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RESEARCH PAPER

Cultivating Efficiency: Streamlining Food Delivery Logistics Through Graphical Networks and Eternal m-Certified Domination number

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ABSTRACT

The increasing demand for food delivery services driven by technological innovations has led to a surge in online shopping and food ordering. Efficient logistics play a crucial role in connecting customers with restaurants seamlessly. In this context, the practical application of graphical networks is explored in this article to streamline food delivery operations. We introduce a novel parameter eternal m-certified domination number denoted by $\gamma_{mcer}^{\infty}(G)$, which represents the minimum number of guards needed to handle any sequence of single orders using multiple-guard movements, ensures that the guard arrangement consistently constitutes a certified dominating set. A case study is presented, illustrating how this concept can be employed to de-crease human resources in a food delivery start-up. This research contributes to optimizing food delivery logistics and reducing operational costs, thereby enhancing the efficiency of the food delivery industry.

KEYWORDS: Domination number; Eternal m-domination number; Certified domination number; Online food delivery; Business logistics.

1. Introduction

The optimization of logistical operations, particularly in the domain of online food delivery services, stands as a critical challenge in contemporary business landscapes. With the burgeoning growth of online commerce and the increasing reliance on graphical networks in logistics, a pressing need emerges for innovative strategies to streamline operations and reduce human resource requirements. This paper introduces a novel parameter, the eternal m-certified domination number, to address this challenge and enhance the efficiency of online food delivery services.

In graph theory, domination numbers are pivotal in modeling and analyzing various real-world scenarios. In a departure from conventional domination parameters, we propose the concept of an eternal m-certified dominating set, wherein a set of sentinels is deemed effective if, under an infinitely long sequence of assaults, their arrangement consistently forms a certified dominating set. Formally, the eternal m-certified domination number, denoted as $\gamma_{mcer}^{\infty}(G)$, represents the minimum number of sentinels required to handle any sequence of individual assaults, ensuring that the sentinels' configuration remains a certified dominating set throughout. Motivated by the imperative to optimize human resources and logistics in online food delivery services, we leverage this novel parameter to devise a strategy to enhance operational efficiency. By analogizing sentinels to delivery personnel and assaults to online food orders, we

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introduce a graphical technique that aids in the planning and managing food delivery logistics. In the graphical representation of a split graph, the vertices are divided into two partitions: one representing restaurants and the other representing customers. Each restaurant is connected to every other restaurant, creating a complete subgraph within the hotel partition.

Additionally, all possible connections exist between restaurants and customers. However, no connections are present within the customer partition, ensuring customers are only linked to hotels. This structure is helpful in modeling scenarios where direct customer interactions are unnecessary, such as in food delivery networks where customers only interact with delivery points (restaurants). This technique capitalizes on the inherent characteristics of graphical networks to optimize route planning, resource allocation, and delivery scheduling, thereby minimizing operational costs and enhancing customer satisfaction.

Efficient logistics are essential for modern businesses, especially in online food delivery. Sorting facilities have emerged as a strategic solution, streamlining processes and maximizing resources. By centralizing sorting, companies can lower costs and improve delivery performance, ultimately enhancing customer satisfaction and competitiveness. Reem Khir (2021) discusses planning two-stage parcel sorting operations, considering time deadlines and sorting capacities [20]. Parcels are initially sorted into groups by a primary sorter, then dispatched to secondary stations for final sorting. The optimization problem uses mixed-integer programming to minimize operational costs while adhering to machine capacity and parcel deadline constraints.

In the subsequent sections, we elaborate on the theoretical foundations of eternal m-certified domination, present our proposed graphical technique for food delivery logistics, and provide empirical evidence to validate the effectiveness of our approach. Furthermore, we discuss the practical implications of our research and outline avenues for future investigation in the domain of logistics optimization using graph-theoretic principles.

1.1. Motivation

In recent years, the integration of graph theory into logistics management, particularly in the context of transportation and environmental impact assessment, has demonstrated significant potential for enhancing operational efficiency and sustainability within the logistics industry. Shuangdong (2019) and Anchal Gupta (2020) have explored the application of graph theory in logistics, showcasing its relevance in designing computer-assisted models for transportation optimization and evaluating the environmental footprint of logistic service providers through graph theory matrices. Additionally, Burger et al. [11, 12] proposed the concept of dynamic domination, later refined as “eternal security” by Goddard et al. [13], further emphasizing the utility of graph theory in addressing complex logistical challenges.

