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RESEARCH PAPER

Efficient Resource Management and Pricing in A Two-Echelon Supply Chain with Cooperative Advertising: A Bi-Level Programming Approach

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ABSTRACT
The growing online marketplace has opened a plethora of opportunities for businesses across various industries. Manufacturers, seeking to bypass intermediaries and directly reach end-users, have been increasingly adopting online sales channels in addition to their traditional retail sales. A key challenge, however, lies in determining optimal pricing strategies and advertising investments for both manufacturers and retailers while considering various constraints. This study contemplates a two-echelon supply chain model involving one manufacturer and two retailers. The manufacturer sells its product both through retailers (offline channel) and directly to consumers via an online channel. The model features both global and local advertising. The influence of global advertising is realized through distinct advertising channels, each with a unique impact on demand. To further motivate retailers, the manufacturer contributes to the cost of local advertising. In response to these challenges, this research formulates a bi-level model and employs the concept of Variational Inequalities to solve it. The model also contends with production capacity and budget constraints, leading to a Generalized Nash-Stackelberg game. The validity of the model and the efficacy of the solution method are assessed through numerical experiments performed. Finally, a set of valuable managerial insights are provided..

KEYWORDS: *Generalized nash-stackelberg game; Variational inequalities; Cooperative advertising; Pricing; Supply chain management.*

1. Introduction

1.1. Context

Advertising plays a crucial role in today's business environment, considering its high cost, varying returns, and sensitivity to financial constraints. It can be categorized into different areas, including global advertising and local advertising. Global advertising is primarily focused on branding and can be conducted through various channels such as TV, satellite, internet, and virtual networks. Each of these channels yields different returns and impacts on the investment made. Conversely, local advertising, a more recent and targeted approach, is carried out by local retailers or distributors within a specific geographic area. An important issue is cooperation in this type of advertising, where a manufacturer wishing to sell their product to a retailer offers a participation rate under a partnership agreement in which the manufacturer agrees to reimburse the retailer for a portion of the cost of local advertising, called the manufacturer's local advertising contribution. One reason for this is the growing desire of the retailer to invest more in local advertising and thereby increase sales and profits for the manufacturer and the retailer as opposed to non-cooperative terms. Conversely, the retailer also accepts certain privileges for the manufacturer, such as considering a certain percentage of the retail space for the manufacturer's product. However, one might ask, why should advertising be divided into two categories: global or branding advertising and local advertising? Moreover, why is there a need for both? Global

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advertising by manufacturers increases potential customers on a relatively large geographic scale whose motivation and willingness to buy a promoted or branded product has increased, but it won't happen immediately. Local advertising complements global advertising and is the link between intrinsic motivation and authentic purchase. Potential customers motivated by global advertising, become real customers, influenced by local advertising and recommendations from retailers and closer product experiences. Therefore, every aspect of the advertising campaign, from the desire to the final stage to the purchase, is essential. While global advertising may draw in customers even without local advertising, and vice versa, the optimal results and high effectiveness we expect from these ads will not materialize without the integration of both. So, it's worth using both ads to complement each other. The supply chain considered in this study consists of a manufacturer selling its product through a number of retailers. Retailers each have their own monopoly market and are not involved in other retailers' markets. For example, regional sales or provincial agencies of Iran Tractor Construction Company can be defined as each having a market with a provincial territory and not authorized to operate in any other province, and also provide special facilities for buyers of this product in a province, the buyers from other provinces have no incentive to purchase the product from other provincial agencies.

Given the delineation of advertisements and supply chain information, both the manufacturer and each retailer have a specific advertising budget at their disposal. From the point of view of production, the manufacturer has a maximum production capacity related to the technical limits of production. Under these conditions, the objective of each retailer or provincial agency is to make more sales and, therefore, more profit. Even though they all cooperate under an advertising contract with the manufacturer, it is natural for everyone to ask for promotional subsidies and more goods from the manufacturer to sell despite limited resources. Consequently, retailers compete for the use of these shared resources provided by the manufacturer. The answer to this competition at the retail level must be sought in the form of a generalized Nash equilibrium.

On the other hand, as the manufacturer first announces its share of each retailer's advertising costs and approved sales prices for offline and online channels, and retailers make decisions about how many products to buy and investments to make in local advertising to consider the manufacturer's decision, there is a Stackelberg game, led by the manufacturer and followed by the retailers, where the leader decides based on the results of his decision on the retailers and their decisions. The manufacturer is also affected by the decisions of these retailers. When considering the overall competition, we observe a generalized Nash Stackelberg game. A distinctive feature of this research is the interconnection of retailers through shared resources. In such a condition, it is not possible to find an equilibrium solution through the Nash equilibrium across retailers, and it is necessary to refer to the generalized Nash format, which has much more technical complexity. Accordingly, the overall structure of the game in this paper is a generalized Nash-Stackelberg game. From a mathematical modeling perspective, in this bi-level programming, the manufacturer is at the top level, and at the bottom is a set of retailers, who have their own optimization problems. All second-level optimization problems are interconnected due to common constraints, and identifying an optimal solution for all retailers presents considerable technical challenges. In summary, the manufacturer, as the leader of the game, would announce the wholesale price, the prices of offline and online sales channels, the local advertising participation rate, and the amount of money it invests in advertising channels. On the other hand, retailers decide how much product they buy from the manufacturer and how much they invest in local advertising, with all these decisions being made in consideration of the manufacturers' and retailers' capabilities and budgetary constraints.

1.2. Global and local advertising

The advertising of marketing channel members serves multiple purposes. The primary objective of manufacturers' global advertising is to enhance brand equity, whereas retailers' local advertising directly impacts customer demand [1]. Global advertising refers to advertising conducted on a broad geographic scale, typically at the international or national level. It utilizes various commonly used tools, such as TV shows and advertisements on platforms like Google, Facebook, and the internet. In contrast, local advertising includes activities like offering free car test drives, food samples, discounts, and participation in community events. One of the main objectives of global advertising is to establish long-term relationships with consumers, contributing to brand recognition. Global advertising helps shape brand identity, builds credibility, fosters customer loyalty, and creates intellectual and emotional connections with potential customers. On the other hand, local advertising aims to generate immediate user action, while global advertising focuses on enhancing overall brand awareness. Consequently, manufacturers rely partly on retailers' advertising, which can sometimes be seen as insufficient in promoting the manufacturer's brand effectively [2].

1.3. Research questions

This study seeks to answer the following key questions:

- What conditions prompt the manufacturer to participate in cooperative advertising?
- How do the manufacturer's and retailers' conditions, such as the manufacturer's capacity and the supply chain members' budgets, influence cooperative advertising?
- For manufacturers planning to introduce an independent online sales channel alongside their traditional, physical retail outlets, how should the allocation of cooperative advertising investment, pricing, and product distribution be handled considering limited capacity?
- In what sequence and to what degree is the manufacturer willing to participate in local retailer advertising?
- How will the competition landscape among retailers form, and which parameters will most significantly impact their competitive advantages and disadvantages in this competition?

1.4. Contributions

This study contributes to the existing literature in two significant ways. Firstly, it presents a novel framework for a generalized Nash-Stackelberg game, which considers the coupling among retailers due to shared limited resources. This framework also includes an online sales channel managed by the manufacturer in collaboration with traditional retailers. Secondly, it underscores the importance of differentiating global advertising channels based on their unique impacts on demand. Furthermore, the conditions and assumptions underpinning the proposed supply chain model mirror those encountered during the ongoing Covid-19 pandemic, which has been ongoing since its inception. For instance, the shortage of electronic chipsets has affected major industries, notably automobile manufacturing.

Given this context, using Nash equilibrium alone is insufficient for determining an equilibrium solution among the participating retailers in such a game. The retailers' optimal decisions involve solving interconnected constrained optimization problems that do not lend themselves to easy solutions. Therefore, a generalized Nash equilibrium must be sought for the more challenging lower-level game. Hence, the game structure described in this paper is a generalized Nash-Stackelberg game.

From a mathematical modeling perspective, the manufacturer occupies the upper level of this bi-level programming model. Conversely, the retailers aim to make optimal decisions by solving nested optimization problems at the lower level of the bi-level model. The constraints of the model encompass the manufacturer's production capacity and the allocation of the manufacturer's contribution to retailers' local advertising costs, which interconnects the retailers' lower-level optimization problems.

As the game's leader, the manufacturer determines the expenditures for global advertising, announces participation rates for local advertising, and decides on the quantity of goods to be supplied to the retailers. As followers in the game, the retailers determine their level of investment in local advertising while simultaneously competing for a larger share of the manufacturer's production capacity. Budgetary constraints on both the leader and followers limit the advertising expenditures.

In summary, the proposed game framework in this study involves a bi-level structure where the manufacturer assumes the role of the upper-level leader, and the retailers act as followers at the lower level. The optimization problems faced by the retailers are coupled due to shared resources and budget constraints, necessitating the exploration of a generalized Nash equilibrium.

1.5. Organization of the paper

The remainder of the paper is organized as follows: Section 2 provides an overview of related studies, Section 3 presents the mathematical model and the methodology employed to solve it, Section 4 includes a numerical examination of the model and managerial insights, Section 5 concludes the study, and Section 6 offers suggestions for future research.

