HEAT TRANSFER IN SEMITRANSPARENT MEDIUM CAUSED BY LASER PULSE

E. Izadpanah, S. Talebi, M. H. Hekmat & A. Akhavan Taheri

Abstract: In this paper, the combination of conduction with radiation into a semitransparent medium which includes absorption, emission and scattering has been investigated. In order to Study the conduction in medium, the Non-Fourier heat conduction has been applied. In this model there is a time delay between heat flux and temperature gradient. Also, in contrast with Fourier heat conduction, the speed of heat propagation is finite. The radiation transfer equation has been solved via P_1 approximate method. Also to solve the energy conservation equation and Non-Fourier heat conduction simultaneously, flux-splitting method has been applied. The results show that the transient temperature responses are oscillatory for Non-Fourier heat conduction. Also the Non-Fourier effect can be important when the thermal relaxation time of heat conduction is large. In the initial times, the difference of transient temperature responses between the Fourier and the Non-Fourier heat conduction is large under this condition. For the laser-flash measurement of thermal diffusivity in semitransparent materials, omitting the Non-Fourier effect can result in significant errors.

1. Introduction

Combined conduction and radiation heat transfer have numerous applications in the area of laser processing of semiconductors, fire protection, manufacturing of glass, fibrous and foam insulation, crystal growth, etc. Therefore, it has been the subject of numerous investigations [1-6]. In the analysis of heat conduction involving extremely short times, high-rate change of temperature or heat flux, the classical heat conduction equation, Fourier's law, breaks down because the validity of infinite heat propagation speed is restricted.

Several investigations have indicated that the finite heat propagation speed becomes dominant [7-11]. Therefore, the traditional heat diffusion equation is replaced with a hyperbolic equation that accounts for finite thermal propagation speed. The use of the hyperbolic equation removes the non-physical infinite

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speed of propagation predicted by the classical parabolic heat conduction equation [12]. Ozisik and tzou [13] gave an excellent review on the research emphasizing engineering applications of the thermal wave theory. Yuen and Lee [14], Tang and Araki [15] analyzed the non-Fourier heat conduction in a solid subjected to periodic thermal disturbances. Glass and Ozisik [16-18] studied the non-Fourier effects on the transient temperature resulting from pulse heat flux in one-dimensional semi-infinite solid with linear or non-linear boundary condition. Recently, the transient temperature response in semitransparent medium has evoked the wide interests of many researchers. Tan et al. [19-20] investigated the temperature response caused by a pulse or a step laser irradiation in semitransparent slabs with generalized boundary conditions. Andre and Degiovanni [21] studied the transient conduction- radiation heat transfer of non-scattering glass specimen.

Hahn et al. [22] applied the three-flux method to calculate the temperature response caused by laser irradiation in an absorbing, isotropic scattering coupled conduction and radiation heat transfer process in semitransparent materials.

Spuckler and Siegel [23-24] used the finite difference method in combination with Green's function to solve the steady-state radiative-conductive heat transfer problems in one-dimensional semitransparent slabs.

Ratzel and Howell [25] concluded that for one dimensional planar problem, the p-approximation method yields result in close agreement with exact solutions.

Due to the complexity of radiative heat transfer phenomenon and the numerical instability of hyperbolic system, the combined effect of non-Fourier heat conduction and radiation is rarely investigated. Glass et al. [26] examined combined conduction and radiation using non-Fourier law with hyperbolic heat conduction model in an absorbing and emitting medium. While the scattering effect was not taken into account in their investigation.

Liu and Tan [27] studied the combined conductive and radiative in absorbing, emitting, and scattering medium. They used flux splitting and discrete ordinate methods to solve the hyperbolic heat conduction and the radiative transfer equations.

The objective of this work is to analyze the non-Fourier effect on the transient temperature response in semitransparent (emitting, absorbing and scattering) medium caused by laser pulse. The coupled conduction and radiation heat transfer in a one dimensional semitransparent slab with black boundaries is studied by numerical simulation, the hyperbolic heat conduction equation being solved by the flux splitting method and the radiative transfer equation being solved by the P_1 approximation method. For the sake of analysis, the transient temperature response obtained from hyperbolic heat conduction equation is compared with those obtained from classical parabolic heat conduction equation. The influence of relating parameters, such as the pulse intensity, and the Relaxation time on the transient temperature response will be analyzed.

2. Physical Model and Equations

Here, the effects of presence of conduction and radiation have been investigated simultaneously in a slab (Fig. 1). We consider an absorbing, emitting and scattering, gray, one-dimensional semitransparent slab with black boundary surfaces initially at thermal equilibrium with surrounding. This slab is affected by laser pulse from left side for a definite time. Also it is in thermal exchanging with environment. By writing energy conservation equation for a slab element we have:

$$\frac{\partial q^c}{\partial x} + \frac{\partial q^r}{\partial x} = -\rho C_p \frac{\partial T}{\partial t} \qquad (1)$$

Cattaneo [28] and Vernotte [29] stated an improved model for Fourier conduction, that was known as Non-Fourier heat conduction, as the following:
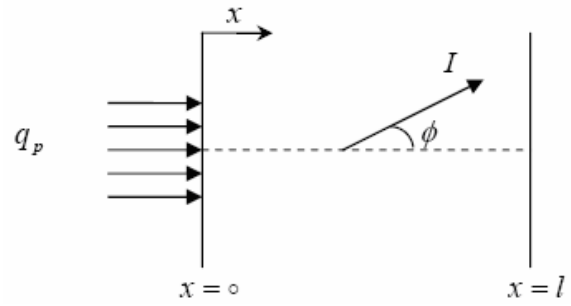
$$q^c(x, t + \tau_r) = -k\nabla T(x,t) \qquad (2)$$

Fig. 1. Schematic of the physical model and coordinates

Using Taylor series expansion, the first order approximation of Eq. (2) gives:

$$q^c(x,t) + \tau_r \left(\frac{\partial q^c(x,t)}{\partial t}\right) = -k\nabla T(x,t) \qquad (3)$$