However, despite these advancements, there remains a critical need for innovative approaches to enhance the efficiency of food delivery logistics, a sector facing unique operational constraints and environmental concerns. Current methods often struggle to optimize delivery routes, minimize environmental impact, and ensure the security of food shipments throughout the supply chain. Therefore, there is a compelling motivation to leverage graphical networks and the concept of eternal m-certified domination to revolutionize food delivery logistics. By applying graph theory principles to model delivery networks, optimize routes, and enhance security measures, this research aims to cultivate efficiency in food delivery logistics, ultimately streamlining operations and minimizing environmental footprint.

1.2. Literature review

Graph theory has applications in transportation problems and shortest distance problems. Graph theoretical concepts play a vital role in operational research (OR). The studies primarily focus on static OR problems, such as route planning or network design. Future research could explore integrating dynamic elements, such as time-sensitive constraints, demand fluctuations, or environmental factors, into graph-based models to capture the evolving nature of real-world logistics and supply chain operations. Graphs, whether directed or undirected, are highly beneficial in solving many OR problems. The purpose of graph theory is to simplify and model complex OR problems [2]. Martin (2006) suggested a method using logistic regression with a 1-constraint to estimate the neighborhood of a node, considering a graphical framework [1]. Gutierrez (2016) proposed a viable solution to the challenging task of route planning by employing an applied graphical network approach [4]. This research focuses on graph-based algorithms’ scalability and efficiency in handling large datasets and complex network structures without sacrificing computational performance. Saurabh Agrawal utilizes graph theory, employing a matrix-based approach to identify the optimal disposition alternative [3]. Shuangdong (2019) researched graph theory's application in Logistics management linked with transportation including Computer-assisted model design [10]. Anchal Gupta (2020) proposed a framework for assessing the environmental impact of logistic service providers through graph theory matrices [6].

However, there is a limited exploration of how graph theory can effectively address environmental or social concerns in logistics and supply chain management. In 2020, Masoud Rabbani Offered a heuristic algorithm grounded in graph theory to tackle a multifaceted responsive supply chain network design challenge encompassing lateral transshipment between retailers [5]. In 2023, Svetlana Dabic-Miletic [22] conducted a study examining the advantages of prevalent autonomous vehicles in fulfilling user requests within the last mile of logistics operations. The research offered practical guidelines for selecting the most appropriate option for logistics companies while highlighting challenges associated with their implementation. In 2023, Svadlenka [21] focused on the third-party logistics (3PL) providers for sustainable Multi-criteria decision-making (MCDM) approaches like the Best-Worst Method (BWM) and the Combined Compromise Solution (CoCoSo) method offer promising solutions. Validation through real-life case studies, such as the application to an e-shop company in Belgrade, Serbia, adds practical relevance. In 2024, a study by Korucuk [23] revealed that the primary obstacle to e-logistics in enterprises is the "inadequate proficiency in managing digital technologies to bolster the system."

The *eternal m-security number* refers to the quantity of sentinels necessary to manage any given series of individual assaults through several sentinels' movements. A well-positioned arrangement of sentinels meeting this requirement is termed an *eternal m-secure set*. Burger et al [11, 12]. Initially proposed a dynamic domination concept, later termed "*eternal security*" in the work of Goddard et al. [13]. In this framework, a predefined number of sentinels are stationed at the vertices of a graph $G = (V, E)$, with a maximum of one sentinels per vertex. If a sentinels positioned at a vertex x can effectively counter an assault targeting a vertex y by relocating through the edge from x to y (provided that x is currently has no sentinels), and this response is feasible regardless of which vertex is targeted, then adjusting the sentinels positions to maintain this capability indefinitely, we designate the sentinels as constituting an eternal secure set.

Klostermeyer et al. [14, 15] introduced the notion of an "*eternal dominating set*" (EDS) within the realm of graph theory. An EDS of graph G is defined as a set D such that for every sequence of assaults $R = r_1, r_2, r_3, \dots$ where $r_i \in V$, there exists a corresponding sequence $H = H_1, H_2, H_3, \dots$ of dominating sets and a sequence of vertices $s_1, s_2, \dots$ where $s_i \in H_i \cap N[r_i]$, ensuring that $h_{i+1} = (H_i - s_i) \cup r_i$. It's worth noting that $s_i = r_i$ is a possibility. In this context, H_{i+1} represents the location of sentinels after successfully defending against the assault at r_i. If $s_i \neq r_i$, it implies that the sentinel at s_i has relocated to r_i. The smallest cardinality among all EDSs defines the "*eternal domination number*" $\gamma^{\infty}(G)$. In the m-EDS problem, the previously described model is adapted to allow several sentinels to relocate in response to a sequence of assaults. The size of the smallest m-EDS of graph G is defined as the *m-eternal domination number* $\gamma_m^{\infty}(G)$. This variant, where several sentinels can move, was introduced by Goddard et al. [12] and is also known as the *"all sentinels move"* model. Further investigations into this parameter have been conducted by Roushini Lleely Pushpam et al. [16, 17].

