Evolutionary Heuristic Algorithms for a Bi-objective Single Allocation Hub Location Problem

Amir Shariati[a], Hamid Pakzadeh[a], Mehdi Seifbarghy[b 1], Wichai Chattinnawat [c 2]

[a] *Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*
[b] *Department of Industrial Engineering, Faculty of Engineering, Alzahra University, Tehran, Iran*
[c] *Department of Industrial Engineering, Chiang Mai University, Chiang Mai, Thailand*

emails: shariati.ie@gmail.com, hamid.pakzadeh@gmail.com, m.seifbarghy@alzahra.ac.ir, chattinw@eng.cmu.ac.th

Abstract

The growing size and complexity of hub location networks make large-scale problems computationally challenging and necessitate efficient solution methods capable of generating high-quality solutions within reasonable computational times. In this paper, two novel metaheuristic algorithms based on the Non-dominated Sorting Genetic Algorithm II (NSGA-II) are developed to solve bi-objective single-allocation hub location problems. Multiple enhancements are introduced to improve the search process, including a heuristic population generation procedure, an adaptive mechanism for determining the number of hubs in the initial population, and an improved mutation operator designed to overcome the frozen-gene issue encountered in previous studies.
The proposed algorithms are applied to two bi-objective hub location models that were previously solved using an interactive approach (IA) only for small instances of the Australian Post (AP) data set. Computational experiments demonstrate that the proposed algorithms are effective in generating high-quality and diverse non-dominated solutions. In addition, they enable the solution of AP instances containing up to 200 nodes, extending the scope of the computational studies previously reported in the literature. The findings confirm the suitability of the proposed approaches for large-scale bi-objective hub location problems.

Keywords: Hub location problem; Bi-objective problem; NSGA-II; Evolutionary algorithms

In press

[1] Corresponding author: Department of Industrial Engineering, Faculty of Engineering, Alzahra University, Tehran, Iran. Email: M.Seifbarghy@alzahra.ac.ir, Tel: +98 912 1232308
[2] Corresponding author: Department of Industrial Engineering, Chiang Mai University, Chiang Mai, Thailand. Email: Chattinw@eng.cmu.ac.th, Tel: +66 (0)53-944126.

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

1. Introduction

The hub location problem (HLP) is widely used in airline, postal, freight, computer, and telecommunication networks. These networks consist of nodes and arcs, with flows transported between origin-destination node pairs. Since direct links between all pairs are often too costly, hub networks route flows through selected hubs for collection, consolidation, transfer, and distribution.

HLP is a location-allocation problem. It determines which nodes should be selected as hubs and how non-hub nodes, known as spokes, should be assigned to them. In single-allocation hub location problems (SAHLP), each spoke is assigned to a single hub, whereas in the multiple-allocation hub problem (MAHLP), spokes may be assigned to more than one hub. Hubs may also be capacitated or uncapacitated, and the number of hubs can be fixed or treated as a decision variable. Figure 1 illustrates a hub-and-spoke network and the flow movements between spokes and hubs.

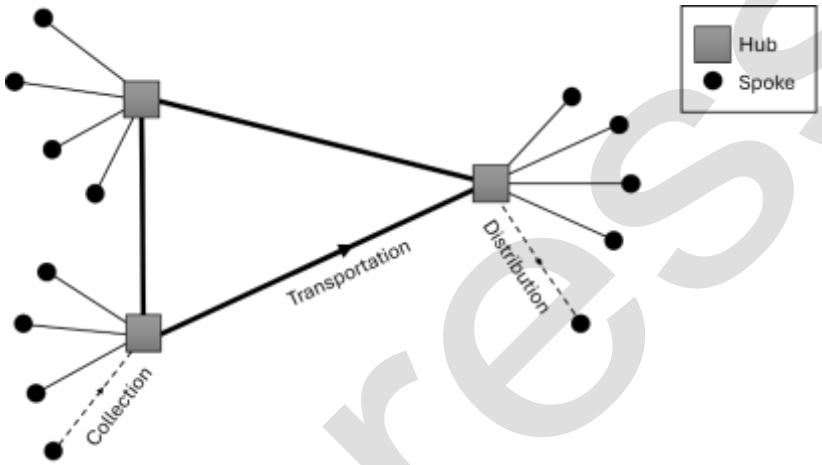


Figure 1. Basic structure of a hub network

Several benchmark data sets have been used in this field, including the Civil Aeronautics Board (CAB) data set introduced by O'kelly [1] and the Australian Post (AP) data set introduced by Ernst and Krishnamoorthy [2]. The AP data set includes 200 postal districts, fixed hub opening costs, and hub capacity levels. The Turkish cargo flow (TR) is another data set introduced by Tan and Kara [3]. The interested reader is referred to Alumur and Kara [4], Campbell and O'Kelly [5], and Farahani et al. [6] for more information on HLP literature.

Early hub location studies mainly focused on cost minimization. However, real-world network design often requires trade-offs among cost, service time, reliability, sustainability, and service quality. This has increased interest in multi-objective hub location models. In their seminal study, da Graça Costa et al. [7] developed a bi-objective single-allocation hub location model that considers cost and service time and solved it using an IA. However, their analysis was limited to small AP instances with up to 40 nodes.

Although subsequent studies have examined various multi-objective hub location formulations, the bi-objective SAHLP introduced by da Graça Costa et al. [7] remains underexplored for medium- and large-sized AP instances. This leaves a gap for solution approaches capable of solving the same model at larger scales, where the problem becomes computationally challenging. To address this gap, this study develops two metaheuristic algorithms based on NSGA-II for the bi-objective SAHLP. The algorithms are tested on small, medium, and large AP instances, extending the analysis of the model introduced by da Graça Costa et al. [7] to larger problem sizes.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the adopted bi-objective models. Section 4 describes the proposed NSGA-II-based

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

In press

heuristic algorithms. Section 5 reports the computational results. Section 6 concludes the paper and outlines future research directions.

2. Literature review

This section reviews the literature on SAHLP. It first covers uncapacitated and capacitated studies, then discusses multi-objective hub location models and evolutionary solution methods for related problems.

2.1 Uncapacitated SAHLP

O'Kelly [8] first introduced the uncapacitated SAHLP as a quadratic integer model with the number of hubs treated as a decision variable, and Campbell [9] developed the first linear programming formulations. Abdinnour-Helm and Venkataramanan [10] later proposed a multi-commodity flow-based quadratic integer formulation and solved it using branch-and-bound and a genetic algorithm heuristic. Their branch-and-bound procedure handled instances with fewer than 20 nodes, while the genetic algorithm solved instances with up to 80 nodes.

Abdinnour-Helm [11] proposed a hybrid genetic algorithm and tabu search heuristic. The genetic algorithm identified the number and locations of hubs, while tabu search improved the allocation decisions from an initial nearest-hub assignment. The method was compared with Abdinnour-Helm and Venkataramanan [10] using the CAB data set. Topcuoglu et al. [12] then proposed a GA-based heuristic that improved on earlier methods. Cunha and Silva [13] later developed a hybrid genetic algorithm and simulated annealing approach, which outperformed the genetic algorithms of Abdinnour-Helm [11] and Abdinnour-Helm and Venkataramanan [10].

Another heuristic for the problem was proposed by Chen [14] in which some approaches were presented for finding upper bounds of the number of hubs. It was shown that the proposed heuristic outperformed the heuristics presented by Topcuoglu et al. [12] in terms of time and solution quality. Filipović et al. [15] proposed a GA-based heuristic for this problem which outperformed the heuristics of Topcuoglu et al. [12] and Chen [14] over the AP data sets. Silva and Cunha [16] developed effective solution methods for the problem. They developed different versions of the tabu search algorithm and solved instances of up to 200 nodes.

Naeem and Ombuki-Berman [17] proposed two GA-based heuristics for the uncapacitated SAHLP, outperforming Abdinnour-Helm [11] and Topcuoglu et al. [12] on the CAB data set. Marić et al. [18] developed a memetic algorithm with two local search heuristics. Abyazi-Sani and Ghanbari [19] proposed a tabu search algorithm that found the best-known solutions for all standard test problems in a shorter computational time than Silva and Cunha [16]. Akbaripour et al. [20] analyzed the solution-space landscape and concluded that effective heuristics should balance exploration and exploitation. They also proposed a new heuristic and compared it with existing methods.

Guan et al. [21] developed a learning-based probabilistic tabu search algorithm for the uncapacitated SAHLP. Tests on benchmark instances showed that the algorithm reached all best-known results in the literature. Guan et al. [22] later proposed a multi-start iterated local search algorithm and tested it on four data sets. Their results showed strong performance in both solution quality and computational efficiency. Ghaffarinasab and Kara [23] proposed Benders' decomposition algorithms for two uncapacitated SAHLP subproblems and tested them on the CAB, TR, and AP data sets. The algorithms reached optimal solutions within reasonable computational times. Čvokić and Stanimirović [24] formulated a version of the problem with price-dependent demand and developed deterministic and robust variants. Alizadeh Firozi et al. [25] proposed a GA-based heuristic that uses population diversity to improve search efficiency.

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

2.2 Capacitated SAHLP

Several studies have examined the capacitated SAHLP, where capacity constraints limit the total incoming and outgoing flows at hubs. Aykin [26] studied this problem with fixed costs and formulated a model that allows both direct transfers and hub-connected transfers. A branch-and-bound algorithm was proposed by them, with lower bounds obtained through Lagrangian relaxation. Aykin [27] later examined a similar problem with fixed costs and a predetermined number of hubs. The study compared two hub location policies: strict and non-strict. Under the non-strict policy, flows may pass through hubs, while under the strict policy, hub routing is mandatory. The problem was solved using an enumeration algorithm and a greedy-interchange heuristic.

