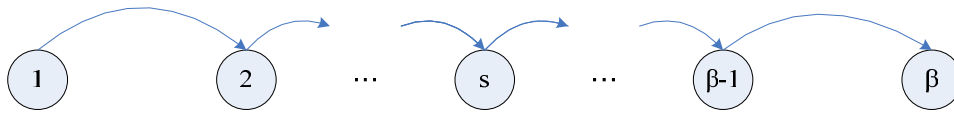


Fig. 1. Representation of gradual deterioration with constant rate

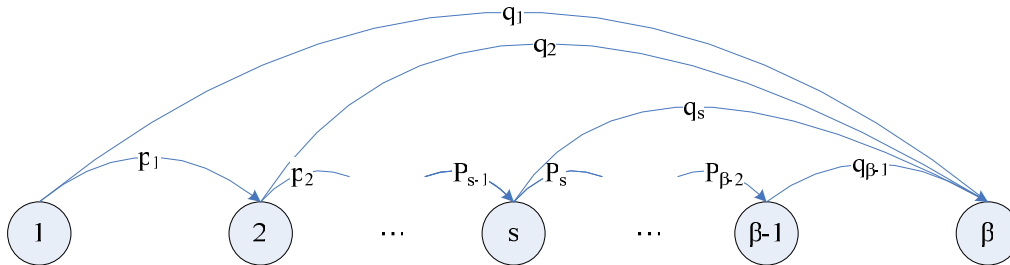


Fig. 2. Representation of stochastic gradual deterioration

The objective of the proposed models is to find optimal values of decision variables being minimized total cost of production process through planning horizon. In order to reduce the fluctuations of arrival rates in consecutive periods a fixed cost is incurred when an arrival rate changes from one period to the following period.

Since the proposed models are defined with a production planning approach, the finished products should satisfy predefined demands of periods, in this way the models are solved to acquire optimal arrival rates of customers and maintenance policy which minimize the total production costs through the planning horizon and try to satisfy demands of periods. In some production and assembling lines and especially in synchronized manufacturing based systems a steady flow of materials with less WIP is of great importance, the proposed models help the system to be fed with an optimal steady rate and in the best operating condition.

Besides balancing the production and assembling lines the obtained steady rate aids the overall system to perform through a reasonable cost.

In order to model the mentioned system and without considering deteriorating conditions, we need to calculate the customers' sojourn time. According to the relations in queuing systems and since this D/M/1 queuing system is a subset of general G/M/1 queuing systems the sojourn time of the system can be calculated as:

$$W = \frac{1}{\mu(1 - x_0)} = \frac{1}{\mu(1 - 1.231\rho^2 + 0.21\rho - 0.002)} = \frac{1}{0.998\mu - \frac{1.23\lambda^2}{\mu} + 0.21\lambda} \tag{5}$$

In the following paragraphs proposed models are described. At first a model without considering maintenance policy is presented then this model is developed to encompass maintenance considerations. These models are non-linear in some constraints. The

$$W = \frac{1}{\mu(1 - x_0)} \tag{1}$$

In which  $\mu$  is the rate of exponentially distributed service time and  $x_0$  is solution of the following characteristic equation:

$$x = e^{-\frac{(1-x)}{\rho}} \tag{2}$$

$\rho$  is utilization factor and is defined as follows:

$$\rho = \frac{\lambda}{\mu} \tag{3}$$

The calculation of  $W$  is related to the solution of equation (1-2). In view of the fact that parametric solution of (1-2) is not possible; an approximate solution of (1-2) is given. A curve fitting approach is employed and characteristic equation (1-2) is approximated with the following relation:

$$x = 1.231\rho^2 - 0.21\rho + 0.002 \tag{4}$$

For different values of  $\rho$  and by a numerical method, equation (1-2) has been solved several times. Then a polynomial regression has been employed and equation (1-4) is output of this procedure. For every  $x$  and  $\rho$  from equation (1-4), equation (1-2) is also held, and vice versa. According to the approximate solution of (1-2)  $W$  is then as follows:

first model which we call it the “basic model” lies under the following assumptions:

1. Sojourn times must not exceed a predefined quantity;
2. Service time is a predefined constant parameter;

3. The inventory holding unit cost is constant and assumes the same value for all periods;
4. The capacity of the queue is neglected;
5. The cumulative available productive capacity during the planning period T meets the total amount of demand;
6. A penalty cost is incurred when an arrival rate varies from one period to the next one;
7. Customers should be served in the same period that they come in.

