

A BI-LEVEL LINEAR MULTI-OBJECTIVE DECISION MAKING MODEL WITH INTERVAL COEFFICIENTS FOR SUPPLY CHAIN COORDINATION

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Abstract: Bi-level programming, a tool for modeling decentralized decisions, consists of the objective(s) of the leader at its first level and that is of the follower at the second level. Three level programming results when second level is itself a bi-level programming. By extending this idea it is possible to define multi-level programs with any number of levels. Supply chain planning problems are concerned with synchronizing and optimizing multiple activities involved in the enterprise, from the start of the process, such as procurement of the raw materials, through a series of process operations, to the end, such as distribution of the final product to customers.

Enterprise-wide supply chain planning problems naturally exhibit a multi-level decision network structure, where for example, one level may correspond to a local plant control/scheduling/planning problem and another level to a corresponding plant-wide planning/network problem. Such a multi-level decision network structure can be mathematically represented by using "multi-level programming" principles. This paper studies a "bi-level linear multi-objective decision making" model in with "interval" parameters and presents a solution method for solving it; this method uses the concepts of tolerance membership function and multi-objective multi-level optimization when all parameters are imprecise and interval .

Keywords: Multi-level programming; Multi-objective decision-making; Multi-level multi-objective decision-making; Fuzzy decision-approach; Linear-programming with interval coefficients.

1. Introduction

1-1. Bi-Level Programming

A bi-level programming problem is formulated for a problem in which two decision-makers make decisions successively. For example, in a decentralized firm, top management makes a decision such as budget of the firm, and then each division determines a production plane in the full knowledge of the budget [1].

Research on multi-level mathematical programming to solve organizational planning and decision-making problems has been conducted widely. The research and application have concentrated mainly on bi-level programming [1]. In the BLP problem, each decision maker tries to optimize its own objective function(s) without considering the objective(s) of the other party, but the decision of each party affects the objective value(s) of the other party as well as the decision space.

The general formulation of a bi-level programming problem (BLPP) is:

$$\begin{aligned} \min_x & F(x, y) \\ \text{s.t.} & G(x, y) \leq 0 \\ & \min_y f(x, y) \\ & \text{s.t.} \quad g(x, y) \leq 0 \end{aligned} \quad (1)$$

The variables of problem are divided into two classes, namely the upper-level variables $x \in R^{n1}$ and the lower-level variables $y \in R^{n2}$. Similarly, the functions $F : R^{n1} \times R^{n2} \rightarrow R$ and $f : R^{n1} \times R^{n2} \rightarrow R$ are the upper-level and lower-level objective functions respectively, while the vector-valued functions $G : R^{n1} \times R^{n2} \rightarrow R^{m1}$ and $g : R^{n1} \times R^{n2} \rightarrow R^{m2}$ are called the upper-level and lower-level constraints respectively. All of the constraints and objective functions may be linear, quadratic, non-linear, fractional,

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Various methods have been proposed to solve these MLP problems. Following taxonomy of methods for multi-level programming problems has given in [3]:

Tab. 1. Taxonomy of methods for multi-level programming problems[3]

| | |
|------------------------------------|-------------------------------------|
| <i>I. Extreme-point search</i> | <i>K_m-best algorithm</i> |
| | <i>Grid-search algorithm</i> |
| | <i>Fuzzy approach</i> |
| | <i>Interactive approach</i> |
| <i>II. Transformation approach</i> | <i>Complement pivot</i> |
| | <i>Branch-and-bound</i> |
| | <i>Penalty function</i> |
| <i>III. Descent and heuristic</i> | <i>Descent method</i> |
| | <i>Branch-and-bound</i> |
| | <i>Cutting plane</i> |
| | <i>Dynamic programming</i> |
| <i>IV. Intelligent computation</i> | <i>Tabu search</i> |
| | <i>Simulated annealing</i> |
| | <i>Genetic algorithm</i> |
| | <i>Artificial neural network</i> |
| <i>V. Interior point</i> | <i>Primal-dual algorithm</i> |