2. Literature Review

Based on previous studies[3], cooperative advertising research can be divided into five main areas:

- **Vertical Co-op Advertising**: This branch of co-op advertising is a popular topic among researchers. The focus is primarily on the relationships between manufacturers (upstream supply chain) and retailers (downstream supply chain), where manufacturers propose a share percentage for retailer advertising costs.

- **Cooperative franchise advertising**: This element resembles Vertical Co-op Advertising, except it involves advertising campaigns run under a cost-benefit sharing agreement between the manufacturer and the retailer [4]–[6].
- **Horizontal cooperative advertising vs. global advertising**: Here, companies at the same level of a supply chain collaborate and compete. Manufacturers jointly promote common products as a group instead of individually [7], [8].
- **Cooperative vs. predatory advertising**: While cooperative advertising typically increases aggregate demand for all contributors, predatory advertising aims to increase a member's market share and demand by reducing the market demand of other competitors.
- **Joint advertising decisions**: The latest research group on cooperative advertising centers on joint advertising decisions, where members cooperate to decide advertising costs to maximize shared benefits, which may be at some level of the supply chain or beyond [9].

This study primarily focuses on the first category, vertical cooperative advertising, and the decisions made regarding the relationship between the manufacturer and the retailer. We aim to review the research conducted in this specific category and subsequently present a comparative table displaying the results of the present study alongside other relevant research papers. The concept of cooperative advertising models was first introduced by Berger [10], utilizing game theory. Over time, these models have evolved to incorporate competitive elements within the games, encompassing competition among manufacturers, among retailers, and between manufacturers and retailers.

Most authors have primarily focused their analysis on determining the optimal advertising decisions of the members [11]. However, a few authors have considered additional factors such as pricing of goods and services, quality considerations, re-manufacturing processes, and inventory control alongside advertising decisions. Most studies in the vertical advertising domain have predominantly centered around the analysis of a single manufacturer at the first level. However, studies conducted by Karry [12], as well as Kim and Staelin [13], have considered the involvement of a second manufacturer. These studies have examined the interplay of both Nash competition among the manufacturers themselves and Stackelberg competition between the manufacturer and retailers. Chutani and Sethi [14] introduced a model that integrates multiple manufacturers and retailers in a Stackelberg game setting. At each level, there is a Nash game involving the manufacturers and a separate Nash game involving the retailers.

Manufacturers often provide diverse promotional partnerships to retailers, who then decide which manufacturer to source from and how much to invest in local advertising. This scenario presents three types of interactions: 1) a Nash game among manufacturers, 2) a Nash game among retailers, and 3) a Stackelberg game between manufacturers and retailers. This paper explores various topics, including global advertising or manufacturer branding. Studies [15] and [16], [17] scrutinize both manufacturer branding and retail store branding, each exerting unique effects on the demand function.

Another vital aspect of cooperative advertising is the approach to financial participation, which can involve cost contributions or discount offers from the manufacturer to the retailer. Alternatively, as suggested in [18], the manufacturer can motivate the retailer to invest more through a profit-sharing contract. Both methods are similar, differing mainly in the type and timing of payment. Within the integrated research areas of vertical and horizontal cooperative advertising, static models exist that consider competition between two manufacturers. These models account for the collaboration between manufacturers within the same supply chain (vertical) and the competition between manufacturers operating in the same market (horizontal). One such model is proposed by Bergon and John [19], involving two manufacturers and two retailers. In this model, each product is characterized by two features: the first associated with the retailers, and the second with the manufacturers. Consequently, there are four different modes in which the demand for each product is influenced by the cooperative advertising efforts between these entities. Research conducted by Kim and Staelin [13] focuses on investigating the most effective participation rate and advertising strategies between manufacturers and retailers. They propose a Stackelberg game model, with two manufacturers acting as leaders and two retailers as followers. The model incorporates the concept of cooperative advertising, represented as a direct payment quota from the manufacturer. The objective of this model is to determine the optimal approach to cooperative advertising to maximize benefits for both manufacturers and retailers. In a study by Karry and Zaccour [12], a Stackelberg model involving two manufacturers and two retailers was examined. The research investigates the combined effects of manufacturer and retailer branding, focusing specifically on "Store Branding" for retailers. The study examines the influence of cooperative advertising programs on the incomes of both manufacturers and retailers. They discovered that when the rate of brand substitution is sufficiently high, cooperative advertising programs present a scenario similar to the well-known "Prisoner's Dilemma" problem. However, if the rate of brand substitution is not high enough, cooperative advertising programs tend to result in increased profits for retailers.

In contrast, Naimi *et. al*, [20] focused on a supply chain consisting of a manufacturer and a retailer, where the manufacturer produced multiple products, and the demand for each product was influenced by both its price and the amount spent on advertising. Two power scenarios were considered: one where the manufacturer had the dominant position and another where the retailer had greater control. This resulted in two distinct games: the Stackelberg-manufacturer game and the Stackelberg-retailer game, respectively. Farshbaf *et al.* [21], aimed to identify a specific timeframe for production that would maximize efficiency and profitability, while taking into account the power dynamics between the manufacturer and the retailer. In the examined supply chain, the manufacturer is responsible for making decisions regarding the production timeframe and wholesale prices of each product. On the other hand, the retailer is in charge of determining the retail price, managing costs, and making investments in advertising for individual products. The research indicates that cooperative advertising plays a role in influencing the willingness to pay (WTP) parameter, as advertising efforts have been shown to contribute to an increase in this parameter. The study considers different game scenarios based on the balance of power. When the manufacturer and retailer have equal power, resulting in a balanced situation, a Nash equilibrium is achieved. Conversely, when the manufacturer holds greater power than the retailer, a Stackelberg game is observed. In a related research conducted by Farshbaf *et al.* [22], an oligopoly market is examined, involving one manufacturer and multiple retailers. Unlike the previous study, the retailers in this market have interdependent effects on each other, and their actions are not exclusive. The pricing decisions made by the retailers directly affect both their own demand and the demand of other retailers. Xiao *et. al.* [23] proposed a supply chain model involving multiple retailers. The demand of each retailer was influenced by both global advertising and local retailer advertising. Notably, the retailers collaborated in their advertising efforts. A key innovation of this paper was the consideration of the advertising effectiveness in relation to the individual environment of each retailer, taking into account the presence of uncertainty. They employed a vertical hybrid multilevel cooperative advertising game, wherein the manufacturer established a vertical partnership with its retailers. Additionally, the retailers formed a union at the lower level, constituting a horizontal partnership among themselves, which determined the total order placed with the manufacturer. In another investigation, Yu *et al.* [24] delved into the challenges of coordinating advertising decisions within a supply chain system. They employed game theory to address the time delay between advertising exposure and its subsequent effect. The study revealed that the implementation of a comprehensive cooperation condition has the potential to amplify advertising investment, boost product demand, and enhance economic profit. Several studies examining supply chain competitions, with a particular focus on horizontal collaborative advertising, have been reviewed. These studies include the works of [25]–[28]. The studies provide a comprehensive understanding of the dynamics within supply chains and offer insights into various strategies for manufacturers and retailers to maximize their profitability.

Another noteworthy trend in the business world is the integration of manufacturers at the retail level, where they directly sell their goods through online platforms. This trend, facilitated by the Internet, has reshaped traditional business models and created opportunities for manufacturers to establish a direct connection with customers. By bypassing intermediaries, manufacturers can effectively promote their products and increase their market presence in a more efficient manner. For instance, Park and Keh [29] asserted that the adoption of mixed sales channels by manufacturers can lead to several benefits. They argued that using such channels would result in lower retail prices, increased demand, and higher profits for both manufacturers and retailers. Dan *et. al.*, [30] examine a conventional two-channel supply chain, with a particular emphasis on the retailer's involvement in providing services and making decisions about service and pricing strategies. Their findings suggest that the retailer should improve the quality of its channel service, leading to a price increase. Additionally, the manufacturer's pricing strategy depends on customer loyalty. In related studies, researchers have incorporated these concepts into discussions on closed-loop supply chains, expanding the research scope to include areas such as product returns, re-manufacturing, cooperative advertising, and subsequent sales [31]. Xie *et. al.*, [32] examine a closed-loop supply chain consisting of one manufacturer and one retailer. The manufacturer in focus uses two different sales approaches. In the first scenario, products are sold to the retailer at a wholesale price, who then sells them to end consumers through the "Offline Channel." In the second scenario, the manufacturer directly sells products to end-users without a retailer, using the “Online Channel” via the internet. The demand function in the study considers the influence of advertising and selling prices for both the offline and online channels, while excluding global and local advertising effects. The paper assumes a certain percentage of customers prefer offline shopping, while the remainder prefer online shopping.

