How to Use MRP in Continuous Production Industries When Order Type is Lot for Lot

Ramin Sadeghian*

Ramin Sadeghian, *Assistant Professor in Industrial Engineering, Engineering Department, Bu Ali Sina University, Hamedan, Iran,*

KEYWORDS

Discrete Materials Requirements
Planning (DMRP),
Continuous Materials
Requirements Planning (CMRP),
Order Type (OT),
Lot for Lot (L4L)

ABSTRACT

The Materials Requirements Planning (MRP) method that is applied in production planning and management has some weaknesses. One of its weaknesses is that the time in MRP method is discrete, and is considered as time period. Hence we are not able to order our requirements at irregular time moments or periods. In this paper, a new form of MRP is introduced that is named Continuous Materials Requirements Planning (CMRP) approach. We discuss the disadvantages of Discrete MRP (DMRP) approach and analyze the conditions, applications and the manner of applying CMRP approach in our problems.

1. Introduction

MRP was first introduced in 1970's; thereafter, many researchers, papers, books, industries, companies and even different sciences have applied it. (Browne J. et al., 1996; Horvat L. et al., 1999; O'Neill L. et al., 2001; Rom W. O. et al., 2002; Dolgui A. et al., 2007).
This system was formed as an approach of production planning and management. Orlicky (1975) introduced this system in a help book in 1975. The introduced MRP was the first version of MRP system, named as MRP I. Later, several MRP systems were extended into other versions including MRP II and ERP. (Browne J. et al., 1996)
In some researches, the advantages and disadvantages of MRPs have been referred to. (see Wallace T., 1980; Latham D., 1981; Hinds S., 1982; Burbidge J., 1985; Safizadeh M. et al., 1986; Harhen J., 1988; Browne J. et al., 1996). In this paper, some other advantages and disadvantages of this system will be pointed out.
Almost in all of related researches, MRP system has been considered as a DMRP; since orders, demands,

scheduled receipts and etc, are defined in *discrete* time or distinguished time periods. (Enns S. T., 1999; Johnny C. Ho. et al., 2001; Gutierrez Y. B. et al., 2008; Pillai V. M. et al., 2008)
In this paper, we explain the weaknesses of a DMRP system and the manner of applying CMRP method in some problems. In the following sections, some needed definitions and the manner of modeling CMRP are pointed out and some problems related to DMRP are solved.
Note that all over this paper, continuous production means a production with continuous time and not continuous supply or demand, or DMRP can also be applied when supply and demand are continuous.
CMRP is preferred over DMRP, because DMRP can not be applied in the industries of Petrochemical, Gas, Oil, Water and Waste and other continuous production industries, while CMRP can be. Also in CMRP approach, instead of Gross Requirements (GR), Scheduled Receipts (SR), On Hand (OH) inventory and other parameters applied in discrete form, some continuous functions such as regression functions, interpolations or extrapolations and even multi rules functions can be defined, which enables us to perform sensitivity analysis and forecast the required parameters of the model.

Corresponding author. Ramin Sadeghian
Email: rsadeghian@basu.ac.ir

2. Definitions

In DMRP approach, generally, the following notations are used, which may vary in some references (Browne J. et al., 1996; Gutierrez Y. B. et al., 2007):

GR_i : Gross Requirements during time period i

SR_i : Scheduled Receipts during time period i

OH_i : On Hand inventory during time period i

NR_i : Net Requirements during time period i

PO_i : Planned Orders during time period i

where GR and SR, during all time periods, and OH, in the beginning of planning horizon, are known, the unknown parameters can be obtained by the known ones.

