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RESEARCH PAPER

A Modified Harris Hawks Algorithm to Solving Optimization Problems

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ABSTRACT
Harris Hawks Optimization (HHO) algorithm, which is a new metaheuristic algorithm that has shown promising results in comparison to other optimization methods. The surprise pounce is a cooperative behavior and chasing style exhibited by Harris' Hawks in nature. To address the limitations of HHO, specifically its susceptibility to local optima and lack of population diversity, a modified version called Modified Harris Hawks Optimization (MHHO) is proposed for solving global optimization problems. A mutation-selection approach is utilized in the proposed Modified Harris Hawks Optimization (MHHO) algorithm. Through systematic experiments conducted on 23 benchmark functions, the results have demonstrated that the MHHO algorithm offers a more reliable solution compared to other established algorithms. The MHHO algorithm exhibits superior performance to the basic HHO, as evidenced by its superior average values and standard deviations. Additionally, it achieves the smallest average values among other algorithms while also improving the convergence speed. The experiments demonstrate competitive results compared to other meta-heuristic algorithms, which provide evidence that MHHO outperforms others in terms of optimization performance.

KEYWORDS: *Metaheuristics; Nature-inspired algorithms; Harris hawks algorithm; Evolutionary algorithms; Global optimization problems.*

1. Introduction

Optimization is the process of finding the best solution for a given system from all possible values in order to maximize or minimize the output. Over the past few decades, as the complexity of problems has increased, new optimization techniques have become necessary to solve optimization problems of both mathematical and combinatorial types [1]. If the optimization problem is simple or if the search space is small, conventional analytical or numerical procedures can be used to solve it. However, if the optimization problem is difficult or if the search space is large, conventional mathematics or numerical induction techniques may not be sufficient [2]. This means that new optimization techniques are needed to effectively solve these more complex and challenging problems.

Meta-heuristic optimization methods are used to solve difficult optimization problems when conventional analytical or numerical procedures are not sufficient. These algorithms include the Genetic algorithm (GA) [3], simulated annealing (SA) [4], ant colony algorithm (ACA) [5], and particle swarm (PS) [6]. Meta-heuristic algorithms do not always guarantee an optimal solution, but in most cases, a near-optimal solution can be obtained in much less time than the computational methods [2]. These algorithms use random search in problem-solving space and rely on random operators to provide suitable solutions to optimization problems [7]. They are simple, flexible, and able to avoid local optima, which makes them widely used in solving various complex optimization problems in the real world. There are three main categories of meta-heuristic algorithms: evolutionary, physics-based and

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swarm intelligence based techniques [8]. Evolutionary meta-heuristics were inspired by biological evolutionary processes such as the evolutionary strategy (ES) [9], evolutionary programming and genetic algorithm (GA) proposed by Holland [10].

Modified Harris Hawks Optimization (MHHO) is proposed using a mutation selection approach. Compared to known feature benchmark optimizations, MHHO showed better performance than the basic HHO [11]. The paper evaluates the proposed algorithm on 23 well-known unimodal and multimodal benchmark functions and compares it with basic HHO as well as several other well-known meta-heuristic algorithms, and several well-known algorithms, including AO [12], WOA [13], GWO [14], ALO [15], MVO [16], and SSA [17], The experimental results show that the proposed MHHO algorithm is superior to other state-of-the-art algorithms.

The original HHO algorithm is introduced in Section 2, while Section 3 describes the proposed modified algorithm. Section 4 illustrates the experiments and results comparison, and finally, Section 5 presents the conclusion of the paper.

2. Harris Hawks Optimization

Harris Hawks Optimization (HHO) algorithm, which was proposed by Heidari et al in 2019 [11]. The HHO algorithm mimics the cooperative behavior and foraging style of Harris Hawks in nature called surprise pounce. It benefits from a small number of controlling parameters setting, simplicity of implementation, and a high level of exploration and exploitation [18].

The HHO algorithm consists of two phases: the exploration phase and the exploitation phase. Each phase of the algorithm represents a different stage in the hawk's predatory behavior. The exploratory phase involves exploring new solutions by generating random values, while the exploitative phase involves moving towards promising solutions based on their fitness values. Figure 1 shows the detailed explanation of the exploratory and exploitative phases of HHO.

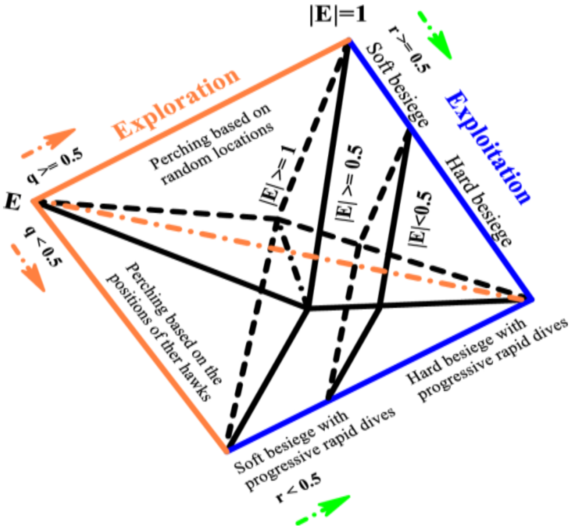

|E|=1
r >= 0.5
Soft besiege
Exploitation
Hard besiege
Exploration
Perching based on random locations
q >= 0.5
E
q < 0.5
Perching based on the positions of ther hawks
|E| >= 1
|E| >= 0.5
|E|<0.5
Hard besiege with progressive rapid dives
Soft besiege with progressive rapid dives
r < 0.5

Fig. 1. Different phases of HHO

2.1. Exploration phase

Harris hawks are known for their ability to perch randomly, wait patiently, and monitor the desert to detect prey. Two perching strategies are chosen according to the random q value and take into account the positions of other family members and the prey. In the exploration phase, let q have an equal chance for each perching strategy. When q is less than 0, Harris's hawks perch according to the positions of their other family members (so they can be close enough to them when attacking) and the rabbit. Alternatively, the Harris's hawks snooze on tall trees that are randomly located within their home range.

The exploration phase's updated position is represented in (1).

$$X(t+1) = \begin{cases} X_{rand}(t) - r_1|X_{rand}(t) - 2r_2X_i(t)|, & q \geq 0.5 \\ X_{best}(t) - X_M(t) - r_3(LB + r_4(UB - LB)), & q < 0.5 \end{cases} \quad (1)$$

The variables and terms used in the model are defined as follows:

- q: $A\ random\ number\ between$ 0 and 1.
- $X(t)$: The position of the i^{th} hawks at the t^{th} iteration.
- $X_{best}(t)$: The position of the rabbit at the t^{th} iteration.

