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RESEARCH PAPER

Using A P Control Chart for Joint Optimization of Maintenance, Quality, and Buffer Stock Policies in Single Machine Production Systems

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ABSTRACT
The aim of this study is to deal with the joint optimization problem of maintenance scheduling, quality control, and buffer stock planning in a single machine production system based on a P control chart. It is assumed that there is a fixed production rate and stochastic machine breakdowns, which directly affect the quality of the product. A buffer is used to reduce production disruptions caused by machine stops and to ensure demand is met during preventive and corrective maintenance. All features of three sub-optimization problems, including maintenance, quality control, and buffer stock policies, are formulated, and the proposed integrated approach is mathematically modeled. In addition, an iterative numerical optimization procedure is developed to provide the optimal values for the decision variables. The proposed method provides the optimal values of preventive maintenance scheduling, buffer stock size, sample size, sampling interval and control chart limits simultaneously, so that the total cost per unit time is minimized. It is found that performing preventive maintenance reduces the overall costs incurred. Moreover, some sensitivity analyses are carried out to identify the key effective parameters. Significant economic benefits can be seen in the proposed joint optimization procedure.

KEYWORDS: *Joint optimization; P periodic preventive maintenance (PM); Corrective maintenance; P control chart; Buffer stock.*

1. Introduction

In today's highly competitive business environment, companies must improve the performance of their production systems while considering demand disruptions, short product lifecycles, rapid technological developments and globalization to achieve a competitive advantage. The performance of a production system is directly related to its ability to satisfy multiple factors simultaneously, such as proper response to demand, high product quality, low production cost, and timely delivery [2]. Shop floor activities consist of maintenance policies, quality control, and buffer stock planning is one of the most important factors affecting the performance of the production system. Previous studies traditionally dealt with the shop floor policies independently. However, these activities are so closely related to one another and optimizing each of them individually without considering their interactions may lead losing the ideal result. Therefore, a joint optimization procedure of these different strategies is necessary in both academic and industrial fields in order to achieve ideal result.

According to the aforementioned reasons, related managers have a common concern that is to optimize the performance of the main activities of their production systems, i.e., maintenance planning, quality control, and buffer control simultaneously to reduce the total costs of the production system. Due to complexity of this approach, various models have been introduced to dealwith the uncertainties in manufacturing systems. The different contributions associated

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with integrated models covered in the literature can be classified as follows

- Integrated policy for production planning and maintenance
- Integrated policy for production planning and quality control
- Integrated policy for maintenance and quality control
- Integrated policy for production planning, quality control, and maintenance

The abovementioned approaches for optimizing related problems can be categorized from another point of view as Figure 1.

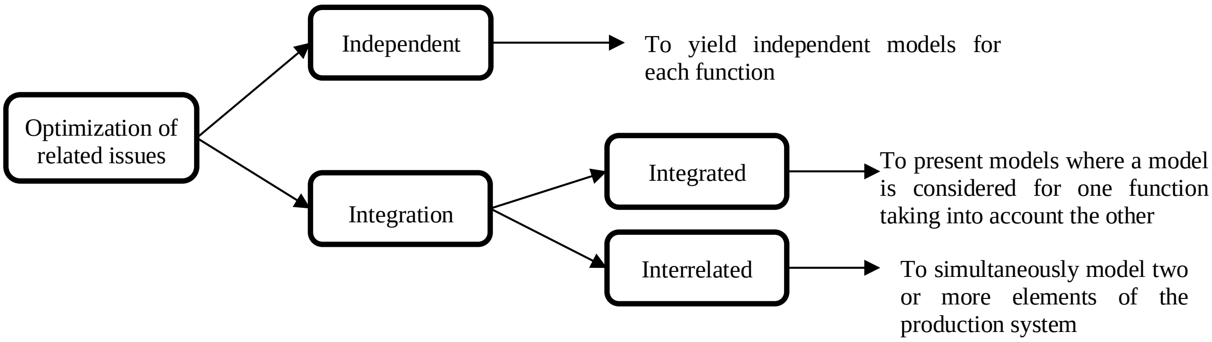


Fig. 1. kinds of approaches for optimizing related problems.

During the last decades, several production control and maintenance policies have been proposed in order to improve manufacturing system performance. Although many researchers have paid attention to integrated policies of two or three issues of maintenance, quality control, and production planning, none of them applied an attribute control chart as a basis for decision making in the integrated policy. In this way, this study aims to introduce an integrated approach for joint optimization of maintenance policies, quality control, and buffer planning based on P chart considering stochastic machine breakdowns in a single machine process. Both preventive and corrective maintenance is considered in this study. Moreover, quality control is used based on the attribute characteristics that we encounter in most production systems and products. To this end, the P control chart is used to control defective ratio and out-of-control states. After each inspection, the decision-maker recognizes whether corrective maintenance is required based on the value of the defective ratio. In addition, the buffer is considered as another variable that should be controlled to overcome the disturbances and variation to meet the production plan.

The remaining sections of the study are as follows. Section 2 provides a comprehensive review of works related to this study. The section 3 is devoted to problem definition. In section 4, the problem at hand is modeled and all possible scenarios are given. The integrated approach is introduces in section 5. Section 6 investigated the performance of the proposed method by providing a numerical instance and result analysis. Finally, section 5 provides a summary of the study and a conclusion with some suggestions for future research. Finally, conclusions are drawn in Section 7.

2. Literature Review

Optimizing the shop floor issues has been received increasing attention in the field of academic research and manufacturing industries due to the vital role of these activities on resource efficiency and cost reduction. Notably, these sub-processes interact with each other and so, joint optimisation of these issues via an integrating approach attracted the attention of researchers during last recent years [12], [24].

One of the earliest studies in the joint optimization field was carried out by [21]. in 1998. They introduced a new integrated model for production planning, inspection schedule, and control chart design simultaneously for a continuous production process. They investigated this problem to minimize the total cost of the considered system that include the setup cost, the holding cost, and the cost of quality [21]. After that, several studies investigated joint optimization problem of shop floor operation in various condition. Ben-Daya and Rahim presented a comprehensive literature review on studies related to integrated models of maintenance, production, and quality published until 2001 [6]. Reviewing the existing studies shows that the joint optimization models in this field can be categorised into two groups. The first group consists of integrated the maintenance and quality design under 100% or sampling inspection and the studies in the second group use control chart and statistical process control (SPC) to optimize maintenance and production-inventory control policy.

