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RESEARCH PAPER

A Robust Optimization Approach for a Discrete Time-Cost-Environment Trade-off Project Scheduling Problem Under Uncertainty

Ali Salmasnia[1], Elahe Heydarnezhad[2] & Hadi Mokhtari[3*]

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ABSTRACT
One of the important problems in managing construction projects is selecting the best alternative for activities' execution to minimize the project's total cost and time. However, uncertain factors often have negative effects on activity duration and cost. Therefore, it is crucial to develop robust approaches for construction project scheduling to minimize sensitivity to disruptive noise factors. Additionally, existing methods in the literature rarely focus on environmentally conscious construction management. Achieving these goals requires incorporating the project scheduling problem with multiple objectives. This study proposes a robust optimization approach to determine the optimal construction operations in a project scheduling problem, considering time, cost, and environmental impacts (TCE) as objectives. An analytical algorithm based on Benders decomposition is suggested to address the robust problem, taking into account the inherent uncertainty in activity time and cost. To evaluate the performance of the proposed solution approach, a computational study is conducted using real construction project data. The case study is based on the wall of the east coast of Amirabad port in Iran. The results obtained using the suggested solution approach are compared to those of the CPLEX solver, demonstrating the appropriate performance of the proposed approach in optimizing the time, cost, and environment trade-off problem.

***Keywords**: Time-Cost-Environment Trade off Problem; Project Scheduling; Multi-Objective Optimization; Robust Optimization; Benders Decomposition.*

1. Introduction

Project managers often face operational challenges when making trade-offs among various conflicting aspects of projects, such as the total project time and cost. To address this challenge, the time-cost trade-off problem (TCTP) has been widely studied in the literature of construction project scheduling. The classical TCTP aims to determine the optimal values of decision variables to achieve a suitable trade-off between project time and cost. However, the construction industry also has significant impacts on the environment, which necessitates considering environmental factors alongside cost and time trade-offs. Recently, an extension of the TCTP, known as the time, cost, and environment trade-off problem (TCETP), has been proposed to simultaneously minimize time, cost, and environmental impacts. The TCETP aims to find a trade-off among project completion time, total project cost, and environmental impacts. The project scheduling problem, as a generalized

* **Corresponding author: Hadi Mokhtari**
mokhtari_ie@kashanu.ac.ir

1. ***Industrial Engineering, Faculty of Technology, and Engineering, University of Qom, Qom, Iran.***
2. ***Faculty of Industrial Engineering, Tarbiat Modares university, Tehran, Iran.***
3. ***Industrial Engineering, Faculty of Engineering, University of Kashan, Kashan, Iran.***

problem of the TCTP, has received considerable attention in the literature in recent years (Peyghami et al., 2013; Noori and Taghizadeh, 2018; Adibhesami et al., 2019; Azimi and Sholekar, 2021).

The time-cost trade-off problem (TCTP) has been extensively studied in the literature (Mokhtari et al., 2010a). Initially, this problem was formulated as a single-objective problem, with minimization of duration, cost, or maximization of resource utilization as the objective function (Robinson, 1975; Falk and Horowitz, 1972; Deckro et al., 1995; Kelley Jr, 1961; Mokhtari et al., 2010b). Kelley Jr (1961) proposed the first research on the TCTP as a type of project scheduling problem. Subsequently, Harvey and Patterson (1979) and Hindelang and Muth (1979) introduced the first studies on a discrete variant of the time-cost trade-off problem (DTCTP). Researchers have also focused on the multi-objective TCTP (Vrat and Kriengkrairut, 1986; Reda and Carr, 1989), aiming to find optimal solutions that minimize both total project time and cost. Moreover, Mokhtari et al. (2012) investigated a relatively new type of scheduling problem that balances project time, cost, and quality. In subsequent years, the time-cost-environmental impact trade-off problem (TCETP) was developed as an extension of the classical DTCTP, aiming to optimize project total time, cost, and environmental impacts simultaneously.

There have been few studies on the TCETP. Marzouk et al. (2008) presented the first paper that added pollutants from construction projects to the project scheduling problem. Recently, Ozcan-Deniz et al. (2011) suggested a project planning framework based on control principles to find the optimal design with project time, cost, and environmental impacts as objective functions. Xu et al. (2012) discussed a multi-objective variant of the time–cost–environment trade-off problem with discrete activities (DTCETP), where the activity durations are assumed to be fuzzy parameters. Liu et al. (2013) presented a PSO in order to minimize project cost and pollutants in a multiple-mode scheduling problem. Cheng and Tran (2014) proposed a differential evolution algorithm to handle trade-off optimization for project cost, project time, and adverse environmental effects to increase project performance.

Most existing research on optimizing construction scheduling problems assumes complete information and deterministic conditions. However, in real-world conditions, there are often sources of uncertainty, such as undesirable procurement, weather conditions, and variations in project scope. These uncertainties usually pose a threat to the successful achievement of project objectives. Hence, we need an appropriate method and algorithm that are invulnerable to the uncertainties created by uncontrollable factors.

Recently, robust optimization, as a modeling approach, has attracted the attention of many researchers to immunize the planning process against uncertain parameters within an optimization framework. The first work in this area was presented by Soyster (1973). Following this study, several robust optimization approaches were developed by Ben-Tal and Nemirovski (1998), Ben-Tal and Nemirovski (1999), Ben-Tal and Nemirovski (2000), Bertsimas and Sim (2003), and Bertsimas and Sim (2004). However, there have been few studies on the implementation of robust methods. Yamashita et al. (2007) tried to minimize the cost of resources with durations under uncertainty. The problem was formulated based on the robust optimization approach, where uncertainty is captured based on a set of possible states. Cohen et al. (2007) presented a robust method for the time and cost trade-off problem by applying the Adjustable Robust Counterpart procedure developed by Ben-Tal et al. (2004). They solved the problem using a conic quadratic programming technique. Hazir et al. (2011) discussed a single objective problem to address DTCTP. To the best of our knowledge, this is the only existing paper on robust optimization based on Bertsimas and Sim's approach where the activity cost uncertainty is considered via interval-type.

A summary of previous studies on DTCTP and DTCETP is presented in Table 1. As shown, Xu et al. (2012) is the only study under uncertainty on DTCETP, while other studies have focused on a deterministic environment. In our paper, three objectives are considered: (1) the project cost; (2) the project completion time; and (3) the project's environmental impacts. The major contributions can be mentioned as:

(1) The proposed model takes into

consideration the adverse environmental impacts as well as project cost and project completion time handled by robust modeling approach.

(2) The proposed method considers interval uncertainty for the activity cost and activity time parameters.

(3) An analytical algorithm based upon a Benders decomposition is presented for solving our model.

(4) A case study regarding the construction of protective wall for the east coast of port in the economic zone of Amirabad is presented.

