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RESEARCH PAPER

Pricing Options based on Volatility Forecasting using A Hybrid Generalized AutoRegressive Conditional Heteroscedasticity Model and Long Short-Term Memory with COVID-19

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ABSTRACT

This paper proposes a data-driven method, using Artificial Neural Networks, to price financial options and compute volatilities, which speeds up the corresponding numerical methods. Prospects of the Stock Market are priced by the Black Scholes model, with the difference that the volatility is considered stochastic. So, we propose an innovative hybrid method to forecast the volatility and returns in Stock Market indices, which declare a model with a generalized autoregressive conditional heteroscedasticity framework .In addition, this research analyzes the impact of COVID-19 on the option, return, and volatility of the stock market indices. It also incorporates GARCH option models network with a traditional artificial neural network and COVID-19 to generate better volatility and option pricing forecasts. We appraise the models' performance using the root second-order quadratic function means of the out-of-sample returns powers. The results illustrate that the autoregressive conditional heteroscedasticity forecasts can serve as informative features to significantly increase the predictive power of the neural network model. Integrating the long short-term memory and COVID-19 is an effective approach to construct proper neural network structures to boost prediction performance. Finally, we interpret the sensitivity of option prices concerning the market or model parameters, which are essential in practice.

KEYWORDS: *Option pricing; Volatility; Stock returns; Artificial neural networks; COVID-19.*

1. Introduction

Risk is an integral part of financial derivatives and risk management is an important angle for Looking at financial derivatives. Over time, various tools such as derivative for risk management is created. One of the reasons for the importance of financial derivatives is their role in it is risk management and transfer and allows economic traders to cover market-related risks. In general, financial derivatives whose value is derived from the value and price of the underlying asset.

Financial options are noticed in the assessment of derivatives, which are the prevalent derivatives applied to manage risk. An option offers the right to buy and sell underlying assets (but not the obligation) [1].

Volatility is an important concept in finance, as it is noticed as a measurement of risk [2]. It plays a significant role in many financial usages, such as derivative pricing and hedging, risk management, and portfolio management. It is one of the essential parameters in option pricing and is often used to quote options instead of their monetary value [3]. Their immense reputation can be credited to the effort of Black - Scholes [4], who suggested a stochastic model for calculating their market value. Results using the Black–Scholes model vary from real-world prices because of facilitating hypotheses of the model. Based on studies and empirical evidence, the classical Black Scholes model in most cases cannot properly express the statistical characteristics of time series. In this case, it is important to state two points: first: the logarithm of returns in all markets does not behave in accordance with the normal distribution. Second: volatility changes

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randomly over time. The volatility has been seen as non-constant, directing to models such as Generalized AutoRegressive Conditional Heteroscedasticity (GARCH) to model volatility variations [1]. The volatility behavior of point and futures prices for goods creates a variant set of issues because of its inherent multidimensionality. Multivariate GARCH procedure estimation methods are vigorous, especially for a large matrix of GARCH covariance. Based on [5-8], the significant GARCH models with several variables like the stable conditional correlation model of [9], correlation matrix that facilitates modeling of the correlation structure, Forecasting conditional covariance matrices of returns, the dynamic conditional correlation model of Engle [10-11] HEAVY model are discussed.

Long short-term memory (LSTM) is an artificial recurrent neural network (RNN) architecture used in the field of deep learning. By recommending a three-gate structure (input gate, forget gate, and output gate), Long Short Term Memory (LSTM) networks [12] have been stated to amend the original RNNs. Unlike standard feedforward neural networks, LSTM has feedback connections. Hierarchical LSTM structures have also been applied in contextual occurrence prediction [13] and activity distinction [14].

For the recent decade, universal financial markets have endured several climatic shocks, including the 911 campaigns in 2001, subprime crisis in the fall of 2007, Lehman Brothers collapsed in September 2008, 2009 European monarch -debt crisis, and 2018-2019 US-China commerce war, etc. However, it is scarce to observe that the contagious disease episodes make the financial market chaotic. In addition, volatility is used in asset pricing and hedge, risk management, portfolio selection, and other economic occurrences. For this reason, we decide to gain whether the COVID-19 pandemic event will operate the dynamic volatility changes.

As mentioned, one of the restrictions of the Black Scholes model is constant volatility, which we consider to be stochastic in this paper. However, in this study, the option is priced by considering the volatility, which combines neural network and time series analysis models with COVID-19. We have tried to achieve a more appropriate model. We also implemented this new model on the Iran stock index. The results show the relative success of hybrid models in prediction.

Our paper is structured as follows. Section 2 presents the literature review. Section 3 describes the theory of the proposed model. Section 4 describes data characteristics and Empirical results. Section 5 presents conclusions.

2. Literature Review

The first time, Black Scholes and Merton [4] valued options. Several conditions are considered for their models, one of these limitations is constant volatility [15]. Following studies, this assumption was updated over time (Hull and White [16-17]. Although these models do not consider volatility constant, in practice their computation is troublous. Implied volatility can be used to solve this problem that changes based on activity in the options marketplace. A GARCH model makes it possible to better explain discrete observations based on market data. The first GARCH model for valuing European trading options was suggested by Duan [18-24] described a closed form for options pricing. Heston and Nandy valued options when asset volatility follows the GARCH model with free latency and correlates with asset returns. Badescu and Kulperger proposed a new method for calculating trading options using a density estimator.

Su et al. indicate that these models give fewer pricing errors than the Practitioner's Black Scholes model. Bahamonde and Veiga extend the robust closed form estimator of the GARCH (1,1) with replacing the estimators of the sample autocorrelations for estimating volatility.