The first three categories i.e., “extreme-point search”, “transformation approach”, and “descent and heuristic”, can be referred to as the traditional approaches. And the last two, i.e., “intelligent computation” (evolutionary approach) and “interior point” approach, are based on more recent developments. The basic concept of extreme-point search, category I, is to seek a compromise vertex by simplex algorithm based on adjusting the control variables. The transformation approach, category II, involves transforming the lower-level problems into constraints for the higher level by the use of various techniques such as Karush-Kuhn-Tucker (KKT) conditions, penalty functions, barrier functions, etc. Category III is developed for solving “discrete or nonlinear MLP problems”, and is based on existing search techniques or heuristic approaches such as gradient techniques, cutting-plane algorithm, and branch-and-bound heuristics. Category IV is based on the recent developments in intelligent computations, which are especially suited for solving NP-hard problems. This category is relatively new and includes taboo search, simulated annealing, and genetic algorithm, artificial neural networks (ANNs) can also be included in this category. The newly developed interior point method (category V) is less sensitive to problem size and would be suitable for solving MLP problems.

1-2. Supply Chain Management

A supply chain may be defined as an integrated process wherein a number of various business entities (i.e., suppliers, manufacturers, distributors, and retailers) work together in an effort to: (1) acquire raw materials, (2) convert these raw materials into specified final products, and (3) deliver these final products to retailers. This

chain is traditionally characterized by a forward flow of materials and a backward flow of information.

In recent years there has been a great interest in enterprise-wide supply chain planning problems because of their impact on substantially improving the overall competitiveness of economic potential of individual organizations. Supply chain planning problems are concerned with synchronizing and optimizing multiple activities involved in the enterprise, from the start of the process, such as procurement of the raw materials, through a series of process operations, to the end, such as distribution of the final product to customers [2].

Supply chain planning has thus obtained a great deal of attention in the open literature. A number of outstanding issues deserve some further attention [2]:

First, supply chain planning problems naturally incorporate multiple decision modeling steps, which are connected in a “hierarchical” way. Since individual activities are often governed by separate supply chain components which have their own, often mutually conflicting, objectives, the operation and control of the entire networks is based on multi-perspectives. Most of the planning models are however, based on the assumption that “all” activities of supply chain networks are governed by a “global organizer” neglecting such multiple perspectives. **Second**, different participant components in the supply chain network may not operate with the same level of information. Some may possess more information, while other may possess less information; this may lead to information distortion. **Finally**, the presence of uncertainty in supply chain planning models further amplifies the complexity of the problem. Uncertainty typically exists in supply chain parameters, such as processing times, performance coefficients, utility coefficients, delivery and inventory costs, supply of raw materials, etc. Here, approaches which have started to appear in the open literature include scenario-based multi-period formulations, stochastic programming formulations and supply chain dynamics and control formulations.

1-3. Interval Linear Programming

In conventional mathematical programming, coefficients of problems are usually determined by the experts as crisp values. But in reality, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of an expert are so precise. Hence, in order to develop good Operations Research methodology fuzzy and stochastic approaches are frequently used to describe and treat imprecise and uncertain elements present in a real decision problem. In fuzzy programming problems the constraints and goals are viewed as fuzzy sets and it is assumed that their membership functions are known. On the other hand, in stochastic programming problems the coefficients are viewed as random variables and it is also assumed that their probability distributions are known. These membership functions and probability distributions play important roles in their corresponding methods.

However, in reality, to a decision maker (DM) it is not always easy to specify the membership function or the probability distribution in an inexact environment. At least in some of the cases, use of an interval coefficient may serve the purpose better [4].

$$A = [a_L, a_R] = \{a \in R : a_L \leq a \leq a_R\}$$

where a_L, a_R are left and right limit of the interval A on the real line, respectively. If $a_L = a_R$ then A is a real number. Interval A is alternatively represented as $A = \langle m(A), w(A) \rangle$ where, $m(A), w(A)$ are the mid-point and half-width (or simply be termed as 'width') of interval A i.e.,

$$w(A) = 0.5(a_R - a_L) \quad m(A) = 0.5(a_R + a_L),$$

The general formulation of interval linear programming problem is[4]:

$$\begin{aligned} \text{Min} \quad & Z = \sum_{j=1}^n [c_{Lj}, c_{Rj}] x_j \\ \text{s.t} \quad & \sum_{j=1}^n [a_{Lij}, a_{Rij}] x_j \geq [b_{Li}, b_{Ri}] \quad \forall i \quad (2) \\ & x_j \geq 0 \quad \forall j \end{aligned}$$