Additionally, a study on cooperative advertising and online price discounts in dual-channel supply chains [33] finds that online price discounts may not always benefit the manufacturer, and that bilateral cooperative advertising may harm the overall supply chain's total profit. The authors suggest that online price discounts be kept relatively low, and that manufacturers and e-platforms avoid bilateral cooperative advertising. Liu and Li [34] present a dynamic Stackelberg game model to investigate the cooperative advertising strategy of a risk-averse manufacturer-led supply chain. The authors use system dynamic theory to analyze the complex dynamics of the game and discover that changes in key parameters can cause the system to become unstable and chaotic. They also find that the manufacturer and retailer can make optimal strategic decisions by adjusting their participation rate, risk tolerance level, and advertising expenditure coefficient. Wu *et. al.*, [35] investigate how cooperative advertising and retailers' demand information sharing affect manufacturers and retailers in dual-channel supply chains. They find that manufacturers benefit from information sharing, while retailers benefit when the retail channel is small. Additionally, the more advertising spillover that occurs, the more likely retailers are to share information voluntarily. Finally, a supplementary effect exists between cooperative advertising and information sharing, meaning that higher manufacturer advertising participation rates increase the probability of signing information-sharing contracts with retailers. Zheng *et. al.*, [36] investigate a closed-loop three-echelon supply chain comprising a manufacturer, a distributor, and a retailer. The supply chain under consideration in this research operates in a closed-loop fashion, where used goods are returned to the original manufacturer for recycling and reproduction. The authors examine both decentralized and centralized scenarios. In the decentralized case, the manufacturer is responsible for producing new products, collecting used products, and managing the recycling process. In the centralized mode, the manufacturer acts as the leader, followed by the distributor and the retailer, resulting in a bi-level programming approach to decision-making within the supply chain. Yu *et. al.*, [37] present a differential game model to investigate cooperative advertising strategies for deteriorating items in a supply chain with a single manufacturer and multiple competing retailers. The model incorporates the impact of reference price on market demand. The authors analyze the equilibrium advertising strategies and supply chain profits under both decentralized and centralized control, and find that centralized control leads to higher profits for both the manufacturer and retailers. The study also offers insights into how managers can design optimal advertising strategies. Shi et al. [38] examined various types of manufacturer's investments in advertising for both green and conventional products. The researchers found that investing in advertising can have a substantial impact on product sustainability and profit margins. Specifically, investing in green product advertising significantly enhanced sustainability and profits. Additionally, as research and development (R&D) costs increase, investing in conventional product advertising becomes more preferable. Taleizadeh *et. al.*, [39] develop a model for a sustainable supply network that allows product returns. The model considers four return policies and four scenarios for offline and online sales channels. The researchers found the best policy is a partial refund offline and a loyalty discount for online returns. This maximizes profits, customer satisfaction, and reduces waste. The proposed policies can help the environment and overall profitability.

Based on the discussed studies, this research focuses on a supply chain with one manufacturer and multiple retailers. The retailers compete based on advertising budgets and manufacturer capacity as constraints. Retailers also have limited budgets. Various advertising channels are considered. The manufacturer sells through offline, online, and retail channels. Table 1 summarizes the configuration to enable comparison.

Based on the discussed studies, this research focuses on a supply chain with one manufacturer and multiple retailers. The retailers compete based on advertising budgets and manufacturer capacity as constraints. Retailers also have limited budgets. Various advertising channels are considered. The manufacturer sells through offline, online, and retail channels. Table 1 summarizes the configuration to enable comparison.

Tab. 1. Summary comparison of similar researches

Reference	Time Frame		Structure		Demand Function	Advertising			Number of			Has constraints?		Separation of advertising channels
	static	dynamic	centralized	decentralized	advertising	cooperative	global	local	products	retailers	manufactures	No	Yes	
[40]	■			■	*D1*	■	■	■	1	1	1	■		
[41]	■			■	*D1*	■	■	■	1	1	1	■		
[42]		■		■	*D2*	■		■	1	1	1	■		
[43]	■			■	*D1*	■	■	■	1	1	1		■	
[44]	■			■	*D3*	■	■	■	1	1	1	■		
[20]	■			■	*D2*			■	p	1	1		■	
[45]	■			■	*D3*	■	■	■	1	1	1	■		
[46]	■			■	*D3*	■	■	■	2	1	1	■		
[32]	■			■	*D4*	■		■	1	1	1	■		
[47]		■		■	*D3*			■	1	n	1		■	
[22]	■			■	*D3*	■	■	■	p	n	1	■		
[14]		■	■	■	*D5*	■			p	n	m	■		
[36]	■		■	■	*D1*	■	■	■	1	n	1	■		
[48]	■		■		*D1*	■		■	1	1	1	■		
[2]	■		■	■	*D3*	■	■	■	1	2	1	■		
[49]	■		■		*D6*	■	■	■	1	2	1	■		
[50]	■		■	■	*D4*	■		■	1	1	1	■		
[51]	■		■		*D4*	■		■	1	n	1	■		
[52]	■		■		*D4*	■		■	1	1	1	■		
[53]	■		■	■	*D4*	■		■	1	1	1	■		
[54]		■	■		*D1*	■		■	1	1	1	■		
[55]	■		■	■	*D4*	■		■	1	1	1	■		

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	static	dynamic	centralized	decentralized	advertising	cooperative	global	local	products	retailers	manufactures	No	Yes	
[56]	■		■	■	*D1*	■			1	1	1	■		
[57]		■	■	■	*D6*	■			1	1	1	■		
[58]	■		■	■	*D1*	■	■	■	1	1	1	■		
[33]	■		■	■	*D6*	■	■	■	1	1	1	■		
[35]	■		■		*D6*	■	■	■	1	1	1	■		
[34]		■	■		*D6*	■	■	■	1	1	1	■		
[39]	■		■		*D4*	■		■	1	1	1	■		
Present Research	■		■		*D6*	■	■	■	1	2	1		■	■

Letters $p, n,$ and m in columns refer to multi-product, multi-retailer, and multi-manufacturer, respectively.

$D1$: $\alpha - \beta\alpha^{-\gamma}A^{-\sigma}$
$D2$: α^{β}
$D3$: $n_1\sqrt{A} + n_2\sqrt{\alpha}$
a: Initial market demand without advertising
K: Advertising impact factor
v: Price impact factor
β, σ: Prices sensitivity parameters

$D4$: $\alpha + K\sqrt{b}$
$D5$: $Kb\sqrt{1-X} - \sigma_{ij}x_{ij}$
$D6$: $a + n_1\sqrt{A} + n_2\sqrt{\alpha}$
A: Global advertising investment
α: Local advertising investment
n_1: Global advertising impact factor
n_2: Local advertising impact factor

3. Mathematical Modeling

The mathematical model considered in this study consists of a manufacturer and two retailers who sell the manufacturer's product. The retailers act as provincial or regional sales representatives who operate in specific geographies and have their own exclusive markets. The manufacturer has also launched a direct online sales channel in addition to selling through its regional representatives. The sale channel through retailers is referred to as the offline sales channel. To increase sales in the supply chain, two types of advertising are used: global and local. Global advertising, which is mainly for branding, is the responsibility of the manufacturer. Local advertising, also done in a specific geography, is the responsibility of the retailers, with the difference that the manufacturer can participate in local advertising by the retailers. Global advertising has three different channels: 1) Television and satellite, 2) Internet and social media, and 3) Billboard and leaflet, each with different customer attraction effects. Some customers prefer the offline channel and some prefer the online channel for purchase, represented in the model with the parameter ρ . Regardless of advertising, there are always customers willing to buy the product of a specific manufacturer, shown with the parameter Q.

The demand function considers the effects of global advertising, local advertising, and online and offline prices. The manufacturer and retailers have budget constraints for relevant advertising channels. Additionally, the manufacturer has a specified production capacity. The manufacturer competes with retailers in deciding the participation level in local advertising and product quantity distribution between them and the online channel. This is because the manufacturer moves first, followed by retailers, due to its upstream position. This game is a Stackelberg game. Retailers also compete due to the shared resources of production capacity and advertising participation budget. This competition is a Generalized Nash game given the shared resources. Overall, the competition between all supply chain members is a Generalized Nash-Stackelberg game. The following figure illustrates the relationships:

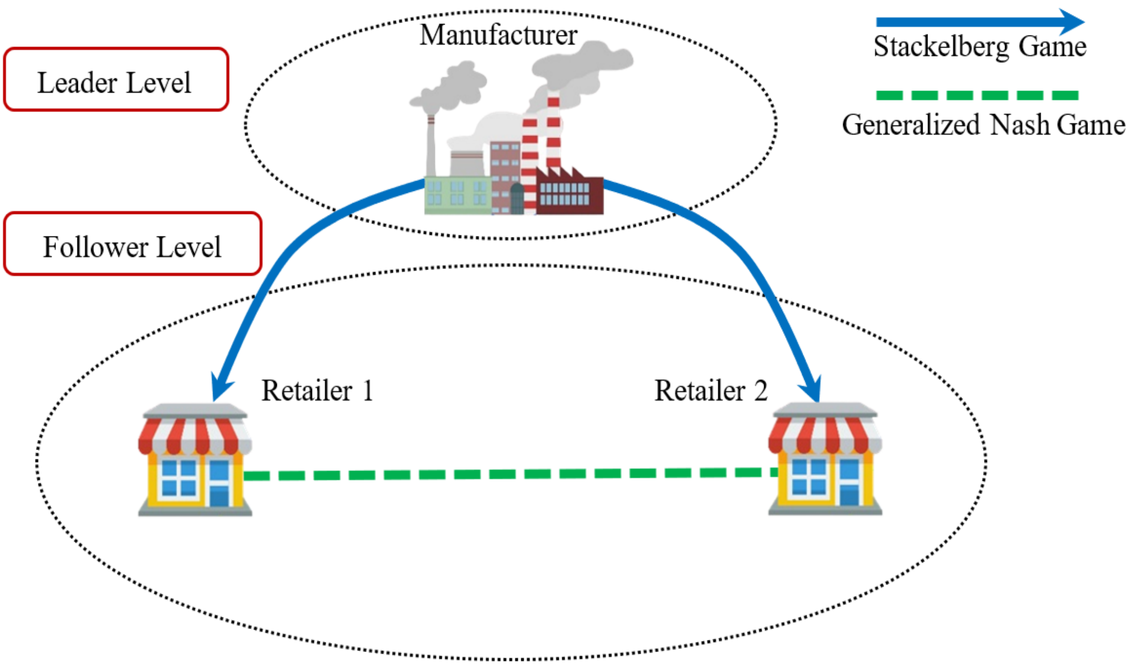

Fig. 1. Games and interactions among supply chain members

Sets

r	Retailers

Parameters

C	The cost incurred by the manufacturer for producing the finished product
Ω	The maximum production capacity of the manufacturer
ϕ	The maximum advertising budget allocated by the manufacturer
ρ	The percentage of customers within the community who prefer to make purchases offline
Q_r	The number of customers of retailer r who typically make purchases from that retailer without being influenced by advertising
n_1	The impact of television and satellite advertising on the demand
n_2	The impact of local advertising on the demand
n_3	The impact of advertising through the internet and virtual networks on the demand
n_4	The impact of advertising through billboards and tracts on the demand
β	The impact of the price of the online/offline channel on the corresponding demand for that channel
σ	The impact of price in the online (offline) channel on the demand in the offline (online) channel
Q_r	The number of customers in retailer r's geographic region who are willing to purchase the manufacturer's product without promotional efforts
γ_r	The allocated advertising budget by retailer r

Decision variables of Manufacturer (Leader)

W	Wholesale price
p_1	Offline price
p_2	Online price
X_2	The quantity of goods dispatched via the online sales channel
θ_r	The participation rate of the manufacturer in retailer r's local advertising
A	The manufacturer's allocated investment in television and satellite advertising

ξ	The manufacturer's allocated investment in internet and virtual networks advertising
ψ	The manufacturer's allocated investment in billboards and tract advertising
X_1^r	The quantity of requested goods by retail r for sale

Decision variables of Retailers (Followers)

α_r	The investment made by retailer r in local advertising

The proposed bi-level model is as follows:

$$Max\, Z_M = \sum_r (W - C) * X_1^r + (p_2 - C) * X_2 - \sum_r \theta_r\, \alpha_r - A - \xi - \psi \tag{1}$$

$$\sum_r X_1^r + X_2 \leq \Omega \tag{2}$$

$$A + \xi + \psi + \sum_r \theta_r\, \alpha_r \leq \phi \tag{3}$$

$$X_2 = (1 - \rho)\left(\sum_r Q_r + n_1\sqrt{A} + \sum_r n_2\sqrt{\alpha_r} + n_3\sqrt{\xi} + n_4\sqrt{\psi}\right) - \beta.(p_2) + \sigma.(p_1) \tag{4}$$

$$Max\, Z_r = (\,p_1 - W) * X_1^r - (1 - \theta_r)\alpha_r \tag{5}$$

$s.t.:$

$$X_1^r = \rho\big(Q_r + n_1\sqrt{A} + n_2\sqrt{\alpha_r} + n_3\sqrt{\xi} + n_4\sqrt{\psi}\big) - \beta.(p_1) + \sigma.(p_2) \tag{6}$$

$$(1 - \theta_r)\alpha_r \leq \gamma_r \tag{7}$$

3.1. Model and constraints descriptions

Expression (1) represents the manufacturer's objective function for profit maximization. The first part of this objective function includes revenues from sales to retailers (offline channel) and online channel sales. It is calculated by multiplying the profit margin of each sales channel (selling price minus production cost) by the quantity sold. The second part includes the manufacturer's costs, which are the expenses for participating in local retail advertising and the costs of the three global advertising channels. Constraint (2) guarantees that the total quantity sent to all retailers (offline sales channel) plus the online channel quantity does not exceed production capacity. Constraint (3) relates to the manufacturer's advertising budget limit, signifying that spending on the three global channels and total participation in local retail advertising cannot surpass the maximum advertising budget. Constraint (4) shows the demand generated for the online sales channel where the manufacturer directly delivers products to consumers. The expression $\left(\sum_r Q_r + n_1\sqrt{A} + \sum_r n_2\sqrt{\alpha_r} + n_3\sqrt{\xi} + n_4\sqrt{\psi}\right)$ represents the total demand that the supply chain has created. This expression includes $\sum_r Q_r$, which represents those customers who are usually willing to buy the manufacturer's product without advertising. In addition, the expression $(\,n_1\sqrt{A} + \sum_r n_2\sqrt{\alpha_r} + n_3\sqrt{\xi} + n_4\sqrt{\psi})$ represents customers who have been attracted through the three global advertising channels as well as customers who have been attracted through local advertising. Customers attracted through local advertising are considered together since the manufacturer may have different participation rates in each retailer's advertising, and accordingly, different investments in local advertising for each retailer. The functions for the advertising channel effects are inspired by previous studies, summarized in Table 1. In Constraint (4), the parameter ρ is used. ρ represents the percentage of customers willing to buy through the offline channel. This group prefers seeing and inspecting products in person. Since this shows online channel demand, the parameter $(1 - \rho)$ is used for the percentage who buy online. The online and offline prices also affect demand for both channels. When one channel's price increases, its customers decrease. These customers switch to the other channel, increasing its demand. So a price increase has a negative effect on that channel's demand but a positive effect on the other's, shown with parameters β and σ. Constraint (5) shows each retailer's profit maximization objective function. The first part is revenue, obtained by multiplying profit margin and quantity sold. The second part covers local advertising costs. Note the manufacturer can participate in each retailer's local advertising.

Note that the manufacturer's participation share in the local advertising of each retailer is shown by θ_r , which is included in the manufacturer's objective function as the manufacturer's cost share $(\sum_r \theta_r \alpha_r)$. The retailer's cost share $((1-\theta_r)\alpha_r)$ in considered in the retailer's objective function. Constraint (6) is for the demand generated for the offline sales channel, or the sales channel related to retailers. All explanations for Constraint (4) also apply to Constraint (6), with the difference that the parameter ρ is used here because the demand here is related to the percentage of customers willing to buy from the offline sales channel. For pricing effects, based on the explanations given, an increase in the offline channel price (p_1) has a negative $(-\beta.(p_1))$ effect and an increase in the online channel price (p_2) has a positive $(\sigma.(p_2))$ effect on the demand generated on the offline sales channel. Finally, Constraint (7) ensures that each retailer's share of their own local advertising does not exceed the maximum budget available for that purpose.

3.2. Variational inequalities for retailers' level

Solving bi-level problems is complex. To simplify, if the necessary conditions are met, we can first convert them to single-level problems and then solve them using conventional methods. One of the appropriate methods for converting bi-level problems to single-level problems is to use the KKT[1] method and the Lagrangian function [59]. However, in the present model, this method cannot be used because at the lower level, due to budget and capacity constraints of the manufacturer, retailers are coupled with each other. For such coupled problems, the variational inequalities method can be used. This formulates the lower-level optimization (objective and constraints) as variational inequalities in the higher-level problem to make it single-level [60]. The concept primarily originates from the Taylor series, which is employed to approximate the value of functions. The Taylor series allows us to approximate a function by using a series expansion, thereby providing an approximation that becomes more accurate as more terms are included.

$$f(y) \approx f(x^*) + \nabla f(x^*)^T (y - x^*) \quad (8)$$

In this context, our objective function aims for minimization. The optimal solution, denoted as x^* , is considered such that when $f(x^*) \le f(y)$ holds true, it represents the optimal solution. In the case of maximization mode, the relationship is reversed. Therefore, we should have the following relationship:

$$\nabla f(x^*)^T (y - x^*) \ge 0 \quad (9)$$

Variational inequalities aim to find the optimal solution x^* that satisfies the aforementioned relationship. The general format of variational inequalities is as follows:

$$G(x^*)^T (x - x^*) \ge 0 \quad (10)$$

From the given relationship, we observe that $G(x^*)^T$ is equivalent to the transpose of the $\nabla f(x^*)^T$. This concept holds true for finding Nash equilibrium. However, in the case of generalized Nash equilibrium, it is not always the same, and different solutions and outcomes may arise. Our problem involves a coupling constraint. When formulating it as a variational inequality, we need to compute $G(x^*)^T$. In generalized Nash problems with constrained player coupling constraints, there are two assumptions that need to be considered:

1. Consider the coupled constraint $\sum_r X^r \le \Omega$, where Ω represents a resource controlled by another agent. This agent provides the resource Ω to all retailers at a fixed price that remains the same for every retailer.
2. In this scenario, we have the coupled constraint $\sum_r X^r \le \Omega$, where player i believes that other players' decisions are beyond their control. This constraint changes to $X^r \le \Omega - \sum_{i \ne r} X^i$ for player i, where the right-hand side of the inequality is considered as a parameter specific to player i.

The research model aligns more with the assumptions of the first case, where the other agent or player is the same manufacturer. The price per unit consumption is equal to the wholesale price for all retailers. In this scenario, it can be shown that the Variational Inequality model for generalized Nash equilibrium will be the same as the ordinary Nash equilibrium, except that the coupled constraint will modify the solution space [60]. In the second case, the concept of Quasi-Variational inequalities should be utilized.

