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A Bayesian Approach for Recognition of Control Chart Patterns

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KEYWORDS

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Pattern Recognition,
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ABSTRACT

Control chart pattern (CCP) recognition techniques are widely used to identify the potential process problems. Recently, artificial neural network (ANN)–based techniques are popular for this problem. However, finding the suitable architecture of an ANN-based CCP recognizer and its training process are time consuming and the obtained results are not interpretable. To facilitate the research gap, this paper presents a simple statistical approach for detecting and identifying control chart patterns. In this method, by taking new observations on the quality characteristic under consideration, the Maximum Likelihood Estimator of pattern parameters is first obtained and then the Beliefs on each pattern is determined. Then using Bayes' rule, Beliefs are updated recursively. Finally, when the amount of a derived statistic falls outside the calculated control interval a pattern recognition signal is issued. The advantage of this approach comparing with other existing CCP recognition methods is that it has no need for training. Simulation results show high accuracy and satisfactory speed of the proposed method.

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1. Introduction

Control chart pattern (CCP) recognition is one of the important tools in statistical process control (SPC) to identify process problems. The observed variation of quality characteristics generally results from either common causes or assignable causes. Common causes are considered to be due to the inherent nature of normal process and assignable causes occur when the process has been changed in materials, machines, operators etc. Assignable causes result in the unnatural variation to the process, which should be identified and eliminated as soon as possible. A normal (N) pattern indicates that the process is operating under natural variation. Unnatural variations are signaled mainly by exhibition of five types of unnatural patterns, i.e., increasing trend (IT), decreasing trend (DT), upward shift (US), downward shift (DS) and cyclic (C) [1], as shown in Fig. 1. There are also other types of unnatural patterns which are either special forms of basic CCPs or mixed forms of two or more basic CCPs.

Recognition of unnatural patterns is a crucial task in SPC for identifying underlying root causes. Traditionally, control chart patterns have been analysed and interpreted manually. Over the years, numerous supplementary rules, like zone test or run rules have been developed to assist the quality practitioners [1-4]. However simultaneous application of several rules could result in excessive numbers of false alarms.

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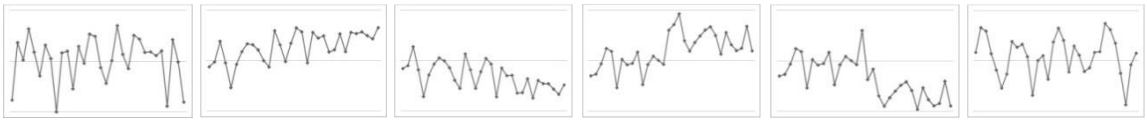
Furthermore, due to the natural random variation in the process, a run would still have a low probability of occurrence. The third problem of using run rules is that the abnormal change of a process that is being looked for can be common to more than one pattern, so identification of the unnatural patterns require considerable experience and skill. Therefore, run rules are not very effective for pattern recognition in control charts [5].

To improve and facilitate the task of CCP recognition, various kinds of expert systems (ES) have been proposed [6-10]. The advantage of a rule-based expert system is that it contains explicit rules that if required, can be modified and updated easily. However, there are also the problems of false or missing alarm by using the expert system to recognize control chart patterns [18].



(a) Normal (b) Increasing Trend (c) Decreasing Trend (d) Upward Shift (e) Downward Shift (f) Cycle

Fig. 1. Six types of basic CCPs

Recently artificial neural networks have been popular for the problem of CCP recognition [11-18]. One disadvantage with ANNs is that the process information of the neural networks is implicit and virtually inaccessible to users. In addition, network topology is complex and correspondent training process is time-consuming.

Some researchers, presented statistical CCP recognizers [19, 22]. For instance, some utilized correlation analysis to develop a control chart pattern recognition tool [20-22]. However this approach does not present a justifiable performance for practical situations.

In this research, we propose a new simple statistical approach for CCP recognition to address the issues of ANN-based systems. By using this approach, the problem of network topological complexity and correspondent training time is eliminated.

The presented statistical mechanism is based on *Decision on Beliefs* (DOB) [23]. In this approach the CCP recognition is looked as a decision making problem to select one member of the alternatives set. The adopted approach of DOB, has already experienced solving some other statistical problems as *Response Surface methodology* [23], *Quality Control* [24] and *Distribution Fitting* [25]. In all the cases, adapted DOB algorithms outperform all the best common ones in many aspects.