In (2) all coefficients are interval. The following LP problem is the necessary equivalent form of (2)[4]:

$$\begin{aligned} \text{Min} \quad & m(Z) = 0.5 \sum_{j=1}^n [c_{Lj} + c_{Rj}] x_j \\ \text{s.t} \quad & \sum_{j=1}^n a_{ij} x_j \geq b_{Li} \quad \forall i \\ & b_{Li} + b_{Ri} - \sum_{j=1}^n [a_{Lij} + a_{Rij}] x_j \geq \alpha (b_{Ri} - b_{Li}) + \alpha \sum_{j=1}^n [a_{Rij} - a_{Lij}] x_j \\ & x_j \quad \forall j \end{aligned} \quad (3)$$

where α may be interpreted as an optimistic threshold assumed and fixed by the DM. It is only when there exists the possibility of multiple solution, that comparative widths ($w(A)$) are required to be calculated and then in favour of a minimum available width, we get the solution[6].

2. A Bi-Level Multi-Objective Decision-Making (BL-MODM) Under Impreciseness

Let $x_i \in R^{n_i} (i=1,2)$ be a vector variables indicating the first decision level's choice and the second decision level's choice $n_i \geq 1 (i=1,2)$. Let $F_1 : R^{n_1} \times R^{n_2} \rightarrow R^{N_1}$ be the first level objective functions and $F_2 : R^{n_1} \times R^{n_2} \rightarrow R^{N_2}$ be the second level objective functions. Let the FLDM and SLDM have N_1 and N_2 objective functions, respectively. Let

G be the set of feasible choices $\{(x_1, x_2)\}$. So the BL-MODM problem may be formulated as follows:

$$\begin{aligned} & [1st\ level] \\ & \text{Max}_{x_1} F_1(x_1, x_2) = \text{Max}_{x_1} (f_{11}(x_1, x_2), \dots, f_{1N_1}(x_1, x_2)) \\ & \text{where } x_2 \text{ solves} \end{aligned} \quad (4)$$

$$\begin{aligned} & [2nd\ level] \\ & \text{Max}_{x_2} F_2(x_1, x_2) = \text{Max}_{x_2} (f_{21}(x_1, x_2), \dots, f_{2N_2}(x_1, x_2)) \\ & \text{s.t} \quad G = \{(x_1, x_2) | g_i(x_1, x_2) \leq 0; i = 1, 2, \dots, m\} \end{aligned}$$

where G is the bi-level convex constraint set. The decision mechanism of BL-MODM is that the FLDM and SLDM adopt the two-planer Stackelberg game. Osman et al.[5] propose a solution method for solving above problem. For more information refer to [5].

3. Model Formulation

Enterprise-wide supply chain planning problems naturally exhibit a multi-level decision network structure, where for example, one level may correspond to a local plant control/scheduling/planning problem and another level to a corresponding plant-wide planning/network problem. Such a multi-level decision network structure can be mathematically represented by using "multi-level programming" principles. Production and distribution models of an enterprise can be mathematically formulated as follows (see Table 2 for the notation used).

3-1. A Production Model

A production part of supply chains is typically subject to the following constraints:

(a) Production amounts from the plants should meet the levels required at the warehouses.

$$\sum_{l=1}^L Y_{lwi} \geq \sum_{r=1}^R X_{wri} \quad \forall w, i \quad (5)$$

(b) Production levels at the plants are limited by individual plant capacities.

$$\sum_{i=1}^I \sum_{w=1}^W [\alpha_{li}^L, \alpha_{li}^U] Y_{lwi} \leq [P_l^L, P_l^U] \quad \forall l \quad (6)$$

(c) Common used resources may be shared by all the plants.

$$\sum_{i=1}^I \sum_{l=1}^L \sum_{w=1}^W [\beta_{li}^L, \beta_{li}^U] Y_{lwi} \leq [Q^L, Q^U] \quad (7)$$

An operating objective of production parts is to minimize their costs, which typically consists of its manufacturing cost and distribution cost between plants and warehouses.

