International Journal of Industrial Engineering & Production Research December 2024 Vol. 35, No. 4: 1-15
DOI: 10.22068/ijiepr.35.4.2094

RESEARCH PAPER

A Nonlinear Model to Solve Multiple Attribute Decision-Making Problems with Interval-Valued Neutrosophic Numbers

Maryam Arshi[1]; A. Hadi-Vencheh[2]; Adel Aazami[3*] & Ali Jamshidi[4]

Received 19 July 2024; Revised 20 September 2024; Accepted 20 October 2024;

ABSTRACT
Cognitive processes can be effectively communicated through linguistic variables (LVs), offering a robust framework for conveying uncertain and incomplete data in multiple attribute decision-making (MADM) problems. This method surpasses conventional techniques in handling such complexities. Recognizing decision experts' (DEs) bounded rationality, particularly their cognitive limitations leading to potential losses, underscores the need for innovative cognitive decision-making strategies in MADM. This study introduces LVs to encapsulate uncertain and hesitant cognitive elements, followed by a mathematical programming approach to tackle MADM problems where attributes or cognitive preferences exhibit interdependence. To enhance this approach within an interval-valued neutrosophic numbers (IVNN) environment, an IVNN multi-attribute group decision-making problem is modeled as a nonlinear programming model. Through variable transformation and aggregation operator application, this model is refined into an equivalent nonlinear programming model. The proposed method empowers decision-makers (DMs) to identify optimal alternatives without relying solely on their expertise, as demonstrated by its successful application in resolving two real-world problems.

KEYWORDS: Multiple attribute group decision making (MAGDM); Interval-valued neutrosophic number (IVNN); Non-Linear programming; Variable transformation; Aggregation operators.

1. Introduction

Today's highly competitive business landscape demands practical and strategic decision-making, often requiring the consideration of diverse and sometimes contradictory evaluation criteria. As a result, many complex decision-making problems can be categorized as multiple attribute decision-making (MADM) [1] problems, particularly when dealing with a finite set of alternatives, each with distinct and potentially conflicting attributes. Attributes can be categorized using two distinct methods. The first method involves grouping attributes into three types: subjective (qualitative and intangible), objective (quantitative and tangible), and critical (those necessary for further processing). The second approach classifies attributes as either benefit-type (where higher values are preferable) or cost-type (where lower values are desirable).

MADM's key strength lies in its ability to offer multiple dimensions, enabling decision-makers to thoroughly assess relevant factors and explore various alternatives across different criteria. Furthermore, to enhance democracy and rational decision-making, many real-world processes occur within group settings. Multiple attribute group decision-making (MAGDM) [2] plays a crucial role in this group decision-making. It is one of the most significant and commonly encountered processes across various important fields such as engineering, economics, management, medicine, and military affairs

Furthermore, in practical group decision-making, owing to the complexity and subjectivity inherent in decision systems, combined with the uncertain nature of human judgment, outputs from decision experts are often not precise numbers. Instead, they may take the form of linguistic terms or represent labels of fuzzy sets [3]. Quantifying uncertain information, especially for qualitative

* Corresponding author: Adel Aazami
adel.aazami@wu.ac.at

1. *Department of Mathematics, Najafabad Branch, Islamic Azad University, Isfahan, Iran.*
2. *Department of Mathematics, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran.*
3. *Department of Global Business and Trade, Institute for Transport and Logistics Management, Vienna University of Economics and Business, Vienna, Austria.*
4. *Department of Mathematics, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran*

attributes, presents significant challenges. This has led to the development of fuzzy methods of multiple attribute group decision-making (FMAGDM) [4] designed to address imprecise, vague, and uncertain data, whether qualitative or quantitative**.** Bakar and Ghani [5] provide a systematic literature review exploring how fuzzy analytic (FA), and multi-criteria decision-making (MCDM) methods contribute to classroom assessment.

In numerous practical FMAGDM problems, hesitation can arise during the evaluation phase or when determining attribute preferences. The intuitionistic fuzzy set (IFS) [6]) proves to be more effective and adaptable in handling the fuzziness and uncertainty that stem from ambiguous information or hesitations. The IFS theory is valuable for addressing the imprecision in human cognitive processes because it incorporates both supporting and opposing information in the decision-making procedure. This theory has been effectively combined with MADM and MAGDM methods [7-10]. Genç et al. [11] explored the challenges related to consistency, missing values, and the derivation of priority vectors in interval fuzzy preference relations. Boran and Akay [12] introduced a bi-parametric similarity measure for intuitionistic fuzzy sets, applying it to pattern recognition. Shen et al. [13] introduced a novel outranking sorting technique for group decision-making utilizing IFSs. Wan et al. [14] created a fresh approach to address MAGDM problems with Atanassov's interval-valued intuitionistic fuzzy values and incomplete attribute weight details. As the decision environment grows more intricate, numerous new expression forms of IFSs are regularly being developed in research**.**

In addition to extending existing models, new methods have been introduced for solving IVIF MADM problems. Li [15] introduced a nonlinear programming approach based on TOPSIS to address MADM problems by considering ratings of alternatives on attributes and weights of attributes represented. Li [16] introduced relative closeness coefficients and developed two non-linear fractional programming models that were converted into two simpler auxiliary linear programming models to determine the relative closeness coefficient of alternatives to the IVIF positive ideal solution. This method was used to establish the ranking order of alternatives. Arshi et al. [17] proposed a novel approach for solving multiple attribute group decision-making (MAGDM) problems using interval-valued intuitionistic fuzzy sets (IVIFS). They solved the MAGDM problems with a non-linear model. Fadhil and Habibnejad [18] conceptualized the problem of determining optimal robot trajectories as a trajectory optimization problem. They developed an iterative linear programming (ILP) method to obtain a numerical solution for this nonlinear trajectory. In recent times, supply chain management (SCM) has become a fascinating problem that has captured the interest of numerous researchers. One crucial aspect of SCM is transportation network design. Khezeli et al. [19] introduced a mixed integer nonlinear programming model (MINLP) aimed at reducing both the transportation time and cost of products

The theory of Intuitionistic Fuzzy Sets (IFS) excels in managing incomplete information in various real-world scenarios but falls short in tackling all forms of uncertainty, particularly indeterminate information. To address this limitation, Smarandache [20] introduced neutrosophic sets (NSs), which approach imprecise, incomplete, and uncertain information from a philosophical perspective. Neutrosophic sets are a broadening of crisp sets, fuzzy sets (FSs), and IFSs. They form part of neutrosophy, which explores the origins, nature, and scope of neutrality and its links to different ideational spectra, Smarandache [21]. Smarandache [20,21] introduced a concept with three independent components: the truth membership function (TMF), the indeterminacy membership function (IMF), and the falsity membership function (FMF).). Subsequently, the single-valued neutrosophic set (SVNS) was developed, as referenced in works by Smarandache [20, 21, 22], Deli and Subas [23], Abdel-Basset and Mohamed [24], and Edalatpanah [25]. Additionally, there have been several generalizations of neutrosophic sets, such as interval neutrosophic sets referenced by Garg [26], Liu and Shi [27], bipolar neutrosophic sets discussed by Deli et al [28], and Ulucay et al. [29], and multi-valued neutrosophic sets explored by Ji, Zhang, Wang [30], Peng, Wang, and Yang [31]. These also include simplified neutrosophic sets (Edalatpanah and Smarandache [32]; Peng et al. [33]) have been presented. Edalatpanah [34] introduced a novel concept in neutrosophic sets called the neutrosophic structured element (NSE) and proposed a decision-making method for multi-attribute decision-making (MADM) problems using NSE information.

Biswas et.al. [35] developed a value and ambiguity-based ranking approach for trapezoidal neutrosophic numbers (TrNNs) and outlined a multi-criteria decision-making (MCDM) strategy.