Ernst and Krishnamoorthy [28] proposed two formulations for the capacitated SAHLP and developed two heuristics based on simulated annealing and random descent. They tested the methods on the AP data set because the CAB data set does not include fixed costs or hub capacities. Tuzun and Burke [29] developed a two-phase tabu search algorithm, where one phase locates hubs and the other routes flows. Labbé et al. [30] studied a version of the problem in which each hub has a fixed traffic capacity. They examined polyhedral properties of the problem and developed a branch-and-cut algorithm. Randall [31] proposed ant colony optimization algorithms for the problem and introduced multiple variants to explore different solution strategies.

Contreras et al. [32] proposed a Lagrangian relaxation algorithm to obtain tight upper and lower bounds for the capacitated SAHLP. Correia et al. [33] revisited the formulation of Ernst and Krishnamoorthy [28] and showed that it was incomplete and could produce infeasible solutions under certain conditions. They strengthened the model by adding a new set of constraints. Contreras et al. [34] later developed a branch-and-price algorithm for the same problem, using Lagrangian relaxation to generate tight lower bounds for the restricted master problem. Their algorithm solved instances with up to 200 nodes. Stanojević et al. [35] proposed a hybrid heuristic combining an evolutionary algorithm and branch-and-bound, and tested it on the AP data set.

Habibzadeh Boukani et al. [36] studied the capacitated SAHLP under parameter uncertainty, including setup costs and hub capacities. They proposed an optimization approach to handle uncertain parameters and evaluated the model using the Iranian Aviation data set. Meier et al. [37] and Meier and Clausen [38] developed row generation algorithms for different variants of the problem, assuming Euclidean transportation costs between hubs. Their methods reached optimal solutions for instances with more than 100 nodes. Momayezi et al. [39] considered hub failure and formulated the problem as a two-stage stochastic program. They proposed an adaptive large neighborhood search metaheuristic to solve the problem.

Seifbarghy et al. [40] developed a hub covering location model with queuing systems and fuzzy transportation times, solved using genetic algorithm and particle swarm optimization metaheuristics. Sangsawang and Chanta [41] studied flood relief facility location and proposed a hybrid variable neighbourhood search and tabu search heuristic. Moeen Moghadas and Fuladi [42] presented several capacitated SAHLP formulations with different fixed-cost structures and objective functions.

2.3. Multi-objective hub location problem (MOHLP)

Single-objective HLPs have been widely formulated and solved by researchers using exact and heuristic algorithms. In the multi-objective setting, da Graça Costa et al. [7] suggested a different approach for the capacitated SAHLP. Instead of considering capacity constraints on the amount of processed flows in the hubs, they considered a second objective function besides the classic cost objective function. The second objective minimizes the processing time of flows in the hubs. They proposed an IA to find non-dominated solutions using the AP data set for small-sized instances of

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

In Press

the problem. Huang and Wang [43] presented a multi-objective optimization GA for locating hubs in a hub-and-spoke airline network. Makui et al. [44] developed a multi-objective formulation for the capacitated p-hub location problem under uncertainty. They presented a scenario-based robust optimization approach to find reliable solutions, where demand and processing time varied under different scenarios. Ghodratnama et al. [45] proposed a fuzzy possibilistic bi-objective nonlinear mixed-integer programming model for the hub covering problem. To address uncertainty, they treated several model parameters as fuzzy. The first objective minimizes total cost, while the second minimizes commodity transportation time from origins to destinations through hub facilities.

Ebrahimi Zade et al. [46] developed bi-objective linear and nonlinear formulations for single- and multiple-allocation maximal hub covering problems. Their objectives were to maximize coverage and maximize the safety of the weakest path in the network. They solved the problem using NSGA-II. Mohammadi et al. [47] proposed a sustainable multi-objective model under mixed uncertainty, incorporating environmental objectives related to air and noise pollution. They developed a fuzzy-stochastic model to handle different types of uncertain input data. Niakan et al. [48] developed a multi-objective mixed-integer model under uncertainty. The objectives were to minimize the maximum accumulated travel time, minimize total cost, including transportation, fuel consumption, and greenhouse gas emission costs, and maximize the minimum service reliability. They used a hybrid uncertainty approach combining interval-valued fuzzy programming and rough interval programming.

Ghodratnama et al. [49] developed a multi-objective mathematical formulation for the SAHLP with three objectives: minimizing total transportation and installation costs, minimizing greenhouse gas emissions and the weighted sum of hub service times, and minimizing flow earliness and tardiness. A multi-objective invasive weed optimization algorithm was proposed to generate near-optimal solutions. Masoumzadeh et al. [50] proposed a model with two objectives: minimizing total costs and maximizing potential flows between origin-destination pairs. Zhalechian et al. [51] developed a multi-objective model under uncertainty, incorporating economic, responsiveness, and social considerations in hub-and-spoke network design. They used a queueing system to estimate waiting times at hub nodes and applied possibilistic programming to handle uncertain parameters. A hybrid two-phase solution method based on possibilistic programming was also proposed to solve the problem.

Madani et al. [52] proposed a bi-objective formulation for the single-allocation p-hub maximal covering problem with backup hubs to address network disruptions. The objectives were to maximize expected covered flows and minimize congestion. They developed an NSGA-II-based solution approach and compared its performance with two other methods. Kahag et al. [53] developed a multi-objective intermodal hub location-allocation problem in which hub facilities were modelled as a queueing system. The problem was formulated as a bi-objective optimization model that minimizes total cost and total system time, including waiting, service, and idle times at hubs.

Mohammadi et al. [54] introduced a bi-objective capacitated p-hub location model under uncertainty in hubs and links. The objectives were to minimize total cost and maximum transportation time. They developed an approximation approach to provide a lower bound for the optimal Pareto frontier and proposed a hybrid metaheuristic algorithm to solve the problem. Soylu and Katip [55] studied a bi-objective multiple-allocation p-hub median problem for airport networks and developed exact and heuristic algorithms to identify the Pareto frontier. Their objectives were to minimize total transportation cost and the cost of two-stop journeys to improve customer satisfaction.

Babashahi et al. [56] examined a hierarchical HLP with possible facility failures. Their model minimized total routing cost and maximized network reliability, and was solved using NSGA-II. Yin et al. [57] developed a three-objective model under uncertainty. In addition to minimizing

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

In press

regular cost, the model considered environmental and customer satisfaction objectives. The environmental objective included noise pollution and carbon emissions, while the customer satisfaction objective reflected delivery time and quality. Khalilzadeh et al. [58] proposed a maximal covering model with two objectives: minimizing time and minimizing risk. Aouam et al. [59] studied a bi-objective model with a fixed number of hubs, minimizing the worst service time and environmental cost. Azzoug et al. [60] developed a two-objective hop-constrained HLP model that minimizes total cost and the number of hops, where hops refer to links between nodes in the network.

2.4. Evolutionary algorithms for MOHLP

Several multi-objective HLP studies reviewed above have employed evolutionary and metaheuristic algorithms to generate Pareto fronts. Among these, genetic algorithms and NSGA-II have been the most widely used due to their ability to generate diverse sets of non-dominated solutions within a single run. Huang and Wang [43] used a multi-objective genetic algorithm for locating hubs in a hub-and-spoke airline network. Ebrahimi Zade et al. [46] applied NSGA-II to bi-objective single- and multiple-allocation maximal hub covering problems. Ghodratnama et al. [49] used a multi-objective invasive weed optimization algorithm for a three-objective SAHLP. Madani et al. [52] developed an NSGA-II-based solution approach for a bi-objective single-allocation p-hub maximal covering problem with backup hubs under network disruptions. Mohammadi et al. [54] proposed a hybrid metaheuristic algorithm for a bi-objective capacitated p-hub location problem under hub and link uncertainty based on self-adaptive NSGA-II. Babashahi et al. [56] applied NSGA-II to a hierarchical HLP with possible facility failures.

The popularity of NSGA-II stems from the development of evolutionary algorithms for multi-objective optimization. Srinivas and Deb [61] introduced the non-dominated sorting genetic algorithm (NSGA), one of the early evolutionary approaches for solving multi-objective optimization problems. Horn et al. [62] later introduced a Pareto domination tournament approach as an alternative to non-dominated sorting and ranking selection. Srinivas and Deb [61] showed that NSGA performed better than the method proposed by Horn et al. [62]. Deb et al. [63] then introduced the elitist non-dominated sorting genetic algorithm, known as NSGA-II, which improved upon the original NSGA through faster sorting, elitism, and diversity preservation mechanisms.

Reviewing the literature shows that HLP studies have progressed from basic cost-based models to formulations that capture capacity, uncertainty, failures, sustainability, reliability, and service performance. Multi-objective models specifically have gained attention because hub network design often involves trade-offs among cost, time, reliability, environmental impact, congestion, and risk. However, most evolutionary and metaheuristic applications focus on covering models, p-hub models, hierarchical networks, disruption settings, uncertainty, or sustainability. Notably, the bi-objective SAHLP introduced by da Graça Costa et al. [7], which minimizes cost and hub processing time, remains underexplored for medium and large AP instances.