Tab. 2. Notation for supply chain planning model

| Indices | |
|---------------------|---|
| i | Product ($1, \dots, I$) |
| l | Plant ($1, \dots, L$) |
| w | Warehouse ($1, \dots, W$) |
| r | Market ($1, \dots, R$) |
| Constant parameters | |
| α_{li} | Capacity coefficient of product i at plant l (interval) |
| β_{li} | Resource coefficient of product i at plant l (interval) |
| γ_{wi} | Resource coefficient of product i at warehouse w (interval) |
| a_{li} | Production cost coefficient for of product i at plant l (interval) |
| b_{lwi} | Transportation cost coefficient for product i from plant l to warehouse w (interval) |
| h_{wri} | Inventory holding cost coefficient for product i at warehouse w for market r (interval) |
| tr_{wri} | Transportation cost coefficient for product i from warehouse w to market r (interval) |
| M_{ri} | Demand of product i at market r (interval) |
| P_l | Production capacity of plant l (interval) |
| Q | Resources available to all the plants for product (interval) |
| Variables | |
| Z_{PC} | Objective function of a production part (cost) |
| Z_{DC}^1 | Objective function of a distribution part (capacity) |
| Z_{DC}^2 | Objective function of a distribution part (cost) |
| Y_{lwi} | Production amount of i at plant l for warehouse w |
| X_{wri} | Inventory of product i at warehouse w for market r |

$$\min_{Y_{lwi}} Z_{PC} = \sum_{l=1}^L \sum_{w=1}^W \sum_{i=1}^I [a_{li}^L, a_{li}^U] Y_{lwi} + \sum_{l=1}^L \sum_{w=1}^W \sum_{i=1}^I [b_{lwi}^L, b_{lwi}^U] Y_{lwi} \quad (8)$$

Operation of the production part can thus be formulated as the following mathematical programming problem:

$$\min_{Y_{lwi}} Z_{PC} = \sum_{l=1}^L \sum_{w=1}^W \sum_{i=1}^I [a_{li}^L, a_{li}^U] Y_{lwi} + \sum_{l=1}^L \sum_{w=1}^W \sum_{i=1}^I [b_{lwi}^L, b_{lwi}^U] Y_{lwi}$$

$$\sum_{l=1}^L Y_{lwi} \geq \sum_{r=1}^R X_{wri} \quad \forall w, i \quad (9)$$

$$\sum_{i=1}^I \sum_{w=1}^W [\alpha_{li}^L, \alpha_{li}^U] Y_{lwi} \leq [P_l^L, P_l^U] \quad \forall l$$

$$\sum_{i=1}^I \sum_{l=1}^L \sum_{w=1}^W [\beta_{li}^L, \beta_{li}^U] Y_{lwi} \leq [Q^L, Q^U]$$

$$X_{wri} \geq 0, \forall w, r, i \quad Y_{lwi} \geq 0, \forall w, l, i$$

3-2. A Distribution Model

A distribution part is typically subject to the following constraints.

(a) Sums of individual warehouses' holding should meet demands in markets.

$$\sum_{w=1}^W X_{wri} \geq [M_{ri}^L, M_{ri}^U] \quad \forall r, i \quad (10)$$

(b) The first objective function indicates capacity of all warehouses.

$$\min_{X_{wri}} Z_{DC}^1 = \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [\gamma_{wi}^L, \gamma_{wi}^U] X_{wri} \quad (11)$$

(c) The following indicates an objective function for the distribution part of the supply chain.

$$\min_{X_{wri}} Z_{DC}^2 = \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [h_{wri}^L, h_{wri}^U] X_{wri} + \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [tr_{wri}^L, tr_{wri}^U] X_{wri} \quad (12)$$

where the first term denotes inventory holding cost including material handling cost at warehouses and the second indicates transportation cost from warehouses to markets.

