# Finding a Common Set of Weights by the Fuzzy Entropy Compared with Data Envelopment Analysis - A Case Study

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## KEYWORDS

## ABSTRACT

*A data envelopment analysis (DEA) method can be regarded as a useful management tool to evaluate decision making units (DMUs) using multiple inputs and outputs. In some cases, we face with imprecise inputs and outputs, such as fuzzy or interval data, so the efficiency of DMUs will not be exact. Most researchers have been interested in getting efficiency and ranking DMUs recently. Models of the traditional DEA cannot provide a completely ranking of efficient units; however, it can just distinguish between efficient and inefficient units. In this paper, the efficiency scores of DMUs are computed by a fuzzy CCR model and the fuzzy entropy of DMUs. Then these units are ranked and compared with two foregoing procedures. To do this, the fuzzy entropy based on common set of weights (CSW) is used. Furthermore, the fuzzy efficiency of DMUs considering the optimistic level is computed. Finally, a numerical example taken from a real-case study is considered and the related concept is analyzed.*

## 1. Introduction

Data envelopment analysis (DEA), which is a very useful management and decision tool, has found surprising development in theory and real-world applications. It was first developed by Charnes, et al [1], in which the traditional DEA model requires crisp input/output data. However, in real-world problems, inputs and outputs are often imprecise. Most of the previous studies dealing with imprecise inputs and outputs in DEA models have simply used simulation techniques [2 and 3]. The final efficiency score for each DMU was derived as a deterministic numerical value less than or equal to unity. In recent years, fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. Sengupta [4] was the first to introduce a fuzzy DEA model. The DEA models with fuzzy data ("fuzzy DEA" models) can more realistically represent real-world problems than the conventional DEA models. The fuzzy set theory also allows linguistic data to be used directly within the DEA models.

Fuzzy DEA models take the form of fuzzy linear programming models. A typical approach to fuzzy linear programming requires a method to rank fuzzy sets and different fuzzy ranking methods may lead to different results [5]. The problem of ranking fuzzy sets has been addressed by many researchers [6 and 7].

The main aim of this paper is to explore the method of finding common set of weights (CSW) based on fuzzy entropy and ranking decision making units (DMUs) by the use of the fuzzy CCR model and fuzzy entropy. Entropy describes the fuzziness degree of the fuzzy set. Many scholars have studied it from different points of view. For example, De Luce and Termini [8]

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introduced some axioms to describe the fuzziness degree of the fuzzy set. Kaufmann [9] proposed a method to measure the fuzziness degree of the fuzzy set by a metric distance between its membership function and the membership function of its nearest crisp set. Another way given by Yager [10] was to view the fuzziness degree of the fuzzy set in terms of a lack of distinction between the fuzzy set and its complement. Some authors have investigated interval valued fuzzy set and its some related topics. For example, Grzegorzewski [11] studied distance between interval valued fuzzy sets based on the Hausdroff metric. Burillo and Bustince [12] and Szmidt, et al [13] researched entropy of interval valued fuzzy set from different point of views, respectively. We can use each of mentioned ways in a particular situation, but the ranking by CSW is the best [14].

In this paper, a procedure is suggested to find a CSW by the fuzzy entropy. In the proposed procedure first, the fuzzy entropy and a CSW are computed, and then the efficiency and ranking of each unit are determined. The rest of this paper is organized as follow. In Sections 2 and 3, we review the fuzzy DEA model and fuzzy entropy. In Section 4, we investigate the method of finding common set of weights (CSW) based on the fuzzy entropy. Finally, an illustrative example is presented in Section 5.