Numerous advanced studies are delving into domination concepts, with one of the latest additions being *certified domination*, introduced by Magda Dettlaff et al. [18] . A *certified dominating set* in a graph G is defined as a set $D \subseteq V$ that dominates every vertex in G, and each vertex in D has either zero or at least two neighbors in the set $V - D$. The *certified domination number* $\gamma_{cer}(G)$ of G is the minimum cardinality of a certified dominating set, and D is referred to as the γ_{cer}-set of G if it is the smallest such set. Further research on this parameter can be found in [19].

1.3. Noveltles

In existing literature, numerous methods leveraging graph theory have been proposed to optimize time and distance in online food delivery services. However, this article presents a pioneering contribution by introducing a novel parameter in graph theory, namely the eternal m-certified domination number, for the first time. The summarized essence of our work is outlined as follows:

1. Introduction of Eternal m-Certified Domination Number: We define the

eternal m-certified domination number, a novel parameter in graph theory, which signifies the minimum number of sentinels needed to maintain a certified dominating set indefinitely.

2. Analysis of Split Graphs: We conduct an analysis of the eternal m-certified domination number for split graphs, utilizing a theorem to elucidate its properties and implications within this specific graph structure.
3. Equivalence between Eternal m-Domination and Certification: We establish conditions under which the eternal m-domination number and the eternal m-certified domination number coincide.
4. Methodology for Time Reduction and Human Resource Optimization: We propose a methodology aimed at reducing time for food pick-up and optimizing human resources in online food delivery logistics. This methodology introduces a sorting facility, which streamlines the delivery process and minimizes resource utilization, thereby improving operational efficiency.
5. The eternal m-certified domination number optimizes manpower allocation by allowing each worker to cover multiple delivery points, thus enhancing efficiency. This approach reduces the total number of workers required, as it leverages the capacity of each worker to cover several delivery points within the delivery network. Consequently, the eternal m-certified domination number is more effective in minimizing manpower than the eternal m-security number, by maximizing the coverage capability of each worker.

1.4. Organization of the article

In this article, we begin with an introduction providing the motivation and background for our study, along with an overview of eternal m-certified domination in graphs, setting the stage for our exploration. Section 2 lays the groundwork by presenting the preliminaries, which include fundamental definitions and concepts in graph theory and an introduction to eternal domination and certified domination. Building upon this foundation, Section 3 delves into the definition of eternal m-certified domination number and the analysis of eternal m-certified domination within split graphs, while also elucidating necessary and sufficient conditions for equal eternal m-domination and certified domination numbers. Transitioning to a practical application, Section 4 presents a case study to reduce the time for food pick-up in online food delivery logistics. This section introduces a methodology designed to streamline the food delivery process, addressing inefficiencies and bottlenecks inherent in the current system. Furthermore, Section 4.1 delves into the concept of human resources reduction by introducing a sorting facility, proposing strategies for its implementation, and discussing potential benefits and challenges. Finally, we conclude our study in Section 5, summarizing key findings and implications while suggesting avenues for future research in this interdisciplinary domain.