Operation of the inventory part can thus be formulated as the following mathematical programming problem.

$$\begin{aligned} \min_{X_{wri}} Z_{DC}^1 &= \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [\gamma_{wi}^L, \gamma_{wi}^U] X_{wri} \\ \min_{X_{wri}} Z_{DC}^2 &= \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [h_{wri}^L, h_{wri}^U] X_{wri} + \\ &\quad \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [tr_{wri}^L, tr_{wri}^U] X_{wri} \quad (13) \\ \sum_{w=1}^W X_{wri} &\geq [M_{ri}^L, M_{ri}^U] \quad \forall r, i \\ X_{wri} \geq 0, Y_{lwi} &\geq 0 \quad \forall w, l, i \end{aligned}$$

Note that the decisions of the distribution part are based on those of the production part: for example, inventory policies are made using the outcome of production decisions. Similarly, decisions on the production part are affected by parameters which are decided by the distribution part: for example, production levels are decided from given information regarding the inventory conditions.

Therefore the overall supply chain planning model can be posed as the following bi-level optimization problem:

$$\begin{aligned} \min_{X_{wri}} Z_{DC}^1 &= \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [\gamma_{wi}^L, \gamma_{wi}^U] X_{wri} \\ \min_{X_{wri}} Z_{DC}^2 &= \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [h_{wri}^L, h_{wri}^U] X_{wri} + \\ &\quad \sum_{w=1}^W \sum_{r=1}^R \sum_{i=1}^I [tr_{wri}^L, tr_{wri}^U] X_{wri} \\ \text{s.t.} \quad \sum_{w=1}^W X_{wri} &\geq [M_{ri}^L, M_{ri}^U] \quad \forall r, i \\ \min_{Y_{lwi}} Z_{PC} &= \sum_{l=1}^L \sum_{w=1}^W \sum_{i=1}^I [a_{li}^L, a_{li}^U] Y_{lwi} + \\ &\quad \sum_{l=1}^L \sum_{w=1}^W \sum_{i=1}^I [b_{lwi}^L, b_{lwi}^U] Y_{lwi} \quad (14) \\ \text{s.t.} \quad \sum_{l=1}^L Y_{lwi} &\geq \sum_{r=1}^R X_{wri} \quad \forall w, i \\ \sum_{i=1}^I \sum_{w=1}^W [\alpha_{li}^L, \alpha_{li}^U] Y_{lwi} &\leq [P_l^L, P_l^U] \quad \forall l \\ \sum_{i=1}^I \sum_{l=1}^L \sum_{w=1}^W [\beta_{li}^L, \beta_{li}^U] Y_{lwi} &\leq [Q^L, Q^U] \\ X_{wri} \geq 0, Y_{lwi} &\geq 0 \quad \forall w, l, i \end{aligned}$$

where the inner problem corresponds to the production optimization problem and the outer problem to the distribution optimization problem.

4. Solution Method

In this section we will present a solution method in flowchart frame work for solving model (14).

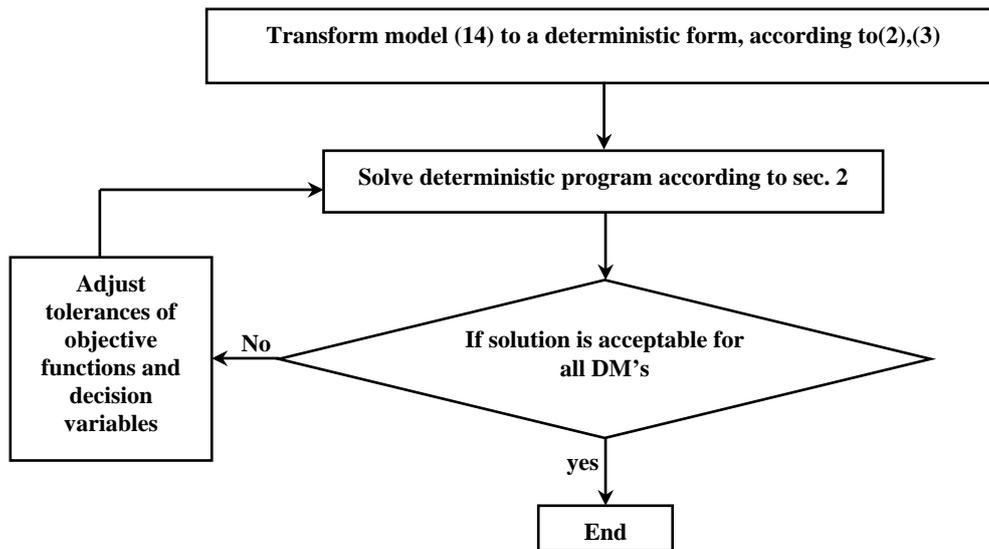


Fig 1. Solution method

5. Numerical Example

Consider following information about a supply chain model with 2 plants, 2 distribution centers and 2 market places.