The reminder of this study is arranged as: In Section 2, a deterministic multi-objective DTCETP is presented and described. Section 3 formulates a robust counterpart for the proposed multi-objective DTCETP. In Section 4, a benders decomposition algorithm to address the proposed robust model is presented. Section 5 consists of a computational study on a construction project in a protective wall case. In Section 6, a sensitivity analysis on model parameters is performed. At final section, some concluding points and future directions for research are given

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Tab. 1. Summary of studies on DTCTP and DECETP

Problem	Paper	Type of objective		Deterministic model	Programming under uncertainty			Parameter(s) under uncertainty	Solution approach
		SO	MO		Robust optimization	Stochastic programming	Fuzzy programming		
	Robinson (1975)	✓		✓				-	
	Hindelang and Muth (1979)	✓		✓				-	
	Demeulemeester et al. (1996, 1998)	✓		✓				-	
	Gutjahr et al. (2000)	✓				✓		time	
	Jin et al. (2005)	✓					✓	time - cost	GA
	Yang (2005)	✓				✓		-	
TCTP	Cohen et al. (2007)	✓			✓			time	
	Eshtehardian et al. (2008)		✓				✓	cost	GA
	Zhu et al. (2008)	✓					✓	time	
	Ghazanfari et al. (2009)		✓				✓	time	GP
	Liang (2009)		✓				✓	time - cost	
	Hazir et al. (2010a)	✓				✓		time	Simulation
	Hazir et al. (2010b)	✓		✓				-	Decomposition
	Klerides and Hadjiconstantinou (2010)	✓				✓		time	Decomposition
	Ke et al. (2010)	✓					✓	time	
	Mokhtari et al. (2011)	✓				✓		time	ACS
	Hazir et al. (2011)	✓			✓			cost	Decomposition
	Ke et al. (2012)	✓				✓		time	GA
	Ke (2014)	✓				✓		time	
	Marzouk et al. (2008)		✓	✓				-	GA
	Ozcan-Deniz et al. (2011)		✓	✓				-	GA
	Xu et al. (2012)		✓				✓	time	(f)GA
TCETP	Liu et al. (2013)		✓	✓				-	
	Cheng and Tran (2014)		✓	✓				-	
	This paper		✓		✓			time-cost	Decomposition

SO: Single objective; MO: Multi-objective; GA: Genetic algorithm; GP: Goal programming; ACS: Ant Colony System; (f)GA: fuzzy adaptive hybrid genetic algorithm

2. Problem Description and Mathematical Formulation Model

As mentioned before, the problem under consideration in this work is to adjust activity modes so that three objective functions of project cost, completion time and adverse environmental impacts are minimized, simultaneously. In this regard, the used assumptions involved in this work are expressed in Section 2.1. Afterwards, the parameters and the variables used throughout the model are introduced. Then a deterministic version of the multi-objective DTCETP is formulated in Section 2.2.

2.1. Assumptions

In order to formulate the DTCETP, the assumptions are utilized as:

(1) The project includes a set of n activities, in which activity 0 and activity n+1 are dummy activities corresponding to the start and end time of project, so we have, $N = \{0,1,2,\ldots,n+1\}$.

(2) Every activity can be carried out by just one of m modes, $m = 1,\ldots,M$.

(3) Each mode is attributed with a time, a cost, and an environmental impact.

(4) The commence time of activities are related to its precedence relation.

(5) Activity preemption is not authorized, i.e. once an activity is begun, the execution of the activity cannot be interrupted.

(6) Minimizing the project completion time, total cost and impacts on environment are the objectives.

According to the above-mentioned assumptions, the project network can be depicted on a graphical graph $G = (N, A)$. In this graph, parameter N denotes a set of nodes, while $A \in N \times N$ is a set of corresponding arcs.

2.2. Deterministic model

As mentioned before, the proposed model takes in to consideration environmental impact as well as project cost and project duration. The notations utilized to model the proposed deterministic problem are shown in Table 2.

Tab. 2. Notations

Index	**Definition**
j	index of activity, where $j = 0, 1, 2, \ldots, n+1$
m	index of modes, $m = 1, 2, \ldots, M$.
k	index of environmental impact score, $k = 1, \ldots, K$.
Parameter	**Definition**
A	the precedence set of relationships for activities
M_j	the activity *j* possible modes
N	the set of nodes
p_{jm}	the nominal value of processing time for mode *m* of activity *j*
c_{jm}	the nominal value of cost for mode *m* of activity *j*
w_j	the weight of activity *j*
w_{jk}	the score *k* weight for environmental impact of activity *j*
e_{jkm}	the environmental impact of mode *m* of activity *j* for score *k*
Variable	**Definition**
T_j	the completion time of activity *j*
x_{jm}	$\begin{cases} 1 & \text{if activity } j \text{ is carried out at mode } m \\ 0 & \text{Otherwise} \end{cases}$
Objective	**Definition**
z_1	the total cost
z_2	the total completion time
z_3	the total environmental impact

In this part, a mathematical programming (mixed integer) formulation is proposed, for our multi objective DTCETP, as follows:

$$Z_1 = Min \sum_{j \in N} \sum_{m \in M_j} c_{jm} x_{jm} \tag{1.1}$$

$$Z_2 = Min \ T_{n+1} \tag{1.2}$$

$$Z_3 = Min\, E = Min \sum_{j \in N} w_j \sum_{m \in M_j} \sum_{k} w_{\mathrm{j,k}} \times e_{\mathrm{jmk}} \times x_{jm} \tag{1.3}$$

$$subject\, to$$

$$\sum_{m \in M_j} x_{jm} = 1 , \forall j \in N \tag{1.4}$$

$$T_j - T_i - \sum_{m \in M_j} p_{jm} x_{jm} \geq 0 \qquad \forall (i,j) \in A \tag{1.5}$$

$$T_j \geq 0 \qquad \forall j \in N \cup \{0, n+1\} \tag{1.6}$$

$$x_{jm} \in \{0,1\} \qquad \forall m \in M_j , \forall j \in N \tag{1.7}$$

In this formulation, the objectives are the minimization of project cost (1.1), project completion time (1.2), and adverse environmental impacts (1.3). Constraint set (1.4) ensures that each sequence position will be allocated by just one mode. The binary control variable x_{jm} denotes the assigned modes to the activities. It is set to 1 if mode *m* is chosen for activity *j*, and 0 otherwise (1.7). Constraint set (1.5) represents the precedence constraint among the activities. T_j is a variable representing the completion time of activity j (1.6).

Generally, the project environment can be classified into two classes: (i) internal project environment, such as the cultural and organizational surroundings, and (ii) external project environment, such as the ecological surroundings, including water, soil, air, and ecological impacts (Liu and Lai, 2009). Some environmental indicators evaluated in this study are air, water, and soil pollution. Although these indicators are quantifiable, decision-makers face two major challenges in estimating and quantifying the environmental impacts. These challenges can be attributed to: (i) the difficulty in assessing the impacts of the activities on the environment, and (ii) the complexity in combining the environmental effects of activities to calculate the total environmental effects of the project. Therefore, we incorporate a new objective function to address the environmental impacts. This objective function is able to consider multiple assessable criteria (environmental) for every project activity. Since the selected environmental criteria are often expressed in different units, it is necessary to transform them into a unified measurement system. In our proposed problem, the results of the measures are converted into a single measure ranging from 0 to 100 to represent the total degree of activity environmental effect.