3. Mathematical formulation

As noted earlier, da Graça Costa et al. [7] developed two bi-objective models for the capacitated SAHLP based on Ernst and Krishnamoorthy [28]. Instead of treating hub capacity as a hard constraint, they modelled service time as a second objective. Model I minimized total hub processing time, while Model II minimized maximum service time. Both models were tested on AP instances with 10, 20, 25, and 40 nodes. The notation is as follows:

Indices

i, j: Indices of origin and destination nodes

k, l: Indices of hub nodes

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

In press

Parameters

W_{ij}: Flow from node i to node j

O_i: Total outgoing flows from node i, where $O_i = \sum_j W_{ij}$

D_i: Total incoming flows to node i, where $D_i = \sum_j W_{ji}$

C_{ij}: Distance between node i and node j

C_{kl}: Distance between hub k and l

F_k: Fixed cost of opening a hub in node k

T_k: Unit processing time in node k

P_k: Fixed time required to initiate processing in node k

α: Cost coefficient for the unit flow transferred between two hubs

χ: Cost coefficient for the unit flow transferred from a non-hub node to a hub

δ: Cost coefficient for the unit flow transferred from a hub to a non-hub node

Decision variables

Y_{kl}^i: Total flow originated from node i and routed via hubs k and l

X_{ik}: Binary variable equal to 1 if node i is allocated to hub k, and 0 otherwise

X_{kk}: Binary variable equal to 1 if node k is selected as a hub, and 0 otherwise

Model I:

$$Min\ Z_1 = \sum_i \sum_k (\chi O_i + \delta D_i)\, C_{ik} X_{ik} + \sum_i \sum_k \sum_l \alpha\, C_{kl} Y_{kl}^i + \sum_k F_k\, X_{kk} \qquad (1)$$

$$Min\ Z_2 = \sum_i \sum_k O_i\, T_k X_{ik} + \sum_k P_k\, X_{kk} \qquad (2)$$

Subject to:

$$\sum_k X_{ik} = 1 \quad \forall i \qquad (3)$$

$$X_{ik} \le X_{kk} \quad \forall i, k \qquad (4)$$

$$\sum_l Y_{kl}^i - \sum_l Y_{lk}^i = O_i X_{ik} - D_i X_{ik} \quad \forall i, k \qquad (5)$$

$$X_{ik}, X_{kk} \in \{0,1\} \quad \forall i, k \qquad (6)$$

In press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

$$Y_{kl}^{i} \geq 0 \quad \forall i,k,l \qquad (7)$$

Model II:

$$Min\ Z_1 = \sum_i \sum_k (\chi O_i + \delta D_i)\, C_{ik} X_{ik} + \sum_i \sum_k \sum_l \alpha\, C_{kl} Y_{kl}^{i} + \sum_k F_k\, X_{kk} \qquad (8)$$

$$Min\ \beta = \max_k \left\{ \sum_i O_i\, T_k X_{ik} + P_k X_{kk} \right\} \qquad (9)$$

Subject to:

$$\beta \geq \sum_i O_i\, T_k X_{ik} + P_k X_{kk} \quad \forall k \qquad (10)$$

$$\sum_k X_{ik} = 1 \quad \forall i \qquad (11)$$

$$X_{ik} \leq X_{kk} \quad \forall i,k \qquad (12)$$

$$\sum_l Y_{kl}^{i} - \sum_l Y_{lk}^{i} = O_i X_{ik} - D_i X_{ik} \quad \forall i,k \qquad (13)$$

$$X_{ik}, X_{kk} \in \{0,1\} \quad \forall i,k \qquad (14)$$

$$Y_{kl}^{i} \geq 0 \quad \forall i,k,l \qquad (15)$$

Equation (1) minimizes three cost components: the transfer costs between non-hub and hub nodes, the transfer costs between hub nodes, and the fixed costs of opening hubs. Equation (2) minimizes the total hub processing time, including both flow-dependent processing time and fixed hub setup time. Constraint (3) ensures that each node is allocated to exactly one hub. Constraint (4) enforces that a node can only be allocated to a hub that is open. Constraint (5) ensures flow conservation at each hub. Constraints (6) and (7) restrict the allocation and hub-opening decisions to binary values and the flow variables to non-negative values.

Equation (8) is identical to Equation (1), minimizing the same flow transfer and hub opening costs. Equation (9) minimizes the maximum processing time among the selected hubs. Constraint (10) linearizes the minimax objective by bounding β above the processing time at every hub. Constraints (11)–(15) are identical to Constraints (3)–(7) in Model I.

4. Solution approach

HLP is generally classified as an NP-hard problem [12], and the same applies to the bi-objective SAHLP considered in this study. Consequently, exact methods become computationally challenging for medium and large instances, making heuristic and metaheuristic approaches attractive alternatives for obtaining high-quality solutions within reasonable computation times.

To solve the proposed models, two NSGA-II-based metaheuristic algorithms are developed. The solution representation is adapted from the chromosome structure proposed by Topcuoglu et al.

In press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

[12]. The first algorithm, denoted NHR (NSGA-II with Random Population Initialization), and the second algorithm, denoted NHH (NSGA-II with Heuristic Population Initialization). The algorithms differ in two main aspects. First, NHR generates initial solutions randomly, whereas NHH uses a heuristic mechanism to improve the quality of the initial population. Second, during offspring generation, NHR reallocates spokes to hubs using a random procedure, while NHH assigns spokes according to distance-based criteria.

4.1. Implementation of NHR

4.1.1. Solution representation

Each solution is represented by two arrays: a hub-array and an assign-array. The length of both arrays is equal to the number of nodes in the network. The hub-array is a binary string in which a value of 1 indicates that the corresponding node is selected as a hub, while a value of 0 indicates that the node is a spoke. The assign-array records the allocation of nodes to hubs. If node i is assigned to hub k, the corresponding entry in the assign-array is k. Hub nodes are assigned to themselves. This two-array representation captures the two key decisions of the SAHLP: hub selection and node allocation.

Since node allocation depends on the selected hubs, the hub-array is generated first, and the assign-array is subsequently constructed using the set of selected hubs. Figure 2 illustrates the representation for a network with nine nodes. According to the hub-array, nodes 3 and 7 are selected as hubs. The assign-array shows that nodes 1, 2, 3, 5, 6, and 8 are assigned to hub 3, while nodes 4, 7, and 9 are assigned to hub 7.

Nodes	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Hub-array	**0**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	**0**
Assign-array	**3**	**3**	**3**	**7**	**3**	**3**	**7**	**3**	**7**

Figure 2. Example of chromosome representation

4.1.2. Initial population generation in NHR

A predefined number of solutions is generated randomly to form the initial population. First, the hub-array is created by randomly assigning each position a value of 0 or 1, indicating whether a node is selected as a hub. Once the hub-array is generated, the assign-array is constructed. Each selected hub is assigned to itself, while the remaining nodes are randomly allocated to one of the selected hubs. This procedure ensures that every solution contains valid hub assignments before the evolutionary search begins.

4.1.3. Selection operator

Binary tournament selection is used to choose parent solutions. Two individuals are selected randomly from the population and compared using the crowded-comparison operator proposed by Deb et al. [63]. The individual with the higher rank is selected. If both individuals belong to the same non-dominated front, the solution with the larger crowding distance is chosen. This process is repeated until the required number of parents is obtained.

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

4.1.4. Crossover operator

Single-point crossover is applied to the hub-array and assign-array of the selected parent solutions. A crossover point is randomly selected along the chromosome, dividing each parent into two segments. Two offspring are then generated by combining the left segment of one parent with the right segment of the other parent. Figure 3 illustrates the crossover process for two parent solutions of length 10. For the first parent (O_1), nodes 2, 6, and 10 are hubs, while hub nodes in the second part (O_2) are 1 and 7. After applying the crossover operator to the hub-arrays, the first offspring (O_1^N) contains nodes 2 and 7 as hubs, whereas the second offspring (O_2^N) contains nodes 1, 6, and 10 as hubs.

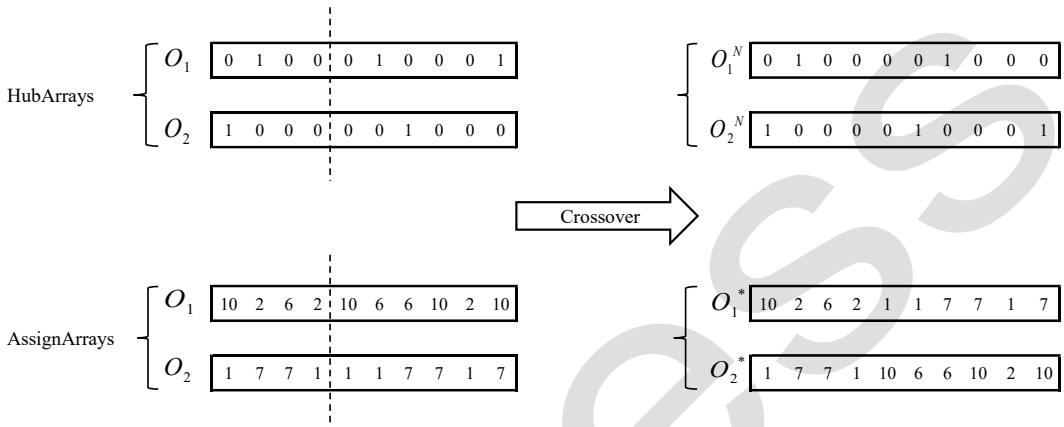


Figure 3. Example of single-point crossover (adapted from Topcuoglu et al. [12])

4.1.5. Mutation operator on assign-array

Following Topcuoglu et al. [12], two mutation operators, namely shift and exchange, are applied to the assign-array.