- $r_1; r_2; r_3; r_4$: *Random numbers between 0 and 1, which are updated in each iteration.*
- LB, UB: *Lower and upper bounds of variables.*
- $X_{rand}(t)$: *A randomly selected hawk from the current population.*

And $$X_M(t) = \frac{1}{N}\sum_{1}^{N} X_i(t) \tag{2}$$

is the average position of the current population of hawks, where N denotes the total number of hawks. The locations of Harris's hawks are all within the group's home range ($LB; UB$).

2.2. Transition from exploration to exploitation phase

The Harris Hawks Optimization (HHO) algorithm incorporates a transition mechanism that switches from an exploration phase to an exploitation phase, depending on the prey's escaping energy. In this algorithm, the prey's energy is represented as gradually decreasing during its escape behavior.

$$E = 2E_0\left(1 - \frac{t}{T}\right) \tag{3}$$

The prey's escaping energy, denoted as E and initialized as E_0, determines the exploration or exploitation phase in the HHO algorithm. If $|E| \geq 1$, the algorithm is in the exploration phase, while $|E| < 1$ indicates the exploitation phase [19].

2.3. Exploitation phase

During the exploitation phase, the HHO algorithm employs four distinct chasing and attack strategies based on the prey's escaping energy and the hawks' chasing behavior. The parameter r is used to select a chasing strategy depending on whether the prey successfully escapes ($r < 0.5$) *or not* ($r \geq 0.5$) before an attack.

i. Soft besiege

When the probability of escape ($r \geq 0.5$) and the escaping energy ($|E| \geq 0.5$), the prey still possesses sufficient energy and attempts to escape. In response, the Harris' hawks softly surround the prey to deplete its remaining energy before launching an attack. The behavior of the Harris' hawks in this phase is modeled as follows:

$$X(t+1) = \Delta X(t) - E|JX_{best}(t) - X(t)| \tag{4}$$
$$\Delta X(t) = X_{best}(t) - X(t) \tag{5}$$
$$J = 2(1 - r_5) \tag{6}$$

Here, $\Delta X(t)$ represents the difference between the prey's position and its current position, and J is a variable representing the random jump strength.

ii. Hard besiege

When the probability of escape ($r \geq 0.5$) and the escaping energy ($|E| < 0.5$), the prey's energy is low, and the Harris' hawks readily encircle it before (2) launching an attack. The positions of the prey and the hawks are updated using the following equations:

$$X(t+1) = X_{best}(t) - E|\Delta X(t)| \tag{7}$$

iii. Soft besiege with progressive rapid dives

When the prey has enough energy to successfully escape ($|E| \geq 0.5$) and ($r < 0.5$), the Harris' hawks perform a soft besiege with several rapid dives around the prey to progressively correct its position and direction. This behavior is modeled using the following equations:

(3)

$$Y = X_{best}(t) - E|JX_{best}(t) - X(t)| \tag{8}$$
$$Z = Y + S \times LF(D) \tag{9}$$
$$X(t+1) = \begin{cases} Y & if\ F(Y) < F(X(t)) \\ Z & if\ F(Z) < F(X(t)) \end{cases} \tag{10}$$

Here, S is a random vector, and the better position between Y and Z is selected as the next position.

iv. Hard besiege with progressive rapid dives

When (|E| < 0.5) and (r < 0.5), indicating that the prey has insufficient energy to escape, the hawks perform a hard besiege by decreasing the distance between their average position and the prey. This behavior is modeled using the following equations:

$$Y = X_{best}(t) - E|JX\_best\ (t) - X_M(t)| \tag{11}$$
$$Z = Y + S \times LF(D) \tag{12}$$
$$X(t+1) = \begin{cases} Y & if\ F(Y) < F(X(t)) \\ Z & if\ F(Z) < F(X(t)) \end{cases} \tag{13}$$

Here, only the better position between Y and Z is selected as the next position for the new iteration.

The flowchart of Harris Hawks algorithm can be seen in Figure 2 [20].

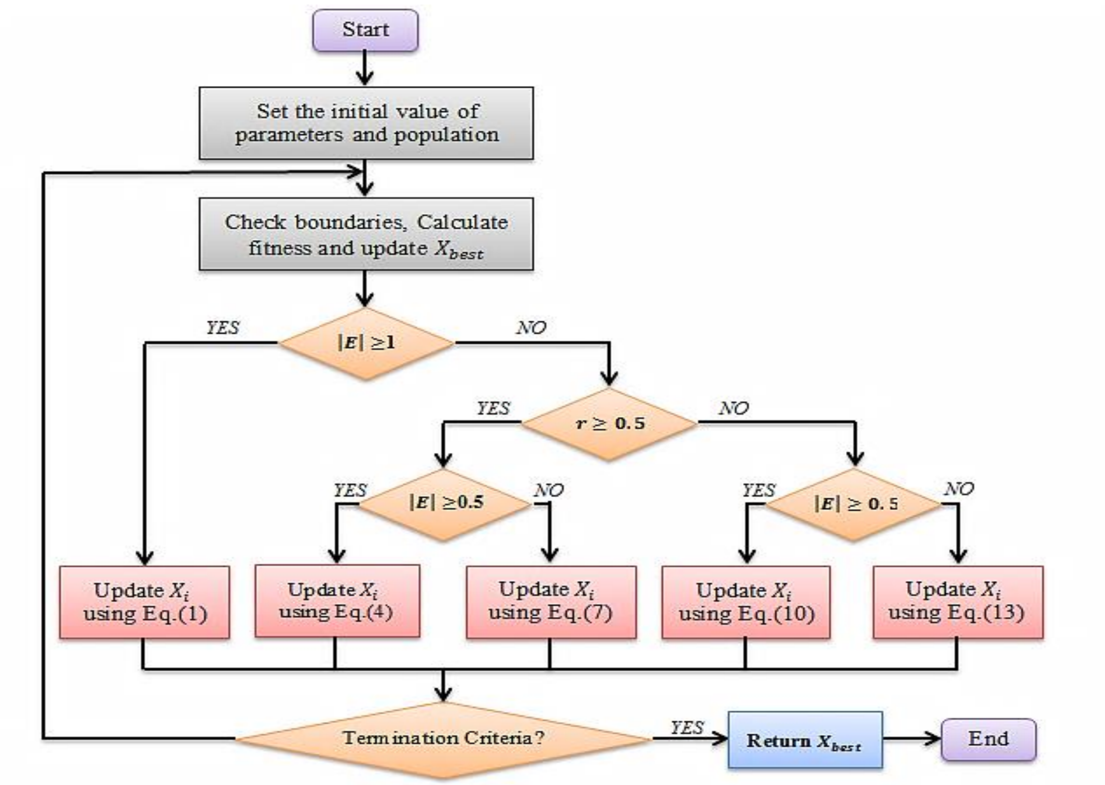

Start
Set the initial value of parameters and population
Check boundaries, Calculate fitness and update X_{best}
YES
$|E| \geq 1$
NO
YES
$r \geq 0.5$
NO
YES
$|E| \geq 0.5$
NO
YES
$|E| \geq 0.5$
NO
Update X_i using Eq.(1)
Update X_i using Eq.(4)
Update X_i using Eq.(7)
Update X_i using Eq.(10)
Update X_i using Eq.(13)
Termination Criteria?
YES
Return X_{best}
End

Fig. 2. The flowchart of HHO the proposed harris hawks optimization (MHHO)

In this study, a modified Harris Hawks Optimizer (MHHO) is proposed that uses a mutation operator to enhance its exploration capabilities and improve the quality of the solutions obtained. The MHHO incorporates mutation into the algorithm by introducing a mutation operator that randomly modifies the position of some of the hawks in the population at each iteration. This is done by randomly selecting a subset of the hawks and adding a small random perturbation to their positions.