By using a weighted method, the proposed objective function is able to aggregate the environmental effect for all of the activities to give us a total environmental effect. To evaluate environmental effect of each activity, this method needs to identify weights: (i) weight of activities (w_j), denoting share of the environmental effect of the activity to the total environmental effect of the project; and (ii) the weight of the score *k* of environmental effect for activity *j* (w_{jk}) to show the relative importance of the indicator among all indicators. These two kinds of weights are utilized to determine the total environmental effect of overall project, as shown in Eq. (1.3).

In the next section, a multi-objective model of DTCETP will be presented to address the uncertainty of activity cost and time.

3. Robust Discrete Time- Cost-Environment Trade Off Problem

Considering the real-world conditions, there are many uncertain factors that have great effects on activity duration and cost. Because of the complexity of uncertain factors, it is hard for project managers to estimate the exact cost and duration of each activity. This study investigates a construction project problem in which activities' cost and time are faced with uncertainties.

There are different modeling techniques for solving problems under uncertainty. One of these approaches is robust optimization which generates an insensitive solution to the changes in parameters. In sequel, we present a model according to the Bertsimas and Sim (Bertsimas and Sim, 2003) to produce a robust project scheduling solution in which uncertainty is modeled via interval cost $[c_{jm}, \bar{c}_{jm} = c_{jm} + d_{jm}]$ and interval time $[p_{jm}, \bar{p}_{jm} = p_{jm} + d'_{jm}]$, where d_{jm} and d'_{jm} represent the maximum possible deviations from the nominal cost (c_{jm}) and the nominal processing time (p_{jm}), respectively.

Since, it is unlikely that all of the activity cost and time parameters deviate from their nominal values simultaneously, the presented model introduces the parameters Γ_0 and Γ_a, $(a \in A)$ with values on the interval $[0,|J_0|]$ and $[0,|J_a|]$, where $|J_0|$ and $|J_a|$ show the number of parameters with uncertainty in the project cost objective and the a^{th} precedence constraint, respectively. The parameter Γ_a adjusts the conservatism level for the a^{th} precedence constraint among the activities. In other words, the presented model assumes that only Γ_a activity time parameters deviate from their nominal values in the a^{th} precedence constraint. In such situation, the presented model guarantees that the solution is feasible. Furthermore, it creates a probabilistic ensure that even if more than the pre-known uncertain parameters alter, the obtained solution will be reasonable, with a high degree of probability.

The parameter Γ_0 is able to control the robustness level in the project cost objective. The presented model aims to obtain a feasible solution to optimize multi-objective DTCETP against all uncertain scenarios under which a number Γ_0 of the activity cost parameters in the total cost objective change. If $\Gamma_0 = 0$, the effect of deviations on the cost are completely ignored. In contrast, if $\Gamma_0 = |J_0|$, maximum possible number of deviated parameters (from their corresponding nominal values) is utilized and then our problem transforms to min-max problem. So, Γ_0 reflects a pessimism level of the decision-maker. It is notable that Γ_0 is assumed to be integer, while Γ_α is not necessarily integers.

According to the above-mentioned descriptions, a robust counterpart of the deterministic DTCETP is proposed as below:

$$Z_1 = Min\ (\sum_{j\in N}\sum_{m\in M_j} c_{jm}x_{jm} + \underset{S_0\subset N,|S_0|\le\Gamma_0}{Max}\{\sum_{j\in S_0}\sum_{m\in M_j} d_{jm}\, x_{jm}\}) \tag{2.1}$$

$$Z_2 = Min\ T_{n+1} \tag{2.2}$$

$$Z_3 = Min\, E = Min\sum_{j\in N} w_j \sum_{m\in M_j}\sum_k w_{\mathrm{j,k}} \times e_{\mathrm{jmk}} \times x_{jm} \tag{2.3}$$

subject to

$$\sum_{m\in M_j} x_{jm} = 1\,, \forall j\in N \tag{2.4}$$

$$T_j - T_i - \{\sum_{m\in M_j} p_{jm}x_{jm} + \underset{\{j|\, j\in S_a, S_a\subset N, |S_a|\le\Gamma_a\}}{Max}\{\sum_{m\in M_j} d'_{jm}\, x_{jm}\}\} \ge 0 \qquad \forall (i,j)\in A \tag{2.5}$$

$$T_j \ge 0 \qquad \forall j\in N\cup\{0,n+1\} \tag{2.6}$$

$$x_{jm}\in\{0,1\} \qquad \forall m\in M_j, \forall j\in N \tag{2.7}$$

where S_0 and S_a are appropriate subsets of activities, such that their elements have cost values and process time values at their corresponding upper bounds. Therefore, the constraint (2.5) can be rewritten as the following non-linear formulation:

$$T_j - T_i - \{ \sum_{m\in M_j} p_{jm} x_{jm} + \max_{\{S_a \cup \{t\} | S_a \subset N, |S_a| \le \Gamma_a, t\in J_a \setminus S_a\}} \sum_{m\in M_j} \sum_{j\in S_a} d'_{jm} |x_{jm}| + (\Gamma_a - \lfloor \Gamma_a \rfloor d'_{tm} |x_{tm}|)\} \ge 0 \;, \forall (i,j) \in A \qquad (2.8)$$

The parameter Γ_a also controls the level of conservatism for activity time parameters that have random nature. Similar to Γ_0, if $\Gamma_a = 0$, the effect of deviations on activity time will be ignored, and then the nominal values based deterministic TCETP is generated. Against, if $\Gamma_a = |J_a|$, maximum possible number of deviated activity time parameters from their corresponding nominal values is considered and problem changes to Soyster's method.