Liang and team [36] defined score, accuracy, and certainty functions for single-valued trapezoidal neutrosophic numbers (SVTrNN) using the center of gravity (COG) concept. Edalatpanah [37] also devised a new algorithm for addressing the single-valued neutrosophic linear programming problem. Elsayed [38] proposed a Multi-Criteria Decision Making (MCDM) framework to evaluate and rank green fuel alternatives aimed at reducing greenhouse gas emissions. This method incorporates the Removal Effects of Criteria (MEREC) to determine criteria weights and uses the TODIM method to rank alternatives, employing triangular neutrosophic numbers. Satyanarayana and Baji [39] introduced the concept of interval-valued neutrosophic N-d ideal, applying it to the d-ideal of algebraic structure d-algebra. Jdid and Smarandache [40] converted several zero-one neutrosophic nonlinear programming problems into linear ones. Khalifa [41] presented a multi-objective assignment (NMOAS) problem utilizing single-valued trapezoidal neutrosophic numbers in cost matrix elements.

Building on the capabilities of interval-valued neutrosophic numbers for describing uncertainty and vague data in decision-making problems, this paper aims to develop a formulation for multi-attribute group decision-making (MAGDM) problems. The MAGDM problem is formulated as a nonlinear programming problem, and an innovative approach is proposed to address it. The contributions of this study are outlined as follows: The decision-making process is fraught with challenges. Typically, individuals face choices where the direction is not well-defined. A significant amount of time and effort is needed to analyze various actions using systematic techniques. However, the area of constrained decision-making problems involving interval-valued neutrosophic numbers (IVNN) has not been fully explored. Since individuals often struggle to quantify their opinions in the context of incomplete fuzzy decision-making problems, IVNNs offer a more effective solution, which is why our focus centers on them; thus, we focus on IVNNs. FSs are limited to handling membership functions and do not contribute to non-membership functions, making them inadequate for addressing haziness and non-deterministic situations. To address this limitation, we have enhanced the current methodology in the NS environment by employing IVNNs. This allows us to manage incomplete information and ambiguous or vague conditions. Consequently, we have structured the MAGDM problem as a nonlinear programming issue and proposed an innovative approach to solve it.

A limitation of the proposed method is its need to solve distinct models for each alternative, which can become cumbersome as alternatives increase. However, various objective functions, as previously noted, can alleviate this difficulty. The absence of a global IVIF scale for MAGDM problems makes it challenging for experts to express their judgments, though this is a common issue across many proposed MAGDM methods. Future research might extend this methodology to other types of neutrosophic numbers. For example, an extension can be explored in the presence of an SVTrNN. Additionally, within the context of this paper, future studies could formulate the LP model using IVNNs. The application of IVNNs could also be investigated to determine the utility function of decision-makers in multi-attribute utility theory

The present study is set as follows**.** Section 2 deals with an overview of NSs and needed concepts. Section 3 deals with the definite problem and its formulations. Section 4 clarifies the presented approach to solve the problem. The proposed technique's applicability is provided by two numerical examples in section 5. Ultimately, in Section 6, some conclusions are made.

2. Neutrosophic Sets

In this section, some basic definitions and calculations for neutrosophic numbers are presented [43,44].

Definition 1. Let X be the space of points (objects) whose common element X is denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the function $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0,1], that is $T_A(x): X \to [0,1], I_A(x): X \to [0,1]$ and $F_A(x): X \to [0,1]$. Therefore, a single-valued neutrosophic set A is defined by $A = \{(x, T_A(x), I_A(x), F_A(x) \mid x \in X\}$ which is called an SVNS. Also, SVNS satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. (Ye [45])

Definition 2. For SVNSs C and D, $C \subseteq D$ if and only if $T_C(x) \leq T_D(x), I_C(x) \geq I_D(x)$, and $F_C(x) \geq F_D(x)$ for every x in X. (Ye [45])

Definition 3. An IVNS (interval-valued neutrosophic set) $\tilde{A}$ on universal set X is defined as:

$$\tilde{A} = \left\{ \langle \varepsilon, ([T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)], [I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)], [F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)] \rangle : \varepsilon \in X \right\} \quad (1)$$

With the condition $0 \le T_{\tilde{A}}(\varepsilon) + I_{\tilde{A}}(\varepsilon) + F_{\tilde{A}}(\varepsilon) \le 3$ Pramanik and Mondal [46]).

Definition 4. Here are some of the arithmetic operations in IVNNs:
Here, we consider two IVNS of $\tilde{A} = \langle [T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)], [I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)], [F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)] \rangle$ and $\tilde{B} = \langle [T_{\tilde{B}^L}(\varepsilon), T_{\tilde{B}^U}(\varepsilon)], [I_{\tilde{B}^L}(\varepsilon), I_{\tilde{B}^U}(\varepsilon)], [F_{\tilde{B}^L}(\varepsilon), F_{\tilde{B}^U}(\varepsilon)] \rangle$ can be defined as follows (Ye [47]):

(i) $\tilde{A} \oplus \tilde{B} = \langle [T_{\tilde{A}^L}(\varepsilon) + T_{\tilde{B}^L}(\varepsilon) - T_{\tilde{A}^L}(\varepsilon).T_{\tilde{B}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon) + T_{\tilde{B}^U}(\varepsilon) - T_{\tilde{A}^U}(\varepsilon).T_{\tilde{B}^U}(\varepsilon)][I_{\tilde{A}^L}(\varepsilon).I_{\tilde{B}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon).I_{\tilde{B}^U}(\varepsilon)], [F_{\tilde{A}^L}(\varepsilon).F_{\tilde{B}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon).F_{\tilde{B}^U}(\varepsilon)] \rangle$ (2)

(ii) $\tilde{A} \otimes \tilde{B} = \langle [T_{\tilde{A}^L}(\varepsilon).T_{\tilde{B}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon).T_{\tilde{B}^U}(\varepsilon)], [I_{\tilde{A}^L}(\varepsilon) + I_{\tilde{B}^L}(\varepsilon) - I_{\tilde{A}^L}(\varepsilon).I_{\tilde{B}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon) + I_{\tilde{B}^U}(\varepsilon) - I_{\tilde{A}^U}(\varepsilon).I_{\tilde{B}^U}(\varepsilon)], [F_{\tilde{A}^L}(\varepsilon) + F_{\tilde{B}^L}(\varepsilon) - F_{\tilde{A}^L}(\varepsilon).F_{\tilde{B}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon) + F_{\tilde{B}^U}(\varepsilon) - F_{\tilde{A}^U}(\varepsilon).F_{\tilde{B}^U}(\varepsilon)] \rangle$

(iii) (3)

(iv) $\lambda\tilde{A} = \langle \left[1 - \left(1 - T_{\tilde{A}^L}(\varepsilon)\right)^{\lambda}, 1 - \left(1 - T_{\tilde{A}^U}(\varepsilon)\right)^{\lambda}\right], \left[\left(I_{\tilde{A}^L}(\varepsilon)\right)^{\lambda}, \left(I_{\tilde{A}^U}(\varepsilon)\right)^{\lambda}\right], [(F_{\tilde{A}^L}(\varepsilon)^{\lambda}, \left(F_{\tilde{A}^U}(\varepsilon)\right)^{\lambda}] \rangle \quad \lambda > 0$ (4)

(v) $\tilde{A}^{\lambda} = \langle [(T_{\tilde{A}^L}(\varepsilon))^{\lambda}, (T_{\tilde{A}^U}(\varepsilon))^{\lambda}], [1 - (1 - I_{\tilde{A}^L}(\varepsilon))^{\lambda}, 1 - (1 - I_{\tilde{A}^U}(\varepsilon))^{\lambda}], [1 - (1 - F_{\tilde{A}^L}(\varepsilon))^{\lambda}, 1 - (1 - F_{\tilde{A}^U}(\varepsilon))^{\lambda}] \rangle \; \lambda > 0$ (5)

Definition 5. Let $\tilde{A} = \langle \varepsilon, [T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)], [I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)], [F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)] \rangle$ be an IVNN, then we define the score function and accuracy function as follows:

$S(\tilde{A}) = \frac{1}{2}([T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)] + (1 - [I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)]) + (1 - [F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)])$ (6)

accuracy $(\tilde{A}) = \frac{1}{2}([T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)] - (1 - [I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)]) - (1 - [F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)]$ (7)