## 2. Fuzzy DEA Model

Fuzzy DEA models take the form of fuzzy linear programming models. Consider $n$ DMUs; each consumes varying amounts of $m$ different fuzzy inputs to produce $s$ different fuzzy outputs. In the model formulation, $\tilde{x}_{i0}$ ($i=1,…,m$) and $\tilde{y}_{r0}$ ($r=1,…,s$) denote the input and output values for DMU0, the DMU under consideration, respectively. The programming statement for the fuzzy duel CCR model is as follows:

**Model 1:**

$$\min z = \theta$$
s.t.
$$\theta \tilde{x}_{ip} \geq \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \qquad \forall i, \tag{1}$$

$$\tilde{y}_{rp} \leq \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \qquad \forall r, \tag{2}$$

$$\lambda_j \geq 0 \qquad \forall j. \tag{3}$$

Among the various types of fuzzy numbers, triangular fuzzy numbers are of the most important. In the sequel, we consider the inputs and outputs of DMUs as triangular fuzzy numbers. Let $\tilde{x}_{ij} = \left( x_{ij}^m, x_{ij}^l, x_{ij}^u \right)$ and

$\tilde{y}_{rj} = \left( y_{rj}^m, y_{rj}^l, y_{rj}^u \right)$. The fuzzy CCR model is as follows:

**Model 2:**

$$\min z = \theta$$
s.t.
$$\theta(x_{ip}^m, x_{ip}^l, x_{ip}^u) \geq \left[ \sum_{j=1}^{n} \lambda_j x_{ij}^m, \sum_{j=1}^{n} \lambda_j x_{ij}^l, \sum_{j=1}^{n} \lambda_j x_{ij}^u \right] \ \forall i, \tag{4}$$

$$(y_{rp}^m, y_{rp}^l, y_{rp}^u) \leq \left[ \sum_{j=1}^{n} \lambda_j y_{rj}^m, \sum_{j=1}^{n} \lambda_j y_{rj}^l, \sum_{j=1}^{n} \lambda_j y_{rj}^u \right] \forall r, \tag{5}$$

$$\lambda_j \geq 0 \qquad \forall j. \tag{6}$$

The -cuts, also known as the -level sets of $\tilde{x}_{ij}$ and $\tilde{y}_{ij}$, are defined by:

$$(\tilde{x}_{ij})_\alpha = \left\{ x \in X \mid \mu_{x_{ij}}(x) \geq \alpha \right\} = \left[ x_{ij}^m, x_{ij}^l, x_{ij}^u \right] \tag{7}$$

$$(\tilde{y}_{rj})_\alpha = \left\{ x \in X \mid \mu_{y_{rj}}(x) \geq \alpha \right\} = \left[ x_{ij}^m, x_{ij}^l, x_{ij}^u \right] \tag{8}$$

Applying the -level of the fuzzy DEA, the following model is achieved.

**Model 3:**

$$\min z = \theta$$
s.t.
$$\left[ \theta(\alpha x_{ip}^m + (1-\alpha) x_{ip}^l, \theta(\alpha x_{ip}^m + (1-\alpha) x_{ip}^u \right] \geq \tag{9}$$

$$\left[ \sum_{j=1}^{n} \lambda_j (\alpha x_{ij}^m + (1-\alpha) x_{ij}^l), \sum_{j=1}^{n} \lambda_j (\alpha x_{ij}^m + (1-\alpha) x_{ij}^u \right] , \forall i$$

$$\left[ (\alpha y_{rp}^m + (1-\alpha) y_{rp}^l, \alpha y_{rp}^m + (1-\alpha) y_{rp}^u) \right] \leq \tag{10}$$

$$\left[ \sum_{j=1}^{n} \lambda_j (\alpha y_{rj}^m + (1-\alpha) y_{rj}^l), \sum_{j=1}^{n} \lambda_j (\alpha y_{rj}^m + (1-\alpha) y_{rj}^u \right] , \forall r$$

$$\lambda_j \geq 0 \qquad \forall j. \tag{11}$$

To rank each unit, the least of inputs level and the much of outputs level compare with the weakest bound of efficiency.

The best part of DMU$_p$ in Equations (9) and (10) is $(\alpha X_p^m + (1-\alpha)X_p^l, \alpha Y_p^m + (1-\alpha)Y_p^u)$ and the weakest part of bound is given by:

$$(\sum_{j=1}^{n} \lambda_j (\alpha X_j^m + (1-\alpha)X_j^u), \sum_{j=1}^{n} \lambda_j (\alpha Y_j^m + (1-\alpha)Y_j^l))$$

Therefore, Model 3 is changed to Model 4.