According to (14) and above data, problem is:

$$M_{X_{wri}} \text{ in } Z_{DC}^1 = [2.5, 4.5](X_{111} + X_{121} + X_{211} + X_{221}) + [7.5, 10](X_{112} + X_{122} + X_{212} + X_{222})$$

$$M_{X_{wri}} \text{ in } Z_{DC}^2 = [15, 20](X_{111} + X_{121} + X_{211} + X_{221}) + [20, 40](X_{112} + X_{122} + X_{212} + X_{222})$$

| Parameters | Lower limit | Upper limit | Parameters | Lower limit | Upper limit |
|--|-------------|-------------|---------------|-------------|-------------|
| $t\tilde{r}_{wri} = 0 \quad \forall w, r, i$ | 0 | 0 | a_{22} | 400 | 500 |
| $\tilde{b}_{lwi} = 0 \quad \forall l, w, i$ | 0 | 0 | α_{11} | 1 | 1 |
| $\tilde{h}_{111} = \tilde{h}_{121} = \tilde{h}_{211} = \tilde{h}_{221} = \tilde{H}_1$ | 2.5 | 4.5 | α_{12} | 1 | 1 |
| $\tilde{h}_{112} = \tilde{h}_{122} = \tilde{h}_{212} = \tilde{h}_{222} = \tilde{H}_2$ | 7.5 | 10 | α_{21} | 1 | 1 |
| $\tilde{\gamma}_{111} = \tilde{\gamma}_{121} = \tilde{\gamma}_{211} = \tilde{\gamma}_{221} = \tilde{\gamma}_1$ | 15 | 20 | α_{22} | 1 | 1 |
| $\tilde{\gamma}_{112} = \tilde{\gamma}_{122} = \tilde{\gamma}_{212} = \tilde{\gamma}_{222} = \tilde{\gamma}_2$ | 20 | 40 | P_1 | 600 | 700 |
| M_{11} | 80 | 90 | P_2 | 700 | 800 |
| M_{12} | 60 | 75 | β_{11} | 1 | 1.5 |
| M_{21} | 40 | 50 | β_{12} | 3 | 4 |
| M_{22} | 85 | 90 | β_{21} | 4.5 | 6.5 |
| a_{11} | 300 | 450 | β_{22} | 0.5 | 1.5 |
| a_{12} | 200 | 250 | Q | 1000 | 2000 |
| a_{21} | 150 | 200 | | | |

$$M_{Y_{lwi}} \text{ in } Z_{PC} = [300, 450](Y_{111} + Y_{121}) + [200, 250](Y_{112} + Y_{122}) + [150, 200](Y_{211} + Y_{221}) + [400, 500](Y_{212} + Y_{222})$$

$$Y_{111} + Y_{211} \geq X_{111} + X_{121}$$

$$Y_{112} + Y_{212} \geq X_{112} + X_{122}$$

$$Y_{121} + Y_{221} \geq X_{211} + X_{221}$$

$$Y_{122} + Y_{222} \geq X_{212} + X_{222}$$

$$[1, 1](Y_{111} + Y_{121}) + [1, 1](Y_{112} + Y_{122}) \leq [600, 700]$$

$$[1, 1](Y_{211} + Y_{221}) + [1, 1](Y_{212} + Y_{222}) \leq [700, 800]$$

$$[1, 1.5](Y_{111} + Y_{121}) + [3, 4](Y_{112} + Y_{122}) + [4.5, 6.5](Y_{211} + Y_{221}) + [0.5, 1.5](Y_{212} + Y_{222}) \leq [1000, 2000]$$

$$X_{wri} \geq 0, Y_{lwi} \geq 0 \quad w, r, i = 1, 2$$

According to (3) equivalent problem is:

$$M_{X_{wri}} \text{ in } m(Z_{DC}^1) = 3.5(X_{111} + X_{121} + X_{211} + X_{221}) + 8.75(X_{112} + X_{122} + X_{212} + X_{222})$$

$$M_{X_{wri}} \text{ in } m(Z_{DC}^2) = 17.5(X_{111} + X_{121} + X_{211} + X_{221}) + 30(X_{112} + X_{122} + X_{212} + X_{222})$$