In the first category, we can cite the study of [9]. that determined the optimal run time for an economic production quantity (EPQ) time based on scrap, reworks, and stochastic machine breakdowns [9]. They also solved a numerical example to present practical usage of the model. Radhoui et al. introduced another integrated problem for optimizing quality control plan and preventive maintenance policy simultaneously for a randomly failing production system that produces conforming and non-conforming units. They developed a new mathematical model combined with simulation to provide the optimal the rate of non-compliant units according to which the preventive maintenance operation should be performed and the size of the safety stock. They considered minimizing the total cost per time unit as the objective function. They used their proposed model for a single machine process to fulfil a constant demand [19]. Radhui et al. developed their study for a machine which must supply another production system considering the just-in-time rules. They indicated a maximum value as l_m to decide requirement for maintenance actions according to the proportion l of non-conforming units determined for each lot. They also used a buffer stock to palliate perturbations caused by the stopping of the machine [21], [10]. investigated the joint optimization issue of an inventory control and preventive maintenance policy for a manufacturing cell. They considered minimizing the total cost of system as the objective function that consists of setup, maintenance, inventory holding, shortage costs, and the cost incurred by producing non-conforming items. They proposed an integrated mathematical model to solve the considered problem [10]. Lopes investigated the effect of a quality inspection policy on an imperfect production system by inspecting a percentage of the produced items. The author formulated the problem to minimize the total expected cost per item while considering an average outgoing quality constraint [13], [14]. studied the integrated problem of optimizing the production quality and condition-based maintenance for a single machine production system that produces a kind of product. They defined an indicator and considered the system is in ‘fail mode’ whenever its degradation level exceeds the indicator in each periodic inspection. They proposed an integrated model to optimize the preventive maintenance costs at the same time reduce production of non-conforming items. Therefore, the indicator should be monitored and optimized so that preventive maintenance is carried out at appropriate time intervals [13], [1]. investigated an aggregated production planning via a bi-level robust optimization model as a leader-follower problem using Stackelberg game. The authors has formulated the problem and proposed an exact method based on the Benders decomposition algorithm for overcoming the computational complexities in large scale [1], [11]. proposed an integrated model for joint optimization of production, maintenance, and process control decisions simultaneously for a single machine production system. In their proposed model, first preventive maintenance scheduling is optimized. Then an integrated model has been developed for production scheduling, inventory holding, and process control to minimize the total cost per unit time [11], [3]. have dealt with a parallel machine scheduling problem considering individual maintenance operations. The authors proposed a new mathematical model to formulate the problem considering scheduling and maintenance operation so that the completion time and the average cost are jointly minimized. They also developed an exact procedure based on the branch and bound (B&B) approach for the problem at hand [3], [15]. Investigated the joint optimization problem of production scheduling, work-in-process (WIP) inventory control, and group preventive maintenance planning in a multi-machine system with multi-components. They proposed a new meta-heuristic named Jaya algorithm (JA) two popular algorithms to obtain optimum production sequence, PM intervals, and grouping of components, which minimize the total expected cost per unit time of the system [15], [25]. studied the joint optimization of condition-based maintenance and spares inventory for a general series–parallel system with two failure modes including hard failures (self-announcing), and soft failures (generally caused by the degradation of components and only be discovered through inspection). The authors proposed a simulation method to minimize the expected average cost per unit time by jointly determining the optimal preventive maintenance and spare parts inventory control policies. They also investigated the performance of the proposed method by solving a numerical instance [25].

In the second category of integrated models, which is related to the use of statistical process control (SPC) tools, the required maintenance operations are determined based on information obtained from SPC techniques. In this field, we can refer to [17]. that tackle three sub-problem preventive maintenance scheduling, quality control, and production scheduling in a joint optimization approach to minimize expected cost per unit time [18]. The same authors presented a literature review on studies that have addressed the joint consideration of three aspects of shop floor operations consist of scheduling, maintenance, and quality. They also highlighted research gaps in this field until 2010. Finally, they suggested a conceptual methodology that can lead to further developments in this field [17]. Bouslah et al. introduced deal the integrated problem of production, quality, and maintenance control in a production line. They supposed that the production line consists of two machines subject to quality and reliability operation-dependent degradation and machines' reliability is correlated due to the level of incoming product quality. Moreover, they formulated the problem with the aim of minimizing the total cost incurred under a constraint on the outgoing quality [8], [4]. dealt with joint optimization of control of production, maintenance, and quality for batch manufacturing systems and developed an integrated approach for it. They used x-bar control chart to monitor the quality of the lot produced and built a buffer stock to maintain production continuity during maintenance actions. The objective function was to minimize the total cost includes setup, inventory, unused products, maintenance, and quality costs. In addition, they considered the buffer stock size, the sample size, the sampling interval, the surveillance, and the control limits as the main decision variables [4]. They continued their study one year later to a new joint production, maintenance, and quality control strategy involving a periodic preventive maintenance policy. In their proposed approach, the maintenance action is performed every "α" inspection of product quality in order to reduce the shift rate to the "out-of-control" state [5]. [22]. proposed an integrated problem of production, maintenance, and quality control planning for a continuous production system with quality deterioration. Their considered system was composed of an unreliable machine that produces one part type satisfying customers demand. They investigated the proposed integrated model considering the effect of such dynamic sampling strategy and relevant interactions with production and maintenance strategies. The objective function was to minimize the expected average incurred cost by determining an appropriate production policy as well as the preventive maintenance and quality control rates [22].

The summary of previous studies shows that many researchers have considered the optimization of manufacturing processes in recent years due to the importance of these activities and their interaction. However, the P control chart is used in this study for the first time for quality-oriented maintenance operations. Therefore, the contribution of this study is twofold: First, proposing an integrated model based on the P control chart for integrating maintenance, quality control, and buffer policies in a single machine production system. Second, developing an iterative numerical procedure for joint optimal policy. The attributes quality characteristics play an important role in production and quality control. We consider all attributes quality characteristics in a P control chart and whenever the chart shows an out-of-control state, the appropriate reactive maintenance action is performed to handle the failure mode and to bring the process back to the "in-control state". In addition, buffer stock size is managed to reduce production disruptions caused by the "out-of-control" and machine stops and to ensure demand providing during the preventive and to ensure demand is met during preventative and corrective maintenance operations.

Table 1 summarizes related studies to identify the research gap and novelty of the present study. As is evident, this is the first effort that deal with three optimization problems including maintenance scheduling, buffer stock planning, and quality control in a single machine production system based on a P control chart.

Tab. 1. Comparison contributions on the related studies.

Ref.	Inventory control		Features of the maintenance				Quality control		Solution approach
			Preventive maintenance						
	Considering the shortage	Regardless of the shortage	Time-based	Predictive	Condition-based	Corrective	Control chart	Sampling	
Radhoui et al., (2009)	✓	✓		✓		✓		✓	Heuristic
Radhoui et al., (2010)	✓	✓		✓		✓		✓	Heuristic
Pandey et al., (2010)			Age based			✓	X-bar chart		Exact
Pandey et al., (2011)			Periodic			✓	X-bar chart		Exact
Dhouib et al., (2012)	✓	✓		✓		✓			Exact
Wang et al., (2018)		✓	Periodic			✓		✓	Meta-heuristic
Bahria et al., (2019)	✓	✓		✓	✓	✓	X-bar chart		Exact
Rivera-Gómez et al., (2020)	✓	✓	Periodic				X-bar chart		Heuristic
Jafarian-Namin et al., (2021)		✓		✓	✓	✓	ARMA control chart		Meta-heuristic
Mishra et al., (2021)		✓	Periodic						Meta-heuristic
Zhang et al., (2022)		✓			✓				Exact
This Paper	✓	✓	Periodic			✓	p bar chart		Exact

3. Problem Definition

Consider a single machine production system with stochastic breakdowns that produce a kind of product to satisfy a constant and continuous demand at rate D. The production system starts under "in-control" state and produces acceptable items with U_{max} production rate $(U_{max} > \mathrm{D})$. Whenever the buffer stock size reaches a maximum amount of h, the production rate is made equal to D. The buffer will be kept during the production until a maintenance action starts. During each maintenance operation, the machine is stopped and demand is provided by buffer stock. After restarting the machine, it starts under "in-control" state again and produces acceptable items with U_{max} production rate. We use a P control chart to monitor the quality of each produced lot. It is supposed that machine breakdown is the only cause for producing non-conforming products and "out-of-control" state and the other causes are omitted. The sample size is shown by n that is inspected in a predefined period. The average and standard deviation of defective products of the process is called as $\mu_{\bar{P}_e}$ and $\sigma_{\bar{P}_e}$ respectively and P_e presents the average of the defective products in a specified sample. So the control limits of the chart are determined as (1) and (2).

$$UCL = \bar{P} + k\sqrt{\frac{\bar{P}(1-\bar{P})}{n}} \tag{1}$$

$$LCL = \bar{P} - k\sqrt{\frac{\bar{P}(1-\bar{P})}{n}} \tag{2}$$

Where, the coefficient k is determined based on the type I error (α) as follows:

$$k = Z_{1-\alpha/2}$$

To more clear, Figure 2 illustrates the quality section in the problem at hand.