For obtaining the parameters in DMRP approach the value of $OH_1, SR_1, \ldots, SR_T$ $GR_1, \ldots, GR_T, T, TT$ should be specified, where the planning horizon is [1,T], *TT* is operations' total time for each piece of product when is defined as sum of four types of time consisting of queue time (QT), process time (PT), carrying time (CT) and setup time (ST) as follows:

$$TT = QT + PT + CT + ST$$

If *OT* is Lot for Lot so:

$$NR_i = GR_i - SR_i - OH_i \tag{1}$$

$$OH_{i+1} = NR_i + OH_i + SR_i - GR_i \tag{2}$$

$$OH(t) = \begin{cases} OH(t_0) + \int_{t_0}^{t} SR(t)dt - \int_{t_0}^{t} GR(t)dt & t_0 \le t \le t_0^* \\ \int_{t_i}^{t} SR(t)dt - \int_{t_i}^{t} GR(t)dt & t_i \le t \le t_i^* \ ; i \in \{1,2,\ldots,p\} \\ 0 & Otherwise \end{cases} \tag{3}$$

where t_0 is the start time of planning horizon and t_0^* is a point that $OH(t_0) + \int_{t_0}^{t_0^*} SR(t)dt - \int_{t_0}^{t_0^*} GR(t)dt = 0$, and $t_1, \ldots, t_p \in [t_0, T]$ are the points in which $SR(t_i) = GR(t_i)$ and $\exists \varepsilon > 0 ; SR(t_i + \varepsilon) - GR(t_i + \varepsilon) > 0$, also $t_1^*, \ldots, t_p^* \in [t_0, T]$ are the points that $\int_{t_i}^{t_i^*} SR(t)dt - \int_{t_i}^{t_i^*} GR(t)dt = 0 ; \forall i \in \{1, \ldots, p\}$.

Also $t_0^*, t_1^*, \ldots, t_p^*$ should be minimum solution obtained from their corresponding equations.

$$PO_i = NR_{i+TT}$$

All of the above definitions and notations are only used in DMRP approach. In CMRP approach they are defined as follows:

$GR(t)$: Gross Requirements in time moment t

$SR(t)$: Scheduled Receipts in time moment t

$OH(t)$: On Hand inventory in time moment t

$NR(t)$: Net Requirements in time moment t

$PO(t)$: Planned Orders in time moment t

3. Modeling

Assume that $GR_1, GR_2, \ldots, GR_T$ are gross requirements in time periods 1, 2, ..., T, so $GR(t)$ function can be obtained in the form of a regression function by $GR_1, GR_2, \ldots, GR_T$. $GR(t)$ can be a polynomial, exponential or any other type of functions. Some assumptions are considered in this paper as follows:

- TT is constant and known
- OT = L4L

Similarly, $SR(t)$ can be defined by the data of past periods in the format of a continuous function.

Lemma 1: In CMRP approach, if OT and SS are equal to L4L and zero respectively, so $OH(t)$ can be determined by the following equation:

Proof:

According to Eq. 2, *OH* in DMRP approach can be written as following:

$OH_{i+1} = NR_i + OH_i + SR_i - GR_i ; \forall i = 1, \ldots, T$

It is clear that, while $OH_1 > 0$, $OH_i + SR_i \ge GR_i$, $NR_i = 0$. In such a case the above equation can be rewritten as follows:

$$OH_{i+1} = OH_i + SR_i - GR_i ; \forall i = 1, \ldots, T$$

Now, use returnable equations for the above equations as follows:

$$OH_2 = OH_1 + SR_1 - GR_1$$
$$OH_3 = OH_2 + SR_2 - GR_2 =$$
$$= OH_1 + SR_1 - GR_1 + SR_2 - GR_2 =$$
$$= OH_1 + SR_1 + SR_2 - GR_1 - GR_2$$
$$OH_4 = OH_3 + SR_3 - GR_3 =$$
$$= OH_1 + SR_1 + SR_2 - GR_1 - GR_2 + SR_3 - GR_3 =$$
$$= OH_1 + SR_1 + SR_2 + SR_3 - GR_1 - GR_2 - GR_3$$
$$\vdots$$
$$OH_{i+1} = OH_i + SR_i - GR_i =$$
$$= OH_1 + SR_1 + SR_2 + SR_3 + \cdots + SR_i - GR_1 - GR_2 - GR_3 - \cdots - GR_i =$$
$$= OH_1 + \sum_{j=1}^{i} SR_j - \sum_{j=1}^{i} GR_j$$

Therefore,

$$OH_{i+1} = OH_1 + \sum_{j=1}^{i} SR_j - \sum_{j=1}^{i} GR_j \qquad (4)$$

All relations of DMRP approach, with some subtle changes, can be used for CMRP approach. For example, summation notations are transformed into integral notations.