2. Preliminaries

Consider a graph $G = (V, E)$, where V represents a finite, simple, undirected, and connected set of vertices, and E denotes the set of edges. In this context, the vertex set is denoted as $V(G)$ and the edge set as $E(G)$. The terminology pertaining to graph theory is drawn from Harary [14]. For any given vertex $r_2 \in V$, the term "open neighborhood" refers to the set $N(r_2)$, defined as $r_1 \in V: r_1 r_2 \in E$. The "closed neighborhood," denoted by $N[r_2]$, encompasses $N(r_1)$ along with r_2. For a subset $A \subseteq V$, $N(A)$ represents the union of open neighborhoods of all vertices in A, and $N[A]$ denotes the closed neighborhood of A. Furthermore, the "private neighborhood" $pn(r_2, A)$ of a vertex $r_1 \in A$ is characterized by $pn(r_2, A) = r_1 \in V - A: N(r_1) \cap A = r_2$. The *external private neighborhood* $epn(r_2, A)$ of a vertex $r_2 \in A$ is defined as the set $epn(r_2, A) = r_1 \in V - A: N(r_1) \cap A = r_2$. A set A qualifies as a *dominating set* if $N[A] = V(G)$, indicating that every vertex not in A is neighboring to at least one vertex in A. The *domination number* $\gamma(G)$ is the smallest size of a *dominating set* in G, and a dominating set A of minimum size is referred to as a γ-set of G. A vertex with a degree of one is termed a *pendant vertex*.

In a graph, a vertex adjacent to a pendant vertex is known as a *support*. A connected graph lacking cycles is defined as a *tree*. A *wheel graph* W_n consists of n vertices, created by linking From individual vertices to every vertex of an $(n-1)$-cycle. A graph G is deemed k-*partite*, where $k \geq 1$, if it's possible to partition $V(G)$ into k subsets $V_1, V_2, \ldots, V_k$ (referred to as partite sets) such that every edge of G connects a vertex of V_i to a vertex of V_j for $i \neq j$. If G is a 1-*partite* graph of order n, then G equals $\overline{K_n}$. When $k = 2$, such graphs are termed *bipartite* graphs. A *split graph* is defined as a graph G in which its vertices can be divided into two sets, R and S. The vertices in set R are independent, while the vertices in set S form a complete graph. For any vertex s_2 in S, $N_R(s_2)$ represents the neighbors of s_2 in set R.

3. Eternal m-Certified Domination for Some Classes of Graphs

In a variation of the parameters mentioned above in the literature, we present the concept of an eternal *m*-certified dominating set. A set qualifies as an *eternal m-certified dominating set,* if, following an infinitely long sequence of assaults, the arrangement of sentinels consistently forms a certified dominating set. The eternal *m*-certified domination number of graph G is defined as the smallest number of sentinels needed to handle any sequence of individual assaults using several sentinels' shifts, ensuring that the sentinel's configuration always forms a certified dominating set. This quantity is denoted by $\gamma_{mcer}^{\infty}(G)$. It's evident that for any graph G, $\gamma_{mcer}^{\infty}(G) \geq \gamma_m^{\infty}(G)$. In this section, we found the values of the m-eternal certified domination number in some standard graphs.

Observation 1. *For paths P_n , $n \geq 2$, $\gamma_{mcer}^{\infty}(P_n) = n$.*

Observation 2. *For cycles C_n , $n \geq 3, \gamma_{mcer}^{\infty}(C_n) = \left\lceil \frac{n}{3} \right\rceil$.*

Observation 3. *For wheel graphs W_n , $n \geq 3$, $\gamma_{mcer}^{\infty}(W_n) = 2$.*

Observation 4. *For complete bipartite graph $K_{m,n}$, $\gamma_{mcer}^{\infty}(K_{m,n}) = \begin{cases} 3 & \text{if } m = 1 \text{ and } n = 2 \\ 2 & \text{otherwise} \end{cases}$*

In all the above observations we see that $\gamma_{mcer}^{\infty}(G) = \gamma_m^{\infty}(G)$, for cycles, wheel graphs and complete bipartitle graphs except $K_{m,n}$ for $m = 1$ and $n = 2$ and paths. Roushini Leely Pushpam et al. have mentioned that $\sigma_m(G) = \gamma(G)$ or $\sigma_m(G) = \gamma(G) + 1$.

Theorem 1. *For any split graph G , $\sigma_m(G) = \gamma(G)$ or $\sigma_m(G) = \gamma(G) + 1$ [18].*

Theorem 2. *For a graph G, $\gamma_{mcer}^{\infty}(G) = \gamma_m^{\infty}(G)$, if and only if there exists no γ_m^{∞}- set A such that $N_{V-A}(v) = 1$ for each $x \in A$.*

Proof. Let $\gamma_{mcer}^{\infty}(G) = \gamma_m^{\infty}(G)$. To prove that there exists no γ_m^{∞} - set A of G such that $N_{V-A}(x) = 1$ for any $x \in A$. Suppose not, in that case there exists a γ_m^{∞} - set A of G such that $N_{V-A}(x) = 1$ for a minimum of one vertex $x \in A$ and let $y \in A \cap N(x)$. Now by the definition of eternal m-certified domination number, we clearly see that $A \cup \{y\}$ is the γ_{mcer}^{∞} - set of G , a contradiction to our assumption.