S.T.

$$X_{111} + X_{211} \geq 82.5$$

$$X_{112} + X_{212} \geq 63.75$$

$$X_{121} + X_{221} \geq 42.5$$

$$X_{122} + X_{222} \geq 86.25$$

$$M_{Y_{lwi}} \text{ in } m(Z_{PC}) = 375(Y_{111} + Y_{121}) + 225(Y_{112} + Y_{122}) + 175(Y_{211} + Y_{221}) + 450(Y_{212} + Y_{222})$$

$$Y_{111} + Y_{211} \geq X_{111} + X_{121}$$

$$Y_{112} + Y_{212} \geq X_{112} + X_{122}$$

$$Y_{121} + Y_{221} \geq X_{211} + X_{221}$$

$$Y_{122} + Y_{222} \geq X_{212} + X_{222}$$

$$Y_{111} + Y_{121} + Y_{112} + Y_{122} \leq 675$$

$$Y_{211} + Y_{221} + Y_{212} + Y_{222} \leq 775$$

$$\begin{aligned}
 &1.5(Y_{111} + Y_{121}) + 4(Y_{112} + Y_{122}) + 6.5(Y_{211} + Y_{221}) + \\
 &1.5(Y_{212} + Y_{222}) \leq 2000 \\
 &2.25(Y_{111} + Y_{121}) + 6.5(Y_{112} + Y_{122}) + 10(Y_{211} + Y_{221}) + \\
 &1.5(Y_{212} + Y_{222}) \leq 3500 \\
 &X_{wri} \geq 0, Y_{lwi} \geq 0 \quad w, r, i = 1, 2
 \end{aligned}$$

According to sec. 4 individual ideal and anti-ideal solutions for each objective function in both levels subject to constraint region are obtained:

$$\begin{aligned}
 Z_{DC}^1 &\rightarrow \min = 1750, \max = \infty \Rightarrow \max_{DM} = 2000 \\
 Z_{DC}^2 &\rightarrow \min = 6687.5, \max = \infty \Rightarrow \max_{DM} = 7000 \\
 Z_{PC} &\rightarrow \min = 0, \max = 558125
 \end{aligned}$$

Membership function of each objective function is:

$$\begin{aligned}
 \mu(Z_{DC}^1) &= \frac{2000 - Z_{DC}^1}{2000 - 1750} = \frac{2000 - Z_{DC}^1}{250} \\
 \mu(Z_{DC}^2) &= \frac{7000 - Z_{DC}^2}{7000 - 6687.5} = \frac{7000 - Z_{DC}^2}{312.5} \\
 \mu(Z_{PC}) &= \frac{558125 - Z_{PC}}{558125}
 \end{aligned}$$

If $\lambda = \text{Min} \{ \mu(Z_{DC}^1), \mu(Z_{DC}^2), \mu(Z_{PC}) \}$ we have:

$$\begin{aligned}
 &\max \quad \lambda \\
 &st. \\
 &3.5(X_{111} + X_{121} + X_{211} + X_{221}) + \\
 &8.75(X_{112} + X_{122} + X_{212} + X_{222}) + 250\lambda \leq 2000 \\
 &17.5(X_{111} + X_{121} + X_{211} + X_{221}) + \\
 &30(X_{112} + X_{122} + X_{212} + X_{222}) + 312.5\lambda \leq 7000 \\
 &375(Y_{111} + Y_{121}) + 225(Y_{112} + Y_{122}) + \\
 &175(Y_{211} + Y_{221}) + 450(Y_{212} + Y_{222}) + 558125\lambda \leq 558125
 \end{aligned}$$

$$\begin{aligned}
 X_{111} + X_{211} &\geq 82.5 \\
 X_{112} + X_{212} &\geq 63.75 \\
 X_{121} + X_{221} &\geq 42.5 \\
 X_{122} + X_{222} &\geq 86.25
 \end{aligned}$$