To modify the HHO using mutation, we first define the best three position vectors: $X_a, X_b\ and\ X_c$, which are based on the fitness function values of the new hawks' position vector $X(t+1)$ among N individual hawks. Then, we use Eq. (14) to calculate the new mutation position vector $X(mut)$ for i$^{\text{th}}$ hawk:

$$X(mut) = X(t+1) + 2 * \left(1 - \frac{t}{t_{max}}\right) * (2 * rand - 1) * (X_a - X_b - X_c) + (2 * rand - 1)\big(X_a - X_b - X(t+1)\big) \quad (14)$$

Where $\boldsymbol{rand}$ is a random number between $(\mathbf{0}, \mathbf{1})$.

Then the position vector for the next generation $\boldsymbol{X(new)}$ can be obtained by the selection process described in the Eq.

$$X(new) = \begin{cases} \mathrm{X(mut)} & \big(F(X(\mathrm{mut}) < F(X(t+1))\big) \\ X(t+1) & (\ F(X(\mathrm{mut}) \geq F(X(t+1)) \end{cases} \quad (15)$$

The flowchart of MHHO is displayed in Figure 3.

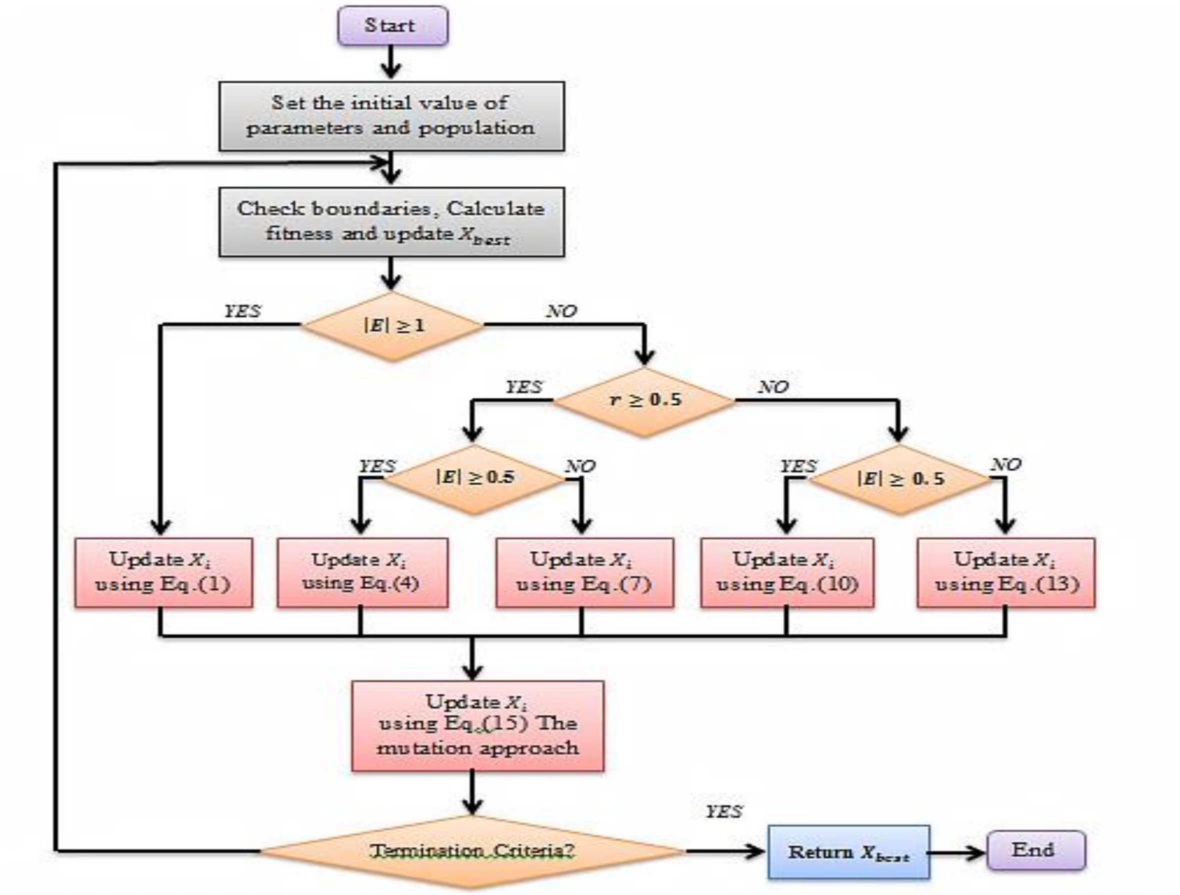

Start
Set the initial value of parameters and population
Check boundaries, Calculate fitness and update X_{best}
$|E| \geq 1$
YES
NO
$r \geq 0.5$
YES
NO
$|E| \geq 0.5$
YES
NO
$|E| \geq 0.5$
YES
NO
Update X_i using Eq.(1)
Update X_i using Eq.(4)
Update X_i using Eq.(7)
Update X_i using Eq.(10)
Update X_i using Eq.(13)
Update X_i using Eq.(15) The mutation approach
Termination Criteria?
YES
Return X_{best}
End

Fig. 3. The flowchart of MHHO

3. Results and Discussion

In this study, the performance of the proposed modified Harris Hawks Optimizer (MHHO) is examined by comparing it with the standard Harris Hawks Optimizer (HHO) using 23 well-known test benchmark functions *($f_1 - f_{23}$)*. The details of the benchmark functions are presented in Tables 1-3. The input parameters for the evaluation are $N = 30$, $T = 500$, and $\beta = 1.5$. The results show that MHHO performs significantly better than HHO on all statistical parameters for unimodal test functions ($f_1 - f_7$) and multimodal test functions with varying dimensions ($f_8 - f_{13}$) for dimension (D) = 30 and 100 are presented in Tables 4 and 5, respectively. All unimodal test functions have shown significant improvement in the MHHO compared to the HHO for all statistical parameters. The statistical results of multimodal test functions with fixed dimension ($f_{14} - f_{23}$) are presented in Table 6. It is observed that while the statistical parameter 'Best' shows similar behavior in both algorithms, the statistical parameters 'Best', 'Avg', 'Worst', and 'Std' of MHHO outperform those of HHO, indicating that MHHO has better overall statistical performance compared to HHO.