Option pricing valuation models with various volatilities are widely spread, which can be referred to the types of models present in Table 1.

Tab. 1. Different literature studies on option pricing

Models	Authors	Volatility
BLACK-SHOLES-MERTON	Black & Sholes & Merton (1973)[4]	Constant
HESTON	Heston (1993) [25]	Stochastic
Developed HESTON	Lee, Min-Ku, and Jeong-Hoon Kim (2018)[26]	Stochastic
Developed GARCH	Hang & others (2017)[27]	Time Series
SIMONATO	Simonato (2019)[28]	Time Series
Zhang and Watada	Zhang &Watada (2019)[29]	Stochastic

HESTON by rational stochastic volatility	Chang, Wang& Zhang(2021)[30]	Stochastic
Stochastic Volatility HESTON	Boukai (2021) [31]	Stochastic
Implicit Stochastic Volatility	Sahlya & Li (2021) [32]	Implicit Stochastic
Our Model		Time Series and Artificial Intelligence with Covid19

[33-43] external factors such as the state of the economy as a whole have studied the systematic market risk for GARCH trading options valuation models. Baron Adsie et al. Valued options using a new model based on GARCH models with filtered historical changes in an incomplete market. Asymmetric and GARCH jump models for market movements were developed by Chiang and Huang. Their model has performed remarkably well in a recessionary economy. Wang's valuation model considers system risk and found that smile volatilities affect valuation. Christoffersen et al offered the empirical document on GARCH option pricing models such as volatility dynamic, multifactor models, nonnormal shock distributions. Monfared and Enke proposed Neural Network models to amend the efficiency of the GARCH model for estimating volatility during 1997-2011. Papantonis proposed the index of volatilities during the evaluation and their results show that the model parameters improve the evaluation in practice. Tang and Diao evaluated trading options using the Black Scholes model and the Markov-GARCH volatility model. Their conclusions show it is desirable to take this manner than the customary GARCH volatility procedure. Paul estimated Value at Risk and Expected Shortfall based on Component GARCH-Extreme Value Theory approach. Rastogi et al. applied implied volatility for options pricing in Indian market and also they compared their model with GARCH family of models. Yoo and Yoon simulated the Chicago Board Options Exchange Volatilities Index Implied in GARCH option pricing models combining the variance and jump premiums. Finaly, Escobar-Anel et al. presented a category of conditional GARCH models that bids notably added flexibility to consistent empirically related features of financial asset returns.

The GARCH(1,1) model (a member of the Garch family with a single time series) has been widely used to predict stock market volatility [44- 47]. Table 2 shows the GARCH models.

Tab. 2. Review of generalized autoregressive conditionally heteroskedastic (GARCH) models

Authors	Data Set	Models	Results
Mawardi [48]	the Indonesia Stock Exchange in the period 2011-2015	GARCH family models	The results of panel data regression analysis showed that the company's stock price volatility in the research samples.
Nguyen [49]	Data including the daily closed price of VN-Index during 2001–2019 with 4375 observations.	GARCH family models	The research results are useful reference information to help investors in forecasting the expected profit rate of the HSX, and also the risks along with market fluctuations in order to take appropriate adjust to the portfolios.
Irfan, Kassim, Shaikh[50]	Major sustainable indices worldwide are gathered to examine the best fit volatility model from 2009 to 2017.	GARCH family models	The findings conclude that there is a significant impact of the regime switch on the price volatility of the sustainability indices with asymmetric behavior that exists in volatility, and positive shocks affect volatility differently than the negative ones.
Chun, Cho, Ryu[51]	The VKOSPI has been published by the Korea Exchange since 2009-2018.	GARCH family models	The VKOSPI exhibits the best forecasting performance among the volatility measures analyzed in this study.
Umar, Mirza, Rizvi, Furqan[52]	The returns in the Pakistan stock exchange (PSX) between 2006 and 2020.	GARCH family models	Asymmetries were observed for daily returns implying that news arriving in the market continuously does impact investors' sentiment and behavior.

The COVID-19 is one of the greatest human challenges of the last century. The impact of this pandemic on the global economy and financial markets cannot be hidden [53-57]. This epidemic due to fear of the future can lead to financial crises. Financial policies are needed to prevent these financial losses. In particular, the

International Monetary Fund forecasted that COVID 19 would be one of the biggest global financial crises ever [58-59].

Our main goal in this article is pricing the options and calculating the volatilities by the GARCH model along with the LSTM model and COVID-19 to accelerate the corresponding numerical methods.

The main contribution of this paper is that it proposes an LSTM model and a GARCH framework with COVID-19 to price financial options and to calculate implied volatilities to accelerate the corresponding numerical methods. LSTM can systematically consider the nonlinearity of volatility at different times. Therefore, our model prices options better.

3. Theory of the Proposed Model

3.1. The black-scholes model

This section derives the Black-Scholes model for a call option on stock as

$$C(S,t) = S_t\boldsymbol{\phi}(d_1) - e^{-r(T-t)}K\boldsymbol{\phi}(d_2)$$

where $$d_1 = \frac{\log(\frac{S_t}{K_t}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

and $$d_2 = d_1 - \sigma\sqrt{T-t}$$

where K and T are strike and maturity. $\Phi(\cdot)$ has the standard normal distribution.