The proposed model is able to simultaneously identify the following set of variables:

- The optimal arrival rates of customers in any time period  $t$  belonging to a planning horizon of time  $T$  (e.g. a year);
- The optimal inventory level in any  $t$ .

The previously defined variables are determined by minimizing the total production and holding costs.

The model is as follows:

$$\text{Min} \sum_{t=1}^T C y_t + \sum_{t=0}^T h I_t + \sum_{t=1}^T p_t a_t \lambda_t \quad (6)$$

s.t.

$$a_t t \lambda_t + I_{t-1} = d_t + I_t \quad \forall t \quad (7)$$

$$\frac{1}{0.998\mu - \frac{1.23\lambda_t^2}{\mu} + 0.21\lambda_t} \leq U_w \quad \forall t \quad (8)$$

$$\frac{\lambda_t}{\mu} \leq 1 \quad \forall t \quad (9)$$

$$y_t \geq |\lambda_{t+1} - \lambda_t| \quad \forall t \quad (10)$$

$$y_t \in \{0,1\} \quad (11)$$

$$\lambda_t, I_t \geq 0 \quad \forall t \quad (12)$$

Where:

$t = 1, \dots, T$  Unit period of time along the planning horizon  $T$ ;

$\lambda_t$  Arrival rate of customers in  $t$ ;

$I_t$  Storage quantity at the end of the time period  $t$ ;

$I_0$  starting storage quantity

$y_t$  1 if  $\lambda$  varies from  $t$  to  $t+1$ , 0 otherwise;

$\mu$  Service rate;

$C$  Fixed penalty cost of variations in arrival rates;

$H$  Unit storage cost which refers to  $t$ . If  $t$  is one month, the cost is the monthly unit storage cost;

$U_w$  Maximum allowed sojourn time;

$p_t$  Production unit cost in time period  $t$ ;

$a_t$  Available hours in a day for time period  $t$ ;

$t_t$  Available days in time period  $t$ ;

$d_t$  Demand in time period  $t$ ;

The objective function is composed of three contributions:

1. Total amount of penalty costs incurred by variations in arrival rates;
2. Total storage cost;
3. Total amount of production cost.

Constraint (1-7) guarantees the satisfaction of demand in each time period with production and inventory quantities. Constraint (1-8) confines sojourn time to an upper limit. (1-9) is a queuing constraint which controls the queue length. This constraint can be ignored considering constraint (1-8). With constraint (1-10) a penalty cost is incurred when consecutive periods are of different arrival rates.

Now, the previous model is developed to consider the first deteriorating condition. Prior to mathematical formulation of model following assumptions should be considered:

1. Initial state of the server is 1 and server fails at state  $\beta$ ;
2. Maintenance operations are performed before periods and out of working shifts;
3. After a maintenance operation state of the server is restored to its initial condition.
4. Service rates are state dependant and are arranged into an increasing order, these service times are predefined parameters;
5. Sojourn times must not exceed a predefined quantity;
6. The inventory holding unit cost is constant and assumes the same value for all periods;
7. The capacity of the queue is neglected;
8. The cumulative available productive capacity during the planning period T meets the total amount of demand;
9. A penalty cost is incurred when an arrival rate varies from one period to the next one;
10. Customers should be served in the same period that they come in.

The proposed model by determination of following set of variables attains a solution:

- The optimal arrival rates of customers in any time period  $t$  belonging to a planning horizon of time  $T$  (e.g. a year);
- Number and locations of maintenance operations during the planning horizon.

- The optimal inventory level in any t.

These variables are determined by minimizing the total production, holding and maintenance costs.

The model is:

$$Min \sum_{t=1}^T C y_t + \sum_{t=0}^T h I_t + \sum_{t=1}^T p_t a_t \lambda_t + \sum_{t=2}^T A y_{t,0} \quad (13)$$

$$s.t. \quad a_t \lambda_t + I_{t-1} = d_t + I_t \quad \forall t \quad (14)$$

$$\left( \frac{1}{0.998\mu_s - \frac{1.23\lambda_t^2}{\mu_s} + 0.21\lambda_t} \right) y_{t,s} \leq U_w \quad \forall t, s \quad (15)$$

$$\frac{\lambda_t}{\mu_s} \leq 1 \quad \forall t, s \quad (16)$$

$$y_t \geq |\lambda_{t+1} - \lambda_t| \quad \forall t \quad (17)$$

$$y_{1,1} = 1 \quad (18)$$

$$\sum_s y_{t,s} = 1 \quad \forall t \quad (19)$$

$$y_{t,s} \leq y_{t+1,s+1} + y_{t+1,1} \quad \forall t, s \quad (20)$$

$$y_{t+1,s+1} \leq y_{t,s} \quad \forall t, s \quad (21)$$

$$y_t, y_{t,s} \in \{0,1\} \quad \forall t, s \quad (22)$$

$$\lambda_t, I_t \geq 0 \quad \forall t \quad (23)$$

Where:

- $S=0, \dots, \beta$  Operating states of the server;
- A Fixed cost of preventive maintenance operation;
- $\mu_s$  Service rate in operating state s;
- $y_{t,s}$  1 if operating state of the server in time period t is s, 0 otherwise;

Other variables and parameters are the same with the previously introduced ones. Since server fails at state  $\beta$ ,  $\mu_\beta$  is an irrational value. In order to simulate the situation where server fails  $\mu_\beta$  is replaced with a zero.

The newly added contribution to the objective function is total amount of maintenance costs. A maintenance cost is incurred just before period t if operating condition of server is restored to state one in period t (i.e.  $y_{t,1}=1$ ).

Constraint (1-15) restricts sojourn time to an upper limit when server is in state s and in period t. (1-18)

states that at the beginning of the first period server is in its best condition.

In each period a server can only operate in just one state (constraint (1-19)). There are just two acceptable shifts in operating state of the server, shift caused by gradual deterioration of the server (from s to s+1) and shift caused by performing a maintenance operation (from s to 1), Constraints (1-20) and (1-21) control this assumption. Remaining constraints are the same with previously introduced ones. The basic model is now developed to consider the second deteriorating condition.

This is a stochastic model and is defined under the same assumptions of the previous model. Decision variables and objective function of the proposed model are identical with the previously mentioned ones of the preceding model.

This system under the second deteriorating condition can be formulated as follows:

$$Min \sum_{t=1}^T C y_t + \sum_{t=0}^T h I_t + \sum_{t=1}^T p_t a_t \lambda_t + \sum_{t=2}^T A y_{t,1} \quad (24)$$

$$s.t. \quad a_t \lambda_t + I_{t-1} = d_t + I_t \quad \forall t \quad (25)$$

$$\left( \frac{1}{0.998\mu_s - \frac{1.23\lambda_t^2}{\mu_s} + 0.21\lambda_t} \right) y_{t,s} \leq U_w \quad \forall t, s \quad (26)$$

$$\frac{\lambda_t}{\mu_s} \leq 1 \quad \forall t, s \quad (27)$$

$$y_t \geq |\lambda_{t+1} - \lambda_t| \quad \forall t \quad (28)$$

$$y_{1,1} = 1 \quad (29)$$

$$\sum_s y_{t,s} = 1 \quad \forall t \quad (30)$$

$$y_{t,s} \leq y_{t+1,s+1} + y_{t+1,1} + (1-z_s)M \quad \forall t, s \quad (31)$$

$$y_{t+1,s+1} \leq y_{t,s} + (1-z_s)M \quad \forall t, s \quad (32)$$

$$y_{t,s} \leq y_{t+1,\beta} + y_{t+1,1} + z_s M \quad \forall t, s \quad (33)$$

$$y_t, y_{t,s} \in \{0,1\} \quad \forall t, s \quad (34)$$

$$\lambda_t, I_t \geq 0 \quad \forall t \quad (35)$$

In which M is a great quantity and  $z_s$  is a state dependent binary random variable with the following probability distribution:

$$P(z_s) = p_s^{z_s} q_s^{(1-z_s)} \quad z_s \in \{0,1\} \quad (36)$$

$p_s$  is transition probability of state  $s$  and  $q_s = 1 - p_s$ . Other variables and parameters are the same with the previously introduced ones and  $\mu_\beta$  is replaced with zero.

With constraints (1-31), (1-32), (1-33) and relation (1-36) only acceptable shifts in consecutive periods occur.

We assumed that with probability  $p_s$  a shift from current state to the following state of the system occurs and also according to relation (1-36) with probability  $p_s$  binary variable  $z_s$  takes one.

With  $z_s = 1$ , constraints (1-31) and (1-32) will be active in the model, these two constraints determine the operating state of the system in the next period that is  $one$  or  $s+1$ .

At the other hand with probability  $q_s$  a failure will occur after state  $s$  and also according to relation (1-36) with probability  $q_s$  binary variable  $z_s$  takes zero.

When  $z_s$  takes zero constraints (1-31) and (1-32) will be inactive and instead constraint (1-33) is considered. Constraint (1-33) controls the state shifts of the system when a shock is imposed to it.