Let $\tilde{A} = \langle [T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)], [I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)], [F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)] \rangle$ and $\tilde{B} = \langle [T_{\tilde{B}^L}(\varepsilon), T_{\tilde{B}^U}(\varepsilon)], [I_{\tilde{B}^L}(\varepsilon), I_{\tilde{B}^U}(\varepsilon)], [F_{\tilde{B}^L}(\varepsilon), F_{\tilde{B}^U}(\varepsilon)] \rangle$

be two arbitrary IVNN, the ranking of $\tilde{A}$ and $\tilde{B}$ by score function is defined as follows:

- If $S(\tilde{A}) < S(\tilde{B})$ then $\tilde{A} < \tilde{B}$
- If $S(\tilde{A}) = S(\tilde{B})$ and if
  accuracy $(\tilde{A}) <$ accuracy $(\tilde{B})$ then $\tilde{A} < \tilde{B}$
  accuracy $(\tilde{A}) >$ accuracy $(\tilde{B})$ then $\tilde{A} > \tilde{B}$
  accuracy $(\tilde{A}) =$ accuracy $(\tilde{B})$ then $\tilde{A} = \tilde{B}$

Definition 6. Let $\widetilde{A_J} = \langle \left[T_{\widetilde{A_J}^L}(\varepsilon), T_{\widetilde{A_J}^U}(\varepsilon)\right], \left[I_{\widetilde{A_J}^L}(\varepsilon), I_{\widetilde{A_J}^U}(\varepsilon)\right], \left[F_{\widetilde{A_J}^L}(\varepsilon), F_{\widetilde{A_J}^U}(\varepsilon)\right] \rangle \; (j = 1, \dots n)$ be an IVNN. The arithmetic average operator is as follows:

$$\text{IVNAA} = \sum_{j=1}^{n} w_j A_j \quad (8)$$

Where $W = (w_1, \dots, w_n)$ is the weight vector of $\widetilde{A_J}$, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 7. For the IVNN weighted arithmetic average operator (IVNNWAA), the aggregated result is as follows:

$$\text{IVNNWA}\;(\tilde{A}_1 \dots . \tilde{A}_n) = \langle [1 - \prod_{j=1}^{n} \left(1 - T_{\widetilde{A_J}^L}(\varepsilon)\right)^{w_j} . 1 - \prod_{j=1}^{n} \left(1 - T_{\widetilde{A_J}^U}(\varepsilon)\right)^{w_i}]. [\prod_{j=1}^{n} (I_{\widetilde{A_J}^L}(\varepsilon))^{w_j} . \prod_{j=1}^{n} (I_{\widetilde{A_J}^U}(\varepsilon))^{w_j}]. [\prod_{j=1}^{n} (F_{\widetilde{A_J}^L}(\varepsilon))^{w_j} . \prod_{j=1}^{n} (F_{\widetilde{A_J}^U}(\varepsilon))^{w_j}] \rangle \quad (9)$$

3. MAGDM Problem Formulation

Supposing a group of *K* decision-makers apprising the alternative set A={A$_1$, A$_2$, …, A$_n$} regard to criteria set C={C$_1$, C$_2$, …, C$_n$}. the evaluations are individually made by *e*ach decision maker along with an individual decision matrix D^k=[$\tilde{x}_{ij}^k$]. The $\tilde{x}_{ij}^k$ elements of D^k are expressed in the form of an IVNN $\tilde{x}_{ij}^k = [\left(\underline{T}_{ij,}^k \overline{T}_{ij}^k\right), \left(\underline{I}_{ij}^k, \overline{I}_{ij}^k\right), \left(\underline{F}_{ij}^k, \overline{F}_{ij}^k\right)]$, $i = 1,2, \dots, m\,; j = 1,2, \dots, n$. The problem is aimed at deciding on ranking or rating alternatives to allow decision-makers to decide on their ultimate alternative(s) or ranking.
An aggregated decision matrix *D* is first created through IVNNWAA (when a predetermined weight is assigned to different experts). The aggregated decision matrix will be acquired as D=[$\tilde{x}_{ij}$] where,

$$\tilde{x}_{ij} = \text{IVNNWA}\;(\tilde{x}_{ij}^1 . \tilde{x}_{ij}^2 . \dots . \tilde{x}_{ij}^k) \quad (10)$$

The prolonged type of aggregated matrix D can be represented as:

$$D = \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix} \quad (11)$$

When the problem ranks alternatives of set A, the flowing formulation is considered for the MAGDM problem:

$$Max \sum_{j=1}^{n} w_j \tilde{x}_{ij}$$

s.t. $\sum_{j=1}^{n}(w_j\tilde{x}_{ij})^2 = \tilde{1} \quad i = 1, \dots, m$ (i)

$w_j \geq 0 \qquad j = 1, \dots, n$ (ii) (12)

in which $\tilde{\mathbf{1}}$ represents an IVNN, like [(0.9, 0.95), (0.01, 0.05), (0.02,0.06)], and $\mathbf{w_j}$, j= 1,2,…,n denotes the criterion j importance weight**.** This data envelopment-based model is initially proposed by Hadi-Vencheh [42] as a weighted nonlinear optimization model for multi-criteria inventory classification problems. Here, the overall score of each alternative I is maximized by the objective function as the criteria's linear function**.** The total scores of the alternatives are restricted by constraint (i) equal to 1, with weights similar to that of alternative i. Constraint (ii) denotes that all the criteria' weights must be positive. Solving this model is repeatedly performed for every alternative and ranked based on their scores' descending order**.**

4. Solving Azzpproach

The MAGDM problems (Eq. (12)) may be regarded as an NLP problem under IVIF information.
An optimization problem is mathematically for finding the supremum or infimum of a certain real-valued function f over a specified set G of a universal set X, i.e.

$$\alpha = \inf\{ f(x) : x \in G \}, \quad G \subseteq X \tag{13}$$

In the optimization problems, obtaining the value of α or equally, an $x_0 \in G$ that $f(x_0) = \alpha$ (Ponstein [48]) is included. A matrix presentation of NLP is found in Eq. (13).

$Max\ CX$

s.t.

$$(AX)^2 \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b\,, \quad X \geq 0 \tag{14}$$

in which X represents the decision variables' $(n \times 1)$ column vector, A represents the $(m \times n)$ technological matrix, C shows the $(1 \times m)$ row vector of cost (profit) coefficients and b shows the $(m \times 1)$ right-hand side vector or resources. Considering the certainty axiom, matrix *A's* all elements, as well as vectors b and C, are determined as deterministic numbers.
Here, a novel technique is evolved to resolve the NLP problems, for which the parameters are described as IVNN. Consider a nonlinear programming with IVN information whose parameters C, A, and b are defined as IVNNs.

$Max\ \tilde{C}X$

s.t.

$(\tilde{A}X)^2 = \tilde{b}$

$X \geq 0$ (15)

In an extended form, IVNN-NLP in Eq. (15), can be stated as:

$$\max \sum_{j=1}^{n} \tilde{c}_j x_j$$

s.t.

$$\sum_{j=1}^{n} (\tilde{a}_{ij}x_j)^2 = \tilde{b}_i \qquad i = 1,2, \dots, m$$

$$x_j \geq 0 \qquad j = 1,2, \dots, n \tag{16}$$

The considered parameters in Eq. (16), are a set of IVNNs as follows:
$\tilde{c}_j = [(c_{1j}, c_{2j}), (c_{3j}, c_{4j}), (c_{5j}, c_{6j})]$, $j = 1,2, \dots, n$ where (c_{1j}, c_{2j}) is truth membership and (c_{3j}, c_{4j}) is indeterminacy membership, and (c_{5j}, c_{6j}) is falsity membership intervals.
$\tilde{a}_{ij} = [(a_{1ij}, a_{2ij}), (a_{3ij}, a_{4ij}), (a_{5ij}, a_{6ij})]$, $i = 1, \dots, m; j = 1, \dots, n$ where (a_{1ij}, a_{2ij}) is truth membership and (a_{3ij}, a_{4ij}) is indeterminacy membership, and (a_{5ij}, a_{6ij}) is falsity membership intervals.
$\tilde{b}_i = [(b_{1i}, b_{2i}), (b_{3i}, b_{4i}), (b_{5i}, b_{6i})]$, $i = 1,2, \dots, m$ where (b_{1i}, b_{2i}) is truth membership and (b_{3i}, b_{4i}) is indeterminacy membership, and (b_{5i}, b_{6i}) is falsity membership intervals.
Now, take into account the objective function $\sum_{j=1}^{n} \tilde{c}\, x_j$. For the reason that objective function coefficients $\tilde{c}_j, j = 1, \dots, n$ are IVNNs; hence, the objective function is denoted as these IVNNs' linear combination by the non-negative coefficients $x_j \geq 0, j = 1, \dots, n$. The results of this linear combination may be yielded via interactive using the multiplication and summation operators, as described respectively in Eqs. (2) and (4). This induction procedure includes n scalar multiplication plus (n-1) IVN summation operation, entirely encompassing (2n-1) operations. The simple variable transformation can be utilized to prevent the number of operations. The variable t is defined as:

$$t = \frac{1}{x_1 + x_2 + \dots + x_n} \tag{17}$$

Now, the objective function $\sum_{j=1}^{n}\tilde{c}_j x_j$ is multiplied by t. Determining the variable $tx_j = y_j, j=1,\ldots,n$, the objective function is converted into:

$$\sum_{j=1}^{n}\tilde{c}_j y_j \tag{18}$$

Since $\sum_{j=1}^{n} y_j = 1$ and $y_j \geq 0, j=1,\ldots,n$, Eq. (18) can be inferred as IVNWA of a set of IVNNs $\tilde{c}_j, j=1,\ldots,n$. According to Eq. (9), Eq. (18) is converted as :

$$\left(\left[\left(1-\prod_{j=1}^{n}(1-c_{1j})^{y_j}\right),\left(1-\prod_{j=1}^{n}(1-c_{2j})^{y_j}\right)\right],\left[\prod_{j=1}^{n}c_{3j}^{y_j},\prod_{j=1}^{n}c_{4j}^{y_j}\right],\left[\prod_{j=1}^{n}c_{5j}^{y_j},\prod_{j=1}^{n}c_{6j}^{y_j}\right]\right) \tag{19}$$

In fact, the application of the variable transformation in Eq. (17) simplicities obtaining a closed form for the objective functions. Primarily regarding score functions explanation, Eq. (6), an IVNN is maximized by increasing its truth membership degree; however, its indeterminacy and falsity membership degree is reduced. Also, an IVNN will be minimized when its truth membership degree is decreased while its indeterminacy and falsity membership degrees are increased. Suppose two interval numbers $A=[\underline{a},\overline{a}]$ and $B=[\underline{b},\overline{b}]$. Then, $A \geq B$ if $\underline{a} \geq \underline{b}$ and $\overline{a} \geq \overline{b}$ (Vahdani, Haji Karim Jabbari, Roshanaei,& Zandieh, [47]), thus, Eq. (19) will be maximized if $\left(1-\prod_{j=1}^{n}(1-c_{1j})^{y_j}\right)$ and $\left(1-\prod_{j=1}^{n}(1-c_{2j})^{y_j}\right)$ are maximized while $\prod_{j=1}^{n}c_{3j}^{y_j}$, $\prod_{j=1}^{n}c_{4j}^{y_j}$, $\prod_{j=1}^{n}c_{5j}^{y_j}$ and $\prod_{j=1}^{n}c_{6j}^{y_j}]$ are minimized. These conditions are satisfied when:
$\prod_{j=1}^{n}(1-c_{1j})^{y_j}$ and $\prod_{j=1}^{n}(1-c_{2j})^{y_j}$ are minimized, and simultaneously,
$\prod_{j=1}^{n}c_{3j}^{y_j}$, $\prod_{j=1}^{n}c_{4j}^{y_j}$ and $\prod_{j=1}^{n}c_{5j}^{y_j}$, $\prod_{j=1}^{n}c_{6j}^{y_j}]$ are minimized.
Subsequently, the single objective function of IVNN-NLP problem is converted as:

$$Min\left(\prod_{j=1}^{n}(1-c_{1j})^{y_j},\prod_{j=1}^{n}(1-c_{2j})^{y_j},\prod_{j=1}^{n}c_{3j}^{y_j},\prod_{j=1}^{n}c_{4j}^{y_j},\prod_{j=1}^{n}c_{5j}^{y_j},\prod_{j=1}^{n}c_{6j}^{y_j}\right) \tag{20}$$

Taking into account *Ln* (logarithm neperien) as an increasing function, minimization of the above IVNN elements is equal to "minimization" of the *Ln* of its elements, as:

$$Min\left(\sum_{j=1}^{n}y_j.Ln\left(1-c_{1j}\right),\sum_{j=1}^{n}y_j.Ln\left(1-c_{2j}\right),\sum_{j=1}^{n}y_j.Ln\left(c_{3j}\right),\sum_{j=1}^{n}y_j.Ln\left(c_{4j}\right),\sum_{j=1}^{n}y_j.Ln\left(c_{5j}\right),\sum_{j=1}^{n}y_j.Ln\left(c_{6j}\right)\right) \tag{21}$$

Since all the elements of the above normalized elements number in [0,1] interval, minimizing it corresponds to "minimization" of the elements summation,

$$Min\ \sum_{j=1}^{n}y_j.Ln\left((1-c_{1j})(1-c_{2j})c_{3j}c_{4j}\,c_{5j}c_{6j}\right. \tag{22}$$

Now, consider i[th] constraint $\sum_{j=1}^{n}(\tilde{a}_{ij}x_j)^2=\tilde{b}_i$, for a given value $i, i=1,\ldots,m$, in Eq. (16). For handling this constraint, their both sides are multiplied by t^2, Eq. (17). Hence, the initial constraint is transformed into $\sum_{j=1}^{n}(\tilde{a}_{ij}y_j)^2 = t^2\tilde{b}_i$. Taking into account the constraint's left side, it is an IVNNWA operator of a set of IVNNs $\tilde{a}_{ij}, j=1,2,\ldots,n$. This IVNNWA can be shown as follows:

$$\left(\left[\left(1-\prod_{j=1}^{n}(1-a_{1ij})^{y_j^2}\right),\left(1-\prod_{j=1}^{n}(1-a_{2ij})^{y_j^2}\right)\right],\left[\prod_{j=1}^{n}a_{3ij}^{y_j^2},\prod_{j=1}^{n}a_{4ij}^{y_j^2}\right],\left[\prod_{j=1}^{n}a_{5ij}^{y_j^2},\prod_{j=1}^{n}a_{6ij}^{y_j^2}\right]\right) \tag{23}$$

The product $t^2\tilde{b}$ can be handled on the right side, in terms of the scalar multiplication in Eq. (4), as:

$$\left(\left[1-(1-b_1)^{t^2},1-(1-b_2)^{t^2}\right],\left[b_3^{t^2},b_4^{t^2}\right],\left[b_5^{t^2},b_6^{t^2}\right]\right) \tag{24}$$

For equality-type constraints, the right side must be equal to the left constraints.

$$\begin{cases} 1-\prod_{j=1}^{n}(1-a_{1ij})^{y_j^2} = 1-(1-b_1)^{t^2} \\ 1-\prod_{j=1}^{n}(1-a_{2ij})^{y_j^2} = 1-(1-b_2)^{t^2} \\ \prod_{j=1}^{n}(a_{3ij})^{y_j^2} = (b_3)^{t^2} \\ \prod_{j=1}^{n}(a_{4ij})^{y_j^2} = (b_4)^{t^2} \\ \prod_{j=1}^{n}(a_{5ij})^{y_j^2} = (b_5)^{t^2} \\ \prod_{j=1}^{n}(a_{6ij})^{y_j^2} = (b_6)^{t^2} \end{cases} \tag{25}$$

The set of constraints in Eq. (25) is transformed into the non-linear form by applying the logarithm Neperien function:

$$\begin{cases} \sum_{j=1}^{n}y_j^2Ln\left(1-a_{1ij}\right) = t^2Ln(1-b_1) \\ \sum_{j=1}^{n}y_j^2Ln\left(1-a_{2ij}\right) = t^2Ln(1-b_2) \\ \sum_{j=1}^{n}y_j^2Ln\left(a_{3ij}\right) = t^2Ln(b_3) \\ \sum_{j=1}^{n}y_j^2Ln\left(a_{4ij}\right) = t^2Ln(b_4) \\ \sum_{j=1}^{n}y_j^2Ln\left(a_{5ij}\right) = t^2Ln(b_5) \\ \sum_{j=1}^{n}y_j^2Ln\left(a_{6ij}\right) = t^2Ln(b_6) \end{cases} \tag{26}$$

Finally, the IVNN-NLP problem in Eq. (16) is transformed into an equivalent NLP problem, as illustrated in Eq. (27). When this problem is solved, the optimum values of t^* and y_j^* , $j = 1,2,\dots,n$ are defined. Using an opposite transformation based on Eq. (17), the optimal values of the original variables $x_j^*, j = 1,\dots,n$ are found.

$$Min \sum_{j=1}^{n} y_j.\ Ln((1-c_{1j})(1-c_{2j})c_{3j}c_{4j}c_{5j}c_{6j})$$

s.t.