**Model 4:**

$\min z = \theta$
s.t.
$$\left[ \theta(\alpha x_{ip}^m + (1-\alpha)x_{ip}^l) \right] \geq \left[ \sum_{j=1}^{n} \lambda_j (\alpha x_{ij}^m + (1-\alpha)x_{ij}^u) \right] \quad , \forall i \quad (12)$$

$$\left[ \alpha y_{rp}^m + (1-\alpha)y_{rp}^u \right] \leq \left[ \sum_{j=1}^{n} \lambda_j (\alpha y_{rj}^m + (1-\alpha)y_{rj}^l) \right] \quad , \forall r \quad (13)$$

$$\lambda_j \geq 0 \qquad \forall j . \tag{14}$$

Model 4 is a parametric linear programming problem, while $\alpha \in [0,1)$ is a parameter. So the efficient units are completely ranked [15].

### 3. Entropy Fuzzy

In physics, the word entropy has important physical implication as the amount of "disorder" of a system, and in mathematics, a more abstract definition is used.

Entropy is as a measure of probabilistic uncertainty. Concept of entropy has penetrated a wide range of disciplines, such as statistical mechanics, business, pattern recognition, transportation, information theory, queuing theory, linear and nonlinear programming and so on. To define entropy, Shannon (1948) proposed some axioms: (1) expansibility, (2) symmetry, (3) continuity, (4) maximum, (5) additivity, (6) monotonicity, (7) branching and (8) normalization. The Shannon's entropy of a variable $A$ (discrete set) is defined by:

$$H(A) = -\sum p(x) \times Ln(p(x)) \tag{15}$$

where, $p(x)$ denotes the probability distribution in the universal set $X$ for all $x$ $X$, and the entropy of a continuous probability distribution with the probability density function $p(x)$ as:

$$H(A) = -\int p(x) Ln(p(x)) dx \tag{16}$$

The first fuzzy entropy formula without reference to probabilities was proposed by De Luca and Termini [8], who defined the entropy using the Shannon's functional form.

$$H(A) = -\sum \mu_A(x) Ln(\mu_A(x)) - (1-\mu_A(x))(Ln(1-\mu_A(x))) \tag{17}$$

The measure of fuzziness $H(A)$ can be regarded as "entropy" of a fuzzy set $A$. At a fixed element $x$, $H(A(x)) = h(A(x))$ where the entropy function $h : [0,1] \rightarrow [0,1]$ is monotonically increasing in $[0,0.5]$ and monotonically decreasing in $[0.5,1]$, moreover $h(u) = 0$, as $u = 0$ and 1; and $h(u) = 1$, as $u = 0.5$. Some well-known entropy functions are shown by:

$$h(u) = \begin{cases} 2u & u \in [0, \frac{1}{2}] \\ 2(1-u) & u \in [\frac{1}{2}, 1] \end{cases} \tag{18}$$

$$h(u) = 4u(1-u) \tag{19}$$

$$h(u) = -u.Ln(u) - (1-u) \times Ln(1-u) \tag{20}$$

where, the last is the Shannon's function.

### 4. Entropy in DEA

In most of the existing methods for possibilistic linear programming, where the -cut is used, the solution is obtained by comparing the interval in the left and right hand side of the constraints.

Different methodologies have been suggested for the comparison of the intervals. In some of these methods simply the end points of interval are considered for justification that makes the model very simple, and hence a lot of information might have been lost. In the others, the complexity of the algorithm may cause computational inefficiency DEA assigns an efficiency score less than one to inefficient DMUs and equal to one to efficient DMUs. So, for inefficient DMUs, a ranking is given; however, for efficient ones no ranking can be given. Some methods for ranking efficient DMUs with crisp data are developed. In this paper, by considering fuzzy DMUs, an alternative ranking method based on entropy of efficiency of DMUs as common set of weights is proposed.

Charnes, et al [1] originally proposed data envelopment analysis (DEA) as a method for determining a set of weights for each decision making unit DMU. These sets of weights are, typically, different for each of the participating DMUs and, some of the weights may be assigned an exceedingly small value.