The boundary and initial conditions are given as follows:

$$q^c(0,t) + q^r(0,t) = f(t) - \sigma\left[T^4(0,t) - T_r^4\right] \qquad (4)$$

$$q^c(l,t) + q^r(l,t) = \sigma\left[T^4(l,t) - T_r^4\right] \qquad (5)$$

$$q^c(x,0) = 0 \ , \ T(x,0) = T_r \qquad (6)$$

Where T_r is the environment temperature, $f(t)$ is laser pulse intensity function and σ is Stefan Bultezman's coefficient. Both sides of the body are exchanging the radiation with environment and the left side is exposed to laser pulse. We consider the Laser pulse represented mathematically in the form:

$$f(t) = \begin{cases} q_p & 0 \le t \le t_p \\ 0 & t > t_p \end{cases} \qquad (7)$$

Where q_p laser is pulse intensity and t_p is the time duration of laser radiation. The equation of radiation transfer (RT) in an absorbing, emitting, scattering, gray, one-dimensional semitransparent slab with black boundaries can be written as:

$$\mu\frac{\partial I(x,\mu,t)}{\partial x} = -\beta I(x,\mu,t) + \frac{\beta w}{2}$$
$$\times \int_{-1}^{1} I(x,\mu',t)\phi(\mu,\mu')d\mu' \qquad (8)$$
$$+ (1-w)\beta I_b(x,t)$$

With boundary conditions:

$$I(0,\mu,t) = \frac{n^2\sigma T^4(0,t)}{\pi}, \quad 0 \le \mu \le 1$$
$$I(l,\mu,t) = \frac{n^2\sigma T^4(l,t)}{\pi}, \quad -1 \le \mu \le 0 \qquad (9)$$

Where I is radiation intensity, w is single-scattering Albdo, n is refraction index of medium, β is extinction coefficient, ϕ is scattering phase function and $\mu = Cos\phi$. Because both boundary surfaces are considered black, radiation equation is like Eq. (9). Non dimensional form of equations and boundary conditions will be become as the following (Eq.(10) to Eq(18)):

$$\frac{\partial \theta(\xi,\tau)}{\partial \tau} + \frac{1}{N}\frac{\partial Q(\xi,\tau)}{\partial \xi} = 0 \tag{10}$$

$$\frac{\partial Q(\xi,\tau)}{\partial \tau} + \frac{N}{\tau_r^*}\frac{\partial \theta(\xi,\tau)}{\partial \xi} +$$

$$\frac{Q(\xi,\tau)}{\tau_r^*} - \frac{\partial Q^r(\xi,\tau)}{\partial \tau} - \frac{Q^r(\xi,\tau)}{\tau_r^*} = 0 \tag{11}$$

Boundary and initial conditions:

$$Q(0,\tau) = f(\tau) - \frac{1}{4n^2}\left[\theta^4(0,\tau) - 1\right] \tag{12}$$

$$Q(\xi_l,\tau) = \frac{1}{4n^2}\left[\theta^4(\xi_l,\tau) - 1\right] \tag{13}$$

$$Q(\xi,0) = 0$$
$$\theta(\xi,0) = 1 \tag{14}$$

Laser pulse:

$$F(t) = \begin{cases} Q_p & 0 \le \tau \le \tau_p \\ 0 & \tau > \tau_p \end{cases} \tag{15}$$

RT equation with boundary conditions:

$$\mu\frac{\partial I^*(\xi,\mu,\tau)}{\partial \xi} = -I^*(\xi,\mu,\tau) + \frac{w}{2}$$

$$\times \int_{-1}^{1} I^*(\xi,\mu',\tau)\phi(\mu,\mu')d\mu' \tag{16}$$

$$+ (1-w)\theta^4(\xi,\tau)$$

$$I^*(0,\mu,\tau) = \theta^4(0,\tau) \quad , \quad 0 \le \mu \le 1 \tag{17}$$

$$I^*(\xi_l,\mu,\tau) = \theta^4(\xi_l,\tau) \quad , \quad -1 \le \mu \le 0 \tag{18}$$

Applied Non-dimensional parameters are presented in the appendix where N is called conduction to radiation coefficient. Eq. (10) and Eq. (11) are coupled together and form a group of non linear differential-integral equations.

3. Method of Solution

Eq. (10) and Eq. (11) are coupled together and form a series of non linear equations. There are various methods to solve these both equations simultaneously. However, the flux splitting method has been applied here [30]. The aim in this method is finding the relation between flux and wave, because in the numerical solution, they (flux and wave) move at various directions. Vector form of Eq. (10) and Eq. (11) can be as the following [27]:

$$\frac{\partial U}{\partial \tau} + \frac{\partial E}{\partial \xi} = S \tag{19}$$

$$U = \begin{bmatrix} \theta \\ Q \end{bmatrix}, \quad E = \begin{bmatrix} \dfrac{Q}{N} \\ \dfrac{N\theta}{\tau_r^*} \end{bmatrix}$$

$$S = \begin{bmatrix} 0 \\ \dfrac{Q^r}{\tau_r^*} + \dfrac{\partial Q^r}{\partial \tau} - \dfrac{Q}{\tau_r^*} \end{bmatrix} \tag{20}$$

It is supposed that vector E (flux) has positive and negative components that are related to direction of signal propagation (thermal waves). Eq. (20) can become linear as the following:

$$\frac{\partial U}{\partial \tau} + [A]\frac{\partial U}{\partial \xi} = S \tag{21}$$