- **Shift operator:** A spoke node is selected randomly and reassigned to another hub selected at random. This operator is not applicable when the solution contains only one hub.
- **Exchange operator:** Two spoke nodes are selected randomly, and their hub assignments are interchanged. This operator requires at least two hubs and two spoke nodes and is therefore not applicable when the solution contains only one hub or one spoke node.

4.1.6. Mutation operator on hub-array

The mutation operators of Topcuoglu et al. [12] only modify node assignments and cannot change a spoke into a hub. Consequently, some hub-array positions may become fixed during the search, referred to as frozen genes, reducing population diversity. To address this limitation, a new hub-array mutation operator is introduced in the proposed heuristics. A randomly selected position changes from 0 to 1 or from 1 to 0, allowing nodes to switch between hub and spoke status.

4.1.7. Arrangement of offspring in NHR

Following crossover and mutation, a feasibility restoration procedure is applied whenever necessary to ensure that all assignments remain consistent with the selected hubs and that the resulting solutions satisfy the problem requirements. If an offspring contains no hub nodes, it is discarded. Infeasible assignments may also occur when a node remains assigned to a hub that is no longer active after crossover or mutation. Therefore, a repair procedure is applied. As illustrated in Figure 4, any assignment associated with an inactive hub is randomly reassigned to one of the active hubs in the offspring. This procedure ensures consistency between the hub-array and assign-array before evaluating the offspring solution.

In press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

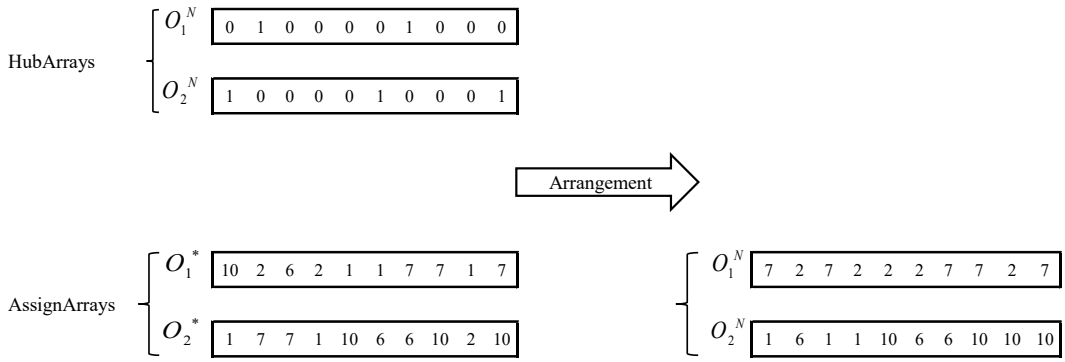


Figure 4. Example of offspring arrangement in NHR (adapted from Topcuoglu et al. [12])

4.2. Implementation of NHH

NHH is based on the same NSGA-II framework as NHR. The two algorithms differ only in the initial population generation mechanism and the offspring arrangement procedure, which are described below.

4.2.1. Initial population generation in NHH

A predefined number of solutions is generated to form the initial population. First, the hub nodes are selected, after which each spoke is assigned to its nearest hub based on the distance matrix. For 75% of the initial population, the number of hubs is selected randomly from the range $[1, an]$, where n is the total number of nodes and a is a random number between 0.001 and 0.5. For the remaining 25% of the population, the number of hubs is selected from the range $[an, n/2]$, using the same range for a. This strategy limits the maximum number of hubs in the network to $n/2$.

After determining the number of hubs, the hub nodes are selected in the hub-array. The total flow associated with each node is then calculated as $O_i + D_i - W_{ii}$. The nodes are subsequently sorted in descending order of total flow to form a list called the flow list. For 75% of the initial population, hub nodes are selected from the top two-thirds of the flow list, giving nodes with larger traffic volumes a higher probability of being selected as hubs. For the remaining 25% of the solutions, hub nodes are selected from the entire set of nodes. The control parameters of NHR and NHH were calibrated using the Taguchi method. The selected parameter values are reported in Appendix A.

The initial population generation procedure of NHH is based on that of Topcuoglu et al. [12]. However, two modifications are introduced to improve the representation of node flows and provide greater flexibility in the hub-selection process:

- In the present study, the flow associated with node i is calculated as $O_i + D_i - W_{ii}$, whereas Topcuoglu et al. [12] used $O_i + D_i$. The latter counts W_{ii} twice, which may be acceptable for the CAB data set where $W_{ii} = 0$, but is less suitable for the AP and TR data sets where $W_{ii} \neq 0$. The revised measure provides a more accurate representation of node flow for data sets with non-zero self-flows.

- Topcuoglu et al. [12] used the fixed ranges $[1, n/4]$ and $[n/4, n/2]$ to determine the number of hubs, irrespective of problem size. In contrast, the proposed approach introduces parameter a to adjust these ranges according to the characteristics of the problem instance. Using the Taguchi method, the best value of a was found to be 0.25 for small and medium-

In press

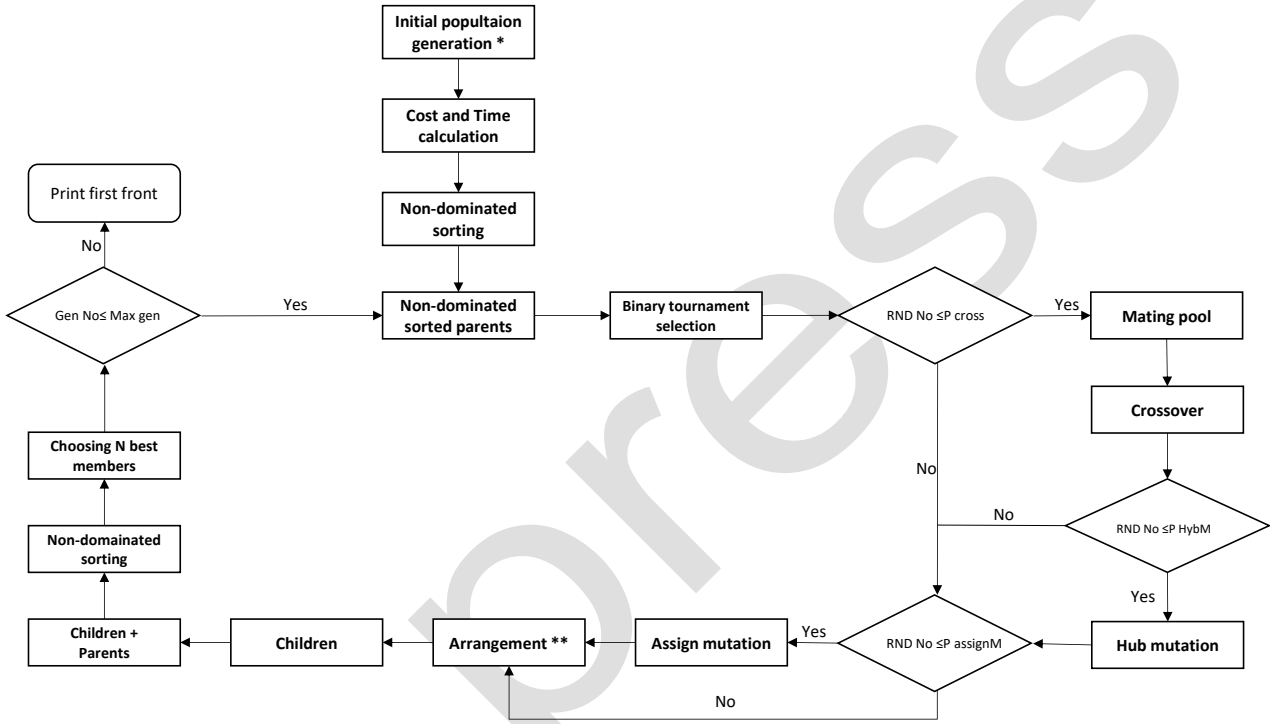
[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

sized instances and 0.20 for large-scale instances. This provides a more adaptive mechanism for generating diverse initial solutions.

4.2.2. Arrangement of offspring of NHH

The offspring arrangement procedure in NHH follows the same framework as NHR, except for the reassignment mechanism. If a hub becomes a spoke after crossover or mutation, all nodes assigned to that hub are reassigned to the nearest active hub according to the distance matrix. This distance-based reassignment maintains solution feasibility while reducing the randomness.

The flowchart of the proposed algorithms is presented in Figure 5.



Notes:
** NHR uses random population initialization, whereas NHH uses heuristic population initialization*
*** NHR repairs offspring through random reassignment, whereas NHH uses distance-based reassignment.*

Figure 5. Flowcharts of the proposed NHR and NHH algorithms

4.3. Evaluation metrics

The performance of the proposed heuristics is evaluated using the quality and diversity metrics adopted from Hyun et al. [64] and da Graça Costa et al. [7], respectively.

4.3.1 Solution quality metric

For the quality metric, a reference set is first constructed by combining all non-dominated solutions generated by the compared heuristics. The combined solution set is then sorted according to non-domination, and the first Pareto front is considered the reference set. The solutions generated by each heuristic are subsequently compared with the reference set. The quality metric is then computed as:

$$QM_h = \frac{| R \cap S_h |}{| R |}$$

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

In press

where QM_h is the quality metric of heuristic h, R is the reference set, and S_h is the set of non-dominated solutions generated by heuristic h.

A larger value indicates better performance. A quality metric value of 1 indicates that all solutions in the reference set have been identified by the heuristic.

4.3.2 Solution diversity metric

The diversity metric is measured by the number of non-dominated solutions generated by each heuristic. A larger number of non-dominated solutions indicates a greater ability to provide alternative trade-off solutions to the decision maker and, therefore, a higher level of solution diversity.