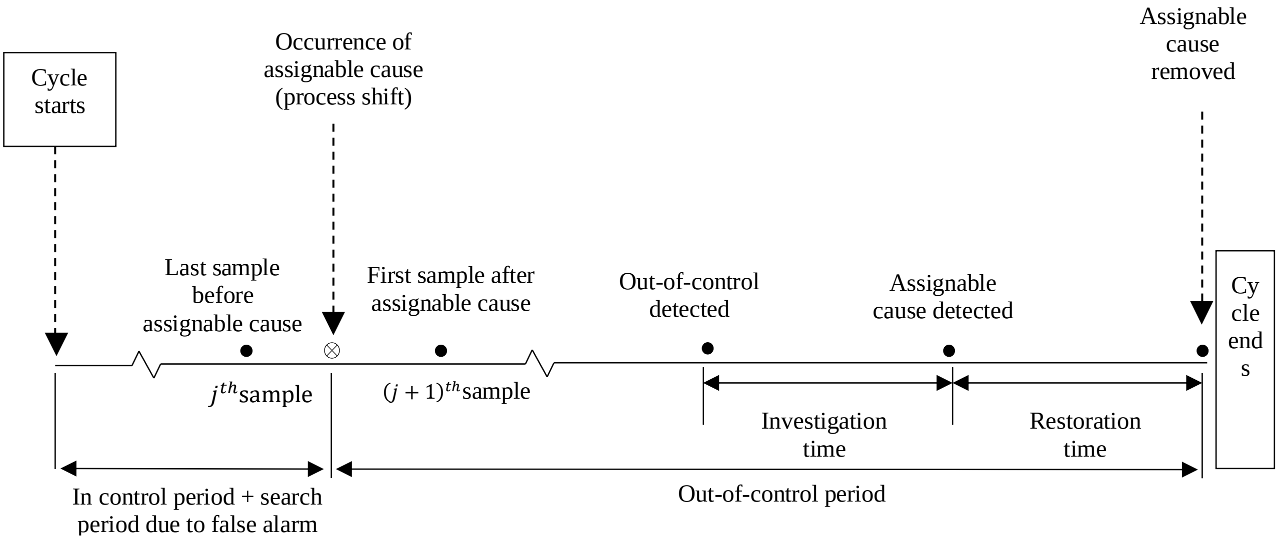


Fig. 2 The process cycle of the quality control in the considered problem.

Three bellows quality cost factors are considered in the joint optimization proposed model to be reduced simultaneously.

- Sampling costs
- False alarm costs
- Non-conforming costs (failure costs)

Preventive maintenance operations requiring negligible durations may be conducted when the process is in-control state to reduce the shift rate to the "out-of-control" state. In addition, it is assumed that preventive maintenance operations are conducted at all $\gamma \times Is$ time intervals, where γ is a nonzero integer and Is is the sampling interval. Therefore, the schematic view of maintenance policy can be presented as Figure 3. Due to this process, during the "in-control" state, preventive actions are conducted as predefined scheduling and whenever we have an "out-of-control" state, the appropriate corrective maintenance action should be performed.

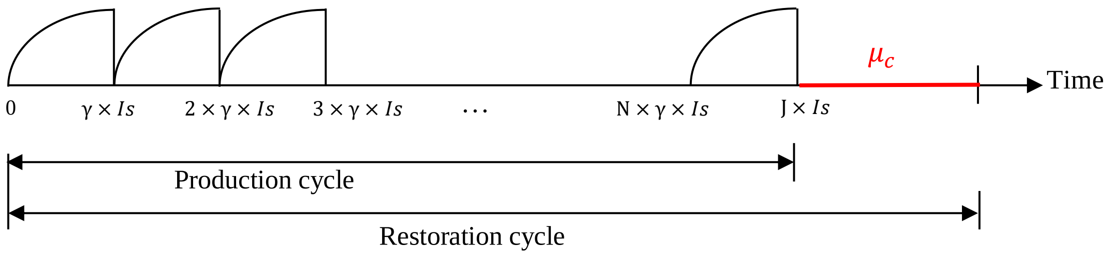


Fig. 3. The maintenance strategy.

Two bellows maintenance cost factors are considered in the joint optimization proposed model to be reduced simultaneously.

- Preventive maintenance
- Corrective maintenance

Figure 4 highlights the integrated approach to maintenance policy and quality control for one preventive maintenance operation during a production cycle. As this figure shows, whenever machine breakdown causes an increasing in defective items ratio, it will lead to an "out-of-control" state, that is detect a point above the upper control limit ($\hat{p} \geq UCL$).

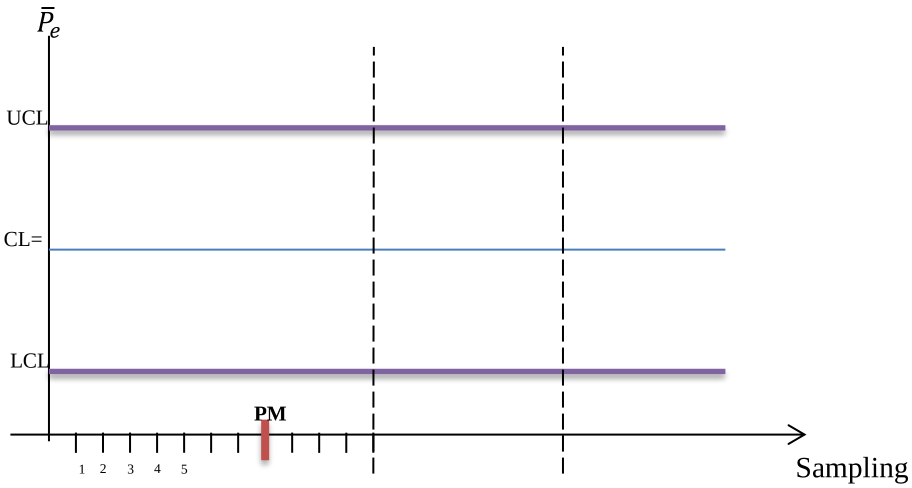


Fig. 4. Integrated quality control and maintenance strategy.

In addition of two operations maintenance and quality control, buffer stock is built-up at the beginning of each production cycle and it will be managed and used to palliate perturbations caused by production interruption during overhaul and to ensure the continuity of supply. As it can be seen in Figure 5, we will have two possible states of the process according to the control chart status.