3.2. The GARCH models: equations and parameters

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) is a spread of the ARCH model which merges a moving average with the autoregressive. The GARCH models are discrete-time models and thus they support changes in the time dependent volatility, such as increasing and decreasing volatility in the same series. The GARCH (1, 1) model was initially demonstrated by Bollerslev [60] that it is given by the following equation:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$
$$\varepsilon \sim N(0, \sigma^2)$$
$$\omega \geq 0, \quad \alpha, \beta \geq , \quad \alpha + \beta < 1$$

σ_t^2 :Prediction of variance for the period t

ε_{t-1}^2 :The remaining squares (sentence error) in the period $t-1$

σ_{t-1}^2 :Predicted variance for the period $t-1$

α, β, ω *:* Model parameters are estimated to predict the variance of future periods.

3.3. Parameters estimation

Given the heterogeneity effects of the GARCH model for daily stock returns residues, the ordinary least square method for estimating model parameters does not seem appropriate. So, the method of maximum likelihood estimation is employed in GARCH models [61].

In the GARCH (1, 1) model with $\alpha + \beta < 1$, $\omega > 0$ the parameters ω, α, β are unique. Let $\theta = (\omega, \alpha, \beta)$. Then one has [77]

$$L(\theta|x_0, \dots, x_n) = f(x_0, \dots, x_n|\theta) = f(x_n|\theta)f(x_{n-1}|\theta)\dots f(x_0|\theta),$$

so

$$-\log L(\theta|x_0, \dots, x_n) = -\sum_{k=0}^{n} \log f(x_k|\theta),$$

where $f(x_0, \dots, x_n|\theta)$ is the joint probability distribution of $\{X_0, \dots, X_n\}$in a GARCH model with parameters θ, resulting in the quasi maximum likelihood function

$$QL(\theta|x_0, \dots, x_n) = -\sum_{k=0}^{n} \log f(x_k|\theta)$$

which is to be minimized. In the case of normally distributed innovations e_t, we have

$$QL(\theta|x_0, \dots, x_n) = \frac{n}{2}\log 2\pi + \frac{1}{2}\sum_{k=1}^{n}\left(\log\sigma_k^2(\theta) + \frac{x_k^2}{\sigma_k^2(\theta)}\right).$$

The parameter $\hat{\theta}_n = (\hat{\omega}_n, \hat{\alpha}_n, \hat{\beta}_n)$ which minimizes this function given observations $X_0 = x_0, \dots, X_n = x_n$, or equivalently which minimizes $l_n(\theta) = \frac{1}{n}\sum_{k=1}^{n}\left(\log\sigma_k^2(\theta) + \frac{x_k^2}{\sigma_k^2(\theta)}\right)$,), is called the Quasi-Maximum-Likelihood estimator. So we solve nonlinear programming

$$\text{Arg}\min_\theta l_n(\theta) = \frac{1}{n}\sum_{k=1}^{n}\left(\log\sigma_k^2(\theta) + \frac{x_k^2}{\sigma_k^2(\theta)}\right),$$

subject to:

$\alpha + \beta < 1$, $\omega > 0$.

3.4. Review on LSTM

The LSTM (Long Short-Term Memory) encompasses specific units named memory blocks in the recurrent hidden layer. They include memory cellules by self-connections reserving the temporary case of the network with the multiplicative blocks nominated gates to direct the current of data. Any memory unit in the main structure comprised an input gate and an output gate. The input gate directs the current of input activations to the memory cellule. The output gate directs the output current of cellule activations to the remainder of the network. After, the forget gate was collected to the memory block [62]. Moreover, the modern LSTM structure includes window links from its internal cells to the gates in the same cell to learn the accurate timing of the outputs [63].

3.5. Hybrid LSTM-GARCH models

We now introduce the prediction models of stock volatility that will formally be tested and analyzed in detail by integrating the aforementioned LSTM networks with the GARCH model.

A combined model of GARCH - LSTM neural network, the amplified model of GARCH with a hidden layer and s neurons, is defined as follows:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + LSTM$$
$$= GARCH(1,1) + LSTM$$

In the first step, the input variables to the network are extracted from the basic GARCH model. After estimating the parameters and ω, α, β and, the following variables are defined as neural network inputs:

$$\varepsilon_{t-1}^{2'} = \alpha\varepsilon_{t-1}^2$$
$$\sigma_{t-1}^{2'} = \beta\sigma_{t-1}^2$$

3.6. Hybrid LSTM-GARCH models with COVID-19

The impact of COVID-19 on average returns and market volatilities is very important. This model considers a dummy variable during the COVID-19 period:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma COVID_t + LSTM$$

The $COVID_t$ is 0 for the pre-coronavirus period (1 January 2019–31 December 2019) and it is 1 during the coronavirus period.

3.7. Forecast comparison methodology

In general, evaluating forecasts is unavoidable and helps to improve performance. One of the evaluation criteria is the loss function. In this paper, the root mean square error function (RMSE) is used as a loss function for evaluation, which is defined as follows:

$$RMSE = \sum_{i=1}^{N} \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\hat{y_i} - y_i)^2}$$

where $\hat{y_i}$ is the predicted value, y_i is the observed value, and N is the number of observations.

4. Data Characteristics and Empirical Results

We focus on the daily closing values of the Iran stock Index during 11/1/2016 –11/3/2021. We analyze market turmoil under the influence of positive and negative stock return news. The statistics of daily percentage returns of the Iran Stock Index are given in Table 3. The kurtosis and skewness statistics show that there had been a deviation from normality in the series, and Jarque–Berra test confirms the result.