$$\begin{aligned}
 Y_{111} + Y_{211} &\geq X_{111} + X_{211} \\
 Y_{112} + Y_{212} &\geq X_{112} + X_{212} \\
 Y_{121} + Y_{221} &\geq X_{211} + X_{221} \\
 Y_{122} + Y_{222} &\geq X_{212} + X_{222} \\
 Y_{111} + Y_{121} + Y_{112} + Y_{122} &\leq 675 \\
 Y_{211} + Y_{221} + Y_{212} + Y_{222} &\leq 775
 \end{aligned}$$

$$\begin{aligned}
 &1.5(Y_{111} + Y_{121}) + 4(Y_{112} + Y_{122}) + \\
 &6.5(Y_{211} + Y_{221}) + 1.5(Y_{212} + Y_{222}) \leq 2000
 \end{aligned}$$

$$\begin{aligned}
 &2.25(Y_{111} + Y_{121}) + 6.5(Y_{112} + Y_{122}) + \\
 &10(Y_{211} + Y_{221}) + 1.5(Y_{212} + Y_{222}) \leq 3500
 \end{aligned}$$

By solving above problem, final solution is:

| Variable | Value |
|----------|-----------|
| LANDA | 0.9003359 |
| X111 | 82.50000 |
| X121 | 0.00000 |
| X211 | 0.00000 |
| X221 | 42.50000 |
| X112 | 63.75000 |
| X122 | 0.00000 |
| X212 | 0.00000 |
| X222 | 86.25000 |
| Y111 | 0.00000 |
| Y121 | 0.00000 |
| Y112 | 63.75000 |
| Y122 | 86.25000 |
| Y211 | 82.50000 |
| Y221 | 42.50000 |
| Y212 | 0.00000 |
| Y222 | 0.00000 |

and objective values are:

$Z_{DC}^1 = 1775, Z_{DC}^2 = 6718.75, Z_{PC} = 55812.5$. We assume obtained solution is acceptable for all DM's ($\lambda = 0.9$) and tolerance adjustment for objective functions and variables is not essential.

6. Conclusions

In mathematical programming, coefficients of problems are usually imprecise and uncertain. Fuzzy and stochastic approaches are frequently used to describe and treat imprecise and uncertain elements present in a real decision problem. Membership functions and probability distributions play important roles in their corresponding methods. However, in reality, to a decision maker (DM) it is not always easy to specify the membership function or the probability distribution in an inexact environment.

This paper was intended to outline some of the features of the association between the participating parties in the leader-follower interaction, and devise them into well-composed, solvable problems for easy follow-up analysis and comparison. We also methodically suggested a solution method for solving a bi-level multi-objective decision making model with interval parameters in supply chain management. A simplified example was used to illustrate the process of interaction. However, even though the method is young, it can be applied to explicit situations by changing certain assumptions to solve the specific problem properly. Although the optimal solution is rarely possible, a compromise solution, which is

acceptable for all parties with conflicting objectives, provides conflict resolution.

Reference

- [1] Emam, O.E., "A Fuzzy Approach for bi-level Integer Non-Linear Programming Problem", Applied Mathematics and Computation, 2005.
- [2] Jun-Hyung Ryu, Vivek Dua, Efstratios N. Pistikopoulos, A bilevel programming framework for enterprise-wide process networks under uncertainty, Computers and Chemical Engineering 28, 2004, PP. 1121–1129.
- [3] Hsu - Shih Shih, Ue - Pyng Wen, E.S., Lee, Kuen-Ming Lan, Han-Chyi Hsiao, A Neural Network Approach to Multi objective and Multilevel Programming Problems, Computers and Mathematics with Applications 48, 2004, PP. 95-108.
- [4] Atanu Sengupta, Tapan Kumar Pal, Debjani Chakraborty; Interpretation of Inequality Constraints Involving Interval Coefficients and a Solution to Interval Linear Programming; Fuzzy Sets and Systems 119, 2001, PP. 129-138.
- [5] Osman, M.S., Abo-Sinna, M.A., Amer, A.H., "A Multi-Level Non-Linear Multi-Objective Decision-Making Under Fuzziness", Applied Mathematics and Computation 153, 2004, PP. 239–252.