Where $[A]$ is the Jacobian matrix and $\partial E/\partial U$ is given as the following:

$$[A] = \frac{\partial E}{\partial U}, \quad E_1 = \frac{Q}{N}, \quad E_2 = \frac{N\theta}{\tau_r^*}$$

$$U_1 = \theta, \quad U_2 = Q$$

$$[A] = \begin{bmatrix} \dfrac{\partial E_1}{\partial U_1} & \dfrac{\partial E_1}{\partial U_2} \\ \dfrac{\partial E_2}{\partial U_1} & \dfrac{\partial E_2}{\partial U_2} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{N} \\ \dfrac{N}{\tau_r^*} & 0 \end{bmatrix} \tag{22}$$

Because the equation is hyperbolic, a similarity transformation exists so that

$$[A] = [T][\lambda][T]^{-1} \tag{23}$$

Where $[\lambda]$ is diagonal matrix of real eigenvalues of $[A]$, and $[T]$ is matrix of eigenvectors $[A]$ and $[T]^{-1}$ is the inverse matrix of $[T]$:

$$[\lambda] = \begin{bmatrix} -\dfrac{1}{\sqrt{\tau_r^*}} & 0 \\ 0 & \dfrac{1}{\sqrt{\tau_r^*}} \end{bmatrix}$$

$$[T] = \begin{bmatrix} 1 & 1 \\ -\dfrac{N}{\sqrt{\tau_r^*}} & \dfrac{N}{\sqrt{\tau_r^*}} \end{bmatrix} \tag{24}$$

Then reverse T matrix is determined as:

$$[T]^{-1} = \frac{1}{|T|}\begin{bmatrix} \dfrac{N}{\sqrt{\tau_r^*}} & \dfrac{N}{\sqrt{\tau_r^*}} \\ -1 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} \dfrac{1}{2} & -\dfrac{\sqrt{\tau_r^*}}{2N} \\ \dfrac{1}{2} & \dfrac{\sqrt{\tau_r^*}}{2N} \end{bmatrix} \tag{25}$$

Positive and negative components of matrix are introduced as the following:

$$[A] = [A^+] + [A^-] = [T][\lambda^+][T]^{-1} + [T][\lambda^-][T]^{-1} \tag{26}$$

Finally E is obtained as the following:

$$E = E^+ + E^- = [A^+]U + [A^-]U \tag{27}$$

Eq. (19) can be written by using flux splitting method as the following:

$$\frac{\partial U}{\partial \tau} + \frac{\partial E^+}{\partial \xi} + \frac{\partial E^-}{\partial \xi} = S \tag{28}$$

Where E^+ and E^- are related to positive and negative signal propagation direction. By applying backward and forward finite difference method for E^+ and E^-, we have:

$$U_i^{n+1} = U_i^n - \frac{\Delta\tau}{\Delta\xi}\left[(E^+)_i^n - (E^+)_{i-1}^n\right]$$
$$- \frac{\Delta\tau}{\Delta\xi}\left[(E^-)_{i+1}^n - (E^-)_i^n\right] + \Delta\tau\, S_i^n \tag{29}$$

Finally, by replacing Eq. (28) in Eq. (29), we will have:

$$U_i^{n+1} = U_i^n - \frac{\Delta\tau}{\Delta\xi}[A^+] \times$$
$$\left[(U^+)_i^n - (U^+)_{i-1}^n\right] - \frac{\Delta\tau}{\Delta\xi}[A^-] \times$$
$$\left[(U^-)_{i+1}^n - (U^-)_i^n\right] + \Delta\tau\, S_i^n \tag{30}$$

Where $\Delta\xi$ and $\Delta\tau$ are the space and the time steps respectively. RT equation has been solved via P1 approximate method. In this method, by applying a series of approximation (spherical harmonious phrases), radiation intensity (that is a function of place and direction) is separated to two distinct parts (as a series product of multiply of these two parts). By replacing these two parts in RT equation, finally, a group of partial differential equations are obtained. This method has been described completely in [31].
In order to comparison, the non dimensional form of the Fourier heat conduction equation in an absorbing, emitting, scattering, gray, one - dimensional semitransparent slab with black boundaries is written as:

$$\frac{\partial\theta(\xi,\tau)}{\partial\tau} = \frac{\partial^2\theta(\xi,\tau)}{\partial\xi^2} - \frac{1}{N}\frac{\partial Q^r(\xi,\tau)}{\partial\xi} \tag{31}$$

With boundary and initial conditions:

$$\frac{\partial\theta(0,\tau)}{\partial\xi} = \frac{1}{N}\times$$
$$\left[Q^r(0,\tau) - F(\tau) + \frac{1}{4n^2}\left(\theta^4(0,\tau)-1\right)\right] \tag{32}$$

$$\frac{\partial\theta(\xi_l,\tau)}{\partial\xi} = \frac{1}{N}\times$$
$$\left[Q^r(\xi_l,\tau) - \frac{1}{4n^2}\left(\theta^4(\xi_l,\tau)-1\right)\right] \tag{33}$$

$$\theta(\xi,0) = 1\ ,\ \tau = 0 \tag{34}$$

An implicit central-difference is used to solve the Fourier heat conduction equation.

4. Results and Discussion

In this section, the obtained results from numerical solution are presented. The Non-Fourier heat conduction is solved by the flux-splitting method, and the radiation transfer equation is solved by P1 method. Also the scattering phase function, $\phi = 1 + 0.5\mu\mu'$, is used. In Non-Fourier heat conduction, the time step is 10^{-6} and the numbers of nodes which have been selected by considering to [32] are 501. For Fourier heat conduction model, time step is given as 2×10^{-6} and the numbers of nodes which have been considered are 501. For homogeneous materials, such as pure liquids, gases and dielectric solids, the values of thermal relaxation time range vary from 10^{-14} to 10^{-6}, though the relaxation time values for heterogeneous solids are bigger than homogeneous ones [33]. To examine the accuracy of the results gained by the used method in this paper, the results in Fig. 2 have been compared with the once in [32]. The comparison shows a good conformity.
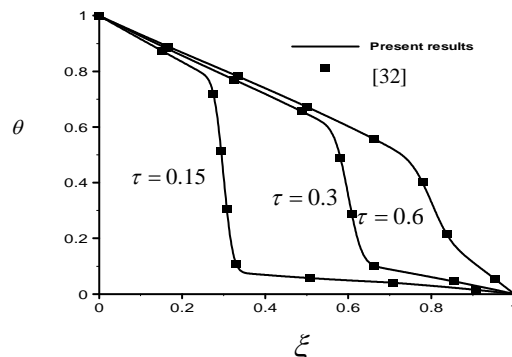