With respect to the above comments, the Eq. 4 for CMRP approach can be rewritten as follows:

$$OH(t) = OH(1) + \int_0^t SR(t)dt - \int_0^t GR(t)dt$$

If the beginning point of planning horizon is t_0, then:

$$OH(t) = OH(t_0) + \int_{t_0}^t SR(t)dt - \int_{t_0}^t GR(t)dt \qquad (5)$$

The above equation is correct if $OH(t_0)$ and cumulative $SR(t)$ are able to satisfy cumulative $GR(t)$, so if t_0^* is the point that

$$OH(t_0) + \int_{t_0}^{t_0^*} SR(t)dt - \int_{t_0}^{t_0^*} GR(t)dt = 0, \quad \text{and} \quad t_0^* \text{ is}$$

minimum solution obtained by the corresponding equation. It means that for the points after t_0^* ; $OH(t_0)$ and cumulative $SR(t)$ are not able to satisfy cumulative $GR(t)$, hence; Eq. 7 is correct only for interval $t_0 \le t \le t_0^*$.

Similarly, assume $t_1, \ldots, t_p \in [t_0, T]$ are the points in which $SR(t_i)$ is equal to $GR(t_i)$, so the previous paragraphs' comments are correct when

$\exists \varepsilon > 0$; $SR(t_i + \varepsilon) - GR(t_i + \varepsilon) > 0$. In this case $NR(t_i)$ will also be equal to zero, the related equation

in discrete case is $OH_{i+1} = \sum_{j=1}^{i} SR_j - \sum_{j=1}^{i} GR_j$,

because OH_1 has already been consumed completely, and in continuous case it can be written as follows:

$$OH(t) = \int_{t_i}^t SR(t)dt - \int_{t_i}^t GR(t)dt \qquad (6)$$

For determining the mentioned intervals, it is sufficient to find the points $t_1^*, \ldots, t_p^*$, subject to

$$\int_{t_i}^{t_i^*} SR(t)dt - \int_{t_i}^{t_i^*} GR(t)dt = 0 \; ; \; \forall i \in \{1, \ldots, p\}, \text{ and } t_1^*, \ldots, t_p^*$$

are minimum solution obtained from their corresponding equations. Thus, Eq. 6 is correct during the intervals $t_i \le t \le t_i^* ; \forall i = 1, \ldots, p$.

On the other hand, since *OT* is *L4L*, in the points that $NR(t)$ is not zero, the quantity of $NR(t)$ is exactly equal to the requirements of system, hence; there is not any extra inventory for stocking. Consequently in these points $OH(t)$ is equal to zero.

Therefore; the proof is completed.

$NR(t)$, $PO(t)$ can also be determined by $GR(t)$, $SR(t)$ and $OH(t)$ in any moment of time as follows:

$$NR(t) = GR(t) - OH(t) - SR(t) \qquad (7)$$
$$PO(t) = NR(t + TT)$$

In DMRP approach, *TT* should be integer, while in CMRP approach *TT* can also be considered as a real number. This fact is the other advantage of CMRP approach over the DMRP one.