A possible answer to these difficulties lies in the notion of a CSW. In this paper, a procedure is suggested to find a CSW in the fuzzy entropy. This is done in six steps. A CSW is determined in the fifth step by computing the entropy [13 and 16].

**Step 1:** Consider $m$ DMUs, each consumes varying amounts of $s$ different fuzzy inputs to produce $r$ different fuzzy outputs. In the presented model formulation, $A_1, A_2, ..., A_m$, $\tilde{x}_{ij}$ $(i = 1, \cdots, s)$ and $\tilde{y}_{i,s+j}$ ($j$=1,…,$r$) denote DMUs, the $j$-th input value for the $i$-th unit, and the $j$-th output value for the $i$-th unit, respectively.

According to the input and output values of the DEA model, the decision making matrix is shown in Table 1. Let $\tilde{x}_{ij} = (a_{ij}, b_{ij})$ and $\tilde{y}_{ij} = (a_{i,s+j}, b_{i,s+j})$ are two positive LR fuzzy numbers. Then the operations $\{+ \_ . \mid\}$ are defined by:

$$\tilde{A}(+)\tilde{B} = (a_1, a_2)(+)(b_1, b_2) = (a_1 + b_1, a_2 + b_2) \quad (21)$$

$$\tilde{A}(-)\tilde{B} = (a_1, a_2)(-)(b_1, b_2) = (a_1 - b_2, a_2 - b_1) \quad (22)$$

$$\tilde{A}(.)\tilde{B} = (a_1, a_2)(.)(b_1, b_2) \quad (23)$$
$$= [\min(a_1b_1, a_1b_2, a_2b_1, a_2b_2), \max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)]$$

$$\tilde{A}(\div)\tilde{B} = (a_1, a_2)(\div)(b_1, b_2) = [\min(a_1/b_2, a_1/b_1, a_2/b_2, a_2/b_1), \quad (24)$$
$$\max(a_1/b_2, a_1/b_1, a_2/b_2, a_2/b_1)]$$

**Step 2:** Calculate the normalized input and output values:

$$\tilde{x}'_{ij} = \frac{\tilde{x}_{ij}}{\sum_{i=1}^{m} \tilde{x}_{ij}} \qquad , \qquad j = 1, ..., r \quad (25)$$

**Tab.1. Decision making matrix with fuzzy inputs and outputs**

|       | $\tilde{x}_1$ | $\cdots$ | $\tilde{x}_s$ | $\tilde{y}_{1+s}$ | $\cdots$ | $\tilde{y}_{r+s}$ |
|-------|---------------|----------|---------------|-------------------|----------|-------------------|
| $A_1$ | $\tilde{x}_{11}$ | $\cdots$ | $\tilde{x}_{1s}$ | $\tilde{y}_{1,s+1}$ | $\cdots$ | $\tilde{y}_{1,s+r}$ |
| $A_2$ | $\tilde{x}_{21}$ | $\cdots$ | $\tilde{x}_{2s}$ | $\tilde{y}_{2,s+1}$ | $\cdots$ | $\tilde{y}_{2,s+r}$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $A_m$ | $\tilde{x}_{m1}$ | $\cdots$ | $\tilde{x}_{ms}$ | $\tilde{y}_{m,s+1}$ | $\cdots$ | $\tilde{y}_{m,s+r}$ |

$$\tilde{y}'_{ij} = \frac{\tilde{y}_{ij}}{\sum_{i=1}^{m} \tilde{y}_{i,s+j}} \qquad , \qquad j = 1, ..., r \quad (26)$$

For the above equations, fuzzy normalized inputs $\tilde{x}'_{ij}$ can be expressed by LR fuzzy numbers, $\tilde{x}'_{ij} = [a''_{ij}, b''_{ij}]$, $j = 1, 2, ..., r$, where,

$$a'_j = \sum_{i=1}^{m} a_{ij} \quad (27)$$

$$b'_j = \sum_{i=1}^{m} b_{ij} \quad (28)$$

According to (24) we can obtain:

$$a''_{ij} = [\min(\frac{a_{ij}}{b'_j}, \frac{a_{ij}}{a'_j}, \frac{b_{ij}}{b'_j}, \frac{b_{ij}}{a'_j})] \quad (29)$$

$$b''_{ij} = [\max(\frac{a_{ij}}{b'_j}, \frac{a_{ij}}{a'_j}, \frac{b_{ij}}{b'_j}, \frac{b_{ij}}{a'_j})] \quad (30)$$