Hence, there exists no such A in G. Converse part of the theorem can be derived directly from the definition of eternal m-certified domination number. □

Theorem 3. *Let G be a split graph with bi-partition (R, S) where R is independent and $G[S]$ is complete and $|R| = m$ and $|S| = n$. Then $\gamma_m^{\infty}(G) = \gamma_{mcer}^{\infty}(G)$, if and only if there exists a γ-set A such that one of the following holds.*

(i) If $\gamma_m^{\infty}(G) = \gamma(G)$, then $A \cap S = \emptyset$ and there exists no vertex r in R such that $deg(r) = 1$.

(ii) If $\gamma_m^{\infty}(G) = \gamma(G) + 1$, then for at least one vertex $r \in R$, $deg(r) = 1$ with $r \in pn(S, A')$ and $|pn(S, A')| = 1$ such that there exists at least two vertices in $S - A'$ where $A' = A \cup \{v\}$ for some $v \in V(G)$.

Proof. Let $\gamma_m^{\infty}(G) = \gamma_{mcer}^{\infty}(G)$. By theorem [3.1], either $\gamma_m^{\infty}(G) = \gamma(G)$ or $\gamma_m^{\infty}(G) = \gamma(G) + 1$. If $\gamma_m^{\infty}(G) = \gamma(G)$, then to prove that there is a γ- set A of G such that $A \cap S = \emptyset$, that is to prove $|A| = m$. Suppose not. Then there is a vertex $s \in S$ such that $|epn(s, A) \cap R| > 1$ and if an assault takes place at $z \in epn(s, A)$, then the sentinels stationed at s reacts to it, while the other members of $epn(s, A)$ are left undefended. This indicates that $\gamma_m^{\infty}(G) > \gamma(G)$, which contradicts our assumption. Hence $A \cap S = \emptyset$.

Now suppose that there is a vertex $w \in A$ such that $deg(w) = 1$. Then $|epn(w, A)| = 1$, which is a contradiction to the definition of eternal m-certified domination number. Hence (i) is true.

Suppose $\gamma_m^{\infty}(G) = \gamma(G) + 1$ and $A' = A \cup \{v\}$ for some $v \in V(G)$ is a $\gamma_m^{\infty}(G)$-set of G. Then $|epn(v, A')| \geq 2$ for at least one vertex $v \in A'$. If all the vertices $r \in R \cap A'$ have at least two external private neighbors, we are through. Otherwise, there is a vertex $r \in R \cap A'$ with $|pn(r, A')| = 1$. Let $s \in pn(r, A')$. Then there exists at least two vertices say $s_1, s_2 \in S - A'$. If not, $epn(r, A') = 1$, a contradiction to the definition of eternal m-certified domination number. Suppose there is exactly one vertex say $s_1 \in S - A'$. Then for an assault at the vertex $s \in R \cap epn(r, A')$, the sentinels at the vertex $r \in A'$ responds to it such that $A_1 = (A' - \{r\}) \cup \{s\}$ is the new configuration of sentinels. Now we clearly see that $|epn(s, A_1)| = 1$ and $r \in epn(s, A_1)$, a contradiction.

Hence there exists at least two vertices $s_1, s_2 \in S - A'$ such that $A'' = A \cup \{r\}$ be a new configuration of sentinels, so that $s_1, s_2 \in epn(r, A'')$. This implies that A'' is a γ_{mcer}^{∞}-set of G. Hence $|A'| = |A''|$. Therefore (ii) is true.

Conversely, suppose (i) is true. Then to prove $\gamma_m^{\infty}(G) = \gamma_{mcer}^{\infty}(G)$. By (i) A is a γ- set of G and

$|A| = m$ (since $A \cap S = \emptyset$ and R is independent). Also, each vertex in R has at least two neighbors in S and $G[N[r]]$ is complete for every r. Hence A is a γ_{mcer}^{∞}-set of G. Suppose (ii) is true. Then $|epn(v, A')| \geq 2$ for each vertex $v \in A'$. Here A' is a γ_{mcer}^{∞}-set of G. Hence $\gamma_{m}^{\infty}(G) = \gamma_{mcer}^{\infty}(G)$. □