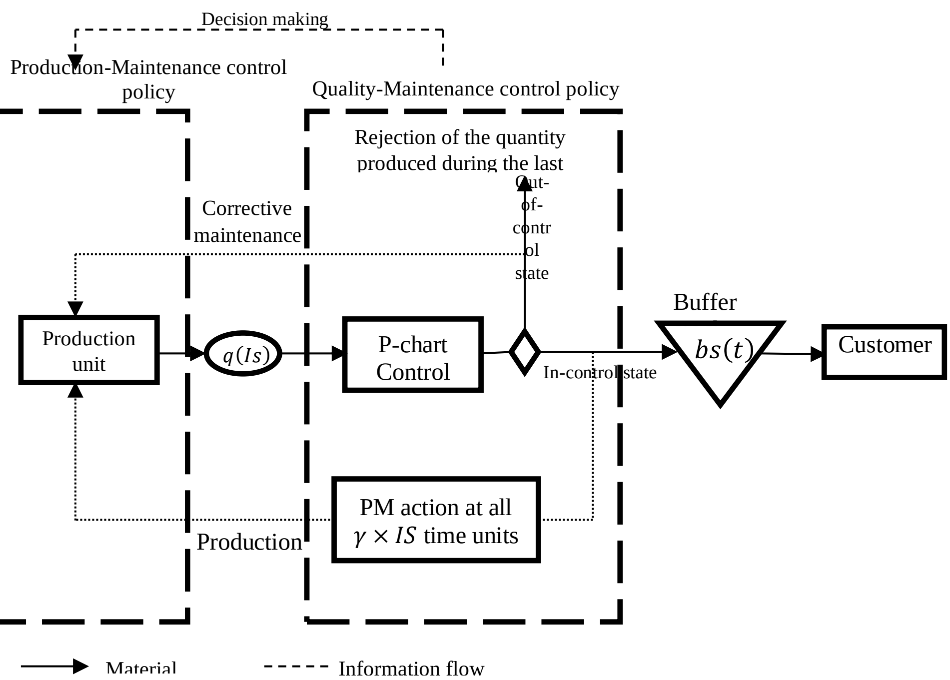


Fig. 5. The integrated production, maintenance and the quality control policy.

According to the abovementioned explanations, research assumptions and problem features can be summarized as follows [7], [4], [23], [5].

- The production unit lifetime cumulative distribution function is not known.
- The produced items are imperishable
- The demands which cannot be satisfied are lost.
- The nonconformity of the products is only due to the degradation of the machine
- The nonconforming items are not reintegrated in the manufacturing process (no rework).
- Maintenance actions restore the production unit to “as-good-as-new”

state.

- The unused buffer stock is not reinjected into production.
- The unit costs related to inventory, maintenance and quality are known and constant.
- The resources needed to perform maintenance actions are available.

4. Problem Modeling and Cost Factors

4.1. Notations and scenarios

The required parameters and decision variables to formulate the problem at hand are defined as follows.

Parameter	Definition
$q(t)$	Batch level at time t
$bs(t)$	Buffer stock at time t
n	The sample size
m	The number of samples
U_{max}	The maximum value of the production rate
D	Demand rate
P_e	The defective ratio in sample e
$\overline{\mathrm{P}}$	The average of defective ratio of process
P_N	The average of defective ratio of process in “out-of-control” state
δ	The magnitude of the shift to the “out-of-control” state compared to the center line
$q(Is)$	Batch level at time Is
J	Average number of samples controlled to detect the “out-of-control” state
μ_c	Average duration of a corrective maintenance action
$P_{in-control}$	Probability that the system is in an “in-control” state
$P_{out-of-cont}$	Probability that the system is in an “out-of-control” state
C_{PM}	Cost of a preventive maintenance action
C_{CM}	Cost of a corrective maintenance action
C_S	Unit holding cost per time unit
C_P	Unit shortage cost
C_r	Unit cost of one defective unit
C_i	Inspection cost of one unit
C_F	Cost of a false alarms
Γ_{Setup}	Average setup cost
: Γ_{S_1}	Average total inventory cost for the first scenario
: Γ_{S_2}	Average total inventory cost for the second scenario
: Γ_{Stot}	Average total inventory cost
: Γ_M	Average total maintenance cost
: Γ_Q	Average total quality cost
: $\Gamma_{Sampling}$	Average total cost of sampling
: Γ_{NC}	Average total cost of nonconforming items
: Γ_{FA}	Average total cost of false alarms
: Γ_{tot}	Average total cost per time unit

Decision variable	definition
γ	The fixed ratio of preventive maintenance
Is	Sampling interval
k	Coefficient in calculating control limits
h	Buffer stock

The main objective of this study is to minimize the average total cost of system per time unit. The proposed integrated model deals with this problem by determining the optimal value for decision variables consist of sampling interval (Is), Coefficient of control limits (k), Buffer stock (h), and the γ coefficient that indicates preventive maintenance intervals ($\gamma \times Is$).

During the production planning horizon, the manufacturing unit can be in one of the two following scenarios depending on the sample J at which the machine shifts to the "out-of-control" state.

Scenario 1:
The first scenario happens when the shift to the "out-of-control" state occurs after reaching the maximum inventory level h. This condition can be determined as (3).

$$J \times Is \geq \frac{h}{U_{max} - D} \tag{3}$$

Scenario 2:
The second scenario occurs when the buffer level h is reached after happening the "out-of-control" state. In this case, a certain level $(h' < h)$ is built. The difference between the two scenarios is that in the first scenario, the machine stores an inventory of h and then the "out-of-control" state occurs, while in scenario 2, the machine enters to the "out-of-control" state before reaching level h. The amount of stock stored in the second cases does not reach the desired amount of stock for the production system and becomes out of control.
The condition of this scenario is determined as (4).

$$J \times Is < \frac{h}{U_{max} - D} \tag{4}$$

Average duration of a restoration cycle
Contrary to the preventive maintenance operations that are undertaken periodically, the corrective maintenance actions are carried out whenever an "out-of-control" state was detected. Therefore, we have a specific duration of the preventive maintenance operations and we should calculated duration of the preventive maintenance operations (μ_c) and so, the average duration of a restoration cycle is determined as (5).

$$RCD = (J \times Is) + \mu_c \tag{5}$$

Where, J is average number of samples taken to detect the passage to the "out-of-control" state the so-called ARL (average run length) that is obtained as (6).

$$J = \frac{1}{1 - \beta} \tag{6}$$

Where, β is the probability of nondetection of the "out-of-control" state. In this case, the defective ratio in sample e (when process is out of control) does not exceed the control limits ($LCL < P_e < UCL$). According to the number of defective items has a binomial distribution, this probability is calculated as (7).

$$\beta = \sum_{d=0}^{[n \times UCL]} \binom{n}{d} . (P_N)^n . (1 - P_N)^{n-d} - \sum_{d=0}^{[n \times LCL]} \binom{n}{d} . (P_N)^n . (1 - P_N)^{n-d} \tag{7}$$

Therefore, the average duration of a restoration cycle is calculated as (8).

$$RCD = \frac{Is}{1 - \left(\sum_{d=0}^{n=n \times UCL} \binom{n}{d} P_N^d (1 - P_N)^{n-d} - \sum_{d=0}^{n=n \times LCL} \binom{n}{d} P_N^d (1 - P_N)^{n-d} \right)} + \mu_c \tag{8}$$

4.2. Average total inventory cost

The aim of production system is to produce a lot size Q at a maximum rate of U_{max}. At the first of each production cycle, we build up a buffer stock of size h at a rate $(U_{max} - D)$. Whenever the control limits of the P control chart are exceeded by defective ratio of a new sample, the production cycle will stop. Therefore, the production cycle will take $(J \times Is)$ time units to end. Equation (9) demonstrates an overall expression of the average total inventory cost according to two possible scenarios.

$$indicator(J \times Is) = \begin{cases} 1 \quad if \quad J \times Is \geq \dfrac{h}{U_{max} - D} \\ 0 \qquad \text{otherwise} \end{cases} \tag{9}$$

Based on relation (9) we can recognize when the size of the buffer stock is reached. Therefore, the average total cost of inventory is determined as (10).

$$\Gamma_{stot} = \left\{\left(indicator(J \times Is) \times \Gamma_{s_1}\right) + \left(\left(1 - indicator(J \times Is)\right) \times \Gamma_{s_2}\right)\right\} \tag{10}$$

The analytical of the average total inventory costs related to the first and second scenarios (that is Γ_{s_1} and Γ_{s_2}) can be presented via two scenarios as follows.