Fig. 2. Thermal distribution inside the slab for several different times

Fig. 3 to Fig. 5 show the transient temperature response in three points ξ=0, 0.5,1. As it is observed, temperature responses are oscillatory for Non-Fourier

heat conduction (in transient temperature response). When laser pulse is interrupted ($\tau/\tau_r^* = 0.2$), a thermal reduction is occurred. At initial times there is a big difference between responses of Fourier and Non-Fourier models. Thus, at the initial times (transient state), it is important to consider the Non-Fourier conduction model. But after passing time $\tau/\tau_r^* = 9$, results of these two models approach to each other.Fig. 3 shows the transient temperature response for left surface. It is observed that after heat flux interruption at time $\tau/\tau_r^* = 0.2$, several thermal waves are created. The first wave is the wave (approximately in time $\tau/\tau_r^* = 1.5$) which was created from right side and reached to this point. The second wave is the wave (approximately in $\tau/\tau_r^* = 3$) which was created from left side and contacted to the right side of slab and returned again. The third wave is the wave which was created from right side surface and after three travels along the slab; it is contacted to the left surface. The speed of heat propagation in non dimensional form is approximately 14.14. Fig. 4 shows transient temperature response in the middle of the slab along time. It is observed that after heat flux interruption, there are several thermal waves. First wave (approximately at $\tau/\tau_r^* = 0.8$) is a wave which was formed by mixing of two thermal waves created from both sides of the slab which reached to each other at the middle of the body. Temperature of this wave is 7.66. Also further waves are similar. Fig. 5 shows transient temperature response in right surface along time. It is observed that there are several thermal waves after heat flux interruption. First wave (approximately at $\tau/\tau_r^* = 1.5$) has been created from left side surface and has reached to this point. First thermal wave created at right side has non dimensional temperature equal to 4.27. But, the first thermal wave created at left side has non dimensional temperature equal to 3.98. Second thermal wave (approximately at $\tau/\tau_r^* = 3$) is the one which is created from right side and transferred to the left side and has returned to the right side again.
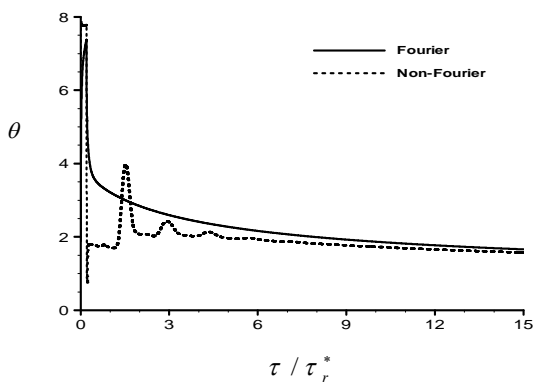

Fig. 3. Transient temperature response in left surface
$N = 5$, $\xi_l = 0.1$, $\tau_r^* = 0.005$, $\tau_P = 0.001$, $n = 1$, $A_1 = 0.5$, $w = 0.5$, $Q_P = 2000$
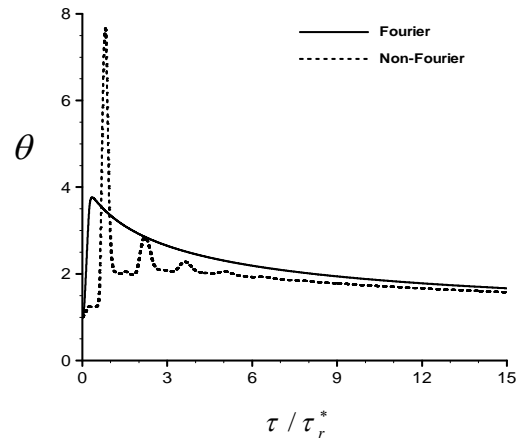

Fig. 4. Transient temperature response in middle of slab
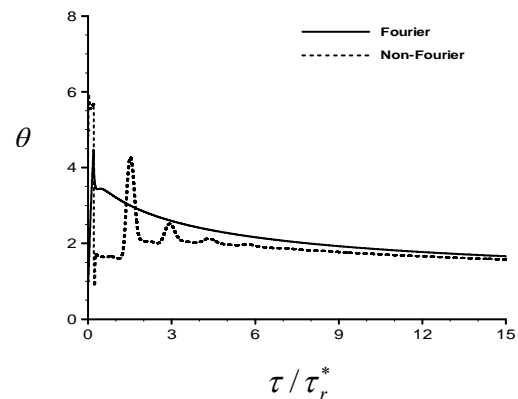$N = 5$, $\xi_l = 0.1$, $\tau_r^* = 0.005$, $\tau_P = 0.001$, $n = 1$, $A_1 = 0.5$, $w = 0.5$, $Q_P = 2000$

Fig. 5. Transient temperature response in right surface
$N = 5$, $\xi_l = 0.1$, $\tau_r^* = 0.005$, $\tau_P = 0.001$, $n = 1$, $A_1 = 0.5$, $w = 0.5$, $Q_P = 2000$