5. Results and discussions

5.1. Comparison with the results of IA

da Graça Costa et al. [7] solved the same bi-objective models using an IA for AP instances with up to 40 nodes. Therefore, the performance of the proposed heuristics was compared with IA using the quality and diversity metrics described in Section 4.3. The algorithms were implemented in C# and executed on a personal computer equipped with a Core i3 2.00 GHz processor and 6 GB of RAM. The parameter values were calibrated using the Taguchi method, and the selected settings are reported in Appendix A.

The comparison results for Model I and Model II are presented in Tables 1 and 2, respectively. In both tables, the first column identifies the problem instance, where T and L denote tight and loose fixed-cost and capacity settings. Columns 2 to 4 report the number of non-dominated solutions generated by each approach, while Column 5 gives the size of the reference set. The remaining columns present the corresponding quality metric values.

Table 1. Performance comparison of NHH, NHR, and IA for Model I on AP instances with up to 40 nodes

Problem type	Number of non-dominated solutions				Solution quality metric		
	IA	NHH	NHR	Reference list	IA	NHH	NHR
10 LL	13	14	14	14	0.93	1	1
10 LT	22	32	30	32	0.69	1	0.94
10 TL	8	8	7	8	1	1	0.87
10 TT	12	12	12	12	1	1	1
20 LL	23	58	49	58	0.40	1	0.84
20 LT	17	22	17	22	0.77	1	0.77
20 TL	19	38	35	38	0.50	1	0.92
20 TT	6	7	1	7	0.86	1	0.14
30 LL	31	183	145	184	0.17	1	0.78
30 LT	38	113	8	116	0.33	0.97	0.07
30 TL	29	175	157	175	0.16	1	0.90
30 TT	25	49	25	49	0.51	1	0.51
40 LL	42	59	2	63	0.67	0.94	0.03
40 LT	52	372	252	381	0.14	0.98	0.66
40 TL	16	105	5	105	0.15	1	0.04
40 TT	26	244	141	248	0.10	0.98	0.57

In press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

Table 2. Performance comparison of NHH, NHR, and IA for Model II on AP instances with up to 40 nodes

Problem type	Number of non-dominated solutions				Solution quality metric		
	IA	NHH	NHR	Reference list	IA	NHH	NHR
10 LL	9	9	8	9	1	1	0.89
10 LT	18	17	17	19	0.95	0.89	0.89
10 TL	11	9	11	11	1	0.82	1
10 TT	12	11	12	12	1	0.92	1
20 LL	36	27	31	53	0.68	0.51	0.58
20 LT	31	32	27	43	0.72	0.74	0.62
20 TL	27	28	25	39	0.69	0.72	0.64
20 TT	30	31	25	58	0.52	0.54	0.43
30 LL	33	37	28	57	0.58	0.65	0.49
30 LT	40	43	36	62	0.64	0.70	0.58
30 TL	32	39	27	59	0.54	0.66	0.45
30 TT	33	49	35	61	0.54	0.80	0.57
40 LL	35	70	22	94	0.37	0.74	0.23
40 LT	47	85	43	108	0.43	0.79	0.40
40 TL	27	50	40	62	0.44	0.81	0.64
40 TT	36	91	40	115	0.31	0.79	0.35

Tables 1 and 2 indicate that NHH generally outperformed IA and NHR in terms of solution quality while maintaining strong diversity performance across most benchmark instances. This highlights the effectiveness of the proposed enhancements in generating high-quality non-dominated solutions.

To statistically compare the quality metric values, one-sided two-sample t-tests were conducted using the 16 AP benchmark instances considered by da Graça Costa et al. [7]. As presented in Table 3, for Model I, NHH achieved the highest mean quality metric (0.992 ± 0.017), compared with NHR (0.629 ± 0.360) and IA (0.523 ± 0.328). The corresponding p-values for the comparisons between NHH and NHR and between NHH and IA were 0.999 and 1.000, respectively. For Model II, NHH again achieved the highest mean quality metric (0.755 ± 0.129), compared with NHR (0.612 ± 0.230) and IA (0.651 ± 0.230). The corresponding p-values were 0.979 and 0.934, respectively.

Table 3. Summary statistics of the quality metric across the 16 AP benchmark instances

Model	Method	Mean	Standard deviation
I	IA	0.523	0.328
I	NHR	0.629	0.360
I	NHH	0.992	0.017
II	IA	0.651	0.230
II	NHR	0.612	0.230
II	NHH	0.755	0.129

To further illustrate the differences among the solution approaches, representative Pareto fronts for 30LL generated by Model I are provided in Appendix B. The Pareto fronts show that NHH generates a broader and more competitive set of non-dominated solutions than IA and NHR, supporting the quality and diversity results reported in Tables 1 and 2.

5.2. Computational time

Since the computational times of IA were not reported by da Graça Costa et al. [7], only NHR and NHH are compared. Table 4 presents the computational times of both heuristics for TT instances, which represent the most challenging problem setting due to the combination of tight capacities and tight fixed costs.

The results show that NHH consistently requires less computational time than NHR across all tested instances. The computational advantage ranges from 13% to 113%, depending on the problem size.

In press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

Although the computational time of both heuristics increases as the problem size grows, NHH maintains its advantage throughout the experiments. Therefore, NHH is computationally more efficient than NHR while also providing higher-quality solutions, as demonstrated in Section 5.1.

Table 4. Computational time comparison of NHH and NHR for TT instances of the AP data set

Problem type	Computational time (s)		Difference (%)
	NHH	NHR	
10TT	8	17	113%
20TT	18	34	89%
25TT	29	51	76%
40TT	286	382	34%
50TT	376	483	28%
100TT	3,695	4,797	30%
200TT	14,344	16,150	13%

5.3. Results of NHH on the AP data set

As demonstrated in Sections 5.1 and 5.2, NHH achieved the best overall performance among the compared solution approaches in terms of solution quality, diversity, and computational efficiency. Therefore, NHH was selected to solve Models I and II for medium- and large-sized AP instances.

Tables 5 and 6 present the results obtained for problem instances containing up to 200 nodes. While da Graça Costa et al. [7] reported results only for instances with up to 40 nodes, the proposed NHH algorithm enables the solution of larger instances containing 50, 100, and 200 nodes. To the best of our knowledge, this is the first time that these larger AP instances have been analysed for the two bi-objective models considered in this study.

For each problem instance, the tables report the solutions with the best cost and the best time, together with the corresponding hub locations and the number of non-dominated solutions obtained. For Model I, the time measure represents the total processing time in the hubs, whereas for Model II it represents the maximum service time among the selected hubs.

Several trends can be observed from Tables 5 and 6. First, the number of non-dominated solutions generally increases with problem size, indicating a richer trade-off structure between cost and service-time objectives. For example, the Pareto set size increases from fewer than 50 solutions in most small instances to several hundred solutions in the 100- and 200-node instances. Second, Model II generally produces more non-dominated solutions than Model I for medium- and large-sized problems, suggesting that minimizing the maximum service time yields a more diverse set of trade-off solutions. Finally, the best-time solutions typically require more hubs than the best-cost solutions, highlighting the trade-off between service performance and network cost.

In Press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

Table 5. Results obtained by NHH for Model I on AP instances with up to 200 nodes

Problem type	Best cost			Best time			non-dominated solutions
	Cost	Time (Second)	Hubs	Cost	Time (Second)	Hubs	
10 LL	224,250	28,011	3,4,7	281,480	18,875	8	14
10 LT	224,250	286,783	3,4,7	300,535	40,510	10	32
10 TL	263,400	24,073	4,5,10	296,319	18,875	8	8
10 TT	263,400	44,930	4,5,10	309,540	40,510	10	12
20 LL	234,691	31,165	7,14	379,687	22,540	5	58
20 LT	234,691	216,618	7,14	275,146	53,083	10	22
20 TL	271,128	27,697	7,19	384,360	22,540	5	38
20 TT	271,128	225,436	7,19	278,168	53,083	10	7
30 LL	236,651	46,546	8,18	746,756	26,416	5	183
30 LT	236,651	1,407,743	8,18	335,851	60,588	25	113
30 TL	295,668	68,887	13	758,158	26,416	5	175
30 TT	295,668	103,151	13	367,146	60,588	25	49
40 LL	240,987	200,220	14,28	278,965	32,086	30	59
40 LT	240,987	138,651	14,28	443,815	70,157	32	372
40 TL	293,165	40,578	19	417,024	32,086	30	105
40 TT	293,165	107,010	19	498,156	70,157	32	244
50 LL	237,422	235,763	15,36	406,599	33,607	31	116
50 LT	237,422	1,806,979	15,36	310,264	72,279	32	142
50 TL	300,421	36,624	24	438,980	33,607	31	183
50 TT	300,421	187,392	24	407,489	72,279	32	4
100 LL	238,016	57,593	29,73	289,946	35,714	46	485
100 LT	238,016	195,454	29,73	359,614	77,392	29	440
100 TL	305,098	54,628	52	387,170	35,714	46	346
100 TT	305,098	272,544	52	523,840	77,392	29	398
200 LL	233,904	64,343	43,148	283,839	36,373	112	419
200 LT	233,904	202,791	43,148	479,546	79,772	27	538
200 TL	272,337	56,705	54,122	536,795	36,373	112	378
200 TT	272,337	136,355	54,122	1,314,074	79,772	27	877

In press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

Table 6. Results obtained by NHH for Model II on AP instances with up to 200 nodes