Scenario 1: The buffer stock level *h* is reached before the transition to the "out-of-control" state

- **Case 1: without shortage**

Figure 6 demonstrates the evolution of the buffer stock when h is reached before the transition to the "out-of-control" state without shortage. This first case is that where the service interruption period μ_c does not exceed the period of consumption of the buffer stock. In order to find the expression of the average inventory cost, Z_i is supposed as the average inventory held in each period. Based on this fact, the average inventory holding cost is determined as (11).

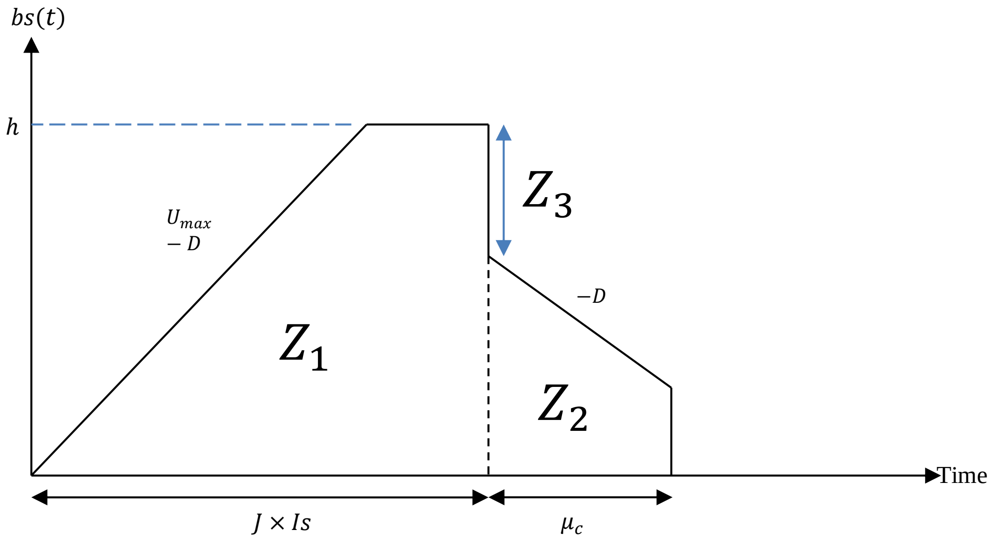


Fig. 6. Evolution of the buffer stock when *h* is reached before the transition to the "out-of-control" state without shortage.

$$\Gamma_{s_{11}} = C_s \times \left\{(p_{in-control} \times Z_1) + \left(p_{out-of-control} \times Z_2\right)\right\} \tag{11}$$

Where, $p_{in-control}$ and $p_{out-of-control}$ are calculated as (12) and (13) respectively.

$$p_{in-control} = Probability\ (LCL \leq P_e \leq UCL) = \sum_{d=0}^{[n \times UCL]} \binom{n}{d} \bar{P}^d (1-\bar{P})^{n-d} - \sum_{d=0}^{[n \times LCL]} \binom{n}{d} \bar{P}^d (1-\bar{P})^{n-d} \tag{12}$$

$$p_{out\_of-control} = Probability(P_e \geq UCL) = 1 - \sum_{d=0}^{[n \times UCL]} \binom{n}{d} \bar{P}^d (1-\bar{P})^{n-d} \tag{13}$$

In addition, we have:

$$Z_1 = \frac{h^2}{2 \times (U_{max} - D)} + h \times \left(J \times Is - \frac{h}{(U_{max} - D)}\right) \tag{14}$$

$$Z_2 = \mu_c \times (h - Z_3) - \frac{\mu_c \times D}{2} \tag{15}$$

In which, quantity produced during the last sampling interval (Is) before exceeding the control limits that is shown as Z_3 is determined as (16).

$$Z_3 = D \times Is \tag{16}$$

And so, we have (16) for Z_2.

$$Z_2 = \mu_c \times (h - D \times Is) - \frac{\mu_c \times D}{2} \tag{17}$$

Finally, the average total inventory holding cost is given as (18) considering all three zones 1, 2 and 3 together.

$$\Gamma_{s_{11}} = C_s \times \begin{Bmatrix} p_{in-control} \times \left(\frac{h^2}{2 \times (U_{max} - D)} + h \times \left(J \times Is - \frac{h}{U_{max} - D}\right)\right) \\ p_{out-of-control} \times \left(\mu_c \times (h - Z_3) - \frac{\mu_c \times D}{2}\right) \end{Bmatrix} \tag{18}$$

- **Case 2: with shortage**

Under shortage condition, the maintenance duration μ_c will exceed the time of using the buffer. In this case, the inventory cost consists of the holding cost and the cost of loss due to the shortage period. The evolution of the stock level for a restoration cycle with shortage has been presented in Figure 7.

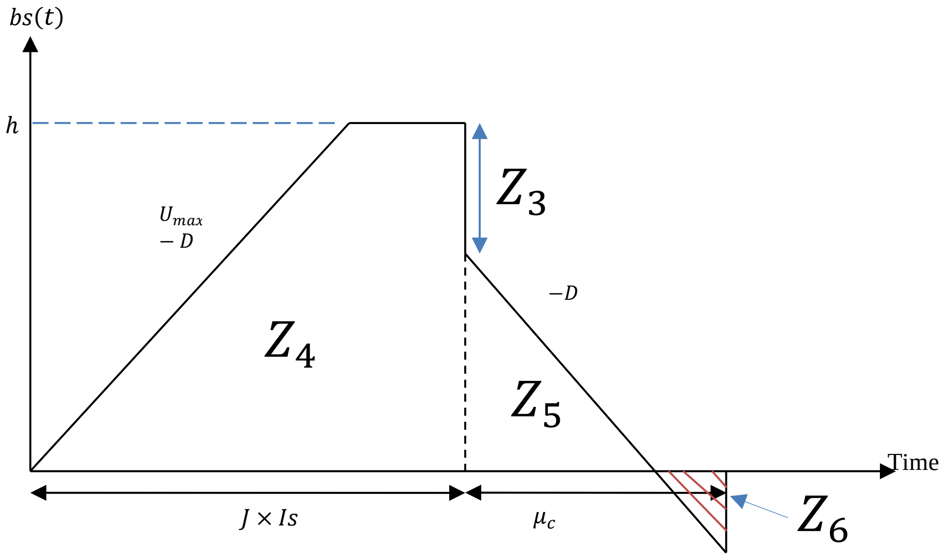


Fig. 7. Evolution of the buffer stock when h is reached before the transition to the "out-of-control" state with shortage.

$$Z_4 = \frac{h^2}{2 \times (U_{max} - D)} + h \times \left(J \times Is - \frac{h}{(U_{max} - D)}\right) \tag{19}$$

$$Z_5 = \frac{(h - Z_3)^2}{2 \times D} \tag{20}$$

$$Z_5 = \frac{(h - D \times Is)^2}{2 \times D} \tag{21}$$

$$Z_6 = \left(\mu_c - \frac{(h - Z_3)}{D}\right) \times D \tag{22}$$

$$Z_6 = \left(\mu_c - \frac{(h - D \times Is)}{D}\right) \times D \tag{23}$$

$$\Gamma_{s_{12}} = \left\{ C_s \times \left(\left(p_{in-control} \left(\frac{h^2}{2 \times (U_{max} - D)} + h \times \left(J \times Is - \frac{h}{(U_{max} - D)} \right) \right) \right) + \left(p_{out-of-control} \times \frac{(h - D \times Is)^2}{2 \times D} \right) \right) + C_p \times \left(p_{out-of-control} \times \left(\mu_c - \frac{(h - D \times Is)}{D} \right) \times D \right) \right\} \tag{24}$$