Tab. 3. Statistics of daily percentage returns, iran stock lndex

	Mean	Maximum	Minimum	Standard Deviation	Skewness	Kurtosis	Jarque -Berra
Stock Index	0.0023996578	0.0581095831	-0.0482772911	0.0140897397	0.0988031621	4.4988046 844	1
Fameli Index	0.0017617245	0.1815736351	-0.5433349023	0.0318348812	-4.9860306047	82.997291 1360	1

Figures 1 and 2 show the daily returns of the total index and stock FAMELI (National Copper Industries of Iran) over 11/1/2016 –11/3/2021. Figure 1 shows the sharp volatilities of the Iran's market indices during the corona virus period. In addition, this chart shows that future volatilities are due to current period volatilities. On average, all return series are reversible, indicating staticity.

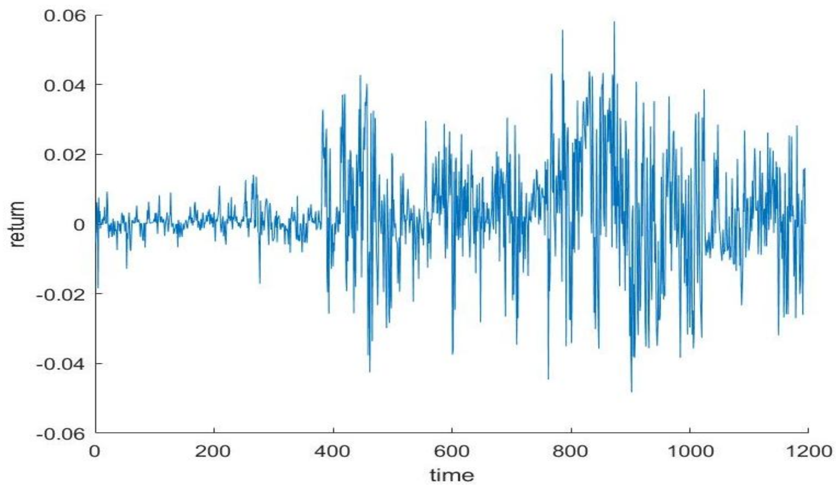

Fig. 1. Iran daily returns.

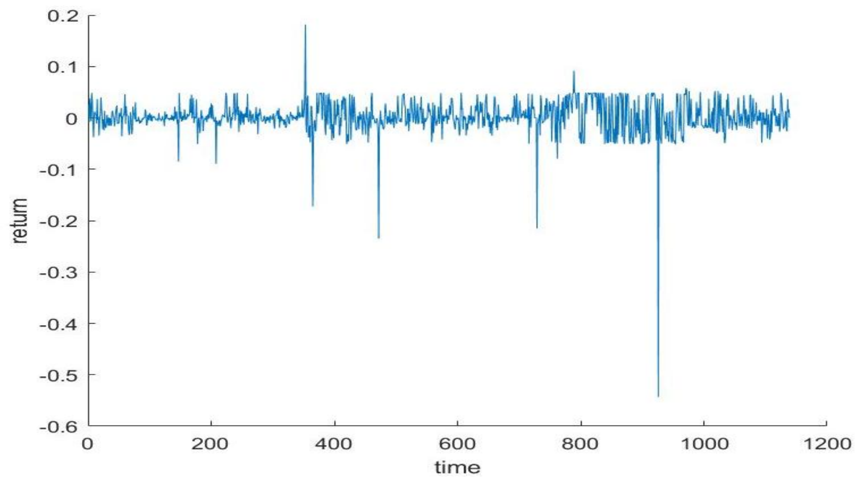

Fig. 2. FAMELI daily returns

In the following part of the study, we calculated GARCH models. The results are given in Table 4. In this table, it is possible to observe that all the series considered have significant ARCH effects and high persistence measured.

Tab. 4. GARCH (1,1) conditional variance model (gaussian distribution) results

		Value	Standard Error	T-Statistic	P-Value
	Constant	8.58394440604408e-07	2.91655495824884e-07	2.94317937735624	0.00324860128492128
Stock	GARCH{1}	0.847626609648645	0.00781918501883388	108.40344711207	0
Index	ARCH{1}	0.152373190351355	0.0135812160746522	11.2194069745891	3.27458915589776e-29
	Constant	2.57367828976314e-05	3.04975337644426e-06	8.43897185143482	3.20141172280979e-17
Fameli	GARCH{1}	0.85250408606682	0.00725654829387469	117.4806604383	0
Index	ARCH{1}	0.14749571393318	0.0106424385803949	13.859202739952	1.11910329433508e-43

Therefore, the GARCH (1, 1) model seems to represent the dynamics of the squared returns series considered adequately. However, it cannot explain the excess kurtosis present in the standardized observations.

In the next step, using the estimated parameters GARCH model, the input data for the calculation model and then the grid are trained. A hidden layer and 200 neurons are provided for network training. On the other hand, in normalizing the data entering the neural network, the data is normalized to reduce the volatility of the results.

We compared the estimated models according to the calculated RMSE. The estimated RMSE of LSTM-GARCH models of the total index and stock FAMELI are given in Figures 3 and 4. As shown in Figure 3, RMSE decreases with increasing iterations, so we regard a model to have an improved forecasting power.