Problem type	Best cost			Best time			non-dominated solutions
	Cost	Time (Second)	Hubs	Cost	Time (Second)	Hubs	
10 LL	224,250	13,478	3,4,7	246,051	6,284	1,4,5,7,8	9
10 LT	224,250	245,685	3,4,7	281,956	14,949	1,4,5,10	17
10 TL	263,400	12,023	4,5,10	345,681	6,283	1,4,5,7,10	10
10 TT	263,400	24,532	4,5,10	300,038	14,949	1,4,5,10	11
20 LL	234,691	21,530	7,14	385,506	6,492	5,7,10,16,19	36
20 LT	234,691	178,903	7,14	330,245	13,999	2,6,9,14,16,19	41
20 TL	271,128	18,062	7,19	426,480	6,492	5,7,10,16,19	41
20 TT	271,128	178,903	7,19	527,149	13,999	9,10,12,14,19	45
30 LL	236,651	32,185	8,18	478,685	6,406	5,7,14,17,19,23	59
30 LT	236,651	1,362,980	8,18	370,332	14,824	4,12,14,19,20,25	59
30 TL	295,668	68,887	13	574,524	7,091	7,9,10,19,23	69
30 TT	295,668	103,151	13	725,820	14,456	1,5,9,12,14,16,19,25	68
40 LL	240,986	183,093	14,28	349,290	7,699	3,10,29,30,36,37	104
40 LT	240,986	108,674	14,28	430,805	13,398	10,14,25,26,30,38,40	111
40 TL	293,165	40,578	19	693,559	6,388	5,10,14,19,30,36,38	115
40 TT	293,165	107,010	19	749,004	13,398	5,14,19,26,31,32,38,40	143
50 LL	237,422	207,540	15,35	382,100	6,552	3,16,28,34,35,38,45,49	177
50 LT	237,422	1,259,284	15,36	351,665	18,572	14,25,32,38,46,47	166
50 TL	300,421	36,624	24	1,577,851	6,552	3,8,16,24,25,34,35,45,49	169
50 TT	300,421	187,392	24	767,198	16,964	1,25,26,32,46,49	122
100 LL	238,016	38,458	29,73	405,054	6,323	24,30,31,64,75,79,91,97	377
100 LT	238,016	173,175	29,73	442,586	13,344	7,29,31,42,66,68,72,76	382
100 TL	305,098	54,628	52	3,011,733	6,804	27,44,50,52,61,73,75,76, 89,91,93,95,99	492
100 TT	305098	272,544	52	2,132,207	12,487	5,13,33,52,62,72,76,87,9 5,96,98	496
200 LL	233,904	50,441	43,159	564,007	6,138	33,46,96,100,111, 133,139, 141,156,159,160, 167,176,	259
200 LT	233,904	116,668	43,148	697,639	12,278	24,32,41,58,73,77,100,1 42,150,156,160,163,166, 170,178,183	297
200 TL	272,327	42,596	54,122	9,796,673	6,138	42,54,84,95,139,156, 158,159, 160,164,170	286
200 TT	272,327	106,983	50,122	2,364,695	13,129	33,49,57,72,84,113,136, 141,168,170,175	252

6. Conclusions and future research

In this research, two NSGA-II-based heuristics, namely NHR and NHH, were developed to solve two bi-objective capacitated single-allocation hub location problems. The proposed heuristics extend the genetic algorithm framework of Topcuoglu et al. [12] by incorporating it within the NSGA-II structure and introducing several enhancements. These enhancements include a hub-array mutation operator to overcome the frozen-gene issue, a heuristic population generation procedure, a distance-based offspring arrangement strategy, a revised node-flow measure for hub selection, and an adaptive mechanism for determining the number of hubs in the initial population.

The proposed heuristics were evaluated using the AP data set and compared with the IA of da Graça Costa et al. [7]. The computational results showed that NHH consistently achieved the highest quality metric values among the compared approaches while maintaining strong diversity performance. In addition, NHH required less computational time than NHR for the tested instances. The proposed algorithm was subsequently applied to Models I and II on AP instances containing up to 200 nodes, extending the computational study of da Graça Costa et al. [7], who reported results for the same bi-objective models only for instances with up to 40 nodes. The results demonstrated

In press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

that NHH was capable in generating a rich set of non-dominated solutions and providing decision makers with a broad range of cost and service-time trade-offs.

From a practical perspective, the proposed solution methods can support hub location and allocation decisions in large-scale service networks. One potential application is the design of healthcare systems in countries such as Thailand, where major hospitals may act as hubs and residential areas or patient populations may represent demand nodes. In such settings, a great number of clients must be assigned to healthcare facilities with limited capacities. Application of the proposed approach enables planners to evaluate alternative network configurations and identify cost-service trade-offs while accounting for capacity constraints. This provides a more systematic basis for decision-making than assigning clients to the nearest available hospital.

Future research could investigate alternative chromosome representations that reduce the need for solution rearrangement after crossover and mutation while preserving feasibility. The proposed heuristics could also be compared with modern multi-objective evolutionary algorithms and evaluated using additional performance metrics, such as hypervolume and inverted generational distance (IGD). Furthermore, future studies could examine the convergence behaviour of the proposed heuristics through convergence plots and convergence-based performance measures to provide additional insight into the search process. The proposed models may be extended to incorporate uncertainty in service rates, customer flows, and hub capacities, particularly in healthcare applications where patient demand is stochastic. Another promising direction is the study of demand surges caused by disruptive events to evaluate the robustness and resilience of hub networks under extreme operating conditions.

References

[1]. O'kelly, M. E. "A quadratic integer program for the location of interacting hub facilities" European Journal of Operational Research, Vol. 32, 1987. P. 393–404. https://doi.org/10.1016/S0377-2217(87)80007-3

[2]. Ernst, A. T., Krishnamoorthy, M. "Efficient algorithms for the uncapacitated single allocation p-hub median problem" Location Science, Vol. 4, 1996. P. 139–154. https://doi.org/10.1016/S0966-8349(96)00011-3

[3]. Tan, P. Z., Kara, B. Y. "A hub covering model for cargo delivery systems" Networks: An International Journal, Vol. 49(1), 2007. P. 28-39. https://doi.org/10.1002/net.20139

[4]. Alumur, S., Kara, B. Y. "Network hub location problems: The state of the art" European Journal of Operational Research, Vol. 190, 2008. P. 1–21. https://doi.org/10.1016/j.ejor.2007.06.008

[5]. Campbell, J. F., O'Kelly, M. E. "Twenty-five years of hub location research" Transportation Science, Vol. 46, 2012. P. 153–169. https://doi.org/10.1287/trsc.1120.0410

[6]. Farahani, R. Z., Hekmatfar, M., Arabani, A. B. et al. "Hub location problems: A review of models, classification, solution techniques, and applications" Computers and Industrial Engineering, Vol. 64, 2013. P. 1096–1109. https://doi.org/10.1016/j.cie.2013.01.012

[7]. da Graça Costa, M., Captivo, M. E., Clímaco, J. "Capacitated single allocation hub location problem-A bi-criteria approach" Computers and Operations Research, Vol. 35(11), 2008. P. 3671–3695. https://doi.org/10.1016/j.cor.2007.04.005

[8]. O'Kelly, M. E. "Hub facility location with fixed costs" Papers in Regional Sciences, Vol. 71, 1992. P. 293–306. https://doi.org/10.1007/BF01434269

[9]. Campbell, J. F. "Integer programming formulations of discrete hub location problems" European Journal of Operational Research, Vol. 72, 1994. P. 387-405. https://doi.org/10.1016/0377-2217(94)90318-2

[10]. Abdinnour-Helm, S., Venkataramanan, M. A. "Solution approaches to hub location problems" Annals of Operational Research, Vol. 78, 1998. P. 31–50. https:// doi.org /10.1023 / A :1018954217758