We can also convert Eq. (2.1) to a linear objective function according to Bertsimas and Sim's approach, as follow:

$$B_0(x^*, \Gamma_0) = \underset{S_0 \subset N, |S_0| \le \Gamma_0}{Max} \{ \sum_{j\in J_0} \sum_{m\in M_j} d_{jm} | x_{jm} | u_{jm} : \sum_{j\in J_0} \sum_{m\in M_j} u_{jm} \le \Gamma_0 \,, 0 \le u_{jm} \le 1, \forall j \in J_0 , \forall m \in M_j \}$$

$$= Min\{ \sum_{j\in J_0} w_{jm} + \Gamma_0 z_0 : z_0 + w_{jm} \ge \sum_{m\in M_j} d_{jm} x_{jm} \;,\; z_0 \ge 0,\; w_{jm} \ge 0,\; \forall j \in J_0 , \forall m \in M_j \} \qquad (2.9)$$

Similarly, the constraint (2.8) can be converted to a linear constraint, as follow:

$$B_a(x^*, \Gamma_a) = \underset{S_a \subset N, |S_a| \le \Gamma_a}{Max} \{ \sum_{m\in M_j} \sum_{j\in J_\alpha} d'_{jm} |x_{jm}| \, u_{jm} : \sum_{j\in J_a} \sum_{m\in M_j} u_{jm} \le \Gamma_a \,, 0 \le u_{jm} \le 1 \;, \forall j \in J_a ,\; \forall m \in M_j \}$$

$$= Min\{ \sum_{m\in M_j} w_{jm} + \Gamma_a z_j : z_j + w_{jm} \ge \sum_{m\in M_j} d'_{jm} x_{jm} \;, z_j \ge 0 \;, w_{jm} \ge 0, \forall j \in J_a ,\; \forall m \in M_j \} \qquad (2.10)$$

The interested authors refer to see Bertsimas and Sim's study (Bertsimas and Sim, 2003) for more information. According to equations (2.9) and (2.10), the linear robust counterpart for DTCETP can be represented by the following formulation:

$$Z_1 = Min \sum_{j\in N} \sum_{m\in M_j} c_{jm} x_{jm} + \Gamma_0 z_0 + \sum_{m\in M_j} \sum_{j\in N} w_{jm} \qquad (3.1)$$

$$Z_2 = Min \;\; T_{n+1} \qquad (3.2)$$

$$Z_3 = Min\, E = Min \sum_{j\in N} w_j \sum_{m\in M_j} \sum_{k} w_{\mathrm{j,k}} \times e_{\mathrm{jmk}} \times x_{jm} \qquad (3.3)$$

subject to

$$\sum_{m\in M_j} x_{jm} = 1 \,, \forall j \in N \qquad (3.4)$$

$$T_j - T_i - \sum_{m\in M_j} p_{jm} x_{jm} - \Gamma_a z_j - \sum_{m\in M_j} w_{jm} \ge 0 \quad \forall i, j \qquad (3.5)$$

$$z_j + w_{jm} \ge d'_{jm} x_{jm} \qquad \forall\, j \in J_\alpha ,\, m \in M_j \qquad (3.6)$$

$$z_0 + w_{0j} \ge d_{jm} x_{jm} \qquad \forall\, j \in J_0 ,\, m \in M_j \qquad (3.7)$$

$$w_{jm} \ge 0 \qquad \forall\, j \in J_{\alpha,0} \;,\, m \in M_j \qquad (3.8)$$

$$z_j \ge 0 \qquad \forall\, j \in J_\alpha \qquad (3.9)$$

$$z_0 \ge 0 \qquad \forall\, j \in J_0 \qquad (3.10)$$

$$T_j \ge 0 \qquad \forall j \in N \cup \{0, n+1\} \qquad (3.11)$$

$$x_{jm} \in \{0,1\} \qquad \forall\, m \in M_j , \forall j \in N \qquad (3.12)$$

In order to aggregate the objective functions, the weighted sum method is employed. To do this, w_1, w_2 and w_3 as the weights of the project cost, time and total environmental impacts are inputted by decision maker. It is notable that the weights must satisfy the equation $w_1 + w_2 + w_3 = 1$.

4. Solution Algorithm

Since the DTCTP is an NP-hard problem (De et al., 1997), the DTCETP is also NP-hard. Many analytical and heuristic solution methods have been employed to solve such problems. Generally, heuristic algorithms do not guarantee global optimality, and their aim is to search the solution space to find near-optimal solutions. On the other hand, exact analytical methods are able to ensure global optimality. In this paper, Benders decomposition, an iterative analytical algorithm, is applied to solve the proposed robust DTCETP. This method was presented by Benders (1962) for solving mixed-integer programming optimization problems. It decomposes the problem into two easier problems, namely the master problem (MP) and the sub-problem (SP). First, the integer variables of the MIP model are temporarily fixed, and the problem is converted to a linear programming (LP) problem. The sub-problem is the dual of the obtained LP problem, which inserts a cut into the MP in each iteration and provides an upper bound (for a minimization problem). The master problem is an integer programming (IP) problem that assigns feasible values to the integer variables and provides a corresponding lower bound for the objective function. The master and sub-problems are then iteratively solved, and this procedure terminates when the lower bound reaches the upper bound.

4.1. Benders decomposition formulation

In order to solve the robust DTCETP using a Benders decomposition algorithm, the MIP model must be transformed to LP model by fixing the integer variables as follow:

$$Z = Min\{(\mathrm{w}_1 \times (\Gamma_0 z_0 + \sum_{m\in M_j}\sum_{j\in N} w_{jm})) + (\mathrm{w}_2 \times T_{n+1})\} \tag{4.1}$$

subject to

$$T_j - T_i - \Gamma_a z_j - \sum_{m\in M_j} w_{jm} \geq \sum_{m\in M_j} p_{jm} x_{jm} \quad \forall i, j \tag{4.2}$$

$$Z_j + w_{jm} \geq d'_{jm} x_{jm} \qquad \forall\, j \in J_\alpha, m \in M_j \tag{4.3}$$

$$z_0 + w_{jm} \geq d_{jm} x_{jm} \qquad \forall\, j \in J_0, m \in M_j \tag{4.4}$$

$$w_{jm} \geq 0 \qquad \forall\, j \in J_{\alpha,0}, m \in M_j \tag{4.5}$$

$$z_j \geq 0 \qquad \forall\, j \in J_\alpha \tag{4.6}$$

$$z_0 \geq 0 \qquad \forall\, j \in J_0 \tag{4.7}$$

$$T_j \geq 0 \qquad \forall j \in N \cup \{0, n+1\} \tag{4.8}$$

Let u_{ij}^1, u_{jm}^2 and u_{jm}^3 indicates the dual variables related to the constraint sets (3.5), (3.6) and (3.7). The SP is obtained as the following formulation in which $su(j)$ and $pr(j)$ indicate the immediate successors and predecessors sets of activity *j*.