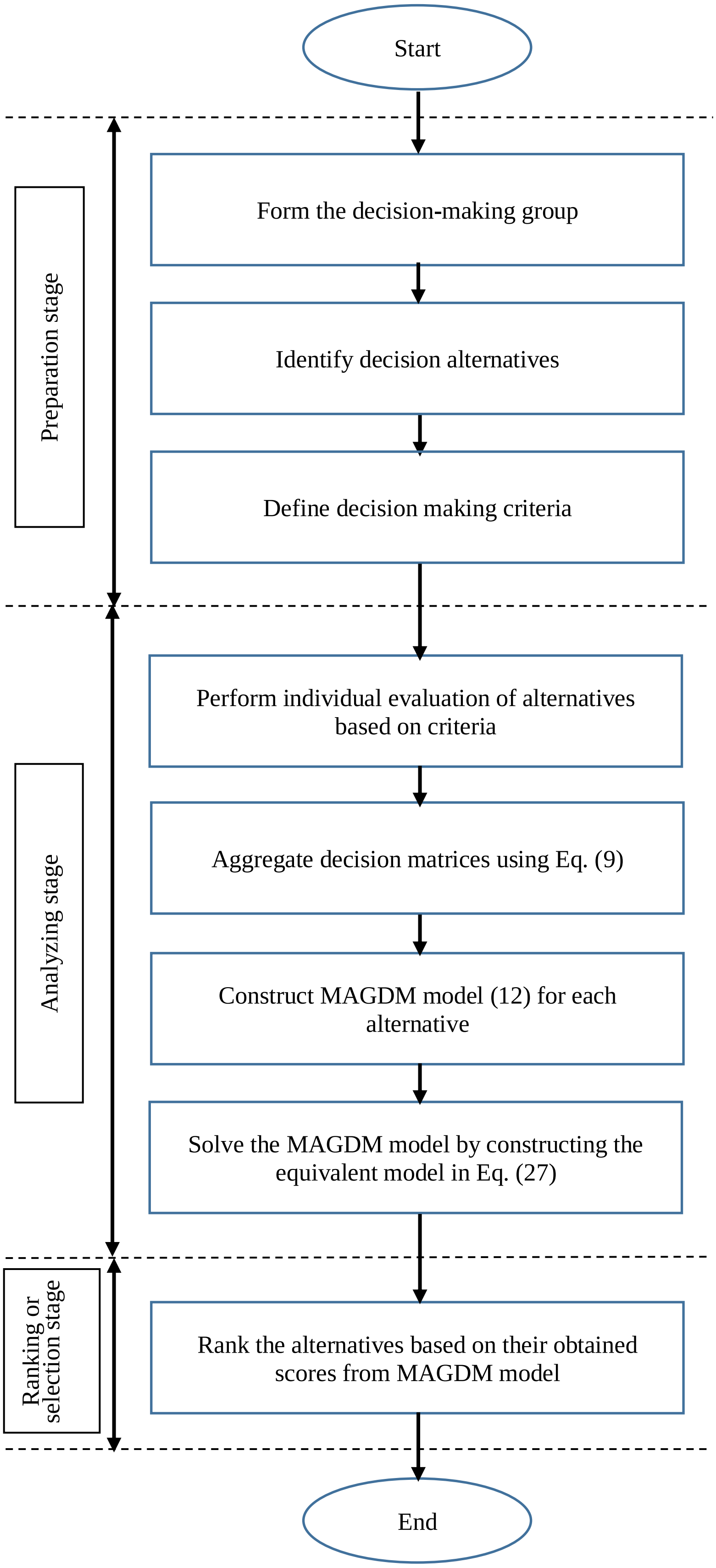
$$\begin{cases} \sum_{j=1}^{n} {y_j}^2 Ln(1-a_{1ij}) = t^2 Ln(1-b_1) \\ \sum_{j=1}^{n} {y_j}^2 Ln(1-a_{2ij}) = t^2 Ln(1-b_2) \\ \sum_{j=1}^{n} {y_j}^2 Ln(a_{3ij}) = t^2 Ln(b_3) \\ \sum_{j=1}^{n} {y_j}^2 Ln(a_{4ij}) = t^2 Ln(b_4) \\ \sum_{j=1}^{n} {y_j}^2 Ln(a_{5ij}) = t^2 Ln(b_5) \\ \sum_{j=1}^{n} {y_j}^2 Ln(a_{6ij}) = t^2 Ln(b_6) \end{cases} \quad y_j \geq 0, t \geq 0 \tag{27}$$

The problem in Eq. (27) is a new nonlinear programming problem that ordinal methods can solve. Such a process can be carried out to resolve the MAGDM problems in Eq. (12).

Remark: In the case that the score and accuracy functions cannot rank the neutrosophic triplets of intervals, one can use the certainty function [47].

4.1. Algorithm of the proposed method

In the previous sections, the MAGDM problem was formulated as an IVNN nonlinear programming model. Then, a method is proposed to solve this problem. The MAGDM algorithm based on the proposed method is shown in Figure 1. This algorithm consists of three steps. The first step is to prepare the decision-making team and solve the problem. In the second step, the problem is analyzed. Finally, the problem of decision was solved by the results of the second round.



5. Numerical Example

In this section, two numerical examples have been solved using the proposed method to illustrate its applicability and efficiency.

Example 1: Suppose 3 DMs as d1, d2, d3, and 4 students (x1, x2, x3, x4) are screened as finalists using the pre-screening method. Using all DMs, it is agreed that these candidates will be assessed on four educational information competencies, A1. A2 level university English test; Ability to work in a group, A3; and potential studies, A4. Here, it is assumed that there is consensus among the group in the evaluation of qualitative features in five linguistic terms. Table 1 also shows the conversion measure between IVNN and linguistic terms. It should be noted that the methodology presented here is insensitive to the scale of the IVNN used to represent decision-makers. So, any arbitrary scale is possible. Decision-makers can use the scale presented in Table 1 to express their uniqueness.

For subjective features, the above scale or any other one is utilized. Regarding objective (quantitative) features, a technique will be needed to convert the precise quantitative values into IVNNs. It is supposed that an objective attribute is assessed as (100,150,120,90,80,70) for a set of four alternatives.

The values of such attribute vectors are divided to normalize their values to the highest value of 150. The obtained normalized vector was (0.7,1,0.8,0.6,0.5,0.4). Then, these values' desirability can be determined by DM based on the linguistic scale.

The IVNN assessments of DM on alternative performances regarding different criteria are shown in Table 1.