Fuzzy normalized outputs $\tilde{y}'_{ij}$ can be expressed by LR fuzzy numbers, $\tilde{y}'_{ij} = [a''_{i,s+j}, b''_{i,s+j}]$, $j = 1, 2, ..., r$, where,

$$a'_{j+s} = \sum_{i=1}^{m} a_{i,j+s} \quad (31)$$

$$b'_{j+s} = \sum_{i=1}^{m} b_{i,j+s} \quad (32)$$

$$a''_{i,j+s} = [\min(\frac{a_{i,j+s}}{b'_{j+s}}, \frac{a_{i,j+s}}{a'_{j+s}}, \frac{b_{i,j+s}}{b'_{j+s}}, \frac{b_{i,j+s}}{a'_{j+s}})]$$
(33)

$$b''_{i,j+s} = [\max(\frac{a_{i,j+s}}{b'_{j+s}}, \frac{a_{i,j+s}}{a'_{j+s}}, \frac{b_{i,j+s}}{b'_{j+s}}, \frac{b_{i,j+s}}{a'_{j+s}})]$$
(34)

**Step 3:** Calculate $\tilde{E}_j$ and $\tilde{E}_{j+s}$ values which are, respectively, the fuzzy entropy values of inputs and outputs.

$$\tilde{E}_j = -\frac{1}{Ln(m)} \sum_{i=1}^{m} \left( \tilde{x}'_{ij} . Ln(\tilde{x}'_{ij}) \right) \quad ; j = 1,...,r \quad (35)$$

$$\tilde{E}_{j+s} = -\frac{1}{Ln(m)} \sum_{i=1}^{m} \left( \tilde{y}'_{i,j+s} . Ln(\tilde{y}'_{i,j+s}) \right) ; j = 1,...,r \quad (36)$$

From Equation (35), the fuzzy entropy values for the LR fuzzy inputs $\tilde{E}_j = [\alpha'_j, \beta'_j]$, $j$=1,2,…,$r$ , as follows:

$$(a_j^*, b_j^*) = \ln(a''_j, b''_j)$$
(37)

$$\alpha_j = [\min(a''_j a_j^*, a''_j b_j^*, b''_j a_j^*, b''_j b_j^*)]$$
(38)

$$\beta_j = [\max(a''_j a_j^*, a''_j b_j^*, b''_j a_j^*, b''_j b_j^*)]$$
(39)

$$\alpha'_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} \alpha_j$$
(40)

$$\beta'_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} \beta_j$$
(41)

Now set $\tilde{E}_{j+s} = [\alpha'_{j+s}, \beta'_{j+s}]$, $j$=1,2,…,$r$, as follows:

$$(a_{j+s}^*, b_{j+s}^*) = \ln(a''_{j+s}, b''_{j+s})$$
(42)

$$\alpha_{j+s} = [\min(a''_{j+s} a_{j+s}^*, a''_{j+s} b_{j+s}^*, b''_{j+s} a_{j+s}^*, b''_{j+s} b_{j+s}^*)]$$
(43)

$$\beta_{j+s} = [\max(a''_{j+s} a_{j+s}^*, a''_{j+s} b_{j+s}^*, b''_{j+s} a_{j+s}^*, b''_{j+s} b_{j+s}^*)]$$
(44)

$$\alpha'_{j+s} = -\frac{1}{\ln(m)} \sum_{i=1}^{m} \alpha_{j+s}$$
(45)

$$\beta'_{j+s} = -\frac{1}{\ln(m)} \sum_{i=1}^{m} \beta_{j+s}$$
(46)