Fig. 6 shows the thermal distribution inside the slab for several different times in Non-Fourier heat conduction. As it is observed at $\tau = 0.001$, two thermal waves are forming on two surface of slab. Since at this time the heat flux is interrupted, thermal wave moves inside the slab after that. At $\tau = 0.002$, two waves, which were created in both sides of solid, move to each other with speed equal to 14.14 (at non dimensional form). At $\tau = 0.004$, these two waves reached to each other at the middle of slab (because they have equal speed) and form a stronger wave. At $\tau = 0.006$, these two waves pass from each other and after some time they far away from each other and approach to the left and right surfaces. At $\tau = 0.01$ these two waves contact to two walls and their movement direction is conversed. Finally at $\tau = 0.035$, wavy behavior of temperature distribution is lost and temperature of all internal points will become equal. The reason of creating these two thermal waves is the presence of the radiation inside the slab which helps to transfer the heat inside that and

results in the thermal variations of right surface from beginning. Here two points are remarkable, first, by passing time, the wave's amplitude is decreased and also it is observed that the created wave is stronger at left side surface, because this surface was subjected to the effect of heat flux. Fig. 7 shows thermal distribution inside slab for several different times at Fourier heat conduction. It is observed that with increasing of time to $\tau = 0.001$, the temperature distribution diagram is moved upward. Therefore, temperature points inside the slab are raised. After heat flux interruption (at $\tau = 0.001$) by increasing time, thermal distribution diagram moves downward until the temperature of all points inside solid become equal. Totally, at Fourier model, thermal variation inside the slab is rapid (speed of heat propagation is infinite) and opposed of Non-Fourier model, the thermal variations don't have wavy behavior. Also duration time to reach the thermal equilibrium is less in Fourier model.
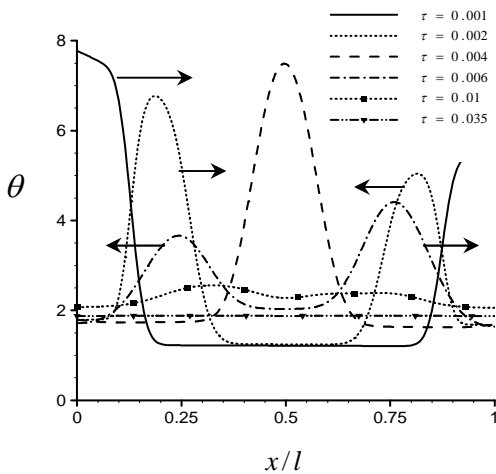
Fig. 8 to Fig. 10 show the influences of thermal relaxation time of heat conduction on the non-Fourier effects of transient temperature responses for three points ($\xi 1 = 0, 0.5, 1$). It is observed that whatever relaxation time increases, the non-Fourier effects of transient temperature responses become more. Therefore the difference between the results of Fourier and Non-Fourier models becomes more and transient time is increased. Also, with increasing of the relaxation time, the time delay increases. Certainly, this matter is more obvious at the middle point. Another point is that by increasing the relaxation time, the speed of heat propagation will decrease. Speed of heat propagation at $\tau_r^* = 0$ (in non-dimensional form) is infinite, at $\tau_r^* = 0.001$ is equal to 31.62 and at $\tau_r^* = 0.01$ is equal to 10. Also it is seen that by increasing the relaxation time, the value of sudden fall becomes more (at $\tau = 0.001$) after thermal flux interruption. The reason of this matter is decreasing of heat propagation term ($\partial\theta / \partial\xi$) in Eq. (11).

Fig. 6. Thermal distribution inside the slab for several different times (Non-Fourier heat conduction)

$N = 5 , \xi_l = 0.1 , \tau_r^* = 0.005 , \tau_P = 0.001 , n = 1 , A_1 = 0.5$

$, w = 0.5 , Q_P = 2000$

Fig. 8. the influences of thermal relaxation time on the transient Temperature responses (Left surface)

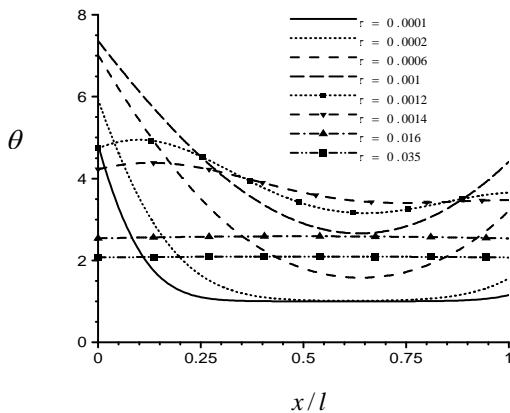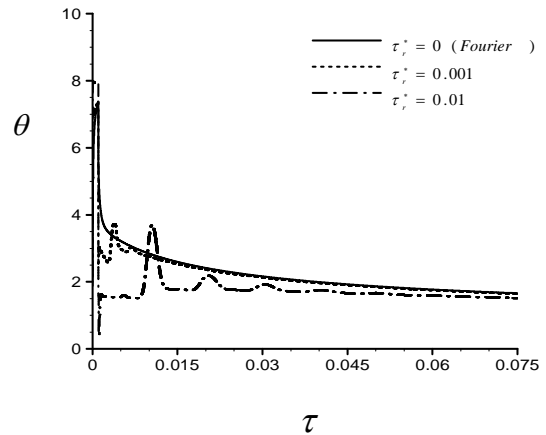$N = 5 , \xi_l = 0.1 , \tau_P = 0.001 , n = 1 , A_1 = 0.5 , w = 0.5 , Q_P = 2000$

Fig. 7. Thermal distribution inside slab for several different times (Fourier heat conduction)

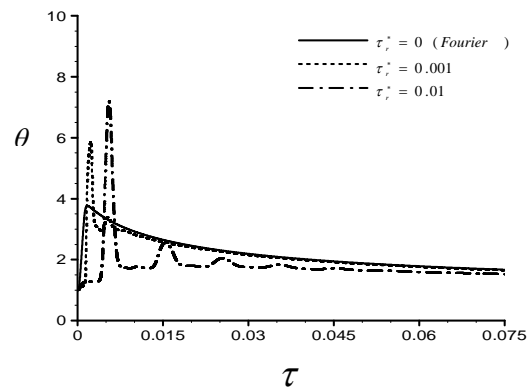$N = 5 , \xi_l = 0.1 , \tau_P = 0.001 , n = 1 , A_1 = 0.5 , w = 0.5 , Q_P = 2000$