Now, let $u_{ij}^{1\,s}$, $u_{jm}^{2\,s}$ and $u_{jm}^{3\,s}$ for $s=1,\ldots,S$ show the polyhedron extreme points generated by the constraints

$$Max \sum_{\{j|j\in J,(i,j)\in A\}} (\sum_{m\in M_j} p_{jm}\, x_{jm}) u_{ij}^{1} + \sum_{j\in J_\alpha} (\sum_{m\in M_j} d'_{jm}\, x_{jm}) u_{jm}^{2} + \sum_{j\in J_0} (\sum_{m\in M_j} d_{jm}\, x_{jm}) u_{jm}^{3} \quad (5.1)$$

$$subject\, to$$

$$\sum_{k\in su(j)} u_{jk}^{1} - \sum_{k\in pr(j)} u_{kj}^{1} \le 0 \qquad \forall j \in A \quad (5.2)$$

$$\sum_{k\in pr(n+1)} u_{(k,n+1)}^{1} \le 1 \quad (5.3)$$

$$-\sum_{(i,j)\in A} \Gamma_\alpha\, u_{ij}^{1} + \sum_{m\in M_j} \sum_{j\in J_\alpha} u_{jm}^{2} \le 0 \quad (5.4)$$

$$-\sum_{(i,j)\in A} u_{ij}^{1} + \sum_{m\in M_j} \sum_{j\in J_\alpha} u_{jm}^{2} + \sum_{m\in M_j} \sum_{j\in J_0} u_{jm}^{3} \le 0 \quad (5.5)$$

$$\sum_{m\in M_j} \sum_{j\in J_0} u_{jm}^{3} \le w_1 \times \Gamma_0 \quad (5.6)$$

$$u_{ij}^{1} \ge 0 \quad (i,j)\in A \quad (5.7)$$

$$u_{jm}^{2} \ge 0 \qquad \forall j \in J_\alpha, m \in M_j \quad (5.8)$$

$$u_{jm}^{3} \ge 0 \qquad \forall j \in J_0, m \in M_j \quad (5.9)$$

of mathematical programming (5). So, the objective function of dual problem can be given as follows:

$$z' = Max \sum_{(i,j)\in A} (\sum_{m\in M_j} p_{jm}\, x_{jm}) u_{ij}^{1s} + \sum_{j\in J_j} (\sum_{m\in M_j} d'_{jm}\, x_{jm}) u_{jm}^{2s} + \sum_{j\in J_0} (\sum_{m\in M_j} d_{jm}\, x_{jm}) u_{jm}^{3s}$$

Therefore, the relaxed master problem can be written by:

$$Min\ z = (\mathrm{w}_1 \times (\sum_{j\in N} \sum_{m\in M_j} c_{jm} x_{jm}) + (\mathrm{w}_3 \times \sum_{j\in N} w_j \sum_{m\in M_j} \sum_{k} w_{\mathrm{j,k}} \times e_{\mathrm{jmk}} \times x_{jm}) + z'$$

$$subject\, to$$

$$z' \ge \sum_{(i,j)\in A} (\sum_{m\in M_j} p_{jm}\, x_{jm}) u_{ij}^{1s} + \sum_{j\in J_j} (\sum_{m\in M_j} d'_{jm}\, x_{jm}) u_{jm}^{2s} + \sum_{j\in J_0} (\sum_{m\in M_j} d_{jm}\, x_{jm}) u_{jm}^{3s} \quad , s=1,\ldots,S$$

$$\sum_{m\in M_j} x_{jm} = 1\,, \forall j \in N$$

$$x_{jm} \in \{0,1\} \qquad \forall m \in M_j, \forall j \in N$$

$$z' \ge 0$$

4.1.1. Benders decomposition algorithm for the robust DTCETP

According to the mentioned explanations, Benders decomposition algorithm for the robust DTCETP can be given as follows:

$\{initialization\}$

$LB := -\infty$

$UB := \infty$

$while\ \mathrm{UB} - \mathrm{LB} > \varepsilon\ do$

$\{solve\ subproblem\}$

$If\ \ (subproblem\ is)\ Unbounded\ then$

$Get\ unbounded\ ray\ u$

Add cut $\sum_{\{j|j\in J,(i,j)\in A\}} (\sum_{m\in M_j} p_{jm}\, x_{jm}) u_{ij}^{1s} + \sum_{j\in J_\alpha} (\sum_{m\in M_j} d'_{jm}\, x_{jm}) u_{jm}^{2s} + \sum_{j\in J_0} (\sum_{m\in M_j} d_{jm}\, x_{jm}) u_{jm}^{3s} \le 0$ to master problem

Else

Get extreme point u

$$\text{Add cut } z' \ge \sum_{(i,j)\in A} (\sum_{m\in M_j} p_{jm}\, x_{jm}) u_{ij}^{1s} + \sum_{j\in J_\alpha} (\sum_{m\in M_j} d'_{jm}\, x_{jm}) u_{jm}^{2s} + \sum_{j\in J_0} (\sum_{m\in M_j} d_{jm}\, x_{jm}) u_{jm}^{3s}$$

$$+ (\mathrm{w}_1 \times (\sum_{j\in N} \sum_{m\in M_j} c_{jm} x_{jm}) + (\mathrm{w}_3 \times \sum_{j\in N} w_j \sum_{m\in M_j} \sum_k w_{\mathrm{j,k}} \times e_{\mathrm{jmk}} \times x_{jm})\ \text{ to master problem}$$

$$UB = min\{UB, \sum_{(i,j)\in A} (\sum_{m\in M_j} p_{jm}\, x_{jm}) u_{ij}^{1s} + \sum_{j\in J_\alpha} (\sum_{m\in M_j} d'_{jm}\, x_{jm}) u_{jm}^{2s} + \sum_{j\in J_0} (\sum_{m\in M_j} d_{jm}\, x_{jm}) u_{jm}^{3s}$$

$$+ (\mathrm{w}_1 \times (\sum_{j\in N} \sum_{m\in M_j} c_{jm} x_{jm}) + (\mathrm{w}_3 \times \sum_{j\in N} w_j \sum_{m\in M_j} \sum_k w_{\mathrm{j,k}} \times e_{\mathrm{jmk}} \times x_{jm})\}$$

$end\ if$

$\{solve\ master\ problem\}$

$\min_{\mathrm{x}_{jm}} \{z\,|\,\mathrm{cuts},\ \mathrm{x}_{jm} \in X\}$

$\mathrm{LB} := \max\{-\infty, \mathrm{z}\}$

$end\ while$

5. Case Study: Robust DTCETP for Protective Wall of the East Coast Construction Project

This section presents a construction project in the Amirabad port zone of Mazandaran, Iran, as a case study problem, namely the construction of a protective wall on the east coast of the port. It has been undertaken to address the increasing sediment brought to the coastline of Amirabad port and to prevent progressive erosion of the coastline.

The location of Amirabad port is crucial due to its exposure to natural habitats, aquatic ecosystems, and adjacent wetlands. In order to prevent any environmental problems caused by activities and projects related to Amirabad port, significant consideration is given to inevitable regional environmental issues. The geographical location of Amirabad port is such that many uncertain factors, especially weather, extensively affect project activities. In such situations, parameters such as cost and time for implementing activities become uncertain, necessitating the use of approaches to deal with uncertainty. From this perspective, this section provides a real case for the robust DTCETP in the construction project of the protective wall on the east coast.

All data for the protective wall on the east coast project are obtained from the technical and engineering unit of Amirabad port. This project has a length of 1.86 km along the east coast of the port, with three execution phases, and consists of 34 actual activities and two dummy activities. The decision-maker aims to optimize the project performance, where the cost and time of activities are random, and seeks to achieve objectives through a more suitable adjustment of the activity sequences. Our proposed model can assist managers in the optimal scheduling of project activities.