Tab. 1. Unimodal benchmark functions

Function	Function name	$\mathbf{f}_{\min}$	Range	Dim
$\boldsymbol{f_1}(\mathrm{x}) = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{x_i^2}$	Sphere	0	[-5.12 5.12]	30,100
$\boldsymbol{f_2(x)} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \lvert \boldsymbol{x_i} \rvert + \prod_{\boldsymbol{i=1}}^{\boldsymbol{n}} \lvert \boldsymbol{x_i} \rvert$	Schwefel 2.22	0	[-10 10]	30,100
$\boldsymbol{f_3(x)} = \sum_{\boldsymbol{i-1}}^{\boldsymbol{n}} \left(\sum_{\boldsymbol{j-1}}^{\boldsymbol{i}} \boldsymbol{x_j} \right)^{\boldsymbol{2}}$	Schwefel 1.2	0	[-100 100]	30,100
$\boldsymbol{f_4(x)} = \boldsymbol{max_i}\{\lvert \boldsymbol{x_i} \rvert, \boldsymbol{1 \leq i \leq n}\}$	Schwefel 2.1	0	[-100 100]	30,100

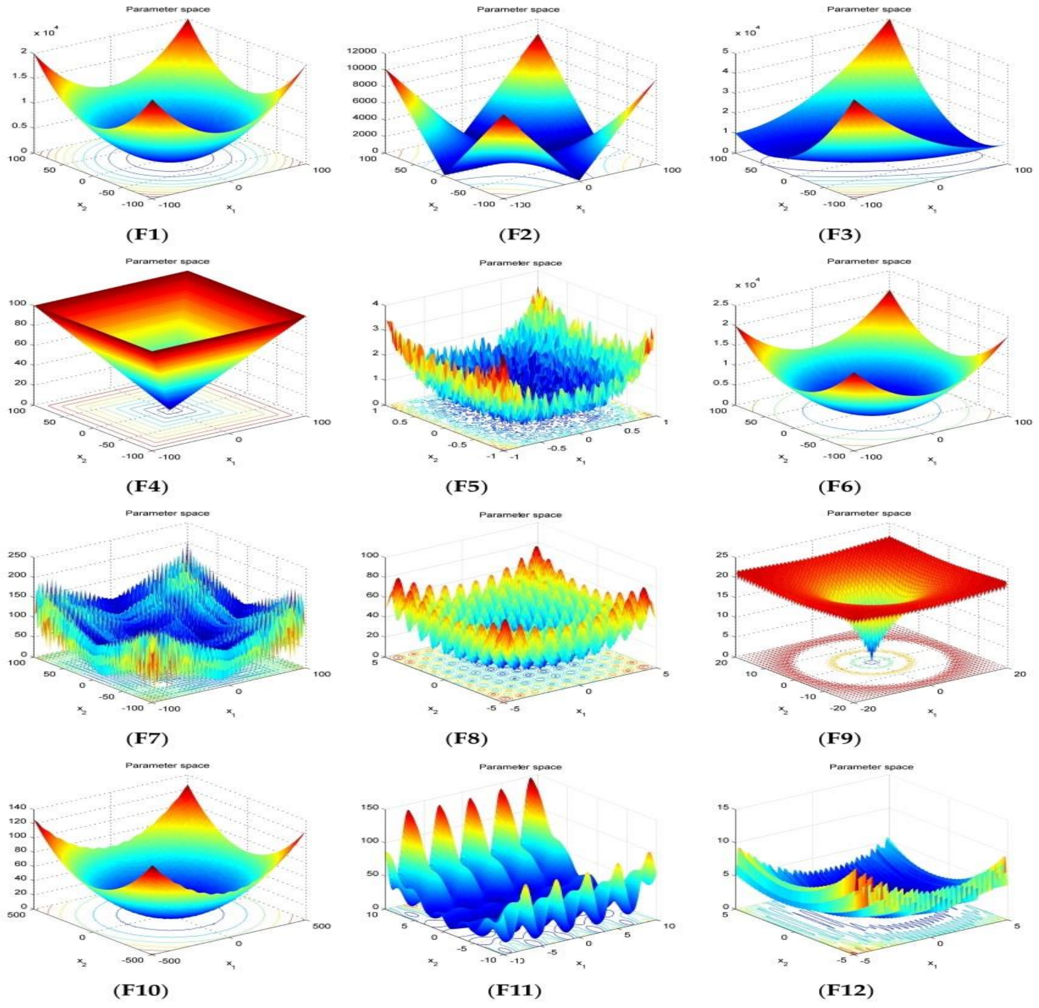
$\boldsymbol{f_5(x) = \sum_{i=1}^{n}[100(x_{i+1} - x_i{}^2)^2 + (x_i - 1)^2]}$	Rosenbrock	0	[-30 30]	30,100
$\boldsymbol{f_6(x) = \sum_{i=1}^{n}(x_i + 0.5)^2}$	Step	0	[-100 100]	30,100
$\boldsymbol{f_7(x) = \sum_{i=1}^{n} ix_i^4} + \text{random}[\boldsymbol{0,1}]$	Quartic	٠	[-1.28 1.28]	30,100

Tab. 2. Multimodal benchmark functions

Function	Function name	f $_{\text{min}}$	Range	Dim
$f_8(x) = \sum_{i=1}^{n} -x_i sin(\sqrt{\lvert x_i \rvert})$	Schwefel 2.26	-12569.5	[-500 500]	30,100
$f_9(x) = \sum_{i=1}^{n}[x_i{}^2 - 10cos\ (2\pi x_i) + 10]$	Rastrigin	٠	[-5.125.12]	30,100
$f_{10}(x) =$ $-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)\right) + 20 + e$	Ackley	0	[-32 32]	30,100
$f_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	Griewank	0	[-600 600]	30,100
$f_{12}(x) = \frac{\pi}{n}\left\{10sin(\pi y_1)\right.$ $+ \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10sin^2(\pi y_{i+1})]$ $\left. + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10,100,4)\ where\ y_i$ $= 1 + \frac{x_i + 1}{4}$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i > -a \end{cases}$	Pendlized	0	[-50 50]	30,100
$f_{13}(x) = 0.1\ \ (sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i - 1)^2[1 + sin^2(3\pi x_i +$ $1)] + (x_n - 1)^2[1 + sin^2(2\pi x_n)]) + \sum_{i=1}^{n} u(x_i, 5,100,4)$	Generalized Pendlized	0	[-50 50]	30,100