To formulate the problem using Variational Inequalities, we need two sets: *L* and *G*. The concept of *G*, as previously explained, represents the coupling constraints. On the other hand, *L*

[1] Karush–Kuhn–Tucker

represents the same solution space based on the constraints of the problem:

$$G = \begin{pmatrix} \frac{\partial f_1}{\partial X_1^1} \\ \frac{\partial f_1}{\partial \alpha_1} \\ \vdots \\ \frac{\partial f_R}{\partial X_1^R} \\ \frac{\partial f_R}{\partial \alpha_R} \end{pmatrix} = \begin{pmatrix} p_1 - W \\ 1-\theta_1 \\ \vdots \\ p_1 - W \\ 1-\theta_R \end{pmatrix} \tag{11}$$

$$L = \left\{ X_1^r, \alpha^r \middle| \begin{array}{c} \sum_r X_1^r + X_2 \leq \Omega, \\ X_1^r = \rho\left(Q^r + n_1\sqrt{A} + n_2\sqrt{\alpha^r} + n_3\sqrt{\xi} + n_4\sqrt{\psi}\right) - \beta.(p_1) + \sigma.(p_2), \\ (1-\theta^r)\alpha^r \leq \gamma^r \\ \forall r \end{array} \right\} \tag{12}$$

Now we can express the Variational Inequalities relationships as follows:

$$G(x^*)^T(x - x^*) = (p_1 - W, \quad 1-\theta_1, \quad \dots \quad p_1 - W, \quad 1-\theta_R) \begin{pmatrix} X_1^1 - X_1^{1^*} \\ \alpha_1 - \alpha_1^* \\ \vdots \\ X_1^R - X_1^{R^*} \\ \alpha_R - \alpha_R^* \end{pmatrix} \geq 0 \tag{13}$$

Which results in the following inequality:

$$\sum_{r=1}^{R} \left\{ (p_1 - W) * (X_1^r - X_1^{r^*}) + (1-\theta_r) * (\alpha_r - \alpha_r^*) \right\} \geq 0 \tag{14}$$

The inequality 14 establishes the conditions under which we can obtain all optimal solution spaces for the retailer level, along with other relevant constraints. In the main bi-level problem, these inequalities are substituted with the retailer level variables while preserving the associated constraints. As a result, the bi-level problem is transformed into a single level problem, which can be expressed as follows:

(Note: variables $X_1^{r^*}$ and α_r^* represent the solutions obtained from variational inequalities)

$$Max\ Z_M = \sum_r (W - C) * X_1^{r^*} + (p_2 - C) * X_2 - \sum_r \theta_r\, \alpha_r^* - A - \xi - \psi \tag{15}$$

$$\sum_r X_1^r + X_2 \leq \Omega \tag{16}$$

$$A + \xi + \psi + \sum_r \theta_r\, \alpha_r \leq \phi \tag{17}$$

$$X_2 = (1-\rho)\left(\sum_r Q_r + n_1\sqrt{A} + \sum_r n_2\sqrt{\alpha_r} + n_3\sqrt{\xi} + n_4\sqrt{\psi} \right) - \beta.(p_2) + \sigma.(p_1) \tag{18}$$

$$X_1^r = \rho\left(Q_r + n_1\sqrt{A} + n_2\sqrt{\alpha_r} + n_3\sqrt{\xi} + n_4\sqrt{\psi}\right) - \beta.(p_1) + \sigma.(p_2) \tag{19}$$

$$(1-\theta_r)\alpha_r \leq \gamma_r \tag{20}$$

$$\sum_{r=1}^{R} \left\{ (p_1 - W) * (X_1^r - X_1^{r^*}) + (1-\theta_r) * (\alpha_r - \alpha_r^*) \right\} \geq 0 \tag{21}$$

4. Computational Results and Analysis

In this section, we examine and analyze the effects of parameter variations on other decision variables in the model. Our numerical experimentation involves a model with one manufacturer and two retailers. The demand decision variables or the number of goods required by the first and second retailer and the online sales channel of the manufacturer are represented by the symbols X_1, X_2 and X_3, respectively. Other initial parameters considered in the model are reported in Table (2).

Tab. 2. Parameters values

Parameters	Value	Parameters	Value
C	*500*	n_2	*1.5*
Ω	*15'000*	n_3	*1.7*
ϕ	*42'000'000*	n_4	*1.5*
ρ	*0.7*	β	*8.5*
Q_1	*900*	σ	*6.5*
Q_2	*400*	γ_1	*1'000'000*
n_1	*1.6*	γ_2	*2'000'000*

In terms of price, the minimum wholesale profit for the manufacturer is 20%. We first examine the influence of changes in the manufacturer's maximum advertising budget on other decision variables.

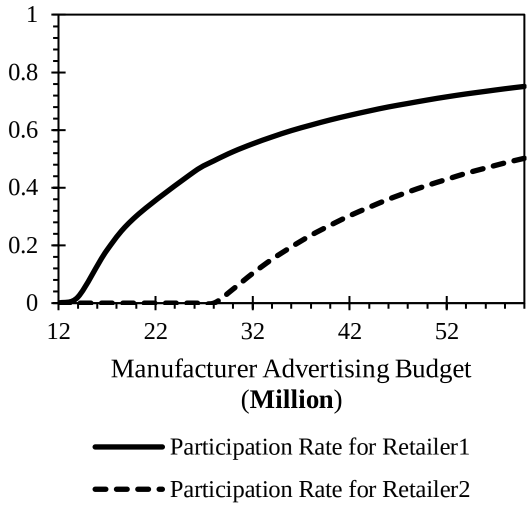

Fig. 2(a). Manufacturer's participation rates based on advertising budget

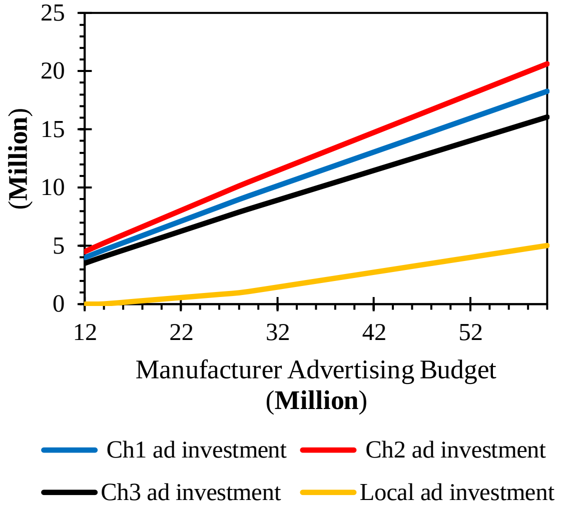

Fig. 2(b). Manufacturer's investment in advertising channels based on advertising budget

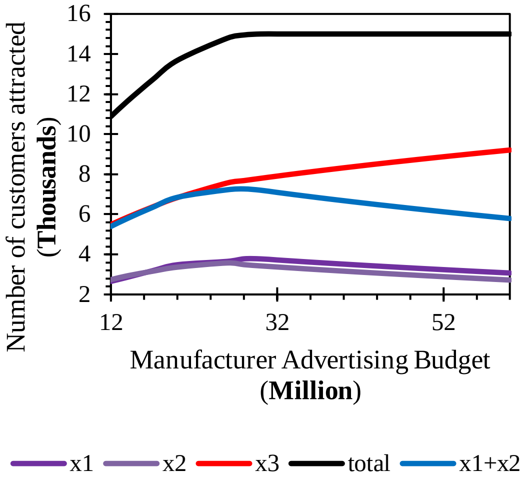

Fig. 2(c). The relationship between advertising budget and customer attraction

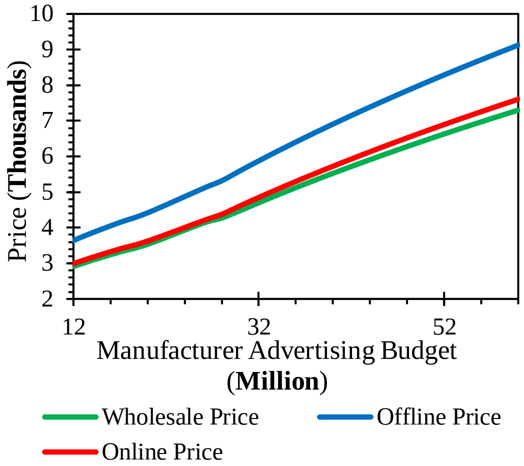

Fig. 2(d). The relationship between advertising budget and prices

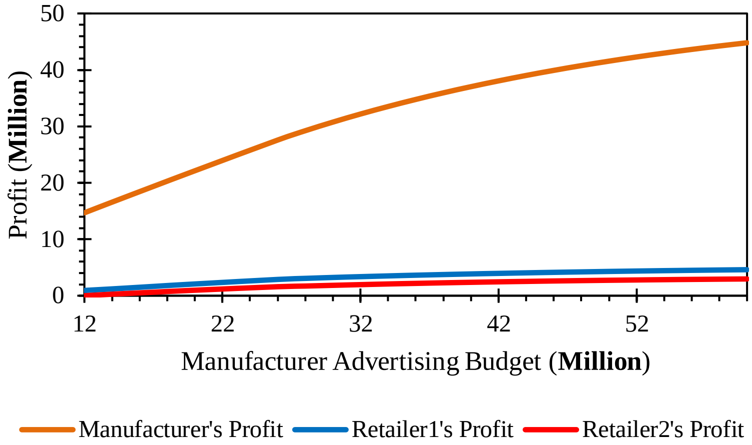

Fig. 2(e). The Relationship between advertising budget and profits of supply chain members