Before the proposed method is described, four research assumptions should be emphasized.

1. Only univariate quality characteristics are used.
2. Only one CCP exists at any given time.
3. When the process is in-statistical-control, the quality characteristic of interest follows a Gaussian distribution with mean μ and variance σ^2.
4. The common-cause standard deviation (σ) is kept constant.

Although assumption 4 must be recognized as a significant simplifying assumption, it provides a close approximation to many types of practical manufacture. The factors that cause changes in process standard deviation tend to be long term compared with the length of the production cycle between resetting events.

The rest of the paper is organized as follows: Section 2, explains the proposed approach in detail. Subsection 2.1 introduces the procedure to estimate patterns parameters. In subsection 2.2, the measure of *Belief* and the recursive method to its improvement is presented. The stopping criterion is explained in 2.3 and the algorithm of method comes in 2.4. Section 3 contains the performance evaluation of the proposed scheme using simulation experiments. Finally, the discussion and conclusions appear in sections 4 and 5 recursively.

2. The Proposed Method

The proposed method looks at the CCP recognition as a decision making problem to select one member of the alternatives set. To do this we assume a quality characteristic is given to be monitored by means of $\bar{X}$ control chart. Although the control chart pattern is unknown, we know it belongs to a candidate set $S = \{P_i; i = 1, 2, 3, 4\}$ where the values of i represent normal, trend, shift and cycle respectively. So the objective is to identify the control chart pattern from the set of four candidate patterns. The presented method selects the candidate with the greatest belief when the stopping criterion is met. At each iteration, we need to calculate the beliefs of the members of S. To calculate and update the beliefs, after obtaining each new observation, we estimate the pattern parameters using the process observations. Using estimated pattern parameters, we attempt to eliminate the effect of considered pattern assignable cause to reach normal distributed data. Then we can update the beliefs on patterns by using the *Bayes' Rule* to calculate the posterior beliefs from the prior ones. After updating the beliefs, by identifying the maximum

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belief, and according to the criterion of stopping condition that will be introduced later, we either signal the maximum belief's regarding pattern or decide to continue and try a new observation.

2.1. Estimation of Patterns Parameters

In first step of the presented method we need to estimate the patterns parameters. We want to obtain the Maximum Likelihood Estimator (MLE) of patterns parameters using several recently gathered process observations. For the sake of simplicity, we assume only one single observation is gathered at each iteration. First, we introduce a required definition.

Definition 1. Let the outcome of *t*th process observation be denoted by x_t. Then, after t observations, we call $O_t = (x_1, x_2, \dots, x_t)$ as *t*th *observations vector*.

The equations along with the corresponding parameters of the six basic CCPs are given in Tab. 1.

Tab. 1. Equations of control chart patterns

CCP	CCP parameter (s)	CCP equation
N	• Mean (μ) • Standard deviation (σ)	$x_t = r_t$
IT, DT	• Slope (k)	$x_t = r_t + kt$; for DT, $k < 0$
US, DS	• Magnitude (s)	$x_t = r_t + s$; for DS, $s < 0$
C	• Amplitude (A) • Period (T)	$x_t = r_t + Asin(2\pi t/T)$; $T > 0$

Note: t is discrete time point at which the pattern is sampled (t = 1,2, ...), r_t is random value of a Gaussian distribution with μ and σ^2 at t^{th} time point, and x_t is sample value at t^{th} time point.

Using equations mentioned in Tab. 1, in case of each pattern, by subtracting the pattern part from the observation data, the remainder value is a Gaussian random variable with known parameters μ and σ^2, as presented in Eq. (1-4):

$$r_{1t} = x_t \quad (1)$$

$$r_{2t} = x_t - kt \quad (2)$$

$$r_{3t} = x_t - s \quad (3)$$

$$r_{4t} = x_t - Asin(2\pi t/T) \quad (4)$$

By knowing the distribution of r_{it}, parameters *k*, *s*, *A* and *T* can be estimated using a point estimation technique like maximum likelihood estimation. MLE of parameters k, s and A ($\hat{k}$, $\hat{s}$ and $\hat{A}$) has been determined in Eq. (5-7) as follows:

$$\hat{k} = 2(\bar{x} - \mu)/(t+1) \quad (5)$$

$$\hat{s} = \bar{x} - \mu \quad (6)$$

$$\hat{A} = \sum_{i=1}^{t} \sin\left(2\pi i/\hat{T}\right)(x_i - \mu) / \sum_{i=1}^{t} \sin^2\left(2\pi i/\hat{T}\right) \quad (7)$$