Tab. 3. Fixed-dimension multimodal benchmark functions

Function	Dim	Range	$f_{\min}$
$F_{14}(x)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}\right)^{-1}$	2	[-65, 65]	1
$F_{15}(x)=\sum_{i=1}^{11}[a_i-\frac{x_1(b_i^2+b_i x_2)}{b_i^2+b_i x_3+x_4}]^2$	4	[-5, 5]	0.00030
$F_{16}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+x_2^4$	2	[-5, 5]	-1.0316
$F_{17}(x)=(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6)^2+10(1-\frac{1}{8\pi})\cos x_1+10$	2	[-5, 5]	0.398
$F_{18}(x)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)]$ $\times[30+(2x_1-3x_2)^2\times(18-32x_2+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	2	[-2, 2]	3
$F_{19}(x)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2)$	3	[-1, 2]	-3.86
$F_{20}(x)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2)$	6	[0, 1]	-3.32
$F_{21}(x)=-\sum_{i=1}^{5}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	[0, 10]	-10.1532
$F_{22}(x)=-\sum_{i=1}^{7}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	[0, 10]	-10.4028
$F_{23}(x)=-\sum_{i=1}^{10}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	[0, 10]	-10.5363



(F1) (F2) (F3)

(F4) (F5) (F6)

(F7) (F8) (F9)

(F10) (F11) (F12)

Fig. 4. Cont.

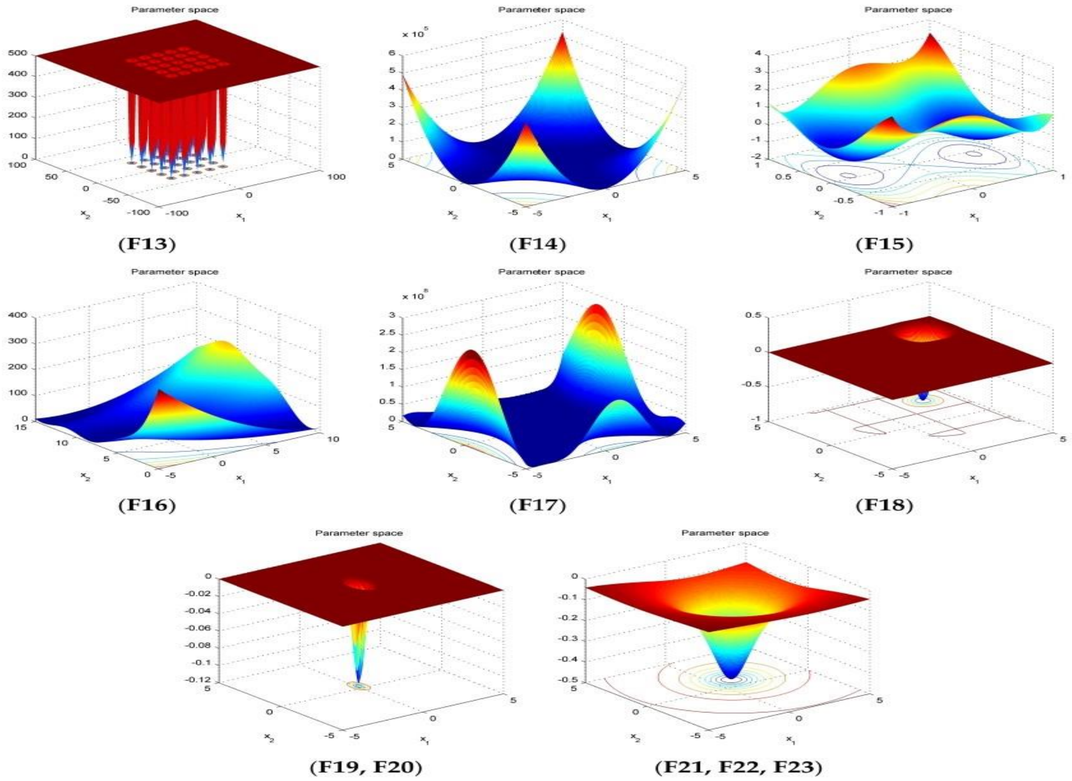


Fig. 4. 2-D versions of the 23 benchmark functions

Tab. 4. Statistical result of unimodal test functions f_{1-7}

F		D=30		D=100	
		MHHO	HHO	MHHO	HHO
f_1	Best	0	6.7688E-112	0	6.4673E-116
	Avg	0	1.5214E-92	0	1.7484E-92
	Worst	0	4.5640E-91	0	5.2453E-91
	Std	0	8.3326E-92	0	9.5765E-92
f_2	Best	3.5666E-200	1.1668E-58	1.4644e-200	9.7128E-58
	AVG	1.9239E-188	9.6485E-52	3.9984e-189	1.2200E-48
	Worst	5.7648E-187	7.5983E-51	1.1223e-187	1.9043E-47
	Std	0	2.2754E-51	0	4.0755E-48
f_3	Best	0	4.4262E-100	0	7.2533E-92
	Avg	0	4.1726E-71	0	2.0715E-55
	Worst	0	1.2517E-69	0	6.2145E-54
	Std	0	2.2853E-70	0	1.1346E-54
f_4	Best	1.8402E-204	1.1086E-57	1.6119e-207	2.9360E-58
	Avg	3.9781E-194	8.1318E-50	1.0054e-190	1.0797E-47
	Worst	1.1451E-192	1.1328E-48	3.0159e-189	2.8627E-46
	Std	0	2.4817E-49	0	5.2204E-47
f_5	Best	4.7753E-06	3.5030E-05	6.8287E-07	1.8748E-05
	Avg	1.8223E-04	7.1157E-03	5.2084E-03	3.7380E-02
	Worst	1.4096E-02	5.4955E-02	1.8227E-02	1.2460E-01
	Std	3.2308E-03	1.1185E-02	5.6396E-03	3.6143E-02
f_6	Best	3.0299E-08	7.3822E-07	7.2627E-09	3.4364E-08
	Avg	1.2310E-05	1.7328E-04	3.6229E-05	4.1778E-04
	Worst	5.0395E-05	1.2593E-03	1.8724E-04	1.8181E-03
	Std	1.4435E-05	2.7062E-04	5.0003E-05	4.7965E-04
f_7	Best	3.9238E-08	2.4104E-06	8.4245e-07	1.0025E-05
	Avg	7.0028E-05	1.4291E-04	9.1565e-05	1.5706E-04
	Worst	2.8093E-04	5.7934E-04	5.1242E-04	6.0222E-04
	Std	6.7320E-05	1.3975E-04	1.0635E-04	1.3876E-04