Lemma 2: In a CMRP system, whose OT is equal to L4L and its SS is zero, $NR(t)$ can be determined by the following equation:

$$NR(t)=\begin{cases}0 & t_i \leq t \leq t_i^* \; ; i \in \{0,1,2,\ldots,p\} \\ GR(t)-SR(t) & Otherwise\end{cases} \quad (8)$$

where t_0 is the start time of planning horizon and t_0^* is a point that $OH(t_0)+\int_{t_0}^{t_0^*} SR(t)dt-\int_{t_0}^{t_0^*} GR(t)dt=0$, and $t_1,\ldots,t_p \in [t_0,T]$ are the points in which $SR(t_i)=GR(t_i)$ and $\exists \varepsilon > 0 \; ; SR(t_i + \varepsilon) - GR(t_i + \varepsilon) > 0$, also $t_1^*,\ldots,t_p^* \in [t_0,T]$ are the points that $\int_{t_i}^{t_i^*} SR(t)dt-\int_{t_i}^{t_i^*} GR(t)dt=0 \; ; \forall i \in \{1,\ldots,p\}$. Also $t_0^*,t_1^*,\ldots,t_p^*$ should be minimum solution obtained from their corresponding equations.

Proof:
It is similar to the proof of lemma 1.

4. Numerical Example

Example: Suppose that functions $GR(t) = -0.2t^2 + 4t + 20$ and $SR(t) = .2t^2 - 5t + 40$; $t \in [1,20]$ represent gross requirements and scheduled receipts in time moment t respectively; moreover, $OH(1)=10$ and $TT=1.5$ time units. Then, by algorithms I and II the functions of $OH(t)$, $NR(t)$ and $PO(t)$ can be determined as follows:

$$t_0 = 1$$

$$\Rightarrow OH(1) + \int_1^t SR(t) - \int_1^t GR(t) = 0$$

$$\Rightarrow 10 + \int_1^t .2t^2 - 5t + 40 - \int_1^t - .2t^2 + 4t + 20 = 0$$

$$\Rightarrow 0.134t^3 - 4.5t^2 + 20t - 5.634 = 0$$

$$\Rightarrow \begin{cases}t^* = 0.302 \notin [1,20] \rightarrow Rejected \\ t^* = 4.906 \rightarrow minimum\,solution \rightarrow Accepted \\ t^* = 28.374 \notin [1,20] \rightarrow Rejected\end{cases}$$

$$\Rightarrow t_0^* = 4.906$$

$$GR(t) = SR(t) =$$
$$\Rightarrow -0.2t^2 + 4t + 20 = 0.2t^2 - 5t + 40$$
$$\Rightarrow 0.4t^2 - 9t + 20 = 0$$
$$\Rightarrow \begin{cases}t_1 = 2.5 \\ t_2 = 20\end{cases}$$

$\nexists \varepsilon > 0 \; ; SR(t_1 + \varepsilon) \ngtr GR(t_1 + \varepsilon)$ and $t_2 = 20$ is the end point of planning horizon, thus calculating t_1^* and t_2^* is not necessary. Hence; the functions of $OH(t)$, $NR(t)$ and $PO(t)$ are obtained as follows:

$$OH(t)=\begin{cases}0.134t^3 - 4.5t^2 + 20t - 5.634 & 1 \leq t \leq 4.906 \\ 0 & Otherwise\end{cases}$$

$$NR(t)=\begin{cases}0 & 1 \leq t \leq 4.906 \\ -0.4t^2 + 9t - 20 & Otherwise\end{cases}$$

$$PO(t)=NR(t+0.5)=\begin{cases}0 & 1 \leq t+0.5 \leq 4.906 \\ -0.4(t+0.5)^2 + 9(t+0.5) - 20 & Otherwise\end{cases}$$

$$\Rightarrow PO(t)=\begin{cases}0 & 1 \leq t \leq 4.406 \\ -0.4t^2 + 8.6t - 15.6 & Otherwise\end{cases}$$