4. Case Study

4.1. Methodology to reduce the time for the food pick-up

The demand for food delivery has seen a notable surge over time, with advancements in technology facilitating online shopping and ordering of food. This industry witnessed remarkable expansion, especially amidst the COVID-19 crisis [7]. In India, the process of online shopping and food ordering is streamlined, benefiting from the widespread use of smartphones among nearly half of the population [8]. The simplicity of options offered by food ordering platforms enhances customer engagement and accessibility. Additionally, the expansion of online businesses has created numerous job opportunities, including roles such as marketing executives, data analysts, sales managers, operation managers, food delivery personnel, and food and beverage trainers [9]. The focus of our study centers on Porur, a locality situated within Chennai, India. Porur holds significance as both a prominent market hub and a densely populated residential area, boasting a plethora of thriving restaurants in its vicinity. Figure 1 is a representation of the Google map showcasing the restaurants in Porur. The ability to provide online food delivery services is imperative for the sustenance of most restaurants. However, the primary challenge faced in the realm of online business pertains to logistics. It is essential to analyze delivery zones to accurately determine delivery distances thoroughly. Ensuring timely delivery and maintaining food quality standards are pivotal for the success of restaurants. Business proprietors often grapple with issues related to vehicle allocation and delivery services. Timely delivery emerges as a pivotal factor influencing customer retention rates. Hence, establishing a successful food delivery business necessitates implementing an effective strategy in vehicle allocation. In our study, the choice of delivery vehicle remains flexible, tailored to specific criteria such as distance, congestion, and order size. While bikes excel in dense urban areas for short distances, scooters and motorcycles offer greater speed and coverage. Cars are preferred for longer distances or larger orders, often utilized by major delivery services like Uber Eats. Vans are utilized for bulk deliveries or when transporting substantial quantities of food. Thus, the selection of vehicle is contingent upon various factors to ensure efficient and effective food delivery.

Fig. 1. Restaurants around porur

In the bustling city of Chennai, various food delivery services like Amazon, Swiggy, Daily Box, and Dunzo have successfully bridged the gap between customers and restaurants. Our study proposes a strategic approach to thriving in the food delivery service industry specifically tailored for Porur. Our strategy revolves around utilizing the concept of eternal m-certified domination numbers. The primary objective is to ensure efficient food pick-up from restaurants within a swift 15-minute timeframe while guaranteeing that delivery vehicles' resulting position remains accessible to any restaurant within a 15-minute radius. To achieve this, we identify key landmarks surrounding Porur, including Lakshmi Nagar, Sakthi Nagar, Mugalivakkam, Ramapuram, Jai Garden bus stop, Valasaravakkam, Karambakkam, Porur toll plaza, Dharmaraja Nagar, Iyappanthangal, Kattuppakkam, Karayanchavadi, Porur EB bus stop, and Vigneshwara Nagar, collectively referred to as L.

In Fig. 2, we present a graphical representation of these landmarks, with vertices denoting each landmark and edges indicating direct routes between them. The distance between any two landmarks in the discussed graph ranges from 2 to 6 kilometers. Suppose an entrepreneur, let's call them Person A, intends to launch a food delivery service, referred to as Service A. By forming partnerships with at least one restaurant situated in every landmark within L, Service A commits to maintaining a delivery boy alignment that represents the certified domination set after each movement. This strategic alignment streamlines operations and reduces the need for extensive human resources. The ultimate aim is to minimize the number of vehicles associated with Service A while maximizing efficiency and customer satisfaction in the food delivery process.

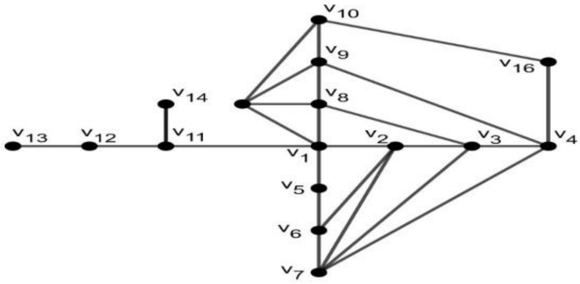

Fig. 2. Graphical representation of the landmarks connected by road way

Let us consider the landmarks as $L = \{v_1, v_2, \ldots, v_{16}\}$. v_1: Porur, v_2: Lakshmi Nagar, v_3: Mugalivakkam, v_4: Ramapuram , v_5: Porur EB bus stop, v_6 : Vigneshwara nagar, v_7 : Madhanandapuram , v_8: Jai Garden bus stop, v_9: Dharmaraja Nagar, v_{10} : Valasaravakkam, v_{11} : Iyappanthangal, v_{12} :Kattupakkam v_{13}:Karayanchavadi, v_{14}: Porur Toll plaza, v_{15}: Karambakkam, v_{16}: Ramapuram. In this scenario, we assume that each hotel can effectively cater to the surrounding area of the landmark where it is situated.