Taking into account the alternative x_1, in terms of Eq. (12), the formulation of the MAGDM model is:

$$\begin{aligned}
&Max\ ([0.1,0.2],[0.2,0.3],[0.4,0.5])w_1 + ([0.2,0.4],[0.3,0.5],[0.1,0.2])w_2 \\
&\qquad + ([0.3,0.4],[0.1,0.2],[0.3,0.5])w_3 + ([0.1,0.3],[0.3,0.4],[0.2,0.3])w_4 \\
&\text{S.t.} \\
&([0.1,0.2],[0.2,0.3],[0.4,0.5]w_1 + ([0.2,0.4],[0.3,0.5],[0.1,0.2])w_2 + ([0.3,0.4],[0.1,0.2],[0.3,0.5])w_3 \\
&\qquad + ([0.1,0.3],[0.3,0.4],[0.2,0.3])w_4)^2 = ([0.9,0.95],[0.01,0.05],[0.02,0.06]) \\
&(([0.2,0.3],[0.2,0.5],[0.4,0.5])w_1 + ([0.2,0.3],[0.2,0.6],[0.4,0.7])w_2 + ([0.3,0.6],[0.1,0.2],[0.1,0.4])w_3 \\
&\qquad + ([0.5,0.6],[0.4,0.5],[0.1,0.3])w_4)^2 = ([0.9,0.95],[0.01,0.05],[0.02,0.06]) \\
&(([0.5,0.7],[0.2,0.3],[0.1,0.2])w_1 + ([0.6,0.7],[0.3,0.4],[0.1,0.3])w_2 + ([0.5,0.6],[0.3,0.4],[0.1,0.3])w_3 \\
&\qquad + ([0.2,0.5],[0.1,0.2],[0.4,0.5])w_4)^2 = ([0.9,0.95],[0.01,0.05],[0.02,0.06]) \\
&(([0.2,0.3],[0.1,0.5],[0.4,0.6])w_1 + ([0,0.1],[0.4,0.6],[0.5,0.7])w_2 + ([0.8,0.9],[0.3,0.4],[0.1,0.2])w_3 \\
&\qquad + ([0.4,0.5],[0.3,0.7],[0.2,0.6])w_4)^2 = ([0.9,0.95],[0.01,0.05],[0.02,0.06]) \\
&\boldsymbol{w_j \geq 0, \qquad j = 1,2,3,4}
\end{aligned}$$

Tab. 1. IVNN scale used to assess alternative

Linguistic terms	IVNNs
Very good (VG)	([0.8,0.9],[0.02,0.05],[0.1,0.3])
Good(G)	([0.7,0.75],[0.2,0.25],[0.5,0.6])
Fair(F)	([0.5,0.55],[0.4,0.45],[0.1,0.3])
Poor(P)	([0.2,0.25],[0.7,0.75],[0.5,0.57])
Very poor(VP)	([0.02,0.05],[0.9,0.95],[0.7,0.8])

Tab. 2. Aggregated decision matrix

	A_1	A_2	A_3	A_4
x_1	([0.1,0.2],[0.2,0.3],[0.4,0.5])	([0.2,0.4],[0.3,0.5],[0.1,0.2])	([0.3,0.4],[0.1,0.2],[0.3,0.5])	([0.1,0.3],[0. 3,0.4],[0.2,0.3])
x_2	([0. 2,0.3],[0.2,0.5],[0.4,0.5])	([0.2,0.3],[0.2,0.6],[0.4,0.7])	([0.3,0.6],[0.1,0.2],[0.1,0.4])	([0.5,0.6],[0.4,0. 50],[0.1,0.3[)
x_3	([0.50.7],[0.2,0.3],[0.1,0.2])	([0.6,0.7],[0.3,0.4],[0.1,0.3])	([0.5,0.6],[0.3,0.4],[0.1,0.3])	([0.2,0. 5],[0.10.2],[0.4,0.5])
x_4	([0.2,0.3],[0.1,0.5],[0.4,0.6])	([0,0.1],[0.4,0.6],[0.5,0.7])	([0.8,0.9],[0.3,0.4],[0.1,0,2])	([0.4,0.5],[0.3,0.7],[0.2,0.6])

This problem is an IVIF linear programming problem, which can be simply solved by converting it into the corresponding Eq. (27) model.

$$\begin{aligned}
&Min - 4.75y_1 - 6.54y_2 - 6.67y_3 - 5.39y_4 \\
&.\text{S.t} \\
&-0.1{y_1}^2 - 0.22{y_2}^2 - 0.35{y_3}^2 - 0.14{y_4}^2 = -2.30t^2 \\
&-0.22{y_1}^2 - 0.51{y_2}^2 - 0.51{y_3}^2 - 0.35{y_4}^2 = -2.99t^2 \\
&-1.6{y_1}^2 - 1.2{y_2}^2 - 2.3{y_3}^2 - 1.2{y_4}^2 = -4.6t^2
\end{aligned}$$

$-1.2{y_1}^2 - 0.69{y_2}^2 - 1.6{y_3}^2 - 0.91{y_4}^2 = -2.99t^2$
$-0.91{y_1}^2 - 2.3{y_2}^2 - 1.2{y_3}^2 - 1.6{y_4}^2 = -3.91t^2$
$-0.69{y_1}^2 - 1.6{y_2}^2 - 0.69{y_3}^2 - 1.2{y_4}^2 = -2.81t^2$
$-0.22{y_1}^2 - 0.22{y_2}^2 - 0.35{y_3}^2 - 0.69{y_4}^2 = -2.3t^2$
$-0.35{y_1}^2 - 0.35y_2^2 - 0.91{y_3}^2 - 0.91{y_4}^2 = -2.99t^2$
$-1.6{y_1}^2 - 1.6{y_2}^2 - 2.3{y_3}^2 - 0.91{y_4}^2 = -4.6t^2$
$-0.69{y_1}^2 - 0.35{y_2}^2 - 1.6{y_3}^2 - 0.69{y_4}^2 = -2.99t^2$
$-0.91{y_1}^2 - 0.91{y_2}^2 - 2.3{y_3}^2 - 2.3{y_4}^2 = -3.91t^2$
$-0.69{y_1}^2 - 0.51{y_2}^2 - 0.91{y_3}^2 - 1.2{y_4}^2 = -2.81t^2$
$-0.69{y_1}^2 - 0.91{y_2}^2 - 0.69{y_3}^2 - 0.22{y_4}^2 = -2.30t^2$
$-1.2{y_1}^2 - 1.2{y_2}^2 - 0.91{y_3}^2 - 0.69{y_4}^2 = -2.99t^2$
$-0.95{y_1}^2 - 0.88{y_2}^2 - 1.15{y_3}^2 - 0.92{y_4}^2 = -4.61t^2$
$-1.2{y_1}^2 - 0.91{y_2}^2 - 0.91{y_3}^2 - 1.6{y_4}^2 = -2.99t^2$
$-2.3{y_1}^2 - 2.3{y_2}^2 - 2.3{y_3}^2 - 0.91{y_4}^2 = -3.91t^2$
$-1.6{y_1}^2 - 1.2{y_2}^2 - 1.2{y_3}^2 - 0.69{y_4}^2 = -2.81t^2$
$-1.2{y_1}^2 - 0.91{y_2}^2 - 0.91{y_3}^2 - 1.6{y_4}^2 = -2.99t^2$
$-3.5{y_1}^2 - 0.1{y_2}^2 - 2.3{y_3}^2 - 0.69{y_4}^2 = -2.99t^2$
$-2.3{y_1}^2 - 0.91{y_2}^2 - 1.2{y_3}^2 - 1.2{y_4}^2 = -4.6t^2$
$-0.69{y_1}^2 - 0.51{y_2}^2 - 0.91{y_3}^2 - 0.35{y_4}^2 = -2.99t^2$
$-0.91{y_1}^2 - 0.69{y_2}^2 - 2.3{y_3}^2 - 1.6{y_4}^2 = -3.91t^2$
$-0.51{y_1}^2 - 3.5{y_2}^2 - 1.6{y_3}^2 - 0.51{y_4}^2 = -2.81t^2$
$y_1, y_2, y_3, y_4 \geq 0$, $t \geq 0$

By solving this model for each of the alternatives, their ranking is determined as $x_3 > x_4 > x_1 > x_2$. Table 3 presents the results obtained from the solving model (27).

Tab. 3. Objective values for alternatives

x_1	-0.821
x_2	-0.837
x_3	-0.106
x_4	-0.795

Example 2: Due to its harmful effects, air pollution has become one of the most visible and serious environmental problems. Tehran is the largest city and capital of Iran with a population of about 8 million people and an area of about 730 square kilometer. Tehran is one of the most polluted cities in the world. The most important thing about him. Risk of heart and lung disease and increased levels of O3, CO and suspended particulate matter. In Tehran, air pollution shortens the life of Tehran residents by an average of 5 years. Here, we analyzed air pollution for three consecutive days in Tehran in 2017, and the data was obtained from different parts of the city. These stations measure particulate matter (PM2.5), PM10, O3 and CO pollution. We considered these locations as DM, 3 consecutive days as options (x1, x2, and x3), and facility and pollutant elimination as options.