Where $\bar{x}$ is the sample mean. To determine $\hat{A}$, first we need to calculate $\hat{T}$ by solving Eq. (8).

$$\sum_{i=1}^{t} icos\left(2\pi i/\hat{T}\right)\left[(x_i - \mu) - \frac{\sum_{i=1}^{t} sin\left(2\pi i/\hat{T}\right)(x_i - \mu)}{\sum_{i=1}^{t} sin^2\left(2\pi i/\hat{T}\right)} \times sin\left(2\pi i/\hat{T}\right)\right] = 0 \quad (8)$$

A numerical search procedure by testing some discrete values (e.g. $4, 5, \dots, 20$) for $\hat{T}$ is used to solve Eq. (8). The value which minimizes the absolute value of Eq. (8) is the solution of this equation.

2.2. Updating the Beliefs

At each iteration of data gathering process beliefs are calculated and updated using a recursive equation based on *Bayes' Rule*. The measure of *belief* is defined as follows.

Definition 2. Assume we have the tth observations vector O_t. The belief on P_i to be the desired pattern, on the basis of the information obtained from the observations up to this point (O_t), is defined as:

$B_i(O_t) := Pr\{CCP \equiv P_i | O_t\}, \ \ i = 1, 2, 3, 4.$

After $(t-1)$ observations, let assume the decision is to continue. At iteration *t*, after taking a new observation x_t, our aim is to improve beliefs based on the observations vector O_{t-1} and the new observation x_t. Let $B_i(x_t, O_{t-1})$, denotes the belief on P_i. Now let $B_i(x_{t-1}, O_{t-2})$ denotes the prior belief on P_i, the posterior belief $B_i(O_t)$ can be determined as follows.

$$\begin{aligned} B_i(O_t) &= B_i(x_t, O_{t-1}) = Pr\{P_i | x_t, O_{t-1}\} \\ &= Pr\{P_i, x_t, O_{t-1}\}/Pr\{x_t, O_{t-1}\} \\ &= Pr\{P_i, O_{t-1}\}Pr\{x_t | P_i, O_{t-1}\}/Pr\{x_t, O_{t-1}\} \\ &= Pr\{O_{t-1}\}Pr\{P_i | O_{t-1}\}Pr\{x_t | P_i, O_{t-1}\}/Pr\{x_t, O_{t-1}\} \\ &= Pr\{O_{t-1}\}Pr\{P_i | O_{t-1}\}Pr\{x_t | P_i, O_{t-1}\}/ \\ &[Pr\{O_{t-1}\}Pr\{x_t | O_{t-1}\}] \\ &= Pr\{P_i | O_{t-1}\}Pr\{x_t | P_i, O_{t-1}\}/Pr\{x_t | O_{t-1}\} \\ &= Pr\{P_i | O_{t-1}\}Pr\{x_t | P_i\}/Pr\{x_t | O_{t-1}\} \end{aligned}$$

Thus by applying the Bayes' Rule, $B_i(O_t)$ can be determined by the following recursive equation.

$$B_i(O_t) = \frac{Pr\{P_i | O_{t-1}\}Pr\{x_t | P_i\}}{\sum_{j=1}^{4} Pr\{P_j | O_{t-1}\}Pr\{x_t | P_j\}} = \frac{B_i(x_{t-1}, O_{t-2})Pr\{x_t | P_i\}}{\sum_{j=1}^{4} B_j(x_{t-1}, O_{t-2})Pr\{x_t | P_j\}} \quad (9)$$

In Eq. (9), we need to calculate $Pr\{x_t | P_i\}$. When pattern P_i exists in the process then it is concluded that