As shown in Figure 2(a), as the manufacturer's advertising budget increases, its participation rate in local advertising and advertising in other advertising channels also increases (Figure 2(b)). However, why the participation rate of the first retailer increases earlier than that of the second retailer is debated. The manufacturer factory has a production capacity of 15,000 units for a specific product. Figure 2(c) illustrates the initial state of the supply chain before the advertising budget of 260,000,000 units is allocated. It is observed that the supply chain has not fully utilized its production capacity and has instead chosen to increase prices to maximize profits (as depicted in Figure 2(d)). Consequently, this pricing strategy has led to a decrease in the number of customers, resulting in a demand lower than the production capacity. The number of retailers, both individually (Retailer 1 and Retailer 2) and their collective sum, initially exhibited an upward trend before the allocation of the 26,000,000 unit budget. However, after the budget allocation, there was a decline in the number of retailers. On the other hand, the number of online customers consistently increased throughout the observed period. While it may appear that the revenue of both retailers is also declining, the price chart in Figure 2(d) reveals that as the number of customers decreases, prices for offline, online, and wholesale channels increase. To make an accurate assessment, it is necessary to consider the profit (or income) chart of the entire supply chain, as shown in Figure 2(e). The revenue of all supply chain members, including the manufacturer and the two retailers, shows an upward trajectory. However, the manufacturer's revenue growth rate is significantly higher compared to that of the retailers, indicating a more substantial increase in the manufacturer's profit relative to the retailers. The analysis of profit charts provides a comprehensive view of the overall performance and financial outcomes of the manufacturer and retailers. Another notable aspect in the profit chart is the presence of a little revenue for the second retailer. This indicates that under these circumstances, the second retailer would not be able to sustain its operations. To ensure the viability of retailers, especially those with higher advertising budgets, it is recommended that they collaborate with a manufacturer who possesses at least half of the total advertising budget allocated to all retailers. In this case, the manufacturer should have a minimum advertising budget of 150,000,000 units to ensure the profitability and sustainability of the retail partners. This balance in advertising investment helps maintain a healthy and mutually beneficial relationship between the manufacturer and retailers. The parameter ρ, which denotes the proportion of customers willing to make purchases through offline sales channels or physical retailers, plays a crucial role in shaping the the supply chain. Exploring the impact of ρ allows for a deeper understanding of various supply chain aspects, including retailer performance, customer demand, pricing strategies, and overall profitability.

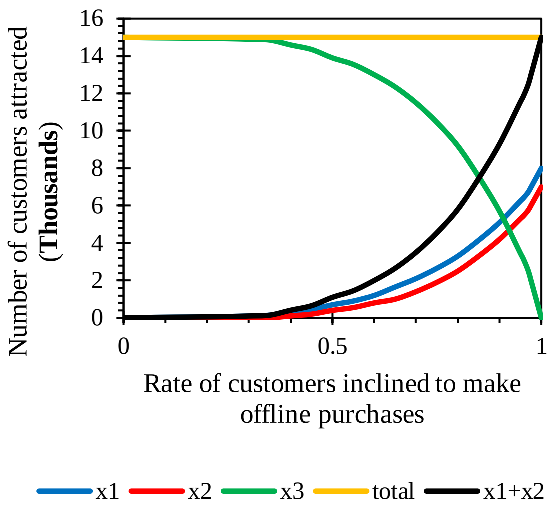

Fig. 3(a). The relationship between customer attraction and ρ

Fig. 3(b). The relationship between prices and ρ

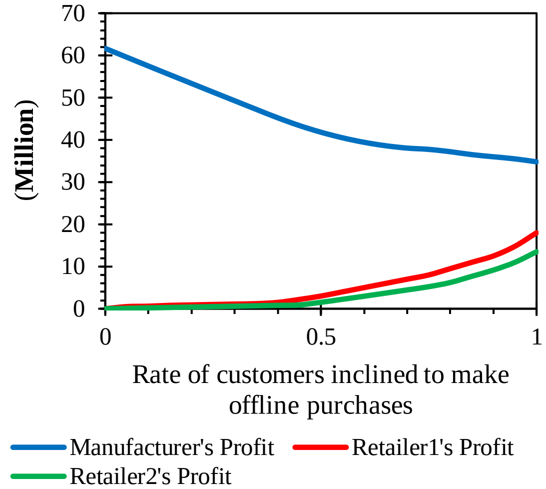

Fig. 3(c). The relationship between ρ and profits of supply chain members

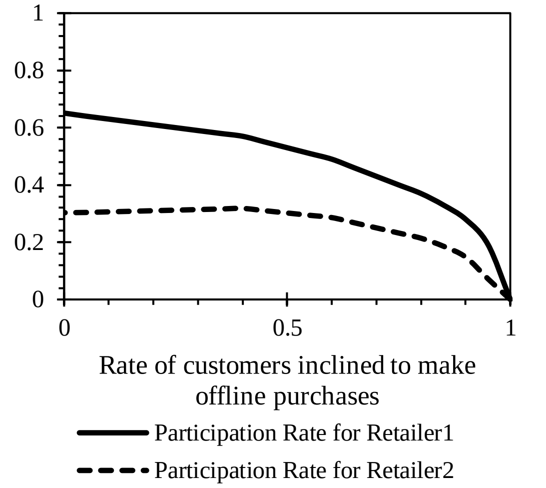

Fig. 3(d). The relationship between ρ and manufacturer's participation rates

As depicted in Figure 3(a), an increase in the rate of customers willing to make offline purchases (from retailers) leads to a decrease in the number of goods shipped online and an increase in the number of goods shipped offline. Furthermore, when the rate of customers willing to buy offline drops below 35%, the demand for retailer 2 becomes completely zero, and the demand for retailer 1 diminishes significantly and tends towards zero. Figure 3(b) complements these findings by displaying the corresponding price changes. As the investigated ratio increases, there is an observed increase in wholesale and offline prices, while online prices decrease. These results highlight the relationship between customer preferences, shipping channels, and pricing strategies. The findings demonstrate that customer preferences for offline shopping significantly impact the allocation of goods, the demand for retailers, and the pricing strategy within the supply chain. The increase in wholesale and offline prices can be attributed to the rise in offline customers and the corresponding decline in online prices due to reduced online customer demand. Figure 3(b) reveals another significant observation, wherein the manufacturer makes efforts to sustain the online channel by aligning wholesale and online prices perfectly. This strategic adjustment aims to attract retailers to act as online channel customers alongside other end-customers. Consequently, the online channel's prices, as well as wholesale prices, are reduced to their lowest possible levels. In Figure 3(c), it is evident that as the ratio increases, the manufacturer's revenue decreases, while retailer revenue increases. This trend can be attributed to the shifting customer preferences towards offline purchases. Retailers, with their physical presence, benefit from the increased demand in offline channels, resulting in higher revenue. On the other hand, the manufacturer's revenue is negatively impacted due to reduced online customer demand and the subsequent decrease in online and wholesale prices. These findings underscore the impact of changing customer preferences on pricing strategies, revenue distribution of the supply chain. Understanding these issues allows supply chain stakeholders to adapt their strategies effectively and optimize their performance in response to evolving customer preferences. When the ratio falls below 50%, the pricing decisions made by the manufacturer can significantly threaten the retailer market. Retailers may face challenges such as increased costs and difficulty competing in the market. Regarding the discrepancy between retailer 1 and retailer 2 in terms of profit loss, the reasons lie within the parameters and variables specific to each retailer. Two crucial parameters for retailers are the number of customers inclined to purchase from the manufacturer without advertising (referred to as Q) and the available budget allocated to each retailer for local advertising. To gain further clarity, we conducted a precise model run using the parameters that put the second retailer at risk. This analysis aimed to examine the impact of these two parameters on the retailer's profit loss. By carefully assessing the influence of Q and the allocated advertising budget on retailer profitability, we can identify the factors that contribute to the retailer's risk and potential profit loss. These findings will provide valuable insights for retailers to strategize their marketing efforts, allocate budgets effectively, and mitigate the risks associated with fluctuating customer preferences.