### 3. Solution Methodology

In this section with regard to each proposed model a solution approach is introduced. Since proposed models are non-linear, GA based solution approaches are employed to solve them. These solution approaches will be described in the following paragraphs.

#### 3-1. Model Under the First Deteriorating Condition

In order to solve this model a simple GA based approach is employed. The main objective regarding to the solution of this model as discussed earlier is to find optimal arrival rates and inventory levels of time periods and also an optimal combination of preventive maintenances which minimizes total cost of the production system. In this manner and in order to reduce the system size, the problem is divided into two independent sections.

To do so, decision variables are not dealt in a parallel manner, instead and initially a random feasible maintenance policy is generated. A maintenance policy is called feasible if there is no maintenance operation at last period of planning horizon and the distance between two consecutive maintenance operations is lesser than or equal to the maximum number of maintenance states.

In fact the later assumption indicates that a maintenance operation should be performed before occurrence of a failure. A randomly generated maintenance policy for a ten-period planning horizon has been shown in Fig. 3. In this figure each element is representative of a time period and each 1 is

representative of a preventive maintenance operation. For example in this case in periods 2, 4, 6 and 8 we have preventive maintenances.



Fig. 3. A randomly generated maintenance policy

After generating a feasible maintenance policy service rates of time periods according to the generated policy should be calculated.

In this case suppose that there are 3 maintenance states and service rates pertaining to these states are  $\mu_1$ ,  $\mu_2$  and  $\mu_3$ . As a matter of this fact that system fails at state three,  $\mu_3$  is replaced with zero. Hence the sequence of service rates for our sample is acquired as Fig. 4.

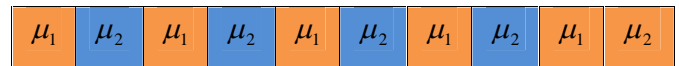


Fig. 4. Sample for sequence of service rates

According to Fig. 4 the service rate in which production system operates in period 1 is  $\mu_1$ , in period 2 is  $\mu_2$  and so on.

Thereupon, the main problem is changed to a problem without considering maintenance parameters. In fact this procedure helps the model to get rid of maintenance parameters and decision variables. Next step is to find solution of the simplified model. In this step and in order to attain an acceptable maintenance policy, the basic model is simplified again. Without considering nonlinear constraints, the basic model is solved for all enumerations of maintenance policies. Eliminating nonlinear constraints increases solution space, therefore no potential solution would be ignored. The simplified model can be solved with one of several commercial linear programming solvers.

Eventually the maintenance policy being followed by minimum total cost among all maintenance policies is considered as final solution of current step. Then with regard to this maintenance policy the main model including nonlinear constraints is solved with genetic algorithm.

#### 3-2. Model Under the Second Deteriorating Condition

The procedure which is employed to solve the second model is also applied through two independent parts. At first a simulation base procedure is employed to simplify the main model and then a genetic algorithm is applied to the simplified model. The difference that makes us to call this model a stochastic model lies under this fact that maintenance state variations and also failures occur according to some predefined probabilities. In other words any change in

maintenance states occurs with uncertainty. In this manner and in order to determine the most probable sequence of failures a simulation base procedure is executed and according to the output of the simulation, the main model is simplified. At first, number of simulation runs is determined ( $N$ ). In each run and according to the predefined probabilities of failures a maintenance policy is generated. In order to generate a maintenance policy, starting from the first period and in each run of simulation a constantly distributed random number is produced for each time period. A failure will be imminent in the next period if the generated random number of a period is greater than given transition probability of that period.

Hence, in order to prevent an undesirable failure a preventive maintenance is scheduled in that period. At first a random number is generated for the first period and the state of the next period is specified. Before generating a random number for second period, transition probabilities of time periods should be updated. Since transition probabilities are state dependant, operating state of a period determines the pertinent transition probability of that period. The act of simulation is repeated for  $N$  times. The output of each run is a string like Fig. 3 in which ones are representatives of maintenance operations. As the number of  $N$  increases some of the failure patterns occur recurrently.

These failure patterns are most probable cases. In order to obtain these cases, strings are arranged according to their frequencies in a decreasing manner. Cases which stand on top of the list are the most probable failure patterns. Some of these patterns are derived from the list and a simplified deterministic model is made from each pattern. A genetic algorithm is applied to each model and maintenance policy being followed by minimum total cost among all derived maintenance policies is considered as final solution. The implementing flow of the proposed procedure can be summarized as follows;

Step 1. Set the number of simulation runs ( $N$ );

Step 2. For  $i=1$  to  $N$

For  $j=1$  to number of periods

a) Calculate service rate of period  $j$  (determine  $s(j)$ , that is, state of period  $j$ )

b) If  $\text{Rand}(0,1) \geq P_{s(j)}(j)$   
A preventive maintenance is scheduled in  $j$ ,

End

Save maintenance policy  $i$ ;

End

Step 3. Determine frequencies of the generated maintenance policies;

Step 4. Choose some of the most repeated maintenance policies;

Step 5. Make a simplified model according to each chosen policy;

Step 6. Execute a genetic algorithm for each simplified model and save the results;

Step 7. Return the GA outputs of the maintenance policy which has the least fitness function.