Furthermore, every hotel has the capability to deliver food to any of the sixteen locations within the defined graph. The delivery personnel are afforded flexibility in their movements, enabling them to traverse from one delivery location to another seamlessly. Following each delivery, they have unrestricted access to food from any of the hotels depicted within the graph.

The task at hand involves determining the minimum number of vehicles required to pick up food orders within a 15-minute timeframe, ensuring that the positions of these vehicles represent a certified domination set after each delivery. This task is equivalent to finding the eternal m-certified domination number, denoted as $\gamma_{mcer}^{\infty}(G)$, of the graph depicted in Figure 2. since $\gamma_{mcer}^{\infty}(G)$ equals 5, it follows that regardless of the final position of the vehicle, a minimum of 5 vehicles is necessary to efficiently pick up food from the hotels within the designated 15-minute window. The existing methodology has mainly addressed a static problem. The upcoming example will illustrate the application of eternal m-security number in a sequence of a single order with multi sentinels' movements. This method will make the food pick-up faster and reduce the logistic expenses. This systematic approach guarantees the timely pick-up of food orders. However, it's important to note that the delivery process is not discussed in this particular section of the analysis.

4.2. Human resources reduction for online food delivery logistic by introducing a sorting facility

The algorithms, such as the Clarke-Wright Savings Algorithm, the Traveling Salesman Problem (TSP), and its variants, like the Vehicle Routing Problem (VRP) and the Capacitated VRP (CVRP), are fundamental in optimizing delivery routes. They determine the most efficient sequence of stops for each vehicle to minimize distance traveled or total time while considering constraints like vehicle capacity and time windows. Alternatively, food delivery enterprises can optimize their logistical operations by introducing a sorting facility or hub. These hubs, situated in proximity to the delivery company's operations, serve as centralized points for receiving food items following each pick-up. Subsequently, the packages are sorted at these hubs before being dispatched to their respective delivery locations. This approach proves particularly advantageous when several deliveries are destined for the same area, as a single delivery personnel can efficiently handle the distribution. Implementing a sorting facility not only reduces delivery time but also offers economic benefits to the company. Eternal m-certified domination number In our case study outlined in Section 4.1, we propose the establishment of four such hubs strategically positioned at locations v_3, v_9, v_{11}, and v_6. These choices were made strategically as they efficiently partition the overall area into four distinct groups, with each group's landmarks positioned approximately 2-3 kilometers away from the nearest hub. Several variables impacted the selection criteria for the sorting hub sites (v_3, v_9 , v_{11} , and v_6) to guarantee operational efficiency and effective service coverage. Traffic patterns, population density, delivery demand, and physical closeness to restaurants and delivery regions were all considered.

- Traffic Patterns: The hubs were intentionally located with reasonably smooth traffic flow and little congestion during peak delivery hours. This approach cuts delivery personnel's travel time while ensuring prompt pick-up and delivery.
- Population Density: Areas with higher population density and more restaurants and delivery locations were preferred for hub locations. This method enabled the effective distribution of orders and optimal resource use.
- Delivery Demand: The hubs were located in high-demand areas, ensuring that they service areas with large orders. This strategy improves the sorting process's efficiency and meets customer delivery expectations.

Geographical Proximity: The chosen hub locations were geographically close to the intended delivery zones, minimizing travel distances and improving order processing and delivery times.

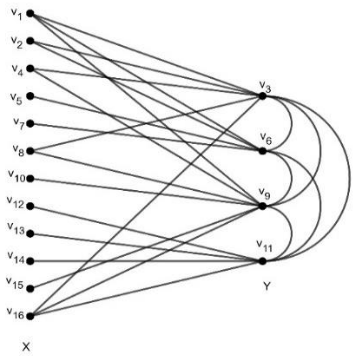

Fig. 3. Graph indicates the transport between hotels, the hub, and hubs.