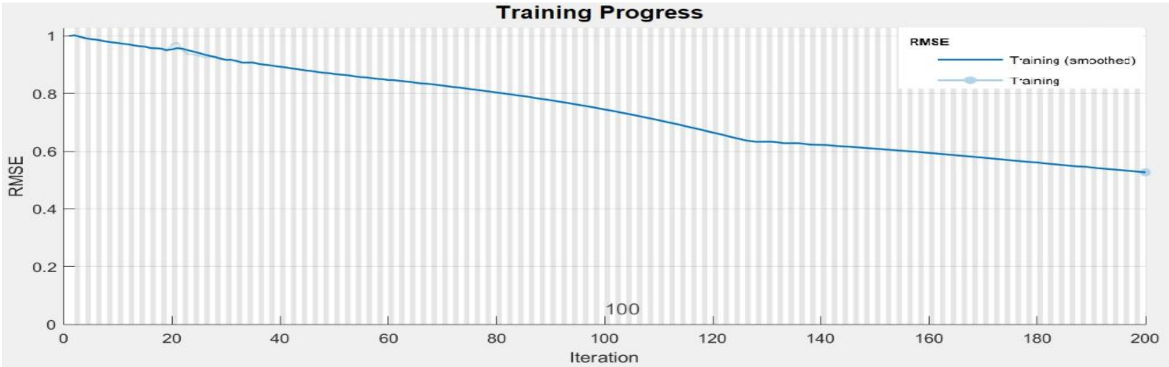

Fig. 3. GARCH-LSTM model RMSE with number of iterations of total index

Fig. 4. GARCH-LSTM model RMSE with number of iterations of stock FAMELI

In the next step, we divide the data into two categories of training (90%) and testing (10%) samples. Ninety percent of the initial data is training and the rest is testing. Figures 5 and 6 show the values trained and tested. Figures sign the prediction results and the actual values for the test data. We find that when prediction errors are measured in terms of RMSE, the best prediction models are GARCH-LSTM.

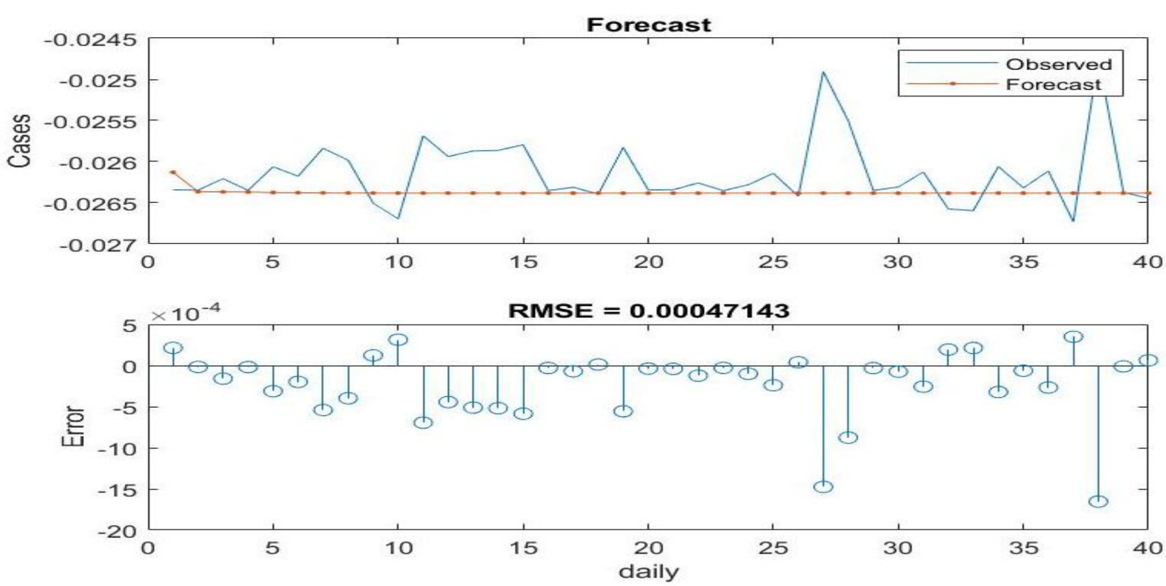

Fig. 5. Predicted and realized volatilities for total Index

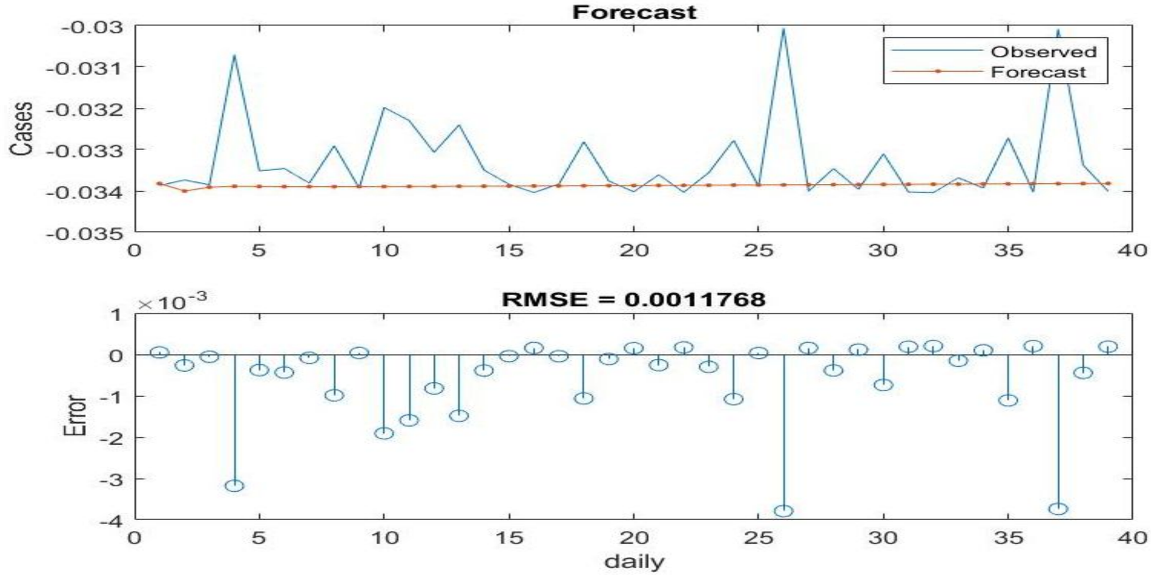

Fig. 6. Predicted and realized volatilities for stock FAMELI

Table 5 shows the numerical results of the GARCH(1,1) model with equations of conditional mean and conditional variance considering the variable COVID-19. The results indicate that COVID-19 increased market volatility.