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

[11]. Abdinnour-Helm, S. "A hybrid heuristic for the uncapacitated hub location problem" European Journal of Operational Research, Vol. 106, 1998. P. 489–499. https://doi.org/10.1016/S0377-2217(97)00286-5
[12]. Topcuoglu, H., Corut, F., Ermis, M. et al. "Solving the uncapacitated hub location problem using genetic algorithms" Computers and Operations Research, Vol. 32, 2005. P. 967–984. https://doi.org/10.1016/j.cor.2003.09.008
[13]. Cunha, C. B., Silva, M. R. "A genetic algorithm for the problem of configuring a hub-and-spoke network for a LTL trucking company in Brazil" European Journal of Operational Research, Vol. 179, 2007. P. 747–758. https://doi.org/10.1016/j.ejor.2005.03.057
[14]. Chen, J.-F. "A hybrid heuristic for the uncapacitated single allocation hub location problem" Omega, Vol. 35, 2007. P. 211–220. https://doi.org/10.1016/j.omega.2005.05.004.
[15]. Filipović, V., Kratica, J., Tošić, D. et al. "GA inspired heuristic for uncapacitated single allocation hub location problem" Applications of Soft Computing, Vol. 58, 2009. P. 149-158. https://doi.org/10.1007/978-3-540-89619-7_15
[16]. Silva, M. R., Cunha, C. B. "New simple and efficient heuristics for the uncapacitated single allocation hub location problem" Computers and Operations Research, Vol. 36, 2009. P. 3152-3165. https://doi.org/10.1016/j.cor.2008.12.019
[17]. Naeem, M., Ombuki-Berman, B. "An efficient genetic algorithm for the uncapacitated single allocation hub location problem" In IEEE Congress on Evolutionary Computation, 2010, (1–8). IEEE.
[18]. Marić, M., Stanimirović, Z., Stanojević, P. "An efficient memetic algorithm for the uncapacitated single allocation hub location problem" Soft Computig, Vol. 17, 2013. P. 445–466. https://doi.org/10.1007/s00500-012-0919-0
[19]. Abyazi-Sani, R., Ghanbari, R. "An efficient tabu search for solving the uncapacitated single allocation hub location problem" Computers and Industrial Engineering, Vol. 93, 2016. P. 99-109. https://doi.org/10.1016/j.cie.2015.12.028
[20]. Akbaripour, H., Masehian, E., Roostaei, A. "Landscape analysis and scatter search metaheuristic for solving the uncapacitated single allocation hub location problem" International Journal of Industrial and Systems Engineering, Vol. 26, 2017. P. 425–459.
[21]. Guan, J., Lin, G., Feng, H. B. "A learning-based probabilistic tabu search for the uncapacitated single allocation hub location problem" Computers and Operations Research, Vol. 98, 2018a. P. 1-12. https://doi.org/10.1016/j.cor.2018.04.020
[22]. Guan, J., Lin, G., Feng, H. B "A multi-start iterated local search algorithm for the uncapacitated single allocation hub location problem" Applied Soft Computing, Vol. 73, 2018b. P. 230-241. https://doi.org/10.1016/j.asoc.2018.08.035
[23]. Ghaffarinasab, N., Kara, B. Y. "Benders Decomposition Algorithms for Two Variants of the Single Allocation Hub Location Problem" Networks Spat Econ, Vol. 19, 2019. P. 83–108. https://doi.org/10.1007/s11067-018-9424-z
[24]. Čvokić, D. D., Stanimirović, Z. "A single allocation hub location and pricing problem" Computational and Applied Mathematics, Vol. 39, 2020. P. 1-24. https://doi.org/10.1007/s40314-019-1025-z
[25]. Alizadeh Firozi, M., Kiani, V., Karimi, H. "Improved genetic algorithm with diversity and local search for uncapacitated single allocation hub location problem" Journal of Decisions and Operations Research, Vol. 6(4), 2022. P. 536-552. https://doi.org/10.22105/dmor.2021.272989.1325
[26]. Aykin, T. "Lagrangian relaxation based approaches to capacitated hub-and-spoke network design problem" European Journal of Operational Research, Vol. 79, 1994. P. 501–523. https://doi.org/10.1016/0377-2217(94)90062-0
[27]. Aykin, T. "Networking policies for hub-and-spoke systems with application to the air transportation system" Transportation Science, Vol. 29, 1995. P. 201–221. https://doi.org/10.1287/trsc.29.3.201
[28]. Ernst, A. T., Krishnamoorthy, M. "Solution algorithms for the capacitated single allocation hub location problem" Annals of Operational Research, Vol. 86, 1999. P. 141-159.

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

https://doi.org/10.1023/A:1018994432663
[29]. Tuzun, D., Burke, L. I. "Two-phase tabu search approach to the location routing problem" European Journal of Operational Research, Vol. 116, 1999. P. 87-99. https://doi.org/10.1016/S0377-2217(98)00107-6
[30]. Labbé, M., Yaman, H., Gourdin, E. "A branch and cut algorithm for hub location problems with single assignment" Mathematical Programramming, Vol. 102, 2005. P. 371-405. https://doi.org/10.1007/s10107-004-0531-x
[31]. Randall, M. "Solution approaches for the capacitated single allocation hub location problem using ant colony optimisation" Computational Optimization and Applications, Vol. 39, 2008. P. 239-261. https://doi.org/10.1007/s10589-007-9069-1
[32]. Contreras, I., Fernández, E., Marín, A. "Tight bounds from a path based formulation for the tree of hub location problem" Computers and Operations Research, Vol. 36, 2009. P. 3117-3127. https://doi.org/10.1016/j.cor.2008.12.009
[33]. Correia, I., Nickel, S., Saldanha-da-Gama, F. "The capacitated single-allocation hub location problem revisited: A note on a classical formulation" European Journal of Operational Research, Vol. 207, 2010. P. 92-96. https://doi.org/10.1016/j.ejor.2010.04.015
[34]. Contreras, I., Díaz, J. A., Fernández, E. "Branch and price for large-scale capacitated hub location problems with single assignment" INFORMS Journal on Computing, Vol. 23, 2011. P. 41-55. https://doi.org/10.1287/ijoc.1100.0391
[35]. Stanojević, P., Marić, M., Stanimirović, Z. "A hybridization of an evolutionary algorithm and a parallel branch and bound for solving the capacitated single allocation hub location problem" Applied Soft Computing, Vol. 33, 2015. P. 24–36. https://doi.org/10.1016/j.asoc.2015.04.018
[36]. Habibzadeh Boukani, F., Farhang Moghaddam, B., Pishvaee, M. S. "Robust optimization approach to capacitated single and multiple allocation hub location problems" Computational and Applied Mathematics, Vol. 35, 2016. P. 45-60. https://doi.org/10.1007/s40314-014-0179-y
[37]. Meier, J. F., Clausen, U., Rostami, B. et al. "A Compact Linearisation of Euclidean Single Allocation Hub Location Problems" Electronic Notes in Discrete Mathematics, Vol. 52, 2016. P. 37-44. https://doi.org/10.1016/j.endm.2016.03.006
[38]. Meier, J. F., Clausen, U. "Solving single allocation hub location problems on euclidean data" Transportation Science, Vol. 52, 2018. P. 1141-1155. https://doi.org/10.1287/trsc.2017.0751.
[39]. Momayezi, F., Chaharsooghi, S. K., Sepehri, M. M. et al. "The capacitated modular single-allocation hub location problem with possibilities of hubs disruptions: modeling and a solution algorithm" Operational Research, Vol. 21, 2021. P. 139-166. https://doi.org/10.1007/s12351-018-0438-6
[40]. Seifbarghy, M., Hemmati, M., Soltan Karimi, S. "Hub Covering Location Problem Considering Queuing and Capacity Constraints" Journal of Optimization in Industrial Engineering, Vol. 11(1), 2018. P. 143-156. https://doi.org/10.22094/joie.2017.351.0
[41]. Sangsawang, O., Chanta, S. "Capacitated single-allocation hub location model for a flood relief distribution network" Computational Intelligence, Vol. 36(3), 2020. P. 1320-1347. https://doi.org/10.1111/coin.12374
[42]. Moeen Moghadas, F., Fuladi, F. "A bi-objective capacitated single-allocation hub location problem with reliability assumption on paths" International Journal of Industrial Mathematics, Vol. 14(1), 2022. P. 59-80. https://doi.org/ 10.30495/ijim.2021.19155
[43]. Huang, J., Wang, Q. "Robust optimization of hub-and-spoke airline network design based on multi-objective genetic algorithm. Journal of Transportation Systems Engineering and Information Technology, Vol. 9, 2009. P. 86-92. https://doi.org/10.1016/S1570-6672(08)60066-5
[44]. Makui, A., Rostami, M., Jahani, E. et al. "A multi-objective robust optimization model for the capacitated P-hub location problem under uncertainty" Management Science Letters, Vol. 2, 2012. P. 525-534. https://doi.org/10.5267/j.msl.2011.12.014
[45]. Ghodratnama, A., Tavakkoli-Moghaddam, R., Azaron, A. "A fuzzy possibilistic bi-objective hub covering problem considering production facilities, time horizons and transporter