Tab. 4. Aggregated decision matrix

	$PM_{2.5}$	PM_{10}	O_3	CO
x_1	([0.10,0.2],[0.2,0.4,[0.2,0.3])	([0.2,0.3],[0.2,0.3],[0.2,0.3])	([0.1,0.2],[0.1,0.2],[0.1,0.1])	([0.1,0.2],[0. 1,0.2],[0.1,0.2])
x_2	([0. 2,0.2],[0.2,0.3],[0.4,0.2])	([0.2,0.3],[0.1,0.6],[0.2,0.7])	([0.3,0.4],[0.1,0.2],[0.1,0.2])	([0.2,0.4],[0.1,0. 30],[0.2,0.3[)
x_3	([0.50.7],[0.1,0.2],[0.2,0.3])	([0.3,0.7],[0.3,0.4],[0.1,0.3])	([0.2,0.6],[0.1,0.2],[0.1,0.3])	([0.2,0. 5],[0.10.2],[0.3,0.5])

Consider the alternative x_1, according to Eq. (27), where the MAGDM model is developed as follows:

$Min - 5.68y_1 - 6.21y_2 - 8.87y_3 - 8.18y_4$
s.t.
$-0.1{y_1}^2 - 0.22{y_2}^2 - 0.10{y_3}^2 - 0.10{y_4}^2 = -2.30t^2$
$-0.22{y_1}^2 - 0.21{y_2}^2 - 0.35{y_3}^2 - 0.22{y_4}^2 = -2.30t^2$

$-0.69{y_1}^2 - 0.35{y_2}^2 - 0.22{y_3}^2 - 0.22{y_4}^2 = -2.30t^2$
$-0.22{y_1}^2 - 0.35{y_2}^2 - 0.22{y_3}^2 - 0.22{y_4}^2 = -2.99t^2$
$-0.22{y_1}^2 - 0.35{y_2}^2 - 0.51{y_3}^2 - 0.51{y_4}^2 = -2.99t^2$
$-1.20{y_1}^2 - 1.20{y_2}^2 - 0.91{y_3}^2 - 0.69{y_4}^2 = -2.99t^2$
$-1.60{y_1}^2 - 1.60{y_2}^2 - 2.30{y_3}^2 - 2.30{y_4}^2 = -4.60t^2$
$-1.60{y_1}^2 - 2.30y_2^2 - 2.30{y_3}^2 - 2.30{y_4}^2 = -4.60t^2$
$-2.30{y_1}^2 - 1.20{y_2}^2 - 2.3{y_3}^2 - 2.30{y_4}^2 = -4.6t^2$
$-0.91{y_1}^2 - 1.20{y_2}^2 - 1.60{y_3}^2 - 1.60{y_4}^2 = -2.99t^2$
$-1.20{y_1}^2 - 0.51{y_2}^2 - 1.60{y_3}^2 - 1.20{y_4}^2 = -2.99t^2$
$-1.60{y_1}^2 - 0.91{y_2}^2 - 1.60{y_3}^2 - 1.60{y_4}^2 = -2.99t^2$
$-1.60{y_1}^2 - 1.60{y_2}^2 - 2.30{y_3}^2 - 2.30{y_4}^2 = -3.91t^2$
$-0.91{y_1}^2 - 1.60{y_2}^2 - 2.30{y_3}^2 - 1.60{y_4}^2 = -3.91t^2$
$-1.60{y_1}^2 - 2.30{y_2}^2 - 2.30{y_3}^2 - 1.20{y_4}^2 = -3.91t^2$
$-1.2{y_1}^2 - 1.20{y_2}^2 - 2.30{y_3}^2 - 1.60{y_4}^2 = -2.81t^2$
$-1.60{y_1}^2 - 0.35{y_2}^2 - 1.60{y_3}^2 - 1.20{y_4}^2 = -2.81t^2$
$-1.20{y_1}^2 - 1.20{y_2}^2 - 1.20{y_3}^2 - 0.69{y_4}^2 = -2.81t^2$
$y_1, y_2, y_3, y_4 \geq 0$, t$\geq$ 0

By solving this model for each of the alternatives, their ranking is determined as $x_1 > x_3 > x_2$. Table 5 presents the results obtained from solving model (27).

Tab. 5. Objective values for alternatives

x_1	-0.10
x_2	-0.95
x_3	-0.11

6. Conclusion

In the field of MCDM problems, many practical problems can be formulated in various administrative, social, economic and engineering fields. Vincke [50] believes that the main problem with these problems is that the facts are not well defined and there is no clear solution to them. This difficulty is compounded by the fact that the concept of uncertainty is an inevitable part of these. Considering the uncertainty of decision-making problems and the ability of interval-valued neutrosophic numbers to describe uncertain data, in this paper we propose a new model based on nonlinear programming to solve the MAGDM problem using IVNN. Motivating from DEA, a robust logical context is traced by the presented formulation. In this model, some models were iteratively solved and an ultimate score was calculated for each alternative. These scores are utilized for ranking and to comparing some alternatives. To compare some m alternatives, the proposed technique solves m models, one for every alternative Though, after formulation of the problem for the first alternative, the objective functions were adjusted based on various alternatives as well as the remained unchangeable feasible space. Thus, it is essential to formulate m various objective functions with the similar set of constraints. Since the proposed model is a nonlinear programming problem with IVNN parameters, an approach is designed to solve this problem.

The main advantage of the proposed method can be summarized as follows: imprimis, there are increasing intends to apply mathematical optimization models in the context of MAGDM problems. Considering this fact, the proposed method provides a basis for deciding the implementation of solutions. Second, the ideas behind the development of the proposed method are simple and acceptable. Increasing the weighted average of each alternative and restricting the score to one method is similar to the concept of DEA, which is a well-known method.

Third: The information requirement of the model is smaller than other methods, because the value of the features does not need to be defined and the model itself defines them. . However, FS cannot identify problems, uncertainties and inaccuracies in the details of actual problems. In this case, some information may be uncertain, uncertain and urgent. Considering the membership functions in truth, error, and uncertainty for each piece of data in a neutrosophic sequence helps in the decision-making process. Ultimately, there is narrow attention paid to mathematical programming with IVNN information. The proposed method can be generalized to solve nonlinear programming problems with IVNN parameters.

References

[1] C.L. Hwang, K.L. Yoon. Multiple Attribute Decision Making: Methods and Applications, Springer Verlag, New York, (1981).

[2] C.L. Hwang, M.J.Lin, Group Decision Making Under Multiple Criteria: Methods and Applications, Springer Verlag, New York, (1987).

[3] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-i, Inf. Sci. Vol. 8, (1975), pp. 199-249.

[4] S.J. Chen, C.L. Hwang, Fuzzy Multiple Attribute Decision Making, Springer Verlag Berlin Heidelberg, (1992).

[5] M Ariffin Abu Bakar, A Termimi Ab Ghani. Capturing the Contribution of Fuzzy and Multi-criteria decision-making Analytics: A Review of the Computational Intelligence Approach to Classroom Assessment Sustainability. International Journal of Industrial Engineering & Production Research Vol. 33, No. 4, (2022), pp. 1-15.

[6] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. Vol. 2, (1986), pp. 87-96.

[7] F.E. Boran, Genc, M.Kurt, D.Akay, A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, Expert Syst, Appl. Vol. 36, (2009), pp. 11363-11368.

[8] F.E. Boran, S. Genc, D. Akay, Personnel selection based on intuitionistic fuzzy sets, Hum. Factors Ergon. Manuf. Serv. Ind. Vol. 21, (2011), pp. 493-503.

[9] F.E. Boran, An integrated intuitionistic fuzzy multi criteria decision making method for facility location selection, Math. . Comput. Appl. Vol. 16, (2011), pp. 487-496.

[10] F.E. Boran, K . Boran, T. Menlik, The evaluation of renewable energy technologies for electricity generation in turkey using intuitionistic fuzzy TOPSIS, Energy Sources Part B. Vol. 7, (2012), pp. 81-90.