**Step 4:** Now calculate the deviation of entropy from one:

$$\tilde{d}_j = 1 - \tilde{E}_j \quad , j=1,2,…,r \quad (47)$$

$$\tilde{d}_{j+s} = 1 - \tilde{E}_{j+s} \quad , j=1,2,…,r \quad (48)$$

For the LR fuzzy numbers, (49) and (50) are presented as follow:

$$\tilde{d}_j = [1 - \beta'_j, 1 - \alpha'_j] = [\alpha''_j, \beta''_j] ; j = 1,...,r \quad (49)$$

$$\tilde{d}_{j+s} = [1 - \beta'_{j+s}, 1 - \alpha'_{j+s}] = [\alpha''_{j+s}, \beta''_{j+s}] ; j = 1,...,r \quad (50)$$

**Step 5:** Calculate the CSW of input and output values:

$$\tilde{w}_j = \frac{\tilde{d}_i}{\sum_{j=1}^{s} \tilde{d}_j + \sum_{j=1}^{r} \tilde{d}_{j+s}} \quad , \quad i = 1, 2,...,r+s \quad (51)$$

By LR fuzzy numbers we can obtain, $\tilde{w}_j = [\delta_j, \theta_j]$, $j$=1,…,$r$, as follows:

$$\alpha^*_j = \sum_{j=1}^{s} \alpha''_j + \sum_{j=1}^{r} \alpha''_{j+s}$$
(52)

$$\beta^*_j = \sum_{j=1}^{s} \beta''_j + \sum_{j=1}^{r} \beta''_{j+s}$$
(53)

$$\delta_j = [\min(\frac{\alpha''_j}{\beta_j^*}, \frac{\alpha''_j}{\alpha_j^*}, \frac{\beta''_j}{\alpha_j^*}, \frac{\beta''_j}{\beta_j^*})]$$
(54)

$$\theta_j = [\max(\frac{\alpha''_j}{\beta_j^*}, \frac{\alpha''_j}{\alpha_j^*}, \frac{\beta''_j}{\alpha_j^*}, \frac{\beta''_j}{\beta_j^*})]$$
(55)

The weights for $i$=1,…,$s$ and $i$=$s$+1,…,$s$+$r$ are the input and output weights, respectively.

**Step 6:** Calculate the efficiency values for each units:

$$\tilde{z}_i = \frac{\sum_{j=1}^{r} \tilde{w}_{j+s} \tilde{y}'_{i,j+s}}{\sum_{j=1}^{s} \tilde{w}_j \tilde{x}'_{ij}} \quad , i = 1, 2,...,m \quad (56)$$

Finally, by LR fuzzy numbers we can obtain $\tilde{z}_j = [\varphi_j, \sigma_j], j=1,\ldots,r$, by:

$$\delta'_{j+s} = [\min(\delta_{j+s} a''_{i,j+s}, \delta_{j+s} b''_{i,j+s}, \theta_{j+s} a''_{i,j+s}, \theta_{j+s}) \quad (57)$$
$$, i = 1,\ldots,m$$

$$\theta'_{j+s} = [\max(\delta_{j+s} a''_{i,j+s}, \delta_{j+s} b''_{i,j+s}, \theta_{j+s} a''_{i,j+s}, \theta_j \quad (58)$$
$$, i = 1,\ldots,m$$

$$\delta'_j = [\min(\delta_j a''_{ij}, \delta_j b''_{ij}, \theta_j a''_{ij}, \theta_j b''_{ij})] , i = 1,\ldots,m \quad (59)$$

$$\theta'_j = [\max(\delta_j a''_{ij}, \delta_j b''_{ij}, \theta_j a''_{ij}, \theta_j b''_{ij})] , i = 1,\ldots,m \quad (60)$$
$$i = 1,2,\ldots,m$$

$$\delta''_{j+s} = \sum_{j=1}^{r} \delta'_{j+s} \quad (61)$$

$$\theta''_{j+s} = \sum_{j=1}^{r} \theta'_{j+s} \quad (62)$$

$$\delta''_j = \sum_{j=1}^{r} \delta'_j \quad (63)$$

$$\theta''_j = \sum_{j=1}^{r} \theta'_j \quad (64)$$

$$\varphi_j = [\min(\frac{\delta''_{j+s}}{\theta''_j}, \frac{\delta''_{j+s}}{\delta''_j}, \frac{\theta''_{j+s}}{\theta''_j}, \frac{\theta''_{j+s}}{\delta''_j})] \quad (65)$$

$$\sigma_j = [\max(\frac{\delta''_{j+s}}{\theta''_j}, \frac{\delta''_{j+s}}{\delta''_j}, \frac{\theta''_{j+s}}{\theta''_j}, \frac{\theta''_{j+s}}{\delta''_j})] \quad (66)$$

So, this method can produce a complete ranking of units that is more practical.