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

In Press

vehicles" International Journal of Advanced Manufacturing Technology, Vol. 66, 2013. P. 187-206. https://doi.org/10.1007/s00170-012-4318-6
[46]. Ebrahimi Zade, A., Sadegheih, A., Lotfi, M. M. "A modified NSGA-II solution for a new multi-objective hub maximal covering problem under uncertain shipments" Journal of Industrial Engineering International, Vol. 10, 2014. P. 185-197. https://doi.org/10.1007/s40092-014-0076-4
[47]. Mohammadi, M., Torabi, S. A., Tavakkoli-Moghaddam, R. "Sustainable hub location under mixed uncertainty" Transportation Research Part E: Logistics and Transportation Review, Vol. 62, 2014. P. 89-115. https://doi.org/10.1016/j.tre.2013.12.005
[48]. Niakan, F., Vahdani, B., Mohammadi, M. "A multi-objective optimization model for hub network design under uncertainty: An inexact rough-interval fuzzy approach" Engineering Optimization, Vol. 47, 2015. P. 1670-1688. https://doi.org/10.1080/0305215X.2014.992891
[49]. Ghodratnama, A., Tavakkoli-Moghaddam, R., Azaron, A. "Robust and fuzzy goal programming optimization approaches for a novel multi-objective hub location-allocation problem: A supply chain overview" Applied Soft Computing, Vol. 37, 2015. P. 255–276. https://doi.org/10.1016/j.asoc.2015.07.038
[50]. Masoumzadeh, S., Solimanpur, M., Abdollahi Kamran, M. "A multi-objective fuzzy goal programming P-hub location and protection model with back-up hubs considering hubs establishment fixed costs" Scientia Iranica E, Vol. 23(4), 2016. P. 1941-1951. https://doi.org/10.24200/sci.2016.3939
[51]. Zhalechian, M., Tavakkoli-Moghaddam, R., Rahimi, Y. "A self-adaptive evolutionary algorithm for a fuzzy multi-objective hub location problem: An integration of responsiveness and social responsibility" Engineering Applications of Artificial Intelligence, 62, 2017. P. 1-16. https://doi.org/10.1016/j.engappai.2017.03.006
[52]. Madani, S. R., Shahandeh Nookabadi, A., Hejazi, S. R. "A bi-objective, reliable single allocation p-hub maximal covering location problem: Mathematical formulation and solution approach" Journal of Air Transportation Management, Vol. 68, 2018. P. 118–136. https://doi.org/10.1016/j.jairtraman.2017.09.001
[53]. Kahag, M. R., Niaki, S. T. A., Seifbarghy, M. et al. "Bi-objective optimization of multi-server intermodal hub-location-allocation problem in congested systems: modeling and solution" Journal of Industrial Engineering International, Vol. 15, 2019. P. 221–248. https://doi.org/10.1007/s40092-018-0288-0
[54]. Mohammadi, M., Jula, P., Tavakkoli-Moghaddam, R. "Reliable single-allocation hub location problem with disruptions" Transportation Research Part E: Logistics Transportation Review, Vol. 123, 2019. P. 90-120. https://doi.org/10.1016/j.tre.2019.01.008
[55]. Soylu, B., Katip, H. "A multiobjective hub-airport location problem for an airline network design" European Journal of Operational Research, Vol. 277, 2019. P. 412–425
[56]. Babashahi, M., Shahanaghi, K. Gholamian, M. R., Yavari, A. "A bi-objective hierarchical hub location model with facility failure" Scientia Iranica E, Vol. 27(4), 2020. P. 2009-2020
[57]. Yin, F., Liu, Y., Chen, Y. "Distributionally robust optimisation model for multi-objective hub location problem via considering ambiguousness" Transportmetrica A: Transport Science, Vol. 19(3), 2023. P. 2094494. https://doi.org/10.1080/23249935.2022.2094494
[58]. Khalilzadeh, M., Ahmadi, M., Kebriyaii, O. "A bi-objective mathematical programming model for a maximal covering hub location problem under uncertainty" SAGE Open, Vol. 15(1), 2025. https://doi.org/10.1177/21582440251324
[59]. Aouam, T., Oudani, M., Ghogho, M. "Efficient meta-heuristic approach for the multi-objective green p-hub centre routing problem. IEEE Transactions on Evolutionary Computation, 2025. https://doi.org/10.1109/TEVC.2024.3410517
[60]. Azzoug, A., Brahami, M. A., Ghomri, L., Bettayeb, B., Sahnoun, M. H. "Multi-objective resolution of hub location problem with hop-constraints" In 2024 International Conference of the African Federation of Operational Research Societies (AFROS), 2024, November.
[61]. Srinivas, N., Deb, K. "Muiltiobjective optimization using nondominated sorting in genetic algorithms" Evolutionary Computation, Vol 2(3), 1994. P. 221-248. https://doi.org/

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

10.1162/evco.1994.2.3.221
[62]. Horn, J., Nafpliotis, N., Goldberg, D. E. "A niched Pareto genetic algorithm for multiobjective optimization. In Proceedings of the First IEEE Conference on Evolutionary Computation" IEEE World Congress on Computational Intelligence (1994), 82–87. https://doi.org/10.1109/ICEC.1994.350037
[63]. Deb, K., Agrawal, S., Pratap, A. et al. "A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II" International Conference on Parallel Problem Solving from Nature, (2000). P. 849–858. https://doi.org/10.1007/3-540-45356-3_83
[64]. Hyun, C. J., Kim, Y., Kim, Y. K. "A genetic algorithm for multiple objective sequencing problems in mixed model assembly lines" Computers and Operations Research, Vol. 25, 1998. P. 675-690. https://doi.org/10.1016/S0305-0548(98)00026-4

In press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

Appendix A. Parameter settings obtained using the Taguchi method

Table A1. Parameter settings for Model I

Problem size	Algorithm	Population size	Generations	Shift mutation	Exchange mutation	Hub-array mutation	Crossover probability	a
Small	NHR	200	200	0.10	0.10	0.20	0.90	-
Medium	NHR	300	400	0.10	0.10	0.25	0.90	-
Large	NHR	350	700	0.20	0.20	0.25	0.90	-
Small	NHH	150	150	0.20	0.20	0.10	0.85	0.25
Medium	NHH	250	300	0.20	0.25	0.15	0.90	0.25
Large	NHH	350	600	0.25	0.25	0.15	0.90	0.20

Table A2. Parameter settings for Model II

Problem size	Algorithm	Population size	Generations	Shift mutation	Exchange mutation	Hub-array mutation	Crossover probability	a
Small	NHR	200	200	0.10	0.10	0.25	0.90	-
Medium	NHR	300	400	0.10	0.10	0.25	0.90	-
Large	NHR	350	700	0.20	0.20	0.25	0.90	-
Small	NHH	150	150	0.20	0.20	0.15	0.85	0.25
Medium	NHH	300	300	0.20	0.25	0.20	0.90	0.25
Large	NHH	350	600	0.25	0.25	0.20	0.90	0.20

Appendix B. Representative Pareto fronts

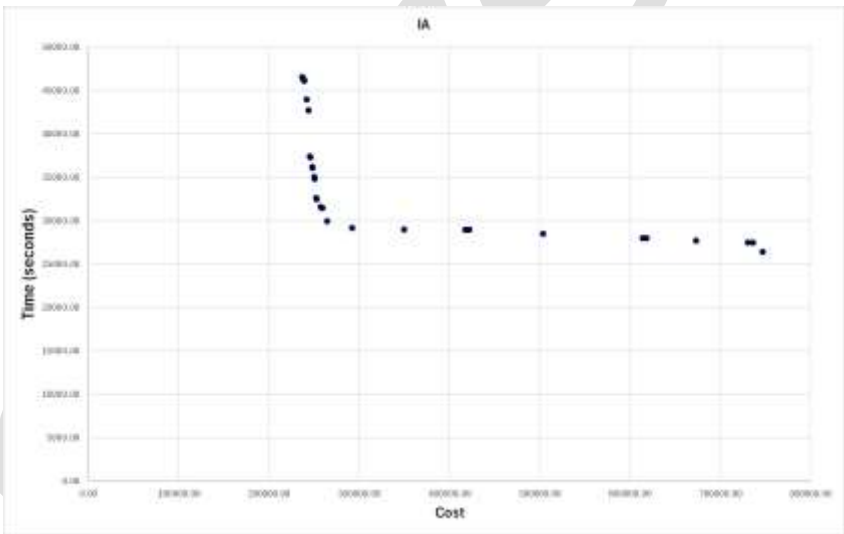


Figure A1. IA Pareto front for 30LL under Model I

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]

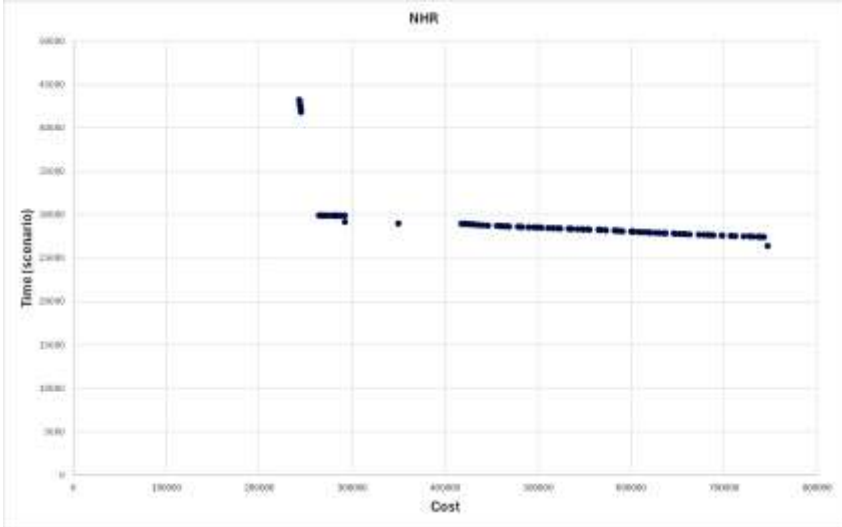


Figure A2. NHR Pareto front for 30LL under Model I

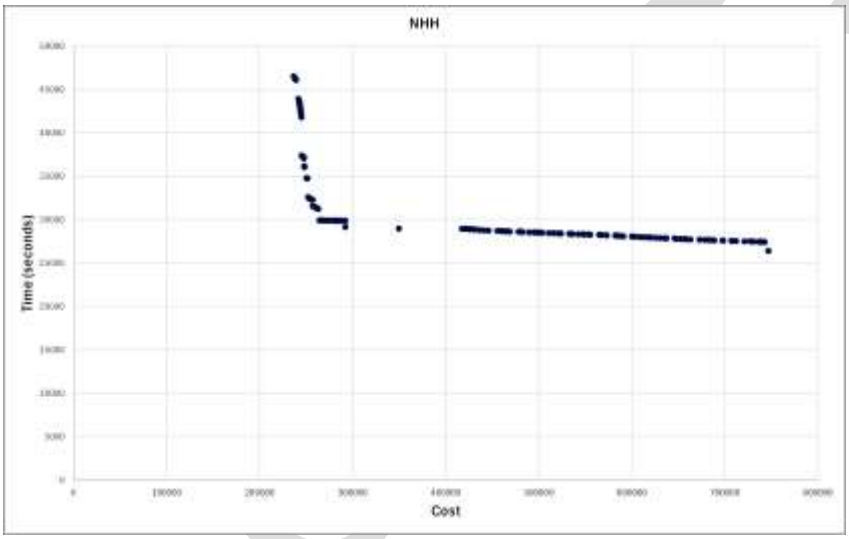


Figure A3. NHH Pareto front for 30LL under Model I

In Press

[Downloaded from ijiepr.iust.ac.ir on 2026-11-02]