Fig. 9. the influences of thermal relaxation time on the transient Temperature responses (Middle of slab)

$N = 5 , \xi_l = 0.1 , \tau_P = 0.001 , n = 1 , A_1 = 0.5 , w = 0.5 , Q_P = 2000$
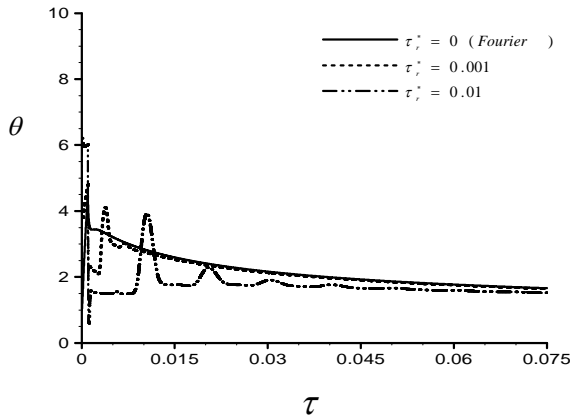
Fig. 10. the influences of thermal relaxation time on the transient Temperature responses (Right surface)

$N = 5$, $\xi_l = 0.1$, $\tau_P = 0.001$, $n = 1$, $A_1 = 0.5$, $w = 0.5$, $Q_P = 2000$

Fig. 11 to Fig. 14 show the influences of laser pulse intensity on non-Fourier effects of transient temperature responses. It is observed that by increasing the laser pulse intensity, the non-Fourier behavior for both sides becomes more. Also with increasing of laser pulse intensity, thermal variation diagram becomes more uneven at Fourier model. Since the speed of heat propagation is equal for all states, then Q_p variation effects on the thermal variations. Also whatever Q_p increases, longer time will be taken to approach the results of Fourier and Non-Fourier model.
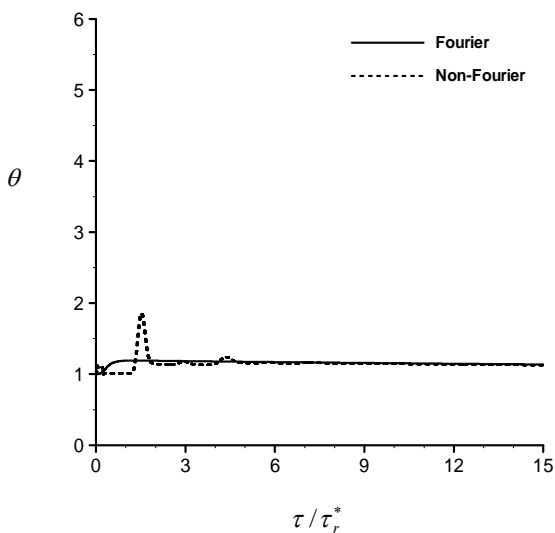

Fig. 11. The influences of laser pulse intensity on transient temperature responses (Right surface)

$N = 5$, $\xi_l = 0.1$, $\tau_r^* = 0.005$, $\tau_P = 0.001$, $n = 1$, $A_1 = 0.5$, $w = 0.5$,

$Q_p = 100$

Fig. 12. The influences of laser pulse intensity on transient temperature responses (Right surface)

$N = 5$, $\xi_l = 0.1$, $\tau_r^* = 0.005$, $\tau_P = 0.001$, $n = 1$, $A_1 = 0.5$, $w = 0.5$,
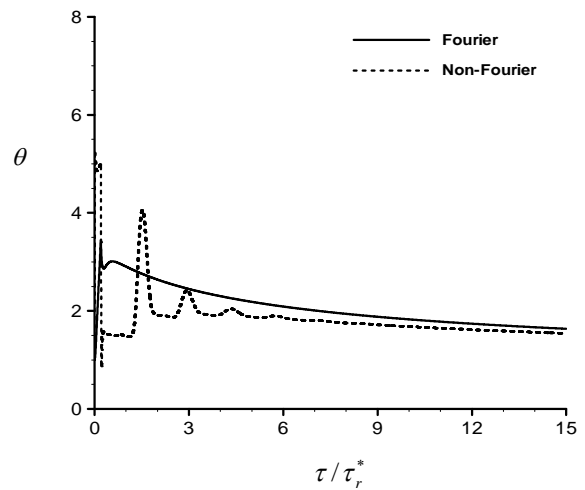
$Q_p = 1500$

Fig. 13. The influences of laser pulse intensity on transient temperature responses (Left surface)

$N = 5$, $\xi_l = 0.1$, $\tau_r^* = 0.005$, $\tau_P = 0.001$, $n = 1$, $A_1 = 0.5$, $w = 0.5$,

$Q_p = 100$

5. Conclusions

In this paper, effects of the Non-Fourier heat conduction on transient temperature responses in semitransparent medium with black boundary surface caused by laser pulse has been studied. The heat transfer in the slab is a compound of conduction and radiation which its equation has been solved numerically. The Radiation transfer equation is solved via the P1 method and the Non-Fourier heat conduction equation is solved via flux splitting method. Meanwhile, transient- temperature responses obtained from Non-Fourier heat conduction was compared with