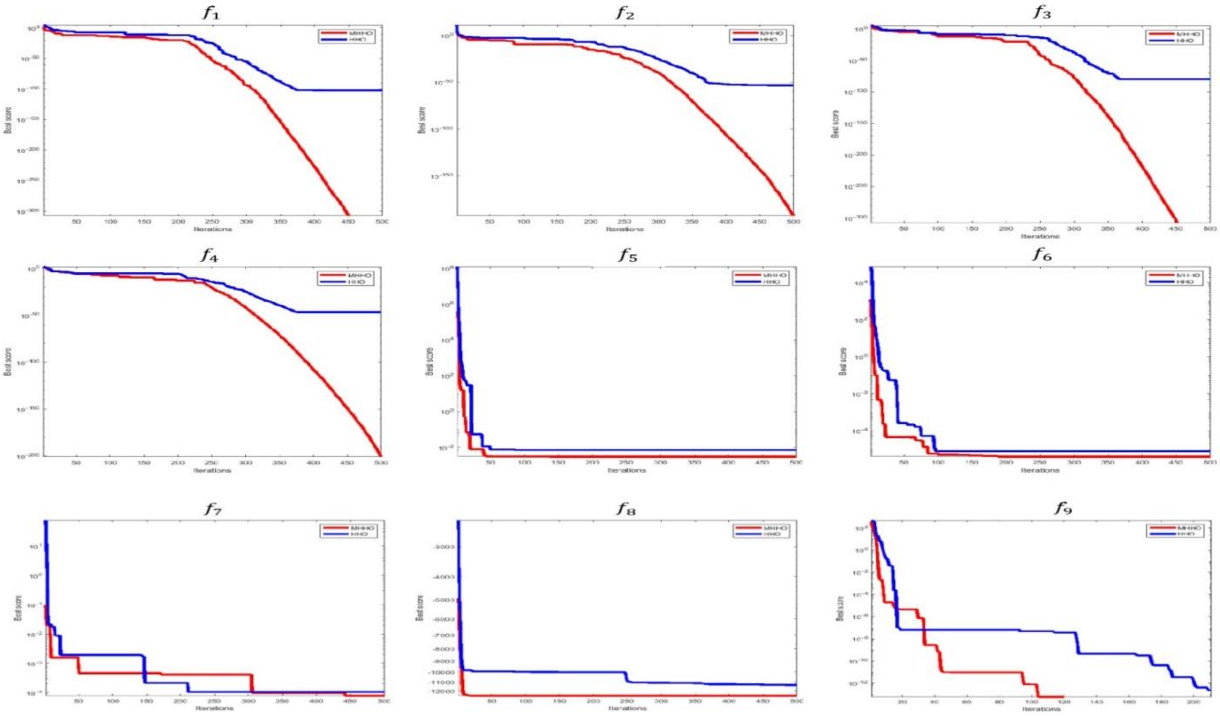
Tab. 5. Statistical result of multimodal test functions with varied dimension f $_{8-13}$

F		D=30		D=100	
		MHHO	HHO	MHHO	HHO
f_8	Best	-12569.4866	-12569.4864	-41898.2887	-41898.2867
	Avg	-12569.3292	-12569.0257	-41897.3629	-41836.5279
	Worst	-12568.8458	-12560.4554	-41890.0012	-40245.5286
	Std	0.16907	1.8515	1.7574	3.01167E+02
f_9	Best	0	0	0	0
	AVG	0	0	0	0
	Worst	0	0	0	0
	Std	0	0	0	0
f_{10}	Best	8.8816E-16	8.8818E-16	8.8816E-16	8.8816E-16
	Avg	8.8816E-16	8.8818E-16	8.8816E-16	8.8816E-16
	Worst	8.8816E-16	8.8818E-16	8.8816E-16	8.8816E-16
	Std	0	0	0	0
f_{11}	Best	0	0	0	0
	Avg	0	0	0	0
	Worst	0	0	0	0
	Std	0	0	0	0
f_{12}	Best	4.4776E-10	7.5036E-08	2.5619E-10	7.9493E-09
	Avg	5.7543E-07	9.4241E-06	3.3978E-07	2.2286E-06
	Worst	2.3659E-06	7.5690E-05	1.8122E-06	1.1854E-05
	Std	6.6886E-07	1.3937E-05	5.3589E-07	2.8315E-06
f_{13}	Best	8.8927E-10	3.1225E-07	4.9960E-09	2.4464E-07
	Avg	9.8098E-06	1.0170E-04	1.3038E-05	1.0596E-04
	Worst	4.9453E-05	7.7131E-04	4.9985E-05	8.9424E-04
	Std	1.2718E-05	1.7000E-04	1.6921E-05	1.9120E-04

Tab. 6. Statistical result of multimodal test functions with fixed dimension f $_{14-23}$

F		MHHO	HHO
f_{14}	Best	0.9980	0.9980
	Avg	0.9980	1.2618
	Worst	0.9980	5.9288
	Std	1.4484E-08	0.9320
f_{15}	Best	3.0752E-04	3.0765E-04
	AVG	3.3533E-04	3.7769E-04
	Worst	4.3460E-04	1.2358E-03
	Std	2.6497E-05	1.6720E-04
f_{16}	Best	−1.0316	−1.0316
	Avg	−1.0316	−1.0316
	Worst	−1.0316	−1.0316
	Std	7.0273E-13	4.3893E-09
f_{17}	Best	0.39789	0.39789
	Avg	0.39789	0.39790
	Worst	0.39789	0.39801
	Std	4.2042E-07	2.3909E-05
f_{18}	Best	3	3
	Avg	3	3
	Worst	3	3.0001
	Std	2.9905E-08	3.4931E-07
F		MHHO	HHO
f_{19}	Best	−3.8628	-3.8628
	Avg	−3.8628	-3.8596
	Worst	−3.8628	-3.8470
	Std	2.1843E-03	3.9540E-03

f_{20}	Best	-3.3178	-3.2710
	Avg	-3.3146	-3.0781
	Worst	-3.0157	-2.7300
	Std	9.2315E-02	1.5224E-01
f_{21}	Best	-10.1526	-9.2618
	Avg	-1.0152E+01	-5.1920
	Worst	-9.7973	-5.0272
	Std	9.0715E-02	0.76867
f_{22}	Best	-10.4028	-10.3887
	Avg	-10.4023	-5.4108
	Worst	-10.0414	-5.0496
	Std	0.10573	1.2519
f_{23}	Best	-10.5360	-10.4379
	Avg	-10.5326	-5.3024
	Worst	-10.5035	-5.1096
	Std	0.067997	0.96994