Tab. 5. Results GARCH (1, 1) with COVID-19 variable.

		Value	P-Value
Total Stock Index	Constant	1.199002703529449	0.002004
	GARCH{1}	0.0004223311982509	0
	ARCH{1}	0.671554392169831	1.04372e-17
	γ(COVID)	-1.172663068640996	0
Fameli Index	Constant	1.318973251256311	0.001203
	GARCH{1}	0.000015706141397	0
	ARCH{1}	0.286354404450375	1.03281e-17
	γ(COVID)	-1.276104687820431	0

The results for the estimated RMSE of LSTM-GARCH models with COVID-19 are given in Figures 7 and 8. As shown in Figure 5, RMSE decreases with increasing iterations. The network specifications are listed in table 6.

Fig. 7. GARCH-LSTM- COVID-19 model RMSE with number of iterations for total index

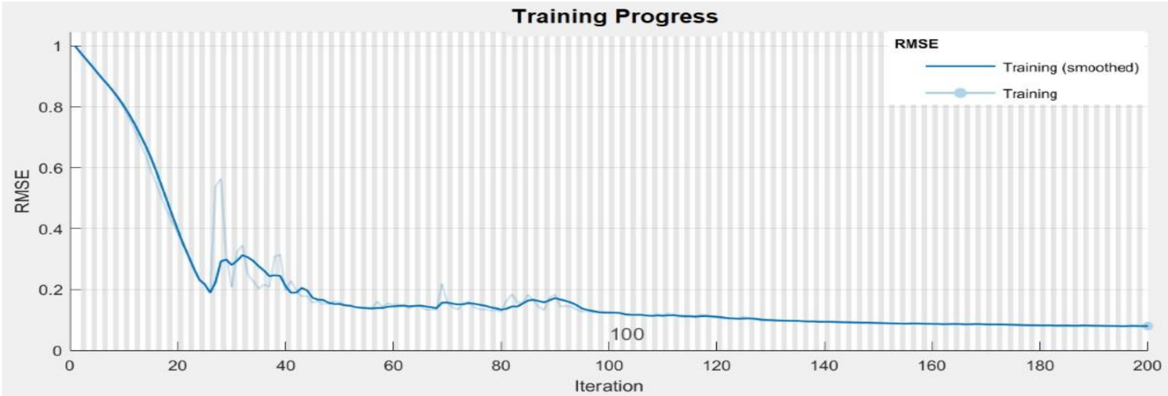

Fig. 8. GARCH-LSTM- COVID-19 model RMSE with number of iterations for stock FAMELI

Further, the predicted and realized volatilities in both the training and testing sets are shown in Figures 9 and 10. Figures show the deviation between the predicted and realized volatilities and have the following key observations. First, we can find that the GARCH model underperforms the network models, and incorporating GARCH with COVID-19 forecasts as inputs can still enhance the prediction power of the network models in general. The following comparisons can illustrate this. Let us compare GARCH-LSTM and GARCH-LSTM with COVID-19 models. The RMSE of GARCH-LSTM-COVID-19 is 0.0006849, smaller than that of the GARCH-LSTM model. Besides, GARCH-LSTM-COVID-19 architecture appears to be better than the other network architectures since the RMSE of hybrid models is always the lowest.