In this case, the average expected inventory cost can be determined as (5).

$$\Gamma_{s_{12}} = \left\{ C_s \times \left((p_{in-control} \times Z_4) + (p_{out-of-control} \times Z_5) \right) + C_P \left(p_{out-of-control} \times Z_6 \right) \right\} \tag{25}$$

Where, we have:
To summarize calculation of this scenario, we consider φ_1 as the index of shortage in this scenario that is determined as (26).

$$\varphi_1 = \begin{cases} 1 & \text{if } \mu_c > \dfrac{h - D \times Is}{D} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

The average total inventory cost Γ_{s_1} taking into account the two cases (without and with shortage) is determined as (27).

$$\Gamma_{s_1} = \Gamma_{s_{11}} \times (1 - \varphi_1) + \Gamma_{s_{12}} \times \varphi_1 \tag{27}$$

$$\Gamma_{s_1} = C_s \times \left(p_{in-control} \left(\frac{h^2}{2 \times (U_{max} - D)} + h \times \left(J \times Is - \frac{h}{(U_{max} - D)} \right) \right) \right) + C_s \times \left(p_{out-of-control} \times \left(\mu_c \times (h - Z_3) - \frac{\mu_c \times D}{2} \right) \right) \times (1 - \varphi_1) + \left(C_s \times \left(\left(p_{out-of-control} \times \frac{(h - D \times Is)^2}{2 \times D} \right) \right) + C_P \times \left(p_{out-of-control} \times \left(\left(\mu_c - \frac{(h - D \times Is)}{D} \right) \times D \right) \right) \right) \times \varphi_1 \tag{28}$$

Scenario 2: The buffer stock level h is reached after the shift to the "out-of-control" state.
Unlike scenario 1, in this condition the buffer stock level reaches a certain level h' ($h' < h$). Moreover, we will have two condition depending on whether shortage occurs or not.

- **Case 1: without shortage**

Figure 8 shows a schematic view of evolution the buffer stock for this case. Moreover, in the situation without any shortage, the average inventory cost ($\Gamma_{s_{21}}$) can be calculated as (29).

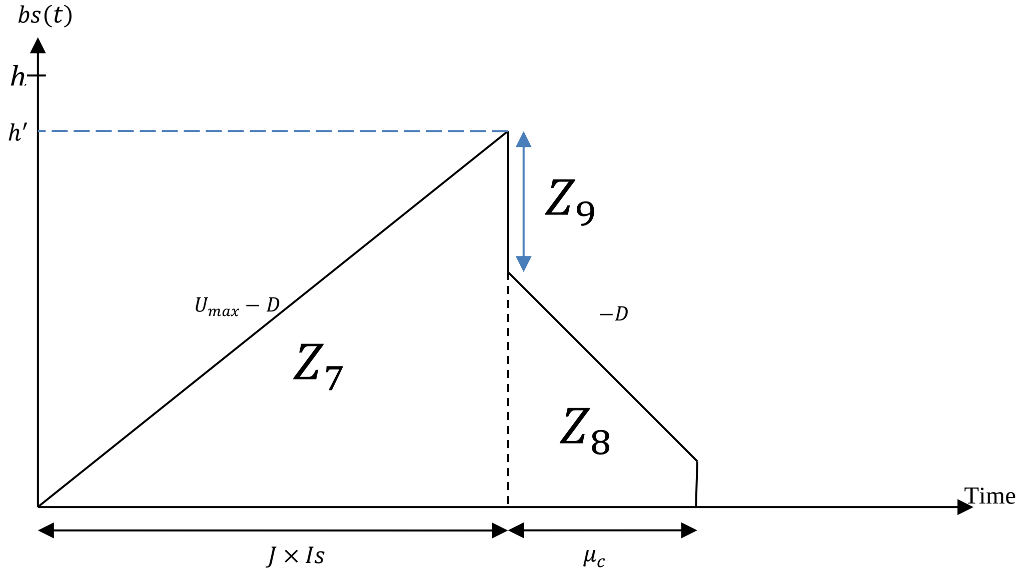


Fig. 8. Evolution of the buffer stock level: the case without shortage.

$$\Gamma_{s_{21}} = C_s \times \{(p_{in-control} \times Z_7) + (p_{out-of-control} \times Z_8)\} \tag{29}$$

Where

$$Z_7 = \frac{J \times Is \times h'}{2} \tag{30}$$

$$h' = J \times Is \times (U_{max} - D) \tag{31}$$

$$Z_8 = \mu_c \times (h' - Z_9) - \frac{{\mu_c}^2 \times D}{2} \tag{32}$$

In which, Z_9 indicates the quantity produced during the last sampling interval (Is) before exceeding the control limits and is determined as (33)

$$Z_9 = U_{max} \times Is \tag{33}$$

Similarly to the abovementioned calculation, the average expected inventory cost is obtained as (34).

$$\Gamma_{s_{21}} = C_s \times (p_{in-control} \times \frac{(J \times Is)^2 - (U_{max} - D)}{2} + p_{out-of-control} \times \left(\mu_c \times \left(\left((J \times Is)^2 \times (U_{max} - D)\right) - U_{max} \times Is\right) - \frac{{\mu_c}^2 \times D}{2}\right) \tag{34}$$

- **Case 2: with shortage**

Figure 9 shows the evolution of the buffer stock in this case. Moreover, the average inventory cost ($\Gamma_{s_{22}}$) is obtained as (35).

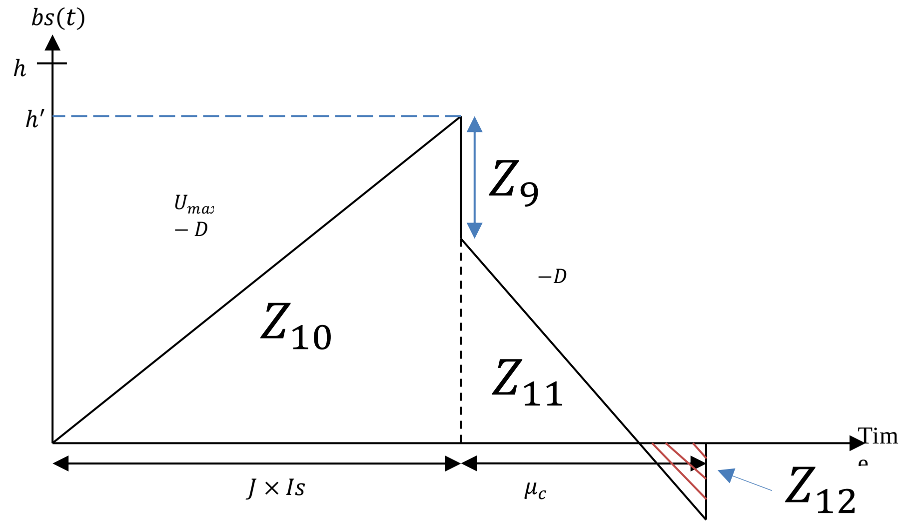


Fig. 9. Evolution of buffer stock: the case with shortage.

$$\Gamma_{s_{22}} = \left\{C_s \times \left((p_{in-control} \times Z_{10}) + (p_{out-of-control} \times Z_{11})\right) + C_p \times (p_{out-of-control} \times Z_{12})\right\} \quad (35)$$

Similarly to the same approach, we calculate $\Gamma_{s_{22}}$ as (36).