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variables r_{it} are independent Gaussian variables with parameters μ and σ^2. Hence by assuming the existence of P_i, we have:

$$Pr\{r_{it}|P_i\} = \left(1/\sqrt{2\pi}\,\sigma\right)e^{-(r_{it}-\mu)^2/2\sigma^2} \qquad (10)$$

2.3. The Stopping Criterion

At each iteration, beliefs are updated and when one of the updated beliefs falls outside the calculated control interval a pattern recognition signal is issued. To calculate the control limits, assume two greatest beliefs are $B_{g1}(O_t)$ and $B_{g2}(O_t)$. Thus we have:

$$Pr\{r_{g1,t}|P_{g1}\} = \left(1/\sqrt{2\pi}\,\sigma\right)e^{-(r_{g1,t}-\mu)^2/2\sigma^2}$$

$$Pr\{r_{g2,t}|P_{g2}\} = \left(1/\sqrt{2\pi}\,\sigma\right)e^{-(r_{g2,t}-\mu)^2/2\sigma^2}$$

Then by defining the statistic Z_t

$$Z_t = \frac{B_{g1}(O_t)}{B_{g2}(O_t)} \qquad (11)$$

Without loss of generality, we assume μ and σ equal to 0 and 1 respectively. Hence:

$$Z_t = \frac{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(r_{g1,t})^2}}{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(r_{g2,t})^2}} \times \frac{B_{g1,t-1}}{B_{g2,t-1}}$$
$$= e^{-\frac{1}{2}(r_{g1,t})^2+\frac{1}{2}(r_{g2,t})^2} \times \frac{B_{g1,t-1}}{B_{g2,t-1}}$$

So the recursive equation will be:

$$Z_t = e^{-\frac{1}{2}(r_{g1,t})^2+\frac{1}{2}(r_{g2,t})^2} Z_{t-1}$$

Hence:

$$\frac{Z_t}{Z_{t-1}} = e^{-\frac{1}{2}(r_{g1,t})^2+\frac{1}{2}(r_{g2,t})^2}$$

In other words:

$$Ln\frac{Z_t}{Z_{t-1}} = LnZ_t - LnZ_{t-1} = -\frac{1}{2}(r_{g1,t})^2 + \frac{1}{2}(r_{g2,t})^2$$

When the process is in state of normal pattern (in-statistical-control), MLEs of all pattern parameters will be approximately equal to zero and therefore all r_{it} approximately follow standard normal distribution and random variables $(r_{g1,t})^2$ and $(r_{g2,t})^2$ follow a chi-square distribution with one degree of freedom. Thus we have:

$$Ln\frac{Z_t}{Z_{t-1}} = LnZ_t - LnZ_{t-1} = -\frac{1}{2}(r_{g1,t})^2 + \frac{1}{2}(r_{g2,t})^2$$
$$\cong -\frac{1}{2}(\chi_1^2(1) - \chi_2^2(1))$$

So

$$LnZ_t - LnZ_{t-1} \cong -\frac{1}{2}\left(\chi_1^2(1) - \chi_2^2(1)\right)$$
$$LnZ_{t-1} - LnZ_{t-2} \cong -\frac{1}{2}\left(\chi_1^2(1) - \chi_2^2(1)\right)$$
$$\vdots$$
$$LnZ_1 - LnZ_0 \cong -\frac{1}{2}\left(\chi_1^2(1) - \chi_2^2(1)\right)$$

Since $\chi^2(\alpha) + \chi^2(\beta) = \chi^2(\alpha+\beta)$, by summing the above equations we have

$$LnZ_t - LnZ_0 \cong -\tfrac{1}{2}(\chi_1^2(t) - \chi_2^2(t))$$

For the initial values of $B_i(O_0)$; for $i = 1,2,3,4$ it is assumed that they have equal values 0.25 hence
$Z_0 = \frac{B_{g1}(O_0)}{B_{g2}(O_0)} = 1$
Thus we have

$$LnZ_t \cong -\frac{1}{2}\left(\chi_1^2(t) - \chi_2^2(t)\right) \qquad (12)$$

Now from Eq. (12) we have

$$LnZ_t \cong -\tfrac{1}{2}\left(\frac{\chi_1^2(t)-t}{\sqrt{2t}} - \frac{\chi_2^2(t)-t}{\sqrt{2t}}\right) \times \sqrt{2t}$$