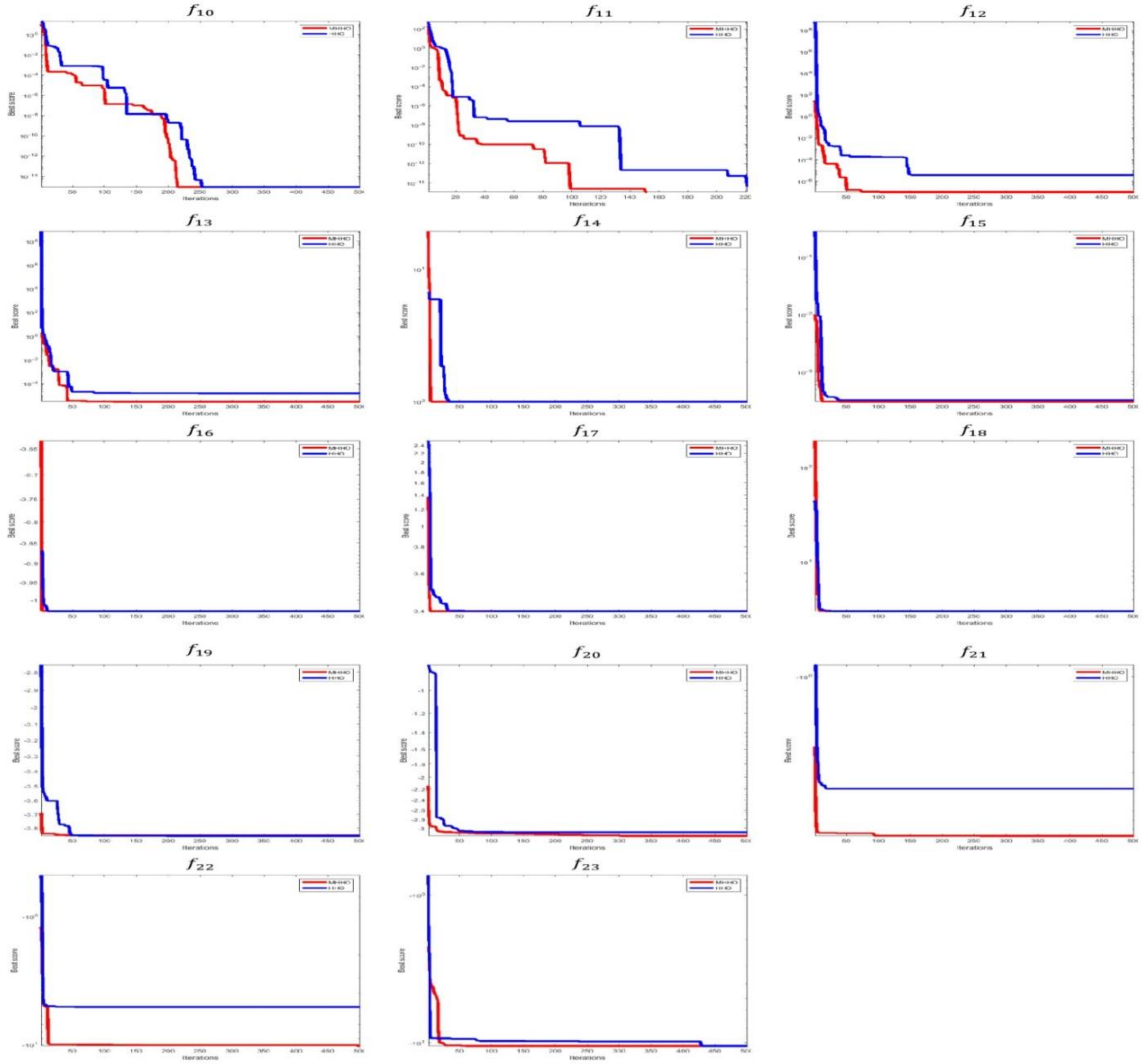


Fig. 5. Convergence curves of 23 benchmark functions.

Tab. 7. Results of algorithms on 23 benchmark functions.

F		MHHO	HHO	AO	ALO	SSA	WOA	MVO	GWO
F1	Avg	0	1.5214E-92	1.1695E-105	1.1199E-08	2.3563E-07	3.8213E-72	3.7529E-03	1.0822E-27
	Std	0	8.3326E-92	6.4056E-105	9.3293E-09	3.5574E-07	1.6267E-71	6.6839E-03	1.6083E-27
F2	Avg	1.9239E-188	9.6485E-52	1.5860E-53	6.1210E-01	1.2152E-03	2.0085E-51	1.0538E-02	9.8655E-17
	Std	0	2.2754E-51	7.7641E-53	1.0284	6.1308E-03	4.4351E-51	1.9189E-02	7.9140E-17
F3	Avg	0	4.1726E-71	2.0944E-100	6.2252E-02	3.4458E-07	4.2326E+04	3.1490E-02	6.4742E-06
	Std	0	2.2853E-70	1.0830E-99	1.4996E-01	8.0916E-07	1.4267E+04	6.7866E-02	1.3896E-05
F4	Avg	3.9781E-194	8.1318E-50	1.1030E-56	3.0504E-03	2.4092E-05	4.3274E+01	2.1050E-02	6.7978E-07
	Std	0	2.4817E-49	4.6931E-56	7.5308E-03	1.0588E-05	2.6122E+01	3.7700E-02	4.7173E-07
F5	Avg	1.8223E-04	7.1157E-03	2.0822E-03	2.0745E+02	1.8119E+02	2.7883E+01	5.7392E+01	2.6983E+01
	Std	3.2308E-03	1.1185E-02	5.2325E-03	4.1838E+02	4.9188E+02	4.6134E-01	2.6324E+02	8.1278E-01
F6	Avg	1.2310E-05	1.7328E-04	1.4464E-04	1.1392E-08	9.5077E-10	3.3461E-01	3.6083E-03	8.1764E-01
	Std	1.4435E-05	2.7062E-04	2.6457E-04	1.5952E-08	2.7909E-10	1.8794E-01	6.4474E-03	3.5552E-01
F7	Avg	7.0028E-05	1.4291E-04	1.1967E-04	2.8147E-02	1.7527E-02	3.3047E-03	1.0585E-03	2.0232E-03
	Std	6.7320E-05	1.3975E-04	1.0827E-04	2.3188E-02	1.2877E-02	3.8494E-03	2.1335E-03	1.3516E-03
F8	Avg	-1.2569E+04	-1.2569E+04	-8.2683E+03	-2.5078E+03	-2.8324E+03	-1.0132E+04	-7.4529E+02	-6.1235E+03
	Std	1.6907E-01	1.8515	3.8186E+03	6.6070E+02	3.0574E+02	1.6136E+03	1.2659E+03	8.7403E+02