$$\begin{aligned}\Gamma_{s_{22}} = C_s \times \Bigg(p_{in-control} \times \frac{(J \times Is)^2 - (U_{max} - D)}{2} \\ + p_{out-of-control} \times \frac{(J \times Is \times (U_{max} - D) - U_{max} \times Is)^2}{2 \times D}\Bigg) \\ + C_p \times \left(p_{out-of-control} \times \left(\mu_c \times D - (J \times Is \times (U_{max} - D) - U_{max} \times Is)\right)\right)\end{aligned} \quad (36)$$

We defined $\varphi_2$as the indicator of shortage in scenario 2 as (36).

$$\varphi_2 = indicator(\mu_c) = \begin{cases} 1 & if \quad \mu_c \geq \dfrac{h' - U_{max} \times Is}{D} \\ 0 & \text{otherwise} \end{cases} \quad (37)$$

Therefore, the average expected inventory cost of the second scenario can be expressed as (38) and (38) by considering two cases (with and without shortage).

$$\Gamma_{s_2} = \Gamma_{s_{21}} \times (1 - \varphi_2) + \Gamma_{s_{22}} \times \varphi_2 \quad (38)$$

$$\begin{aligned}\Gamma_{s_2} = C_s \times p_{in-control} \left(\frac{(J \times Is)^2 - (U_{max} - D)}{2}\right) \\ + C_s \times \left(p_{out-of-control} \times \left(\mu_c \times \left(\left((J \times Is)^2 \times (U_{max} - D)\right) - U_{max} \times Is\right) - \frac{{\mu_c}^2 \times D}{2}\right)\right) \\ \times (1 - \varphi_2) + C_s \times \left(p_{out-of-control} \times \frac{(J \times Is \times (U_{max} - D) - U_{max} \times Is)^2}{2 \times D}\right) \\ + C_p \times \left(p_{out-of-control} \times \left(\mu_c \times D - (J \times Is \times (U_{max} - D) - U_{max} \times Is)\right)\right)) \\ \times \varphi_2\end{aligned} \quad (39)$$

4.3. Average total maintenance cost

The total expected cost of maintenance for all scenarios is expressed as (40).

$$\Gamma_M = C_{PM} \times \left[\frac{J}{\gamma}\right] + C_{CM} \quad (40)$$

4.4. Average total quality cost

The total expected quality costs consist of sampling cost, false alarm, and nonconforming items is expressed as (41).

$$\Gamma_Q = \Gamma_{FA} + \Gamma_{Sampling} + \Gamma_{NC} \tag{41}$$

The sampling cost and the false alarm are calculated as (42) and (43) respectively.

$$\Gamma_{Sampling} = C_i \times n \times J \tag{42}$$

$$\Gamma_{FA} = C_F \times E(f) \tag{43}$$

Where, $E(f)$ shows the number of false alarms during a restoration cycle.
The Average cost of nonconforming items can be expressed as two below scenarios.

- **Scenario 1**

In this scenario, the number of produced items during the last sampling interval (before the transition to the "out-of-control" state) will be equal to Z_3 as was illustrated in Figures 6 and 7. The average cost of nonconforming items for this scenario is determined as (44).

$$\Gamma_{NC_1} = C_r \times D \times Is \tag{44}$$

- **Scenario 2**

In the second scenario, the number of defective items at the shift to the "out-of-control" state is equal to Z_9 as was illustrated in Figures 8 and 9. Therefore, the average cost of defective units for this scenario is calculated as (45).

$$\Gamma_{NC_2} = C_r \times U_{max} \times Is \tag{45}$$

By considering three equations (9), (43), and (44), the total cost of nonconforming unit produced ca be provided by (46).

$$\Gamma_{NC} = (indicator(J \times Is) \times C_r \times D \times Is) + \big((1 - indicator(J \times Is) \times C_r \times U_{max} \times Is)\big) \tag{46}$$

Subsequently, the average cost of quality is given as (47).

$$\begin{aligned}\Gamma_Q = 2 \times C_F \times \left(\sum_{i=0}^{m} \left(i \times (p_{in-control})^i \times (1 - p_{in-control})\right) \times F(-k)\right) + C_i \times n \times J \\ + (indicator(J \times Is) \times C_r \times D \times Is) \\ + \big((1 - indicator(J \times Is) \times C_r \times U_{max} \times Is)\big)\end{aligned} \tag{47}$$

4.5. Average total cost per time unit

The average total cost per time unit can be obtained by dividing the sum of three related cost factors by the average duration of the restoration cycle as (48).

$$\Gamma_{tot} = \frac{\Gamma_{Setup} + \Gamma_{Stot} + \Gamma_M + \Gamma_Q}{RCD} \tag{48}$$

5. The Proposed Solution Approach

In this section, the proposed solution approach is presented. This method is an iterative numeric technique that is developed for the considered problem based on the proposed algorithm in [5]. The proposed algorithm is coded with MATLAB R2014a software and some analysis are carried out using MINITAB 16 software. The algorithm was run on a PC with Intel Core i7-8550U CPU, 2.10 GHz and 8GB RAM. In order to evaluate the performance of the proposed algorithms, data of was used and new required parameters were added. Moreover, some random instances were used for sensitivity analysis and more investigations.
The proposed solution procedure has been presented as Figure 10. This procedure integrates an initialization module to introduce the input data for
μ_c, Γ_{setup}, Γ_{FA}, C_{CM}, C_{PM}, C_S, C_P, C_i, C_r, C_F, $\bar{P}$, δ, U_{max},D, n, ΔIs, Δk, Δh, $\Delta\gamma$, Is_{max}, h_{max}, k_{max}, γ_{max} and then four decision variables Is ،k ،h، α

take the initial values as Is_i ،k_i ،h_i ،α_i . After that, the average duration of the restoration cycle and the average total cost per time unit are calculated as (7) and (47) respectively. Then, the decision variables are incremented, respectively, by the corresponding increments ΔIs, Δk, Δh, $\Delta\alpha$ to the limits Is_{max}, h_{max}, k_{max}, α_{max}. Moreover, all combinations are considered to calculate the average total cost time unit.