5.1. Computational results

To evaluate the sensitivity of the robust model to the number of uncertain parameters, a set of problems is designed by changing the conservatism and robustness parameters. These problems are classified into three categories: (1)

problems with uncertain activity cost, (2) problems with uncertain activity duration, and (3) problems with uncertain activity cost and duration. Furthermore, to demonstrate the effectiveness of the suggested Benders approach, the robust model is solved using the CPLEX solver and the Benders decomposition technique with GAMS 24.1 Software on a Core i5, 2.50 GHz clock pulse with 5.90 GB memory. After running the code, the results are compared. For this purpose, the degree of conservatism in the constraints and the level of robustness in the weighted sum aggregate objective function must be specified by setting the number of uncertain parameters. As mentioned earlier, the maximum number of uncertain parameters is equal to the total number of activity cost parameters. Hence, Γ_0 takes integer values from [0, 34]. Since, there is a parameter p_{jm} in each constraint, so conservatism level for each constraint is defined in the interval [0,1]. Uncertainty rate α is considered 20% of the nominal value by decision maker. It implies the maximum rate where d_{jm} and d'_{jm} can alter around c_{jm} and p_{jm}, so that $d_{jm} = \alpha c_{jm}$ and $d'_{jm} = \alpha\, p_{jm}$.

Table 3 demonstrates sensitivity of the model to the number of cost coefficients can vary in the first category problems and also the effectiveness of the Benders algorithm in the reduction of run time. In this Table, the first column shows the number of cost activities are permitted to deviate from their nominate values, while the second and third columns show total objective function and total project cost, respectively. It is notable that since time activities and environmental impact of activities are considered fixed values in the first category of problems, total project completion time and total project environmental impact are constant. The third column of Table 3 respresents the percentage deviation of total objective function in terms of (from) its optimal value in the deterministic model. If the influence of cost activity deviations $\Gamma_0 = 0$ completely are neglected which equals to solving the deterministic model, while $\Gamma_0 = n$ considers all possible cost deviations and we have worst possible state of problem. Fig. 1 depicts that increasing Γ_0 leads to the worse value for the total objective and cost objective. Furthermore, it is depicted that as the value of Γ_0 increases, the growth rate of the percentage deviation of total objective function decreases. Fig. 2 shows trend of percentage deviation in first category where uncertainty is only considered in the activities' cost.

The amounts of run time corresponding to each problem using CPLEX solver and Benders decomposition technique are indicated in the columns 5 and 6 of Table 3. The obtained results support this claim that Benders decomposition technique reduces run time dramatically. The column 4 represents value of total project cost deviation (in terms of percentage) from its value in deterministic problem.

Tab. 3. Computational results for the first category problems

Γ_0	Total objective function	Total project cost	Deviation (%)	CPLEX CPU time (sec)	Benders CPU time (sec)
0	31687.639	92451	0	0.394	0.187
[0.1n]	34835.291	101708.8	10	1.304	1.205
[0.2n]	36154.967	105590.2	14.21	1.603	1.516
[0.3n]	36867.471	107685.8	16.48	14.189	2.257
[0.4n]	37481.783	109492.6	18.43	14.373	2.453
[0.5n]	37770.919	110343	19.35	386.062	2.253
[0.6n]	37849.187	110573.2	19.6	469.02	2.258
[0.7n]	37938.131	110834.8	19.88	560.695	2.129
[0.8n]	37974.307	110941.2	20	598.392	2.512
[0.9n]	37974.307	110941.2	20	14.179	1.638
n	37974.307	110941.2	20	1.965	1.710

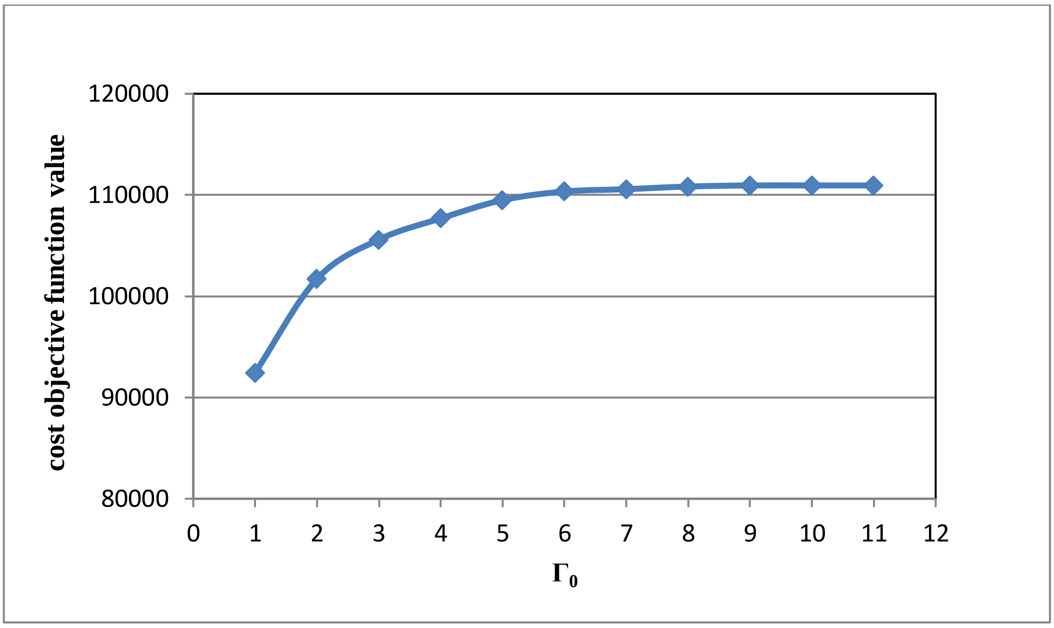

Fig. 1. The impact of parameter Γ_0 on cost objective value

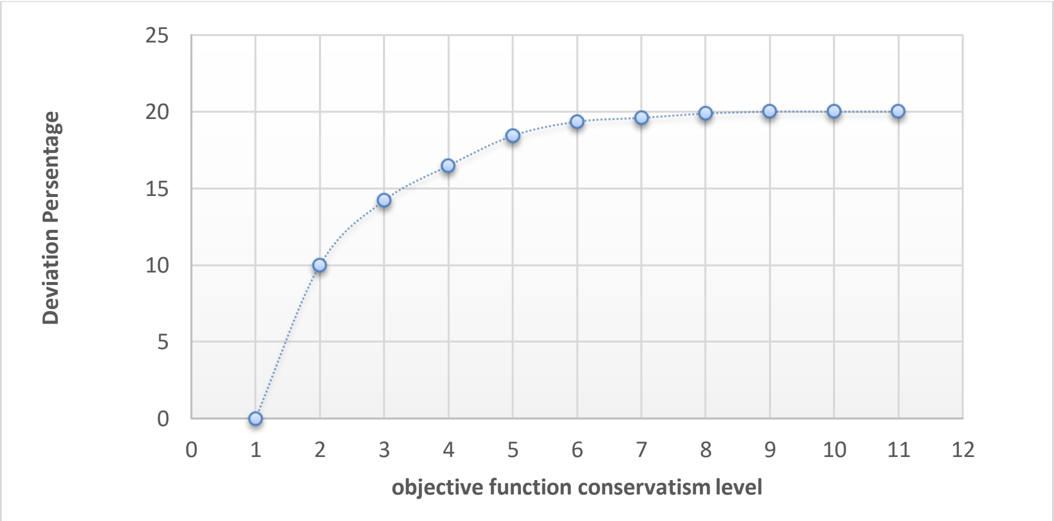

Fig. 2. The impact of parameter Γ_0 on deviation percentage (first category)