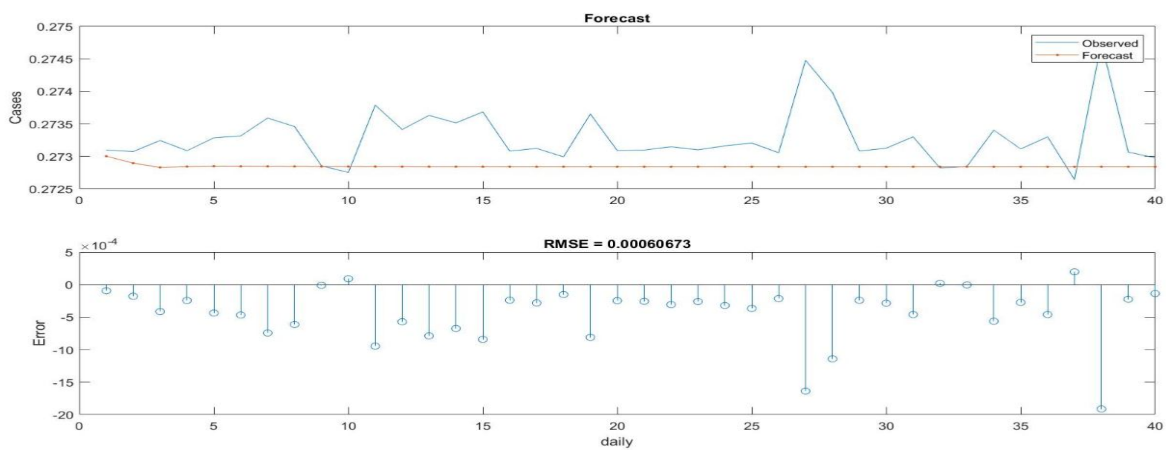

Fig. 9. Predicted and realized volatilities for total Index

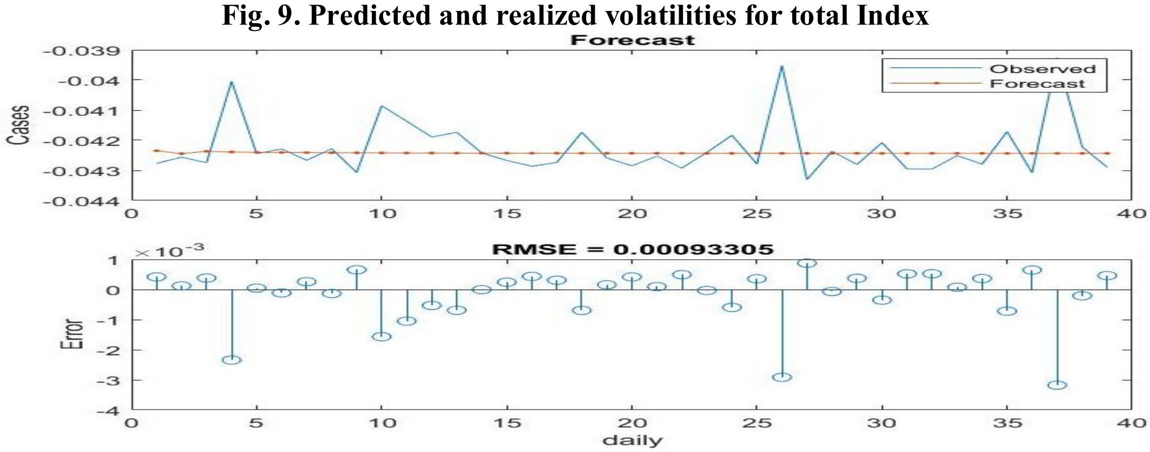

Fig. 10. Predicted and realized volatilities for stock FAMELI

We calculate the corresponding European call option prices $V(S, t)$ of Equation (9). The results contain five variables $\{St, K, r, T, \sigma\}$. Hybrid LSTM-GARCH models with COVID-19 train the volatility. The input includes $\{St, K, r, T, \sigma\}$ and the output is the call option price (Table 6). In Table 6, the option value for the four stocks of FAMELI, KHODRO, AKHABER, and SHATRAN is gained according to the total market volatility and interest rate of 21%. Table 7 shows the option price for the FAMELI stock by FAMELI volatility and interest rate 21%.

Tab. 6. Call option by r=0.21 and σ= 0.273539502500000

stock	St	K	T	Call option
FAMELI	13500	10000	78/365	3.940232493979609e+03
KHODRO	1802	2000	53/365	27.46728081051621
AKHABER	7240	8000	71/365	180.89696354750970
SHATRAN	4585	6776	87/365	1.24255125148970

Tab. 7. FAMELI Call option by r=0.21 and σ= -0.042424700000000

stock	St	K	T	Call option
FAMELI	13500	10000	78/365	3.938846481978527e+03

4.1. Sensitivity analysis of option price

Now, we analyze the sensitivity of the option price to several parameters for the FAMELI stock. The Black Scholes model is used to obtain these sensitivities, against the Black-Scholes formula being a poor approximation to actuality. So, we introduce the Greeks [2]:

Delta: Evaluates the sensitivity of an option's value to the directional movement of the underlying.

Gamma: Evaluates the rate of change in the delta relative to the underlying asset.

Vega: Evaluates rate of change of premium based on change in volatility.

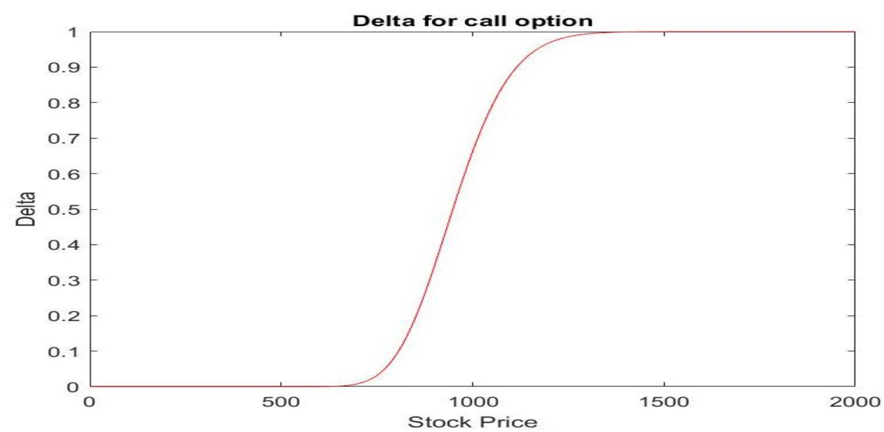

Fig. 11. Delta for FAMELI european call options

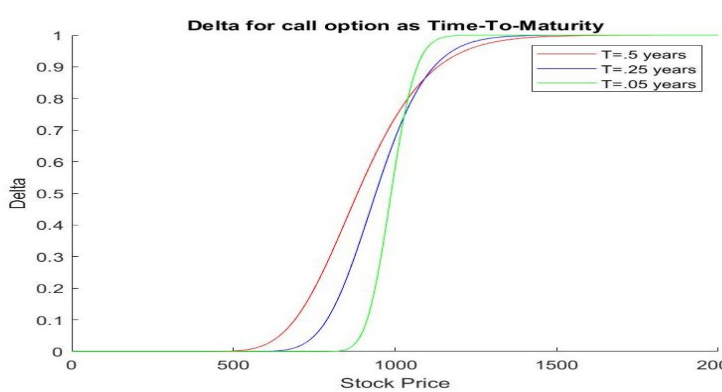

Fig. 12. Delta for FAMELI call options as time-to-maturity varies

Figure 11 demonstrates the delta for a call option as the underlying FAMELI stock price function. Figure 12 shows the delta for three variant times to maturity. Delta around the strike price becomes sharper by reducing the maturity time. Call option delta varies between 0 and 1. If delta is 0.5, the call option is in at the money. If Delta is close to 1, the call option is in the money. If Delta is close to 0, the call option is out the money.