Fourier conduction. By studying the results, it is seen that Non-Fourier conduction behavior is oscillating behavior.
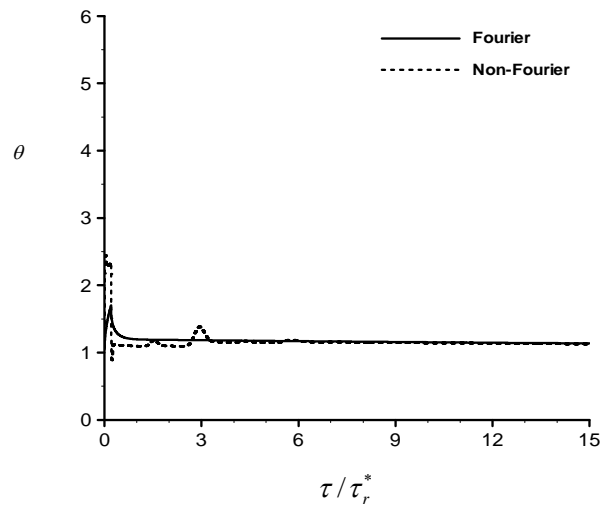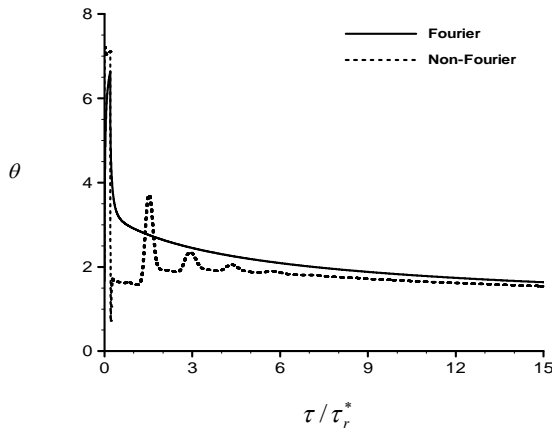

Fig. 14. The influences of laser pulse intensity on transient temperature responses (Left surface)

$N = 5$, $\xi_l = 0.1$, $\tau_r^* = 0.005$, $\tau_P = 0.001$, $n = 1$, $A_1 = 0.5$, $w = 0.5$, $Q_P = 1500$

At the initial times of laser pulse radiation, the difference of the results of Fourier and Non-Fourier heat conduction is bigger; therefore neglecting the Non-Fourier effects at the beginning times is incorrect. Also, by passing time, the Non-Fourier results are approached to the Fourier results. By becoming greater of laser pulse intensity and relaxation time the Non-Fourier effects become more (the difference of the Fourier and the Non-Fourier becomes great) and omitting the Non-Fourier effect may result in significant errors. At Non-Fourier model, it is observed that two thermal waves created at both sides of the slab and move inside that. The reason of creation of the second wave is the presence of radiation inside the slab that helps the heat propagation inside the slab.

References

[1] Ozisik Mn., *Radiative Transfer and Interactions With Conduction and Convection*, New York, Wiley-Interscience, 1973.

[2] Siegel, R., *"Transient Thermal Effects of Radiant Energy in Translucent Materials,"* ASME J. Heat Transfer, Vol. 120, 1998, pp. 4-23.

[3] Fernandes, R., Francis, J., *"Combined Conductive and Radiative Heat Transfer in Absorbing, Emitting, and Scattering Cylindrical Medium,"* ASME J. Heat Transfer, Vol. 104, 1982, pp. 594-601.

[4] Chu, HS, Tseng, C.J., *"Thermal Performance of Ultra-Fine Powder Insulations at High Temperatures,"* J. Thermal Insulation, Vol. 12, 1989, pp. 298-312.

[5] Tseng, C.J., Chu, H.S., *"Transient Combined Conduction and Radiation in an Absorbing, Emitting, and Anisotropically Scattering Medium with Variable Thermal Conductivity,"* Int. J. Heat Mass Transfer, Vol. 35, 1992, pp. 1844-1847.

[6] Lazard, M., Andre, S., Maillet D., *"Transient Coupled Radiative-Conductive Heat Transfer in a Gray Planar Medium with Anisotropic Scattering,"* JQSRT, Vol. 69, 2001, pp. 23-33.

[7] Glass, D.E., Tamma, K.K., *"Non-Fourier Dynamic Thermo Elasticity with Temperature-Dependent Thermal Properties,"* J. Thermophys Heat Transfer, Vol. 8, 1994, pp. 145-151.

[8] Tzou, D.Y., *Macro- to Microscale Heat Transfer: the Lagging Behavior*, Washington, DC: Taylor & Francis, 1996.

[9] Tan, Z.M., Yang, W.J., *"Propagation of Thermal Waves in Transient Heat Conduction in a Thin Film,"* J. Franklin Institute, Vol. 336B, 1999, pp. 185-197.

[10] Wu, J.P., Chu, H.S., *"Propagation and Reflection of Thermal Waves in a Rectangular Plate,"* Numer. Heat Transfer Part A, Vol. 36, 1999, pp. 51-74.

[11] Lor, W.B., Chu, H.S., *"Hyperbolic Heat Conduction in Thin-Film High T_C Superconductors with Interface Thermal Resistance, Cryogenics,"* Vol. 39, 1999, pp.739-750.

[12] Joseph, D.D., Preziosi, L., *"Heat Waves,"* Rev. Modern Phys., Vol. 61, 1989, pp. 41-73.

[13] Ozisik, M.N., Tzou, D.Y., *"On the Wave Theory in Heat Conduction,"* ASME J. Heat Transfer, Vol. 116, 1994, pp. 526-535.

[14] Yuen, W.W., Lee, S.C., *"Non-Fourier Heat Conduction in a Semi-Infinite solid Subjected to Oscillatory Surface Thermal Disturbances,"* ASME J. Heat Transfer, Vol. 111, 1989, pp. 178-181.

[15] Tang, D.W., Araki, N., *"Non Fourier Heat Conduction in a Finite Medium Under Periodic Surface Thermal Disturbance,"* Int. J. Heat Mass Transfer, Vol. 39, 1996, pp. 1585-1590.

[16] Glass, D.E., Ozisik, M.N., *"Non-Fourier Effects on Transient Temperature Resulting From Periodic on-off Heat Flux,"* Int. J. Heat Mass transfer, Vol. 30, 1987, pp. 1623-1631.

[17] Glass, D.E., Ozisik, M.N., Vick, B., *"Hyperbolic Heat Conduction with Surface Radiation,"* Int. J. Heat Mass Transfer, Vol. 28, 1985, pp. 1823-1830.