So

$$\frac{-2LnZ_t}{\sqrt{2t}} \cong \left(\frac{\chi_1^2(t)-t}{\sqrt{2t}} - \frac{\chi_2^2(t)-t}{\sqrt{2t}}\right) \qquad (13)$$

Since random variables $\chi_1^2(t)$ and $\chi_2^2(t)$ have mean equal to t and variance equal to *2t*, thus for sufficiently large *t* according to the Central Limit Theorem it is concluded that random variables $\frac{\chi_1^2(t)-t}{\sqrt{2t}}$ and $\frac{\chi_2^2(t)-t}{\sqrt{2t}}$ follow standard normal distribution. In other words, $\left(\frac{\chi_1^2(t)-t}{\sqrt{2t}} - \frac{\chi_2^2(t)-t}{\sqrt{2t}}\right)$ follows a normal distribution with mean 0 and variance 2Therefore, the standard control charting limits for this random variable, will be $(-\sqrt{2}c, +\sqrt{2}c)$.
Where *c* (a nonnegative value) is the parameter of control chart. Now from Eq. (13) it is concluded that random variable $\frac{-2LnZ_t}{\sqrt{2t}}$ should be in the interval $(-\sqrt{2}c, +\sqrt{2}c)$ to be accepted as an in control variable. So we have

$$\frac{-2LnZ_t}{\sqrt{2t}} \in \left(-\sqrt{2}c, +\sqrt{2}c\right)$$

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Hence
$Z_t \in \left(e^{-\sqrt{t}c}, e^{+\sqrt{t}c}\right)$

If Z_t falls out of this control interval, we conclude that pattern *g1* exists in the process.
When no abnormal pattern exists in the process, the MLEs related to each abnormal pattern will be approximately zero and the values of residuals for all patterns will be approximately equal.
Consequently the values of$B_{g1}(O_t)$and$B_{g2}(O_t)$will be approximately the same and the values of Z_t will be approximately one and it never falls out of the control limits. Hence if for a predetermined value (q) of sample observations, they all fall in the control limits, we conclude that no abnormal pattern exists in the process. The parameters *c* and *q* are adjusted through experiment.

2.4. The Algorithm of Proposed Method

In the proposed approach, at each iteration of the data gathering process, a recursive equation is used to update the values of $B_i(O_t)$.
Then, whenever the stopping condition is satisfied, a pattern recognition signal is issued and the procedure is terminated. Steps of the proposed method are as the following algorithm.
Step 0: Start with equal values for Beliefs ($B_{1,0} = B_{2,0} = B_{3,0} = B_{4,0} = 0.25$).

Step 1: Receive a new observation.

Step 2: Using the recent *t* observations and according to Eq. (5-8), calculate the MLEs.

Step 3: For each pattern, using obtained MLEs from step 2, determine the residuals, r_{it} (Eq. (1-4)).

Step 4: For each pattern, using r_{it}, calculate and update the beliefs using $B_{i,t} = \frac{B_{i,t-1}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(r_{it})^2}}{\sum_{j=1}^{4} B_{j,t-1}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(r_{jt})^2}}$.

Step 5: By finding $B_{g1,t}$ and $B_{g2,t}$ (the greatest belief and the second maximum belief), calculate Z_t.

Step 6: If *t* is smaller than *q* and Z_t is in the control interval $(e^{-\sqrt{t}c}, e^{+\sqrt{t}c})$, go to step 1.

Step 7: If *t* is smaller than *q* and Z_t is not in the control interval $(e^{-\sqrt{t}c}, e^{+\sqrt{t}c})$, then conclude that the unnatural pattern P_{g1} exists and finish the process. If g equals to 2 or 3, then look at the sign of the related MLEs and decide whether the pattern is increasing (upward) or decreasing (downward). For example if $g1 = 2$ and $\hat{k} = -0.085$, conclude that decreasing trend exists in the process..

Step 8: If *t* is greater than *q*, the process is in state of normal. (*c* and *q* are constants and the best combination of them could be determined based on type I and type II errors using simulation experiments. We set the values of *c* and *q* equal to 0.9 and 50 respectively).
The diagram of presented method is shown in Fig. 2.