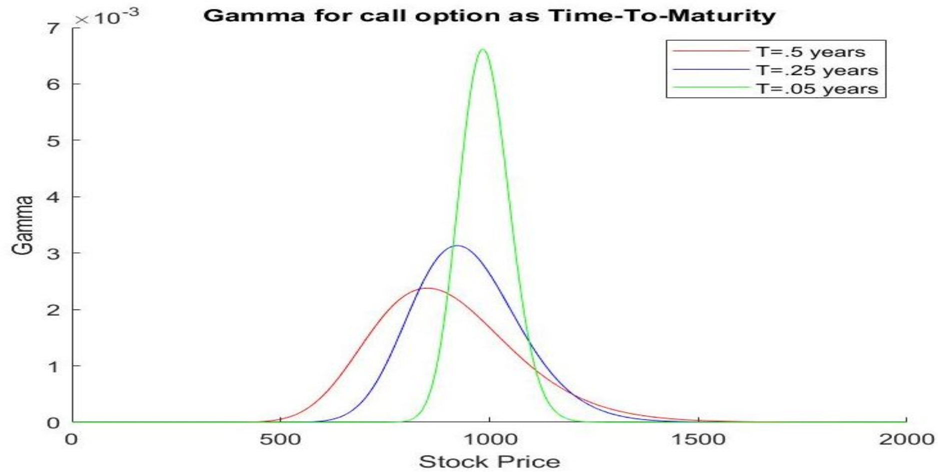

Fig. 13. Gamma for FAMEI stock price

In Figure 13, we indicate the gamma of a European FAMELI stock call option for three variant maturities time. Gamma is positive for call option. If the call option is out the money, gamma is longer. Gamma goes lower for both in the money and out the money options.

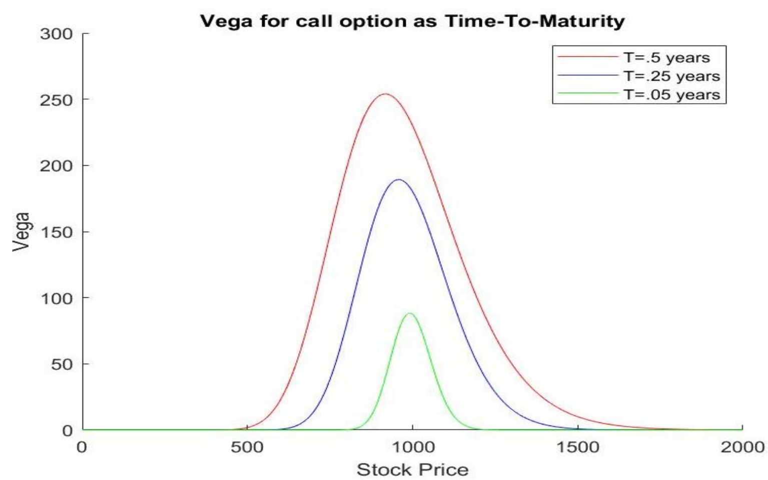

Fig. 14. Vega for FAMEI stock price

In Figure 14, we plot Vega as the underlying stock price function. Vega grows by maturity time. Vega is sharpened around K. Typically, as implied volatility grows, the value of options will increase. A growth in implicit volatility proposes an increased range of potential movement for the stock.

5. Conclusions

We started with the Black-Scholes model, which closed-form option prices with stochastic volatility. We also trained the RNN to learn the volatility based on the hybrid GARCH–LSTM-COVID-19 network. We have combined the integration of deep learning methodology and GARCH model with COVID-19 to improve the Iran Stock Market option prediction. Encountering the long memory circumstance in time series data, recurrent neural networks (RNN) is essentially suitable to instill information from sequences of inputs. In addition, GARCH models are trained, and their best forecasts are used as further inputs to amplify the training data for the hybrid neural networks.

Through the empirical analyses, we have several significant findings. First, we find that the GARCH forecasts can serve as informative features to promote the volatility prediction significantly. We also find that incorporating RNN (LSTM) into the hybrid GARCH–LSTM-COVID-19 network can further improve the volatility prediction implementation. We priced the option by substituting this volatility in the Black-Scholes model. Our numerical results show that our method can compute option prices and volatilities efficiently and accurately in a robust way.

Furthermore, the option Greeks, representing the sensitivity of option prices to the market or model parameters, are essential in practice (i.e., for hedging purposes). As RNN approximates the solution to the financial PDEs, the related derivatives can also be recovered from the trained RNN. Alternatively, a trained RNN may be interpreted as an implicit function, which can help calculate the derivatives accurately.

Extended studies in several valuable research directions will be done in the future. One important extension is to introduce option market prices into the estimation of option pricing model parameters to estimate parameters by treating minimum error between option market prices and model prices as objective function so as to increase the accuracy of parameters estimated. Another valuable extension is to introduce the Markov Switching Volatility Model into the description of volatility of asset returns. However, this method needs more complicated estimation technology, for example, Bayesian estimation. Therefore, it is hard to use this method, and our next works would be oriented in some simple and effectiveness methods, such as the state-of-the-art grey system models [55].

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