In the situation where uncertainty is only considered in the activities' duration (second category), the total cost and total environmental impact remain constant. The results obtained in Table 4 indicate that increasing the conservatism level for constraints leads to an increase in both the total objective function and the project completion time. As Figure 3 illustrates, under a zero-protection level of constraints, the optimal values for the total objective function and the total project time are 31,678.639 and 728, respectively. However, with full protection, the objective values increase to 31,735.687 and 873.6. Furthermore, columns 5 and 6 in Table 4 demonstrate the superior performance of the Benders technique compared to the CPLEX solver.

Fig. 4 shows trend of percentage deviation in second category where uncertainty is only considered in the activities' duration.

Tab. 4. Computational results for the second category problems

Γ_a	Total objective function	Total project time	Deviation %	CPLEX CPU time (sec)	Benders CPU time (sec)
0	31687.639	728	0	0.394	0.187
0.1	31692.443	742.56	2	11.357	1.207
0.2	31697.248	757.12	4	11.226	1.169
0.3	31702.053	771.68	6	13.191	2.124
0.4	31706.858	786.24	8	13.306	2.140
0.5	31711.663	800.8	10	13.224	2.161
0.6	31716.467	815.36	12	14.071	2.211
0.7	31721.272	829.92	14	14.229	2.205
0.8	31726.077	844.48	16	14.409	2.682
0.9	31730.882	859.04	18	13.483	2.439
1	31735.687	873.6	20	13.193	1.606

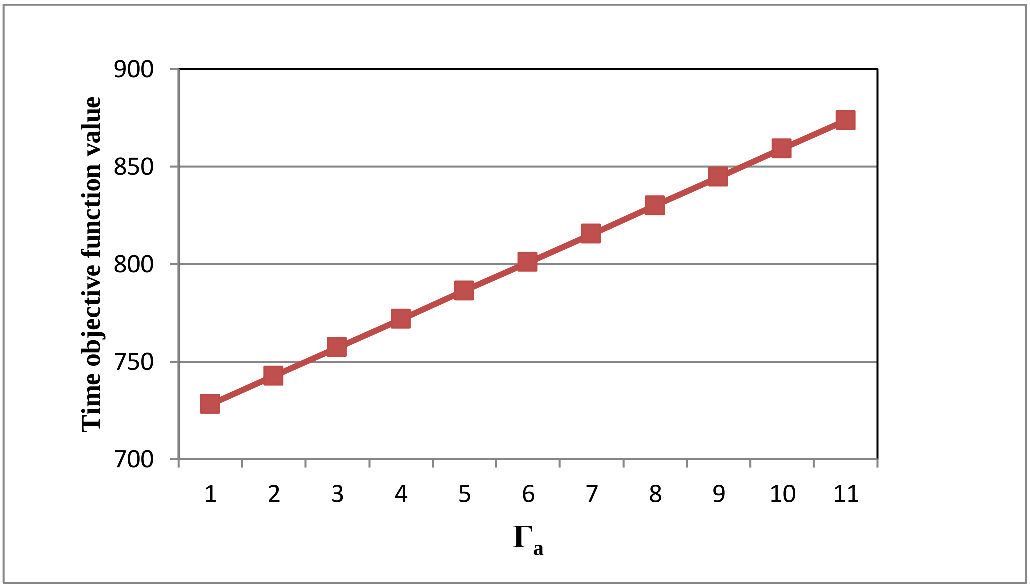

Fig. 3. The effect of parameter Γ_a on time objective value

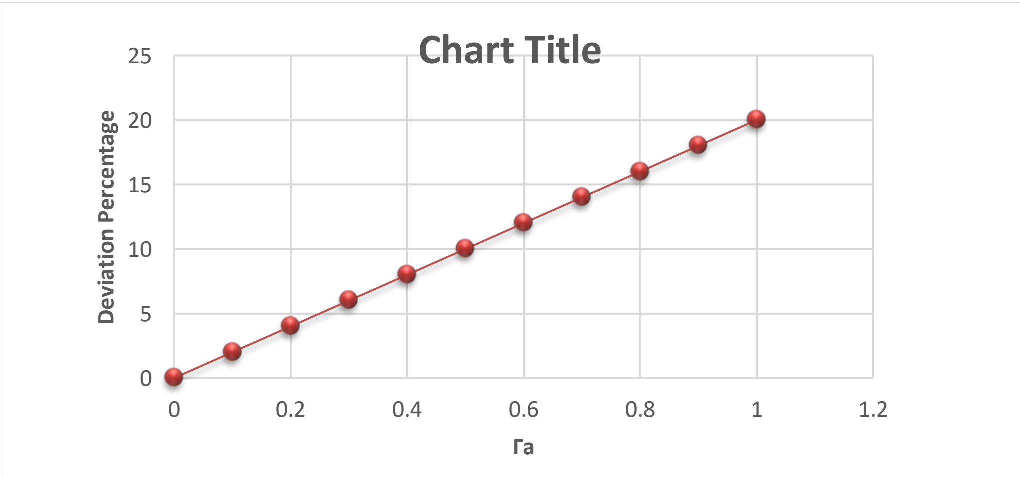

Fig. 4. The effect of parameter Γ_a on deviation (second category)

In the third category of problems, uncertainty in both of cost and process time activity is considered. Table 5 and Figure 5 depict behavior of total objective as a function of robustness and conservatism parameters. It is seen that as the values of Γ_a and Γ_0 increase, the total objective also increases. According to the results, determination of suitable value for the robustness and conservatism parameters, in order to guarantee that the obtained solution is appropriate and reasonable (feasible), is important.

Fig. 6 shows trend of percentage deviation in third category where uncertainty is considered in both the activities' cost and duration.