3. Performance Analysis

Generally, a CCP recognition method has two performance indices: detecting the underlying pattern as soon as possible, and classifying the type of the detected pattern as accurately as possible. That is, shorter pattern average run length (PARL) and higher pattern recognition accuracy (RA) are the two main goals of CCP recognition. Besides out-of-control PARL (recognition speed) and RA (recognition accuracy), the recognition speed stability (i.e. the standard deviation σ of the pattern run length) is also considered an important criterion for CCP recognition in this study, because it is helpful to spot the starting point of the detected CCP. Therefore, this study employed three performance indices (RA, out-of-control PARL, and σ of the pattern run length) to evaluate the proposed statistical CCP recognition method.
In order to verify the performance of the presented approach, a large amount of testing data were generated by Monte Carlo method, which included six basic control chart patterns described in section "Introduction".
We used MATLAB® 2008 to simulate the performance of method. The sample numbers in our simulations was chosen 100. We implemented 1000 runs for each pattern and the proportion of test-set examples that were classified correctly to the total samples was calculated as estimation of RA.
RA of presented method is introduced through confusion matrix in Tab. 2.
It can be seen that the six basic patterns have been recognized accurately. There is a small tendency for the trend and shift patterns to be confused with each other. For instance the presented recognizer misclassifies about 0.54% of the increasing trend patterns as upward shift patterns.
Trends and shifts of different directions could be detected equally, regardless of the value of c. This fact shows the directional invariance property of proposed method.
Results for classification of normal patterns (100.00%) suggest that the type I error probability for proposed recognizer is very good.
RA is governed by the slope for the trend, the magnitude for the shift pattern and the amplitude for the cycle. For example there is a higher probability for the proposed method to misclassify cycle patterns with small amplitude as normal.

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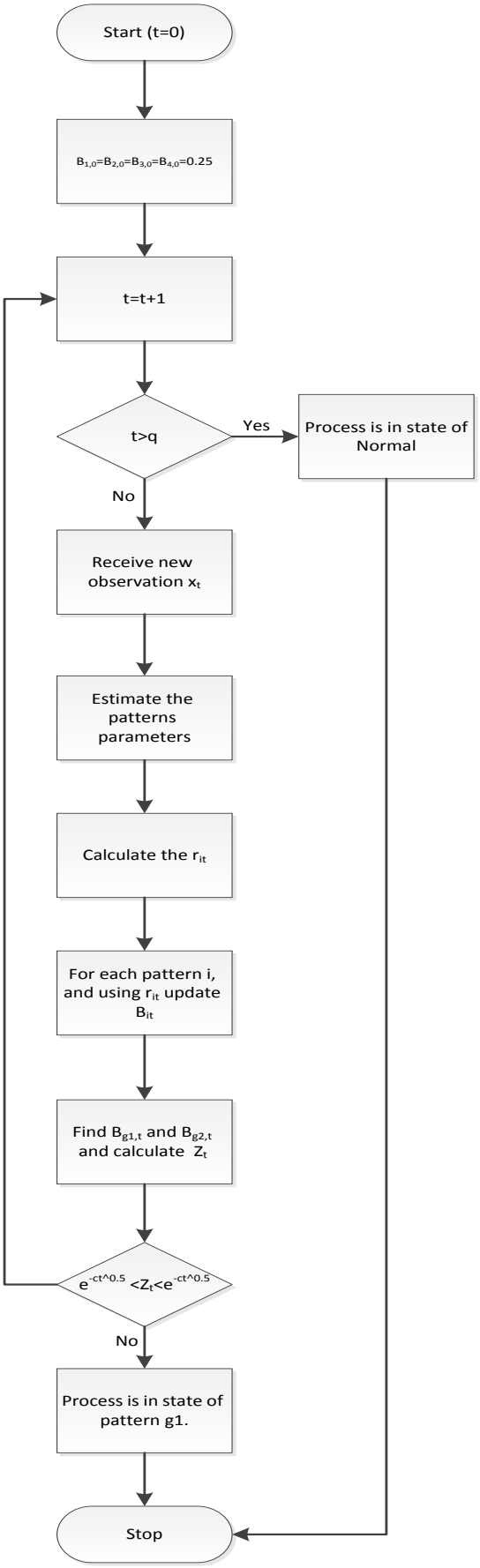

Fig. 2. Diagram of proposed method

In order to obtain the overall recognition accuracy of method, one needs to compute the average value of the performances of the CCPs (the diagonal values of the confusion matrix). This method achieves an overall RA of about 99.43% for recognition of six types of CCPs.