## 5. Numerical Example

Table 2 illustrates an example of a metallurgy factory located in Iran with two fuzzy inputs and two fuzzy outputs. Table 3 lists the fuzzy efficiencies obtained from Model 4 for different   values.

**Tab. 2. DMUs with two fuzzy inputs and two fuzzy outputs**

| DMU | $X_1$ | $X_2$ | $Y_1$ | $Y_2$ |
|---|---|---|---|---|
| 1 | (2.2,3.2,4.2) | (2.6,3.6,4.6) | (1.7,2.7,3.7) | (2.3,3.3,4.3) |
| 2 | (1.7,2.7,3.7) | (2.1,3.1,4.1) | (0.8,1.8,2.8) | (1.4,2.4,3.4) |
| 3 | (1.6,2.6,3.6) | (2.5,3.5,4.5) | (1.6,2.6,3.6) | (2,3,4) |
| 4 | (2.4,3.4,4.4) | (2.4,3.4,4.4) | (1.1,2.1,3.1) | (2.4,3.4,4.4) |
| 5 | (2.8,3.8,4.8) | (2,3,4) | (1,2,3) | (3,4,5) |
| 6 | (1.7,2.7,3.7) | (3,4,5) | (0.4,1.4,2.4) | (2.8,3.8,4.8) |
| 7 | (0.5,1.5,2.5) | (1,2,3) | (0.5,1.5,2.5) | (1,2,3) |
| 8 | (1,2,3) | (2,3,4) | (1,2,3) | (2,3,4) |
| 9 | (3,4,5) | (2,3,4) | (3,4,5) | (3,4,5) |

**Tab. 3. Efficiency of DMUs**

| DMU | =0.1 | =0.5 | =0.9 |
|---|---|---|---|
| 1 | 2.3592 | 1.8424 | 1.1480 |
| 2 | 3.4174 | 2.6086 | 1.4096 |
| 3 | 2.0994 | 1.6680 | 1.1283 |
| 4 | 2.0092 | 1.6211 | 1.1289 |
| 5 | 1.9171 | 1.5795 | 1.1282 |
| 6 | 1.8465 | 1.5399 | 1.1273 |
| 7 | 2.8115 | 2.25 | 1.356 |
| 8 | 2.1717 | 1.7777 | 1.1900 |
| 9 | 1.9385 | 1.6024 | 1.3617 |

To evaluate the entropy of illustrated DMUs, we make interval inputs and outputs and then Equations (35) and (36) are used. Table 4 lists the results the computed fuzzy entropy.

Entropy figures out the uncertainty and fuzziness of score efficiency of DMUs; therefore, the sums concluded of entropy will be more for DMUs having higher or lower efficiency. Thus value entropy is not able to rank DMUs; however, when entropy uses to make CSW results suitable ranking for DMUs. The preference expectations of the fuzzy efficiency of DMUs with use CSW, Equation (56), and different valuation of optimistic levels are listed in Table 5.

Ranking of DMUs by the fuzzy entropy method and fuzzy CCR model is shown in Tables 6 and 7, respectively. By comparing these tables, the fuzzy entropy method presents the unit 5 as the most efficient unit in level  =0.1 and  =0.5; while under fuzzy CCR model, the unit 2 in all level is the most efficient unit.