F9	Avg	0	0	0	2.1756E+01	1.8340E+01	0	5.2426	3.3673
	Std	0	0	0	1.0193E+01	8.4896	0	1.1715E+01	4.8953
F10	Avg	8.8816E-16	8.8818E-16	8.8818E-16	5.5640E-01	4.8988E-01	4.5593E-15	5.0653E-02	1.0605E-13
	Std	0	0	0	7.8005E-01	8.7595E-01	2.1847E-15	2.1050E-01	2.2345E-14
F11	Avg	0	0	0	2.0832E-01	2.2219E-01	8.8107E-03	8.9057E-02	6.0027E-03
	Std	0	0	0	1.2038E-01	1.4458E-01	3.3561E-02	1.6148E-01	1.1077E-02
F12	Avg	5.7543E-07	9.4241E-06	5.0134E-06	3.2575E+00	8.8263E-01	2.9138E-02	1.8440E-04	5.1800E-02
	Std	6.6886E-07	1.3937E-05	8.4100E-07	2.3476E+00	1.1391E+00	3.7196E-02	3.8146E-04	2.8313E-02
F13	Avg	9.8098E-06	1.0170E-04	2.3524E-05	1.7998E-03	2.1975E-03	5.4662E-01	1.2867E-03	6.4978E-01
	Std	1.2718E-05	1.7000E-04	4.1595E-05	4.9381E-03	4.4701E-03	2.2510E-01	2.4803E-03	2.2023E-01
F14	Avg	9.9800E-01	1.2618	3.8977E+00	1.922	1.2622E+00	2.1757E+00	2.6613E-01	4.2353E+00
	Std	1.4484E-08	0.932	4.3742E+00	1.4149	8.1878E-01	2.7266E+00	4.4888E-01	3.7741E+00
F15	Avg	3.3533E-04	3.78E-04	5.3877E-04	2.90E-03	3.4757E-03	7.0357E-04	8.9679E-04	5.0556E-03
	Std	2.6497E-05	1.67E-04	1.4734E-04	5.97E-03	6.7446E-03	4.9718E-04	3.7016E-03	8.5896E-03
F16	Avg	−1.0316	−1.0316	-1.0311E+00	-1.0316	-1.0316	-1.0316	-2.7510E-01	-1.0316
	Std	7.0273E-13	4.39E-09	7.4847E-04	2.19E-13	1.4473E-14	3.5231E-09	4.6400E-01	1.7450E-08
F17	Avg	3.9789E-01	3.9790E-01	3.9811E-01	3.9789E-01	4.5243E-01	3.9790E-01	3.9789E-01	3.9792E-01
	Std	4.2042E-07	2.39E-05	3.2169E-04	1.00E-13	6.2542E-14	3.7443E-05	1.7896E-01	1.5465E-04
F18	Avg	3	3	3.0336	3	3	3.9002	3	3
	Std	2.9905E-08	3.49E-07	3.9347E-02	5.16E-13	1.8926E-13	4.9304	2.6080E-06	4.9586E-05
F19	Avg	−3.8628	-3.8596	-3.8573	-3.8628	-3.8628	-3.8548	-1.0301	-3.862
	Std	2.1843E-03	3.9540E-03	3.3361E-03	4.3984E-13	6.4900E-09	1.4728E-02	1.7374	2.1230E-03
F20	Avg	-3.3146	-3.0781	-3.1529	-3.2742	-3.2309	-3.2432	-8.6955E-01	-3.2653
	Std	9.2315E-02	1.5224E-01	8.0784E-02	5.9542E-02	6.1223E-02	1.1629E-01	1.467	7.3036E-02
F21	Avg	-10.1521	-5.192	-10.1395	-6.4541	-7.4774	-8.5732	-1.6184	-8.9725
	Std	9.0715E-02	0.76867	1.5383E-02	3.0026	3.4131	2.6024	3.2348	2.1735
F22	Avg	-10.4023	-5.4108	-10.3949	-6.428	-8.7129	-8.1278	-2.1665	-10.4014
	Std	1.0573E-01	1.2519	1.2478E-02	3.1889	2.9088	3.0898	3.9831	8.7213E-04
F23	Avg	-1.0536E+01	-5.3024	-1.0527E+01	-7.9175	-8.4981	-6.1469	-2.6309	-10.5348
	Std	6.7997E-02	0.96994	1.3473E-02	3.3649	3.2270	3.3436	4.534	9.3091E-04

The results presented in Tables 4-6 demonstrate the clear superiority of the proposed modified Harris Hawks Optimizer (MHHO) over the standard Harris Hawks Optimizer (HHO) algorithm when compared with 23 well-known benchmark functions for function optimizations. Specifically, MHHO obtains the smallest best, average, and worst values and standard deviations compared to HHO, indicating its better overall performance. Figure 5 also shows that MHHO has a very fast convergence speed.

To verify the results obtained with MHHO, it is compared with several other well-known meta-heuristic algorithms such as AO, HHO, ALO, SSA, WOA, GWO, and MVO. All tests are conducted with a population size of $\boldsymbol{N = 30}$, dimension size $\boldsymbol{D = 30}$, maximum number of iterations $\boldsymbol{T = 500}$, and run 30 times independently. The average and standard deviation results of these test functions are shown in Table 7. Overall, the results from this comparison further highlight the superiority of MHHO over other meta-heuristic algorithms in terms of performance.

Algorithm performance on unimodal test functions ($\boldsymbol{f_1 - f_7}$) was evaluated to assess exploitation capabilities. Unimodal functions have a single global optimum and no local optima. Table 7 shows that the proposed modified Harris Hawks Optimizer (MHHO) outperforms other selected algorithms in terms of both average and standard deviation values for all unimodal functions. MHHO achieves the smallest average and standard deviation values among all algorithms, except for $\boldsymbol{f_6}$. This indicates the high accuracy and stability of MHHO, making it highly competitive in terms of exploitation capability when compared to other metaheuristic algorithms.

Moving to multimodal test functions ($\boldsymbol{f_8} - \boldsymbol{f_{13}}$), the performance of the modified Harris Hawks Optimizer (MHHO) was also evaluated. Table 7 reveals that MHHO performs better than other algorithms in most multimodal and fixed-dimension multimodal functions. Specifically, for multimodal functions ($f_8 - f_{13}$), MHHO achieves the smallest average values compared to other algorithms. Additionally, MHHO demonstrates the smallest standard deviations, further highlighting its superior performance. Furthermore, for ten fixed-dimension multimodal functions ($f_{14} - f_{23}$), MHHO obtains the smallest average values among the algorithms. These findings emphasize the exceptional exploration capability of MHHO, which is crucial for solving complex optimization problems with multiple local optima.

4. Conclusions

In this work, a Modified Harris Hawks optimization (MHHO) is proposed. The MHHO algorithm uses a mutation-selection approach and is compared with well-known benchmark functions for function optimizations. The results show that the MHHO performs significantly better than the basic HHO algorithm in all metrics. Specifically, the MHHO shows complete superiority over HHO in terms of average and standard deviation values, indicating its better accuracy and stability. Additionally, the MHHO improves convergence speed compared to HHO.

To further evaluate the performance of the MHHO algorithm, it is compared with other recent and famous optimization algorithms such as AO, ALO, SSA, WOA, MVO, GWO, and original HHO. The experimental and evaluation results demonstrate that the proposed MHHO algorithm outperforms all these algorithms in all metrics. This indicates that the modified Harris Hawks optimization algorithm is a highly efficient and effective optimization method that can be used to solve complex problems in various fields. Overall, these results showcase the superiority of the proposed MHHO algorithm over other state-of-the-art algorithms in terms of performance and effectiveness.

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