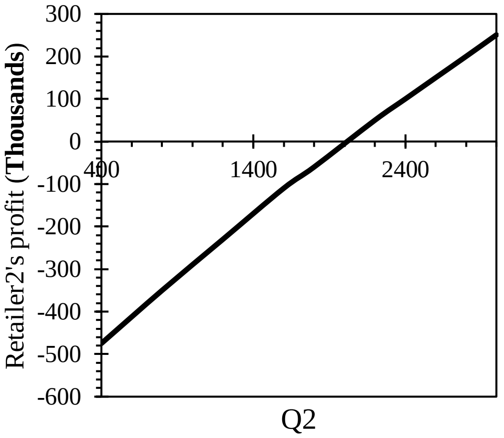

Fig. 4(a). The relationship between Q_2 and retailer 2's profit

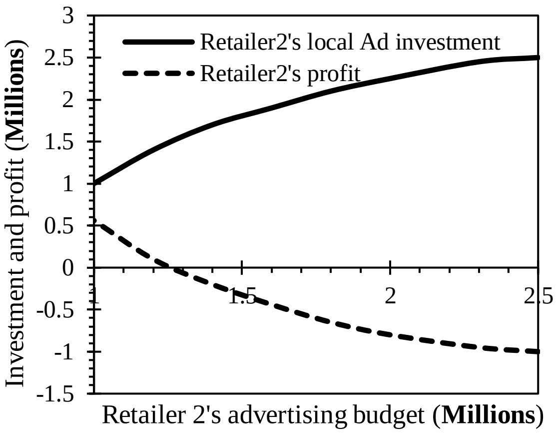

Fig. 4(b). The relationship between retailer 2's advertising investment, profit and advertising budget

Figure 4(a) demonstrates that under constant conditions, increasing the value of Q positively affects retailer earnings. On the other hand, Figure 4(b) illustrates that as the advertising budget of the second retailer increases, their inclination to invest in local advertising also increases. However, it is noteworthy that as the investment in local advertising increases, the retailer's losses also increase. In the case of a 1,250,000 unit investment, Figure 4(b) indicates that retailer 2 incurs losses. Further increasing the investment leads to a steeper slope of loss, while decreasing the investment by the same slope helps mitigate the losses. Having a high value of Q, combined with lower investment in high-risk scenarios such as when the ratio of offline channel customers is 50% or lower, can enhance retailer resilience against losses. As the retailer increases local advertising, the number of customers in their area may indeed increase. However, the retailer faces more losses despite the expectation of attracting more customers. However, the manufacturer's pricing policy is influenced by these advertising efforts and adjusts prices to balance the resulting customer demand. This price adjustment may shift customers towards online shopping channels, making it difficult for retailers with remaining offline customers to cover costs, even with increased prices in the offline channel. Generally, with the given input parameters, the model proves profitable within the range greater than 55% for all supply chain members. However, at lower ratios, one or more members of the supply chain face the risk of losses. Now, it is crucial to examine the impact of Q, specifically the number of customers inclined to purchase the manufacturer's product without advertising.

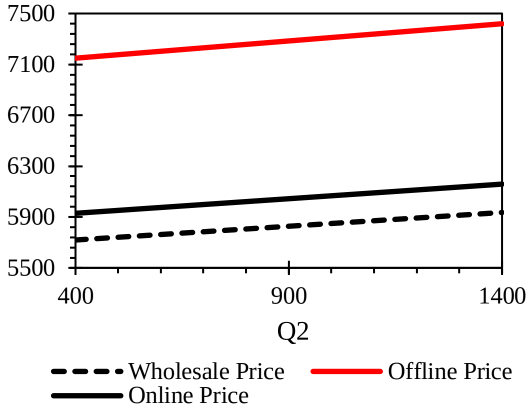

Fig. 5(a). The relationship between Q_2 and prices

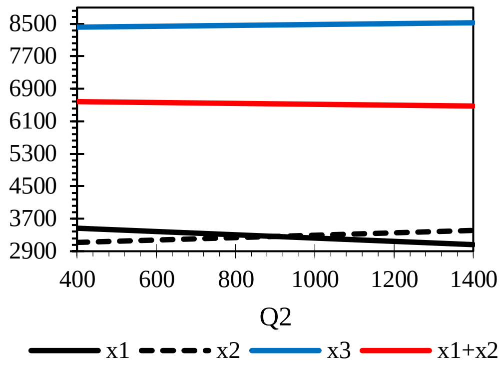

Fig. 5(b). The relationship between Q_2 and customer attraction

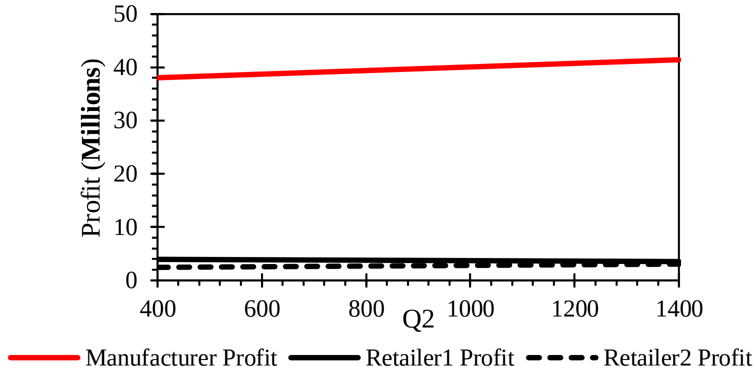

Fig. 5(c). The relationship between Q_2 and profits of supply chain members

Figure 5(a) illustrates an upward trend in offline, online, and retail prices as Q increases for retailer 2. However, it raises the question of why prices are rising as customer numbers increase. The phenomenon can be explained by the law of supply and demand. As customer numbers increase, the demand for products also increases. This surge in demand naturally leads to an increase in selling prices. Additionally, the manufacturing plant has a limited production capacity that remains constant. With more customers vying for this limited capacity, competition intensifies, driving bid prices higher. Figure 5(b) displays the number of customers for both retailer 1 and retailer 2, as well as the total number of retailers and online channel customers. This data provides insights into customer distribution and preferences across different channels. Analyzing these numbers helps understand customer behavior and the market landscape, enabling supply chain stakeholders to make informed decisions and devise effective strategies to optimize customer reach and profitability. Figure 5(c) portrays the profit or income generated by the manufacturer and retailers. As the value of Q increases, the profits of both the manufacturer and retailer 2 demonstrate an upward trend, while the profit of retailer 1 experiences a decline. This can be attributed to the downward trend observed in the customer numbers of retailer 1. Another factor contributing to the decline in retailer 1's profit is the limited production capacity of the manufacturer. The manufacturer aims to channel customers towards the online channel, as it typically yields higher profits, and allocates the remaining customer demand between the retailers. As retailer 2's Q (number of customers inclined to purchase without advertising) grows, a larger portion of the customer base is drawn towards retailer 2 and the online channel. This shift in customer preferences and allocation results in a decrease in customer demand for retailer 1. With a reduced number of customers, retailer 1 experiences a decline in sales and revenue, ultimately leading to a decrease in profit.

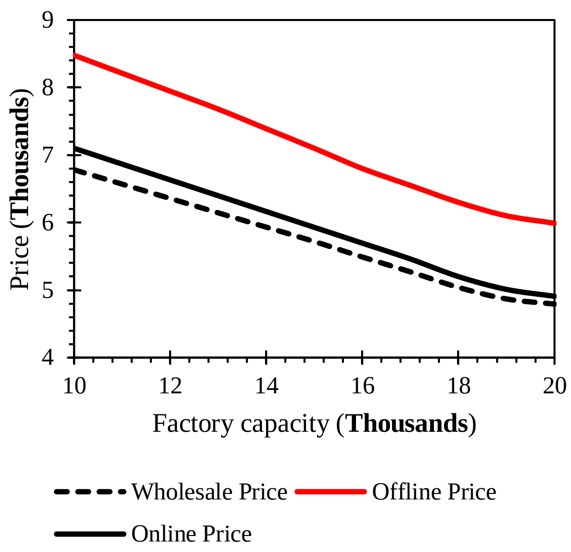

Fig. 6(a). The relationship between factory capacity and prices

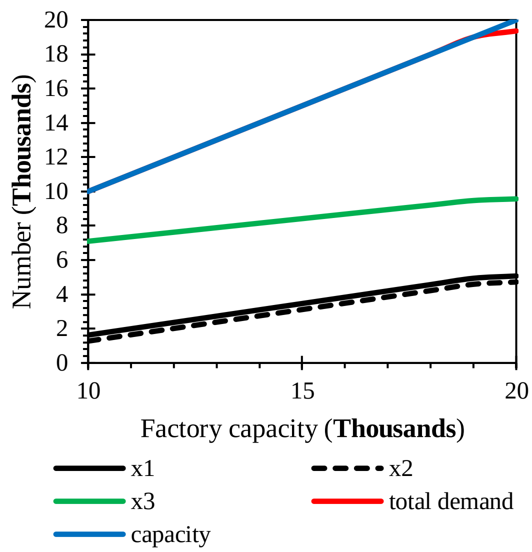

Fig. 6(b). The relationship between factory capacity and customer attraction

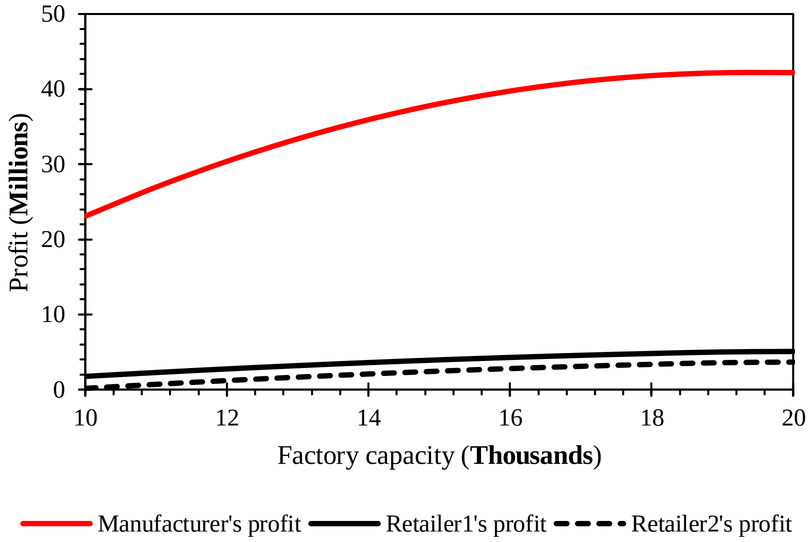

Fig. 6(c). The relationship between factory capacity and profits of supply chain members

By augmenting the factory's capacity, as depicted in Figure 6(a), the supply chain confronts a scenario where supply surpasses relatively constant demand. Consequently, this surplus supply exerts downward pressure on prices. Up to a capacity of 19,000 units, the supply chain adeptly utilizes all available capacity. However, beyond this threshold, the supply chain decides to release some of the capacity due to cost considerations, as illustrated in Figure 6(b). At the endpoints of the diagram, the aggregate demand curve lags behind the factory capacity curve. These strategies are implemented with the objective of maximizing profit, evident in Figure 6(c).