[11] S. Genc, F.E. Boran, D. Akay, Z. Xu, Interval multiplicative transitivity for consistency, missing values and priority weights of interval fuzzy preference relations, Inf. Sci. Vol. 180, (2010), pp. 4877-4891.

[12] F.E. Boran, D. Akay, A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition, Inf. Sci. Vol. 255, (2014), pp. 45-57.

[13] F. Shen, J. Xu, Z. Xu, An outranking sorting method for multi-criteria group decision making using intuitionistic fuzzy sets, Inf. Sci. (2016), pp. 338-353.

[14] S.P. Wan, G.L. Xu, F. Wang, J. Y. Dong, A new method for attanassov's interval-valued intuitionistic fuzzy MAGDM with incomplete attribute weight information , Inf. Sci. Vol. 316, (2015), pp. 329-347.

[15] D. F. Li, TOPSIS-based nonlinear-programming methodology for multiattribute decision making withinterval-valued intuitionistic fuzzy sets, IEEE Transactions on Fuzzy Systems. Vol. 18, No. 2, (2010a), pp. 299-311.

[16] D. F. Li, Linear programming method for MADM with interval-valued intuitionistic fuzzy sets, ExpertSystems with Applications. Vol. 37, No. 8, (2010b), pp. 5939-5945.

[17] M. Arshi, A. Hadi-Vencheh, M. Nazari, A. Jamshidi, A nonlinear programming model to solve MADM problems with interval valued intuitionistic fuzzy numbers, International Journal of Industrial Engineering. Vol. 30, No. 3, (2023), pp. 750-726.

[18] A Fadhil Al-Maliki, M Habibnejad Korayem. Trajectory optimization of

Biped Robots with Maximum Load Carrying Capacities: Iterative Linear Programming Approach. International Journal of Industrial Engineering & Production Research Vol. 34, No. 3, (2023), pp. 164-184.

[19] M. Khezeli, E. Najafi, M. Haji Molana, M. Seidi, AMathematical Model for Sustainable and Resilient Supply Chain by Considering Synchronization in the production and Distribution Network, International Journal of Industrial Engineering & Production Research, Vol. 33, No. 2, (2022).

[20] F. Smarandache, A unifying field in logics. Neutrosophy; neutrosophic propability, set and logic. American Research Press. (1998), pp. 01-157.

[21] F. Smarandache, Neutrosophy and neutrosophic logic. First international conference on neutrosophy, neutrosophic logic, set, probability and statistics, University of New Mexico, Gallup, USA. (2002), pp. 01-147.

[22] F. Smarandache, Neutrosophic set, ageneralisation of the intuitionistic fuzzy sets. J New Theory. Vol. 29, (2019), pp. 01-35.

[23] I. Deli, Y Subas, A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. Int J Mach Learn Cyb, Vol. 8, (2017), pp. 1309-1322.

[24] M. Abdel-Basset, M. Mohamed, The role of single valued neutrosophic sets and rough sets in smart city: Imperfect and incomplete information systems. Measurement. Vol. 124, (2018), pp. 47-55.

[25] S.A. Edalatpanah, Neutrosophic perspective on DEA. Journal of Applied Research on Industrial Engineering. Vol. 5, No. 4, (2018), pp. 339-345.

[26] H. Garg, Non-linear programming method for multi-criteria decision making problems under interval neutrosophic set environment. Applied Intelligence. Vol. 48, No. 8, (2018), pp. 2199-2213.

[27] P. Liu, L. Shi, The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making. Neural Computing and Applications. Vol. 26, No. 2, (2015), pp. 457-471.

[28] I. Deli, S. Yusuf, F. Smarandache, & M. Ali, Interval valued bipolar neutrosophic sets and their application in pattern recognition. In IEEE World Congress on Computational Intelligence. Vancouver, Canada: IEEE. (2016).

[29] V. Ulucay, I. Deli, M. Sahin, Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making. Neural Computing and Applications. Vol. 29, No. 3, (2018), pp. 739-748.

[30] P. Ji, H. Y. Zhang, J.Q. Wang, A projection-based TODIM method under multi-valued neutrosophic environments and its application in personal selection. Neural Computing and Applications. Vol. 29, No. 1, (2018), pp. 221-234.

[31] J. J Peng, J.Q. Wang, W.E. Yang , A multi-valued neutrosophic qualitative flexible approach based on are likelihood for multi-criteria decision making problems. International Journal of Systems Science. Vol. 48, No. 2, (2017), pp. 425-435.

[32] S.A. Edalatpanah, F. Smarandache. Data envelopment analysis for simplified neutrosophic sets. Neutrosophic Sets and Systems. Vol. 29, (2019), pp. 215-226.

[33] J.J. Peng, J.Q. Wang, J. wang, H.Y. Zhang and X.H. Chen. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. International Journal of Systems Science. Vol. 47, No. 10, (2016), pp. 2342-2358.

[34] S.A. Edalatpanah. Neutrosophic

structured element. Expert Systems. Vol. 37, No. 5, (2020), pp. 1-17.

[35] P. Biswas, S. Pramanik and B.C. Giri, Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets Syst. Vol. 12, (2016), pp. 127-138

[36] R.X. Liang, J.Q. Wang and H.Y. Zhang, A multi-criteria decision making method based on single valued trapezoidal neutrosophic preference relation with complete weight information. Neural Comput. Appl. Vol. 30, (2018), pp. 3383-3398.

[37] S.A. Edalatpanah. A Nonlinear Approach for Neutrosophic Linear Programming, Journal of Applied Research on Industrial Engineering. Vol. 6, (2019), pp. 367-373.

[38] A. Elsayed, Multi-Criteria Decision-Making framework for evaluating green fuels alternatives: A hybrid MEREC-TODIM approach. Neutrosophic optimization and intelligent systems. Vol. 3, (2024), pp. 41-56.

[39] B. Satyanarayana, SH, Baji, Interval-valued Neutrosophic N-d-ideal in d-algebra. Neutrosophic optimization and intelligent systems. Vol. 3, (2024), pp. 57-70.

[40] M. Jdid, F. Smarandache, Converting some Zero-one Neutrosophic nonlinear programming problems into Zero-one. Neutrosophic optimization and intelligent systems. Vol. 1, (2024), pp. 39-45.

[41] H. Khalifa, An approach for optimization of multi-objective assignment problems with neutrosophic numbers, International Journal of Industrial Engineering & Production Research, Vol. 31, (2020), No. 2.

[42] A. Hadi-Vencheh, An improvement to multiple criteria ABC inventory classification, European Journal of Operational Research. Vol. 201, (2010), pp. 962-965.

[43] F. Smarandache, M. Abdel-Basset (editors), Neutrosophic Operational Research Methods and Applications, Springer, (2021).

[44] F. Smarandache, M. Abdel-Basset (editors), Optimization Theory Based on Neutrosophic and Plithogenic Sets, Elsevier, (2020).

[45] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. J Intell Fuzzy Syst. Vol. 26, (2014), pp. 165-172.

[46] S. Pramanik, K. Mondal, Interval neutrosophic multiattribute decision-making based on grey relational analysis. Neutr Sets Syst. Vol. 9, (2015), pp. 13-22.

[47] F. Smarandache, The score, accuracy, and certainty functions determine a total order on the set of neutrosophic triplets (TI F). Infinite Study. Vol. 38, (2020).

[48] J. P. Ponstein, Approaches to the theory of optimization, Cambridge: Cambridge University Press, (2004).

[49] J. P. VahdaniHaji, A. Karim Jabbari, V. Roshanaei, M. Zandieh, Extension of the ELECTRE method for decision-making problems with interval weights and data, The International Journal of Advanced Manufacturing Technology, Vol. 50, No. 5-8, (2010), pp. 793-800.

[50] P. Vincke, Multicriteria decision-aid, Chichester: John Wiley, (1992).

Follow this article at the following site:

Maryam Arshi; A. Hadi-Vencheh; Adel Aazami & Ali Jamshidi. A Nonlinear Model to Solve Multiple Attribute Decision-Making Problems with Interval-Valued Neutrosophic Numbers. IJIEPR 2024; 35 (4) :1-15
URL: http://ijiepr.iust.ac.ir/article-1-2094-en.html