PARL is the average number of observations that must be taken before a recognition signal. To get PARL of each pattern, the mean of 1000 run length was calculated.

PARLs and standard deviations of pattern run length are introduced in Tab. 3. From the experiment results of the PARL calculation, it can be concluded that presented pattern recognizer has a good recognition speed.

Tab. 2. Confusion matrix of proposed method (%)

Testing Pattern		Recognition Results					
		N	IT	DT	US	DS	C
	N	100.00					
k=0.12	IT		99.46		0.54		
k=-0.12	DT			99.44		0.56	
s=1.5	US		1.41		98.59		
s=-1.5	DS			0.88		99.12	
A=1.5, T=8	C						100.00

Tab. 3. PARL and standard deviation of pattern run length

	Unnatural Pattern	PARL	Standard Deviation of Pattern Run Length
k=0.12	Trend	19.46	4.52
s=1.5	Shift	16.13	5.28
A=1.5, T=8	Cycle	16.37	2.48

Tab. 4. Performance of the presented method and some other methods

Reference	Recognition Accuracies (%)				Overall PARL
	Normal	Trend	Shift	Cycle	
[11]	-	78.95	82.30	90.00	No PARL Analysis
[12]	-	67.30	72.80	98.70	12.14
[13]	-	79.00	94.00	94.00	17.41
[15]	84.00	98.00	97.20	94.00	No PARL Analysis
[16]	93.62	89.48	84.42	96.38	No PARL Analysis
[17]	98.50	84.80	76.50	87.00	No PARL Analysis
[26]	51.95	98.00	100.00	100.00	No PARL Analysis
Presented method	**100.00**	**99.45**	**98.85**	**100.00**	**17.32**

Direct comparison with other works is difficult because there is no single unified test data set available. Tab. 4 compares the relative performances of the proposed method and some other methods in case of the RA and overall PARLs. It can be seen that the presented method has exciting results. For instance the proposed

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method outperforms the ANN-based CCP recognizer by Cheng & Cheng [17]. The RAs of Cheng & Cheng are 98.50 percent, 84.80 percent, 76.50 percent, 87.00 percent for normal, trend, shift and cycle patterns respectively (extracted from Tab. 4 of [17]). Compared to these papers, the proposed simple method provides satisfactory interpretable results without requiring to training.

4. Discussion

It is a common problem that trend and shift patterns are often confused in control chart pattern recognition approaches. However, this proposed approach may well solve this problem. In addition, this method does not require any training process and has a simple structure against the complex topology of NNs.
We assumed that the process follows normal distribution. In practice it is possible that observations follow any other distribution. In this case there are two alternative solutions:

1. We can use subgroups for data gathering and then calculate the average value of observations in each subgroup that follows normal distribution according to the *Central Limit Theorem*.
2. We can estimate the pattern parameters by applying the maximum likelihood method on the regarding probability distribution.

5. Conclusions

This paper described a new statistical approach for recognizing control chart patterns. This method employs an iterative approach based on Maximum Likelihood Estimation and Bayes' Rule to signal the pattern with the greatest Belief, when the stopping condition is met.
The obtained results from simulation experiments have shown the effectiveness of proposed approach. The proposed framework gives highly accurate results with satisfactory speed to detect the abnormal patterns. This method has a simple structure against the complex topology of ANNs. In addition, recognition results reached by this system are not based on training samples and will be more reliable than that reached by neural networks. There are several directions for future research:

- Only six types of single control chart patterns were studied in this research. The other types of single patterns and concurrent patterns could be studied in future researches.
- It was assumed that the process follows normal distribution. In practice it is possible that the quality characteristic follows any other distribution. The other probability distributions are desired for future research.
- The effect of patterns parameters magnitude, in the performance of method is another research direction.
- This model can provide the quantified parameters of the control chart patterns, which can be used to diagnose the root causes of abnormal change. This method can be combined with a rule-based expert system to diagnose the assignable causes of the recognized abnormal CCP.

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