[18] Glass, D.E., Ozisik, M.N., McRae DS, Vick, B., *"On the Numerical Solution of Hyperbolic Heat Conduction,"* Numerical Heat Transfer, Vol. 8, 1985, pp. 497-504.

[19] Tan, H.P., Maestre, B., Lallemand, M., *"Transient and Steady-State Combined Heat Transfer in Semitransparent Materials Subjected to a Pulse or a Step Irradiation,"* ASME, J. Heat Transfer, Vol. 113, 1991, pp. 166-173.

[20] Tan, H.P., Ruan, L.M., Tong, T.W., *"Temperature Response in Absorbing, Isotropic Scattering Medium Caused by Laser Pulse,"* Int. J. Heat Mass Transfer, Vol. 43, 2000, pp. 311-320.

[21] Andre, S., Degiovanni, A., *"A Theoretical Study of the Transient Couple Conduction and Radiation Heat Transfer in Glass: Phonic Diffusivity Measurements by Flash Technique,"* Int. J. Heat Mass Transfer, Vol. 38, 1995, pp. 3401-3414.

[22] Hahn, O., Raether, F., Arduini-Schuster, M.C., Fricke, J., *"Transient Coupled Conductive/Radiative Heat Transfer in Absorbing, Emitting, and Scattering Media: Application to Laser-Flash Measurements on Ceramic Materials,"* Int. J. Heat Mass Transfer, Vol. 40, 1997, pp. 689-698.

[23] Spuckler, C.M., Siegel, R., "*Refractive Index Effects on Radiative Behavior of a Heated Absorbing-Emitting Layer,*" J. Thermophys Heat Transfer, Vol. 6, 1992, pp. 596-604.

[24] Spuckler, C.M., Siegel, R., "*Refractive Index and Scattering Effects on Radiative Behavior of a Semitransparent Layer,"* J. Thermophys Heat Transfer, Vol. 7, 1993, pp. 302-310.

[25] Ratzel, A.C., Howell, R., *"Heat Transfer by Conduction and Radiation in One-Dimensional Planar Media Using the Differential Approximation,"* ASME J. Heat Transfer, Vol. 104, 1982, pp. 388-391.

[26] Glass, D.E., Ozisik, M.N., McRae, D.S., *"Hyperbolic Heat Conduction with Radiation in an Absorbing and Emitting Medium,"* Numer. Heat Transfer, Vol. 12, 1987, pp. 321-333.

[27] Liu, L.H., Tan, H.P., *"Non-Fourier Effects on Transient Coupled Radiative-Conductive Heat Transfer in One-Dimensional Semitransparent Medium Subjected to a Periodic Irradiation,"* J. Quantitative Spectroscopy and Radiative Transfer, Vol. 71, 2001, pp. 11-21.

[28] Cattaneo, C., "*A form of Heat Conduction Equation Which Eliminates the Paradox of Instantaneous Propagation,"* Comput Rendus, Vol. 247, 1958, pp. 431-433.

[29] Vernotte, P.*, "Les paradoxes De La Theorie Continue De La Chaleur,"* Comput Rendus, Vol. 246, 1958, pp. 3154-3155.

[30] Tannehill, J.C., Anderson, D.A., Pletcher, R.H., *Computational fluid mechanics and heat transfer, second edition, Taylor and Francis*, Washington, DC, 1997, pp. 375-386.

[31] Modest, M.F., *Radiative Heat Transfer*, McGraw-Hill,Inc , 1993.

[32] Hsin-Sen, C., Senpuu, L., Chia-Hui, L., *"Non-Fourier Heat Conduction with Radiation in an Absorbing , Emitting and Isotropically Scattering Medium,"* J. Quantitative Spectroscopy and Radiative Transfer, Vol. 73, 2002, pp. 571-582,

[33] Kaminski, W.J., "*Hyperbolic Heat Conduction Equation for Materials with a Non Homogeneous Inner Structure,"* J. Heat Transfer, Vol. 112, 1990, pp. 555-560.

Appendix

List of symbols

C_P Specific heat, J/kg K

f Laser intensity function, W/m²

F Non-dimensional laser intensity, $F = f / 4n^2\sigma T_r^4$

I Intensity of radiation, W/ m² sr

I^* Non-dimensional intensity of radiation, $I^* = \pi I / n^2\sigma T_r^4$

k Thermal conductivity, W/ m² K

L Thickness of Slab, m

n Refractive index of medium

N Conduction to radiation parameter, $N = k\beta / 4n^2\sigma T_r^2$

q^c Conduction heat flux, w/ m²

q^r Radiation heat flux, w/ m²

q_p Laser pulse intensity, w/ m²

Q Non dimensional heat flux, $Q = (q^c + q^r)/4n^2\sigma T_r^2$

Q^c Non dimensional conduction heat flux, $Q^c = q^c / 4n^2\sigma T_r^2$

Q^r Non dimensional radiation heat flux, $Q^r = q^r / 4n^2\sigma T_r^2$

Q_p Non dimensional laser pulse intensity, $Q_p = q_p / 4n^2\sigma T_r^2$

t_p Time duration of laser pulse radiation, s

t_{tr} Heat conduction relaxation time of, s

T Temperature of medium, K

T_r Temperature of environment, K

x Space coordinate, m

Greek symbols

α Thermal diffusivity, m²/s

β Extinction coefficient, 1/m

θ Non dimensional temperature, $\theta = T / T_r$

μ Direction cosine

ξ Optical coordinate, $\xi = x\beta$

ρ Density, Kg/ m³

σ Stefan boltezman stant, W/ (m K²)²

τ Non dimensional time, $\tau = \alpha\beta^2 t$

τ_r^* Non dimensional thermal relaxation time, $\tau_r^* = \alpha\beta^2\tau_r$

ϕ Scattering phase function

w Single scattering albedo