Several key points merit attention in this figure. Firstly, the profit curves for all supply chain members initially exhibit steeper slopes, gradually decreasing over time. This concept can be leveraged in models to determine the optimal plant capacity. The advantage of setting the factory capacity in this manner is that despite the decline in prices, revenue continues to increase at varying rates. Reducing final prices can enable the manufacturer to outperform competitors and establish a competitive advantage. By meticulously managing factory capacity and considering the relationship between supply and demand, the supply chain strategically positions itself to maximize profitability.

5. Conclusion

In supply chains where the manufacturer aims to establish or has already launched an online sales channel, and retains control over pricing, understanding the manufacturer's intention in this decision-making process is crucial. The primary objective for the manufacturer is typically to eliminate intermediaries and maximize their own profits. Initially, the manufacturer might consider solely establishing an online sales channel without an offline counterpart. However, several factors come into play, including the proportion of customers who prefer offline sales channels, varying across the supply chain depending on the nature of the products or services.

For example, in markets such as footwear and apparel, the proportion of customers who prefer offline purchases tends to be higher compared to the mobile phone market. This is primarily because the specifications of a particular mobile phone model remain consistent regardless of the sales channel, while the specifications of shoes (excluding branded ones) may vary, requiring customers to physically examine the product before making a purchase decision. Consequently, the manufacturer cannot disregard the profitability associated with selling to this group of offline customers. With this in mind, the manufacturer may implement strategies to gradually drive more customers towards the online channel over time. One such strategy is the concept of "showrooming," where the manufacturer establishes physical locations for customers to visit and experience the products before making purchases through online stores [61]. This approach allows customers to benefit from the convenience of online shopping while still having the opportunity to inspect the product in person. Other important considerations for the manufacturers and retailers in supply chains that have the online sales channel include:

- The decision to create an online business for the manufacturer or establish a partnership with a retailer as a selling agent may not always guarantee profitability. The findings of this study indicate that the supply chain was profitable for all its members within a specific range of customer preferences, particularly for ratios exceeding 55%. However, it is important to note that outside this range, at least one member of the supply chain experienced financial losses. These findings highlight the sensitivity of profitability to the preferences of customers and the necessity of aligning business strategies accordingly. Supply chain stakeholders must carefully assess the market conditions and customer preferences before embarking on online business initiatives or collaborations with retailers. Understanding the target customer segment and their preferences is crucial for determining the viability and profitability of such endeavors. By conducting thorough market analysis and feasibility studies, stakeholders can identify the optimal range of customer preferences within which their online business or partnership is likely to be financially viable and successful.
- Retailers must consider several factors to maximize their profits in a profitable environment. Firstly, having a high Q (the number of customers inclined to purchase without advertising) can significantly contribute to higher profits. Secondly, a higher advertising budget can also positively impact profitability by attracting more customers and increasing sales. However, it is important to note that when the interests of offline retailers are compromised, such as when the proportion of customers willing to buy offline falls below 50%, the situation changes. In such cases, the opposite becomes true, and the retailer with the

seemingly advantageous conditions (higher Q and higher advertising investment) may experience worse performance compared to another retailer. This highlights the level of risk and rewards associated with different market conditions. Retailers with higher Q and higher advertising investments face higher risks, particularly when the market conditions become critical. In such scenarios, these retailers may incur more significant financial penalties compared to their counterparts. These findings emphasize the need for retailers to carefully assess the market conditions, customer preferences, and the balance between offline and online sales channels. It is crucial for retailers to weigh the potential risks and rewards of their strategic decisions, including investments in advertising, to optimize their profitability and mitigate potential financial challenges in dynamic market environments.

- While a high advertising budget can bring benefits to a retailer, it is essential to consider market conditions and other constraints. As observed, increasing the advertising budget by a single unit may yield fewer benefits for a retailer compared to the losses experienced by another retailer. This imbalance in returns can create a disadvantageous situation for the retailers as they compete against each other. In a cooperative setting between retailers, it is crucial to strive for a balanced and equitable distribution of investment in local advertising. This ensures that no retailer inflicts significant losses on their competitors in pursuit of marginal profits. Imbalanced investment strategies can lead to intense competition among retailers, ultimately benefiting the manufacturer more than any individual retailer. By maintaining a close alignment in the investment levels of local advertising, retailers can work together to create a more harmonious and cooperative environment. This approach minimizes the risks of excessive competition and maximizes the potential for mutual profitability. Ultimately, it strengthens the collective position of the retailers and enables them to negotiate with the manufacturer from a position of unity and strength.
- Increasing advertising channels can indeed bring benefits to all members of the supply chain. Continually seeking new advertising channels can be more profitable than investing more in existing channels. This concept is clearly demonstrated in Figure 7, which illustrates the profit per unit increase in advertising capital across different numbers of channels.

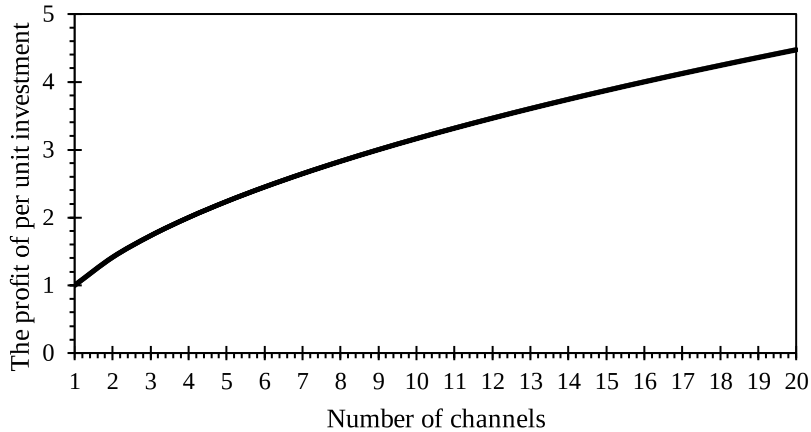

Fig. 7. The profit of per unit investment in advertising channels

6. Future Studies

Building upon the insights garnered from the current studies, the subsequent section outlines promising avenues for future research.

- Investigating the interplay of customer trust and perceived risk in shaping preferences for offline or online channels presents an intriguing avenue for future research. Trust, a pivotal factor in customer-retailer relationships, profoundly influences purchasing behavior. Understanding how trust impacts channel choices and online transaction willingness can furnish crucial insights for retailers and manufacturers. Research in this realm may explore factors contributing to online trust-building, including website security, transparent policies, customer reviews, and reliable customer support. Additionally, delving into customers' perceptions of risks associated with offline and online purchases can illuminate their decision-making processes, encompassing product quality, delivery delays, returns, refunds, and privacy concerns influencing channel preferences.

- A crucial aspect for future research is the incorporation of production capacity adjustment within the model. While the current focus is on pricing and competition, integrating multiple manufacturers and analyzing their interactions provides a more realistic market insight. Introduction of competing manufacturers enables exploration of competitive dynamics and their impact on various supply chain variables, offering valuable insights into market share, pricing decisions, product differentiation, and overall supply chain performance. Furthermore, studying the effects of production capacity adjustments on factors like production efficiency, delivery lead times, and customer satisfaction optimizes supply chain operations. Expansion of the model in these dimensions allows researchers to comprehend the implications of production capacity adjustments and competition for industry practitioners and policymakers.
- In the context of advertising channels, an important consideration is the absence of prerequisites and advertising requirements in the current models. Recognizing that customers' decisions to purchase through local advertising are often preceded by exposure to global advertising is crucial. Global advertising campaigns play a pivotal role in raising awareness and generating interest. Therefore, the impact of local advertising should be seen as a complementary effort that reinforces messaging and value proposition from global advertising.
- Future research aiming for pricing models closer to actual market dynamics can explore diverse supply chain models involving competitive interactions among multiple players. Analyzing how competitive influences impact offline and online prices through the consideration of competing supply chains offers a more realistic market representation. This approach provides insights into how pricing decisions are shaped by competitive pressures and the actions of various supply chain participants, moving closer to real-world market scenarios.

Undoubtedly, addressing these topics through modeling and analysis holds significant potential for providing valuable insights and benefits to market players. By investigating customer preferences, advertising strategies, production capacity adjustments, and competition strategies, researchers can offer practical solutions and recommendations tailored to the diverse needs of industry stakeholders.

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