**Tab. 4. Entropy of DMUs**

| DMU | E1 | E2 | E3 | E4 |
|---|---|---|---|---|
| =0.0 | (0.3103,2.7603) | (0.3611,2.4881) | (0.9161,3.8214) | (0.3626,2.4634) |
| =0.1 | (0.3548,2.4764) | (0.4047,2.2635) | (0.2414,3.2866) | (0.4061,2.2426) |
| =0.5 | (0.5755,1.6342) | (0.6166,1.5662) | (0.4815,1.8868) | (0.6167,1.556) |
| =0.9 | (0.8864,1.0891) | (0.9055,1.0889) | (0.8539,1.1152) | (0.9031,1.0846) |

**Tab. 5. Fuzzy efficiency of DMUs with different**

| DMU | =0.1 | =0.5 | =0.9 |
|-----|------|------|------|
| 1 | (-3.06135,1.6472) | (-2.0699,1.4005) | (-1.482,1.2782) |
| 2 | (-3.0865,1.4070) | (-1.7202,1.1639) | (-1.1852,1.0222) |
| 3 | (-4.1154,1.8760) | (-2.3934,1.6793) | (-1.7466,1.5064) |
| 4 | (-2.8712,1.3088) | (-1.5956,1.0795) | (-1.0979,0.9469) |
| 5 | (-2.5393,1.1575) | (-1.3915,0.9415) | (-0.9405,0.8111) |
| 6 | (-2.6293,1.1985) | (-1.4211,0.9615) | (-0.9357,0.8070) |
| 7 | (-4.1154,1.8760) | (-2.3934,1.6193) | (-1.7466,1.5064) |
| 8 | (-4.1154,1.8760) | (-2.3934,1.6193) | (-1.7466,1.5064) |
| 9 | (-4.1154,1.8760) | (-2.3934,1.6193) | (-1.7466,1.5064) |

**Tab. 6. Ranking order for the nine DMUs by the fuzzy entropy method**

| DMU | =0.1 | =0.5 | =0.9 |
|-----|------|------|------|
| 1 | 5 | 5 | 5 |
| 2 | 4 | 4 | 4 |
| 3 | 8 | 8 | 8 |
| 4 | 3 | 3 | 3 |
| 5 | 1 | 1 | 2 |
| 6 | 2 | 2 | 1 |
| 7 | 6 | 6 | 6 |
| 8 | 9 | 9 | 9 |
| 9 | 7 | 7 | 7 |

**Tab. 7. Ranking order for the nine DMUs by the fuzzy CCR model**

| DMU | =0.1 | =0.5 | =0.9 |
|-----|------|------|------|
| 1 | 3 | 3 | 4 |
| 2 | 1 | 1 | 1 |
| 3 | 5 | 5 | 7 |
| 4 | 6 | 6 | 6 |
| 5 | 8 | 7 | 8 |
| 6 | 9 | 8 | 9 |
| 7 | 2 | 2 | 2 |
| 8 | 4 | 4 | 3 |
| 9 | 7 | 7 | 5 |

Since the data base is in fuzzy so we should use the fuzzy system. From another point of view, the less fuzziness, the less ambiguity will be made. Thus, it is better to choose the shorter interval in the fuzzy entropy because of the decreasing of fuzziness (according to this, the ranking of units is made in Table 6). When α-cuts increase in this model, the interval of data will be shorter; so, $\alpha$=0.9 is the optimistic and recommended level to managers (see Table 5 in Section 5). Also in the CCR model, the sets of the weights are typically different for each of the participating DMUs, and in some cases it may be considered unacceptable where the same factor is

accorded widely differing weights. Thus, it is important to find a common set of weights (CSW). According to this entropy, the model is proposed. So by comparing of two methods, the entropy method is more stable.

## 6. Conclusion

Since the efficiency is fuzzy in standard data envelopment analysis models with fuzzy data, it is difficult to rank the efficiencies. In this paper, two methods to rank efficient and inefficient DMUs are suggested, which one of them is more stable comparing with the other methods. The first method based on the fuzzy CCR model, by a little modification, could measure the efficiency of units and rank them completely. The second method was the fuzzy entropy based on the common set of weights (CSW). Defining the CSW, the efficiency was measured and contrasted. One of the most significant advantages of this method was the compatibility and stability of which in ranking. Regarding the     index, the manager's opinion to measure efficiency has been considered as well. From the other point of view if the manager's opinion is changed and considered as a parameter in the presented model, we can examine the opinion in future research.

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