#### 4. Computational Results

This section describes the computational tests which are used to evaluate the performance of the presented algorithms. Previously proposed algorithms are applied to a sample problem with 10 periods and 5 maintenance states. In this sample,  $C$ ,  $H$ ,  $U_w$ ,  $P_t$ ,  $a_t$ ,  $t_t$ , and  $A$  are respectively set to 500, 50, 0.5, 10, 8, 20 and 1000 for all time periods. Remaining required information are according table 1. The algorithms have been coded in MATLAB and implemented on a personal computer with 2.8 GHz CPU and 256 MB RAM.

Tab. 1. Sample parameters

Maintenance state	Service rate (per hour)	period	demand	period	demand
1	10	1	1200	6	2000
2	9.8	2	1500	7	1900
3	9.6	3	1400	8	1600
4	9.4	4	1600	9	1200
5	0	5	1900	10	1400

Model under the first deteriorating condition and according to described parameters is solved, and resulted solution is as table 2.

Tab. 2. Results of first model

period	Arrival rate	Storage quantity
1	9.99	2261.1
2	9.78	2326.7
3	9.99	2526.6
4	9.79	2493.5
5	9.79	2193.3
6	9.99	1792.8
7	9.99	1491.2
8	9.99	1491
9	2.63	711.5
10	4.3	0

Starting storage quantity=1861.7  
Total cost= 1106361.68

The maintenance policy by which GA is applied to the model is as table 3. Ones are representatives of maintenance operations. It's obvious that arrival rates and storage quantities are integer values, hence the results should be rounded in a way that total cost is minimized.



**Tab. 3. Maintenance policy of first model**

period	1	2	3	4	5	6	7	8	9	10
maintenance	0	1	0	1	1	1	1	0	1	0

In addition to the previously mentioned parameters a transition probability array as table 4 is defined too and model under the second deteriorating condition is solved.

**Tab. 4. Transition probability array**

state	1	2	3	4	5
probability	0.7	0.6	0.4	0.8	0.6

Simulation size is set to 1000 and five policies of most frequented ones are chosen. Models simplified according to these five policies are solved through GA. The results are as table 5 and 6.

**Tab. 5. Results of second model**

period	Arrival rate	Storage quantity
1	9.99	2287.2
2	9.79	2354.2
3	9.99	2553.2
4	9.79	2519.7
5	9.6	2155.7
6	9.99	1754.5
7	9.79	1421.6
8	9.59	1356.2
9	3.47	711.5
10	4.3	0

Starting storage quantity=1887.2  
Total cost= 1095713.04

**Tab. 6. Maintenance policy of second model**

period	1	2	3	4	5	6	7	8	9	10
maintenance	0	1	0	0	1	0	0	0	1	0

It's necessary to mention that arrival rates and storage quantities are integer values, hence the results should be rounded in a way that total cost is minimized.

## 5. Conclusion

We examined a D/M/1 queue with a multi-state deteriorating server subject to gradual deterioration and a preventive maintenance policy. System is considered under two different deteriorating conditions, that is, deterioration with constant rate and stochastic deterioration. Since, in most assembling lines and synchronized manufacturing based systems a constant and optimal flow rate of materials is needed, a model was proposed for each deteriorating condition whereby optimal definite arrival rates of parts(customers) are

determined. In order to preserve the system in an appropriate condition, the model was developed to consider a maintenance policy. The developed models are also able to determine number and locations of preventive maintenance operations during the planning horizon.

The proposed models settle the introduced variables through minimizing total cost of production, holding and maintenance in order to satisfy predefined demands of time periods. In addition to production, holding and maintenance costs a penalty cost will be incurred if the arrival rates vary from one period to the next one. Another feature which has been considered in this model is ability to control sojourn times of customers during planning horizon. A GA based algorithm is proposed for each model. Finally an illustrative example is given to clarify the solution procedures.

For future research, control on sojourn times can be applied by introducing another objective function. System's functionality can be increased by minimizing both system total cost and customers' sojourn times. MODM approaches are applicable to these models.

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