In the first rule of *PO(t)* function, *t* can be placed in the interval [0.5,4.406]. Since the planning horizon is [1,20], the mentioned interval is written as [1,4.406].
For comparing the obtained functions and their curves, see figures 1 to 4: (Note that in all charts of this paper, the horizontal axis shows time unit and the vertical axis shows the specified parameters in each chart.)
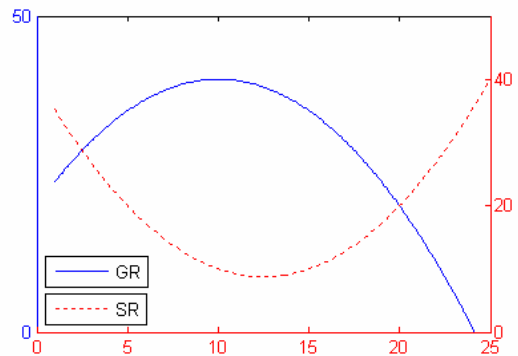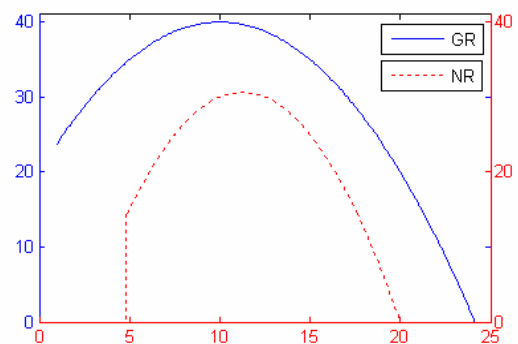

Fig. 1. The curves of GR and SR

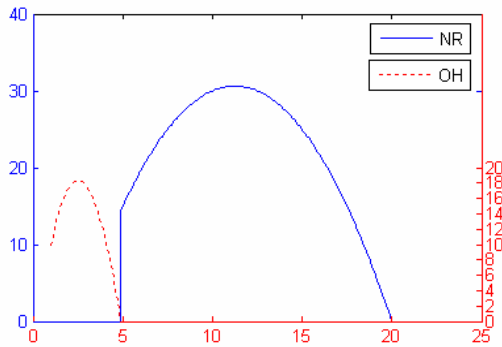**Fig. 2. The curves of GR and NR**
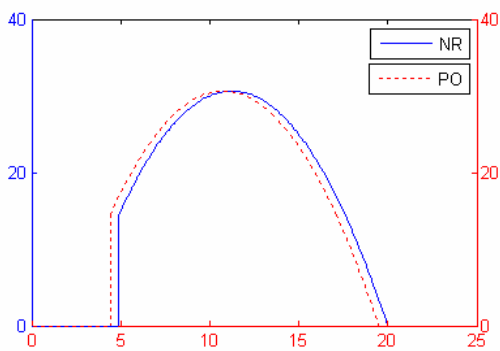
Fig. 3. The curves of NR and OH

Fig. 4. The curves of NR and PO

A significant advantage of CMRP approach is that it enables us to determine the required amount of product or order in each time interval or moment. For example, if the present time is t_1, the required quantity of product during interval $[t_1, t_2]$ is determined by the following equation:

Required quantity of product during interval $[t_1, t_2]$ =

$$\int_{t_1}^{t_2} NR(t)dt$$

Also the required quantity of order during interval $[t_1, t_2]$ can be determined by the following equation:

Required quantity of order during interval $[t_1, t_2]$ =

$$\int_{t_1}^{t_2} PO(t)dt = \int_{t_1}^{t_2} NR(t+TT)dt$$

While, the required quantity of product or order in any arbitrary time interval can not be ever determined by DMRP approach.

5. Conclusion

In this paper, CMRP approach was introduced and some advantages of this approach over DMRP one were explained. Three lemmas with their proofs and three algorithms were also described. Later, two numerical examples were solved completely. Some

priorities of CMRP approach over DMRP one are as follows:

- DMRP can not be usually applied in some industries such as Petrochemical, Gas, Oil, Water and Waste and other continuous production industries, while CMRP can be.

- In CMRP approach, for *GR*, *SR*, *OH* and other parameters, some continuous functions can be defined, whose applications enables us to perform sensitivity analysis and forecast the required parameters of the intended model.

- In DMRP approach, *TT* have to be integer, while in CMRP, *TT* can also be a non-integer number.

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