Figure 3 illustrates the logistics network connecting hotels and hubs and inter-hub connections. The split graph design is used in Figure 3 because it relates to the logistical issues encountered in food delivery operations. A split graph is one in which the vertices may be separated into two sets, one of which is an independent set and the other a complete graph. This modeling technique visually depicts how sorting facilities are strategically positioned to optimize supply routes and lower operational expenses.

In this depiction, vertices labeled X denote hotel locations, while Y represents hub locations. The edges signify roadways that can be traversed within a 15-minute, assuming no traffic delays. Despite hotels being situated within the landmark denoted by Y, the brief transfer time renders food delivery from hotels to hubs insignificant. The graph in Figure 3 exhibits a split graph configuration. According to Theorem 3.2, both the eternal m-domination number and the eternal m-certified domination number are determined to be '4'. This implies that a team of at least four individuals can efficiently manage prepared food distribution from hotels to hubs, ensuring domination and certified domination through their positions. Once the food reaches the hubs, subsequent customer delivery can be executed effectively.

This case study presents a graphical methodology aimed at reducing human effort and time required for food pick-up and organization at hubs, thus expediting logistics, a critical factor for success in the online food delivery business. Our study disregards sorting time as it isn't directly linked to the graph depicted in Figure 3. To enhance clarity, we plan to utilize a weighted graph representation, where each landmark is represented as a vertex with edge weights corresponding to the distance/time between landmarks and vertex weights indicating the waiting time for future analyses.

4.3. Enhancing clarity and accuracy with weighted graph representation

The study suggested employing a weighted graph representation to increase the clarity and accuracy of future investigations in food delivery logistics. This model distributes edge weights proportional to the distance or time between landmarks and vertex weights representing waiting time, providing numerous advantages:

- Visual Clarity: The weighted graph, which includes edge weights denoting distance or time, gives a visual representation that directly represents real distances or transit times between locations. This clarity helps to grasp the distribution network's geographical linkages and logistical problems.
- Quantitative Analysis: Including numerical edge weights enables quantitative examination of delivery routes and efficiency. Algorithms can improve routes by lowering the distance traveled or overall delivery time, resulting in more effective logistics operations.
- Resource Allocation: Vertex weights reflecting wait times at sorting hubs or delivery points aid in resource allocation choices. By considering waiting periods, logistics managers may optimize staffing levels at hubs, properly distribute delivery workers, and reduce idle time, thus increasing resource utilization and lowering costs.
- Scenario Planning: Weighted graphs facilitate scenario planning by simulating various traffic circumstances, demand levels, and delivery priorities. This capacity facilitates decision-making by assessing the impact of proposed modifications on logistical performance and customer service.

Integrating this weighted graph representation into our study improves the clarity and accuracy of logistical assessments, giving a solid foundation for future investigations and optimization efforts.

5. Conclusions

The utilization of the graph's eternal $\boldsymbol{m}$-domination, $\boldsymbol{\gamma_m^\infty(G)}$, holds practical implications in logistics. In our article, we introduced $\boldsymbol{\gamma_{mcer}^\infty(G)}$ to minimize the number of sentinels required for graphical protection or service. We explored the necessary condition for the equality of $\boldsymbol{\gamma_m^\infty(G)}$ and $\boldsymbol{\gamma_{mcer}^\infty(G)}$. The article presents a generalized approach for determining $\boldsymbol{\gamma_{mcer}^\infty(G)}$ in split graphs. It advocates for leveraging the eternal $\boldsymbol{m}$-domination number methodology to streamline food pick-up and delivery logistics by introducing sorting facility which is represented as a split graph such as cost reduction and shorter delivery times.

5.1. Limitations

The study admits shortcomings such as omitting sorting time, ignoring traffic variability's influence on delivery timings, assuming equal demand, and failing to consider hub scalability. These omissions may impact delivery estimate accuracy, operational responsiveness, and long-term cost-effectiveness. To improve practical application, future studies should include real-time traffic data, dynamic demand predictions, and scalability evaluations for sorting hubs.

5.2. Future scope

Further studies might expand the suggested logistics model to include bigger urban areas and analyze how factors such as longer distances and different traffic patterns affect operational efficiency and cost-effectiveness. Furthermore, investigating the usability of sorting facilities in diverse operational contexts, such as suburban or rural locations, may provide insights into how to adapt the strategy to differing delivery demands and infrastructure circumstances. Evaluating the scalability and cost-effectiveness of sorting hubs in these scenarios will give valuable insights for optimizing food delivery logistics in a broader range of settings.

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