Tab. 5. Computational results of the third category of problems

$\Gamma_0 = [\alpha \times n]$	Γ_a	Total objective	Total project cost	Total project time	Deviation %	CPLEX CPU time (sec)	Benders CPU time (sec)
0	0	31687.639	92451.0	728.00	0	0.394	0.187
[0.1n]	0.1	34840.095	101708.8	742.56	10	11.427	1.351
[0.2n]	0.2	36164.576	105590.2	757.12	14.12	12.431	2.103
[0.3n]	0.3	36881.885	107685.8	771.68	16.42	14.744	2.111
[0.4n]	0.4	37501.000	109492.6	786.24	18.34	10.687	2.165
[0.5n]	0.5	37794.943	110343.0	800.8	19.27	14.871	2.251
[0.6n]	0.6	37878.010	110573.2	815.36	19.54	38.792	2.380
[0.7n]	0.7	37971.764	110834.8	829.92	19.83	1000.884	2.641
[0.8n]	0.8	38012.745	110941.2	844.48	19.96	1002.009	2.238
[0.9n]	0.9	38017.557	110941.2	859.04	19.99	100.649	2.029
n	1	38022.355	110941.2	873.6	20	2.066	1.717

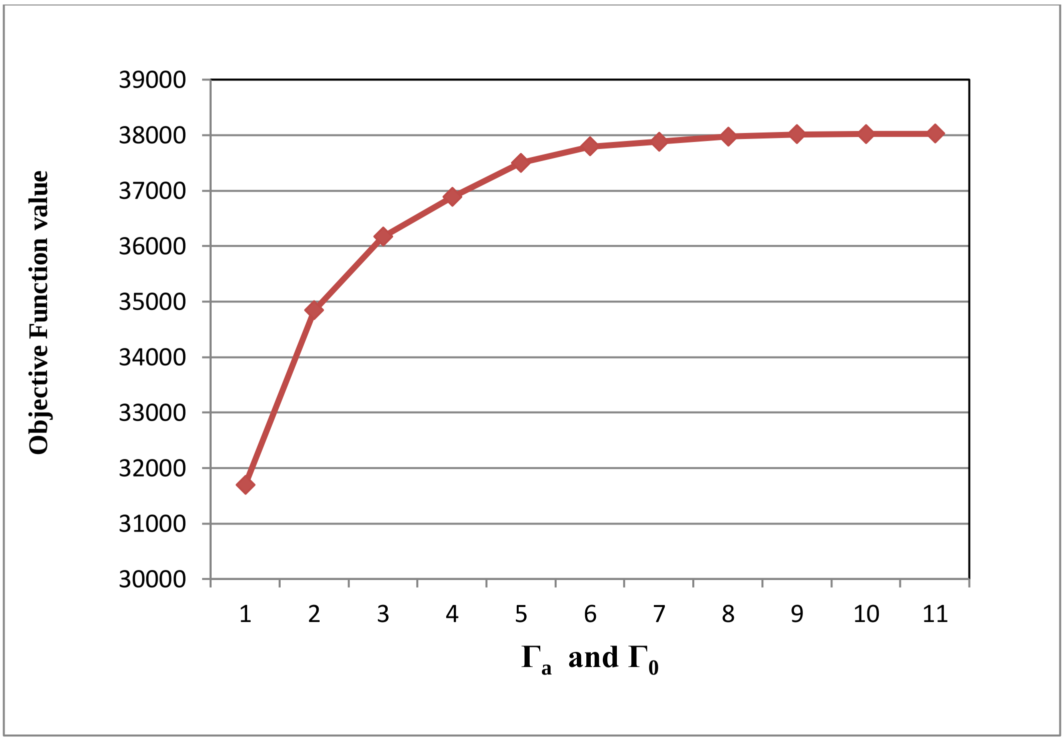

Fig. 5. The impact Γ_a and Γ_0 on total objective function value

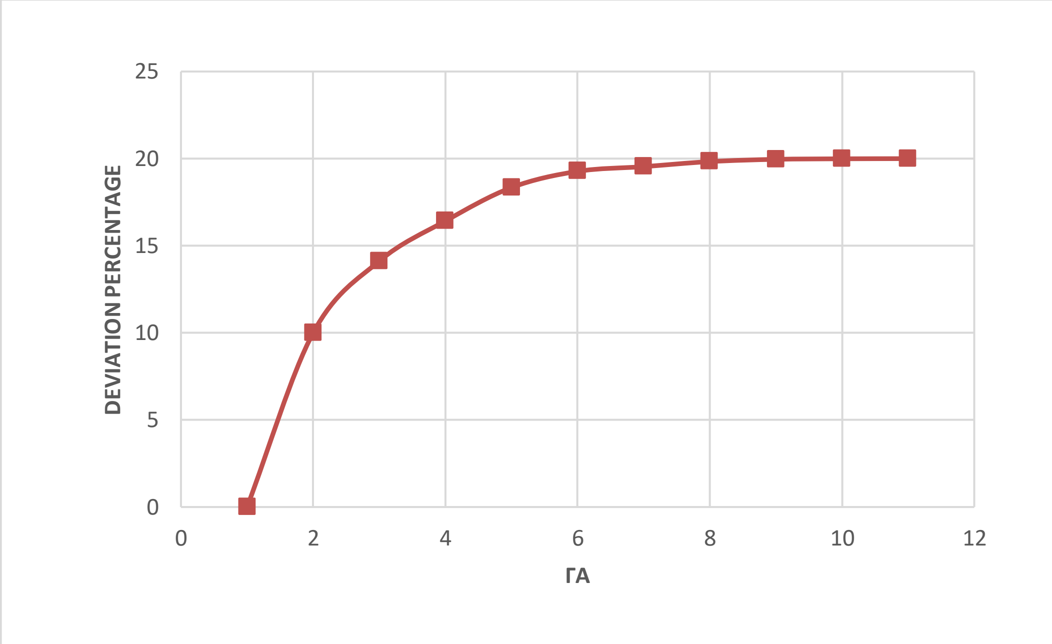

Fig. 6. The effect of parameter Γ_a on deviation (second category)

6. Conclusions and future research

This study presents an efficient approach for robust project scheduling problems that are least vulnerable to disruptions caused by environmental uncontrollable factors. The approach aims to concurrently optimize the project cost, total project time, and environmental impacts while considering interval uncertainty for the activity cost and activity time parameters. The construction project of the wall on the east coast of Amirabad port in Iran is used as a numerical case to evaluate the efficiency of the proposed approach. To assess the sensitivity of the presented model to the number of uncertain parameters, three sets of problems were designed by varying the number of uncertain activity costs and uncertain activity durations. The obtained results for each set of problems demonstrate the superior performance of the presented solution approach compared to the CPLEX solver. A robust modeling approach and Benders decomposition technique were utilized to handle and solve the three stochastic problems. All three categories of problems were programmed using CPLEX and the Benders approach within GAMS. Consequently, the three stochastic categories were solved analytically to obtain optimal solutions. Furthermore, the efficiency of both approaches was evaluated by assessing the CPU time. The results indicate that our approach (Benders decomposition) is superior to CPLEX in terms of efficiency as well.

As future research, two directions are suggested to extend this paper. First, developing a robust model to address other types of uncertainty, such as ellipsoidal uncertainty. Second, considering generalized precedence constraints in the robust DTCETP model.

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