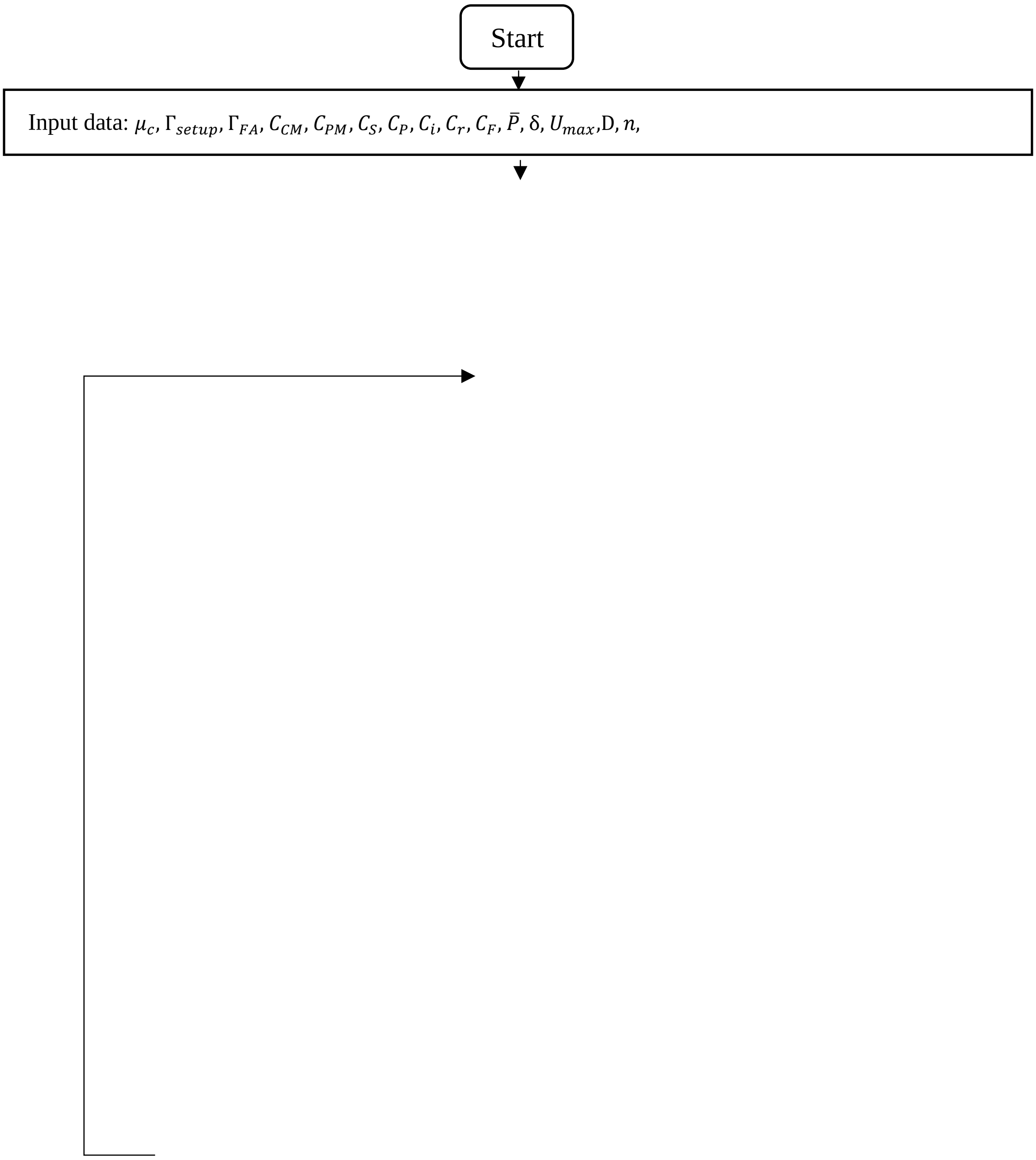


Fig. 10. The diagram of the numerical optimization procedure.

6. Numerical Example and Result Analysis

A numerical example is presented and solved in this section to illustrate and investigate the performance of the proposed solution approach. Required data for the numerical instance has been adapted from the and new parameters have been added. Table 2 represents all parameters of the problem at hand and their value.

Tab. 2. Characteristic of the numerical example

Parameter	Value
μ_c	2 h
$\bar{P}$	0.2551
n	50
δ	0.4
D	20 units/h
U_{max}	30 units/h
C_S	0.5$/unit/h
C_{CM}	1000$
C_{PM}	200$
C_P	25$/lost unit
Γ_{setup}	500$
Γ_{FA}	0.082$
C_r	70$/nonconforming unit
C_i	5$/inspected unit
ΔIs	0.1 h
Δk	0.1
Δh	1
$\Delta\gamma$	0.1
Is_{max}	30 h
h_{max}	2000
k_{max}	4
γ_{max}	50

Table 2 shows the result of solving the abovementioned numerical example using the proposed joint optimization approach. According to this result, the best strategy consists of taking a sample of size 52 every 2.9 h. The optimal distance between control limits and centre line (CL) has been determined as 2.6 number of standard deviations. In addition, preventive maintenance operations should be performed for every 7 samples on average. Moreover, the transition to the "out-of-control" state would occur in average after 59 samples ($J = 59$). Therefore, the expected time to shift to the "out-of-control" state is 52.2 ($J \times Is$).

Figure 11 represents the evolution of the buffer stock level for the proposed optimal policy according to the first scenario. Due to the result, it is necessary to build a buffer stock of 124 units, in order to continue to satisfy the demand when production is stopped to perform an overhaul. Finally, the total expected cost per time unit is equal to 215.1$/h.

Tab. 3. The obtained results with the optimal integrated policy

Is^*	k^*	h^*	γ^*	J	RCD(h)
2.9	2.6	124	7	18	54.2
Γ_{stot}($)	Γ_M($)	Γ_{NC}($)	$\Gamma_{sampling}$($)	Γ_{tot}($/h)	
1198.3	1400	4060	4500	215.1	

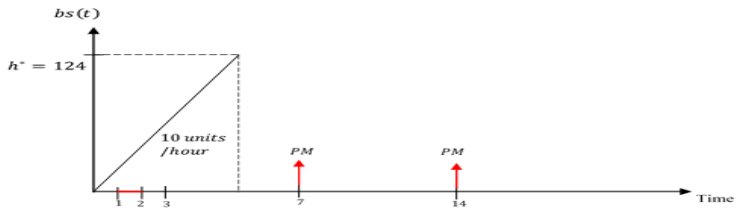


Fig. 11. Evolution of the buffer stock level.

Several supplementary analyses have been performed to measure effect of the main parameters and the sensitivity of the proposed integrated policy. The purpose of these analyses is to validate the simulation results and to study the reaction of the optimal solution in response to changes of input model parameters. Table 4 presents the cases of variation of these parameters (unit costs, maintenance, and quality parameters). In the following, the effects of these changes on the optimal solution and the total expected cost are illustrated.

- **Changes of holding cost**

When the holding cost per time unit increases, the buffer stock h^* decreases in order to reduce the average total inventory cost. In addition, the average total inventory cost decreases as the sampling interval (IS^*) decreases and consequently the average required sampling to detect an “out-of-control” state (J^*) will reduced.
For that reason, the average time to perform a CM action decreases by decreasing in J^* and inspection cycle will reduce due to the constant value of the μ_C. Moreover, table 4 highlights that reduction in k^*, the control chart becomes tighter and the accuracy of the chart increases. Therefore, the average total cos of nonconforming items will also reduce. Furthermore, the sampling cots decreasing due to the constant value of n and C_i, and reducing the J^*. However, although it is expected to decrease the total cost, it will increase due to decreasing in inspection cycle.

- **Changes of inspection cost**

As the inspection cost per time unit increases, the model increase sampling interval (IS^*) to reduce the total inspections. Moreover, α^* increase to increase PM frequencies ($\alpha \times IS$) and less PM operations will be required to performed. This causes increasing in interval time to the next CM operation (increase in J^*). On the other hand, by increasing C_i and J^*, the total sampling cost will increase. Totally, by considering all changes in this case, the average total cost is reduced.

Tab. 4. The sensitivity analysis for the problem parameters

case	parameter	variation	Is^*	k^*	h^*	α^*	J	RCD	Γ_{stot} ($)	Γ_M ($)	Γ_{NC} ($)	$\Gamma_{sampling}$ ($)	Γ_{tot} ($/h)
basic پایه			2.9	2.6	124	7	18	54.2	1198.3	1400	4060	4500	215.1
1	C_S	-0.2	6.4	2.9	189	11	27	174.8	1942.7	1400	8960	6750	111.85
2		-0.1	5.5	2.7	141	9	21	117.5	1538.4	1400	7700	5250	139.47
3		0.1	2.3	2.5	114	6	16	38.8	1060.9	1400	3220	4000	262.39
4		0.2	1.8	2.3	108	5	12	21.6	960.3	1400	1820	3000	355.57
5	C_i	-2	1.5	2.2	215	3	8	14	983.6	1400	2100	1200	441.69
6		-1	2.3	2.3	168	6	15	36.5	1019.1	1400	3220	3000	250.38
7		1	3.3	2.7	107	10	24	81.2	1530.5	1400	4620	7200	187.81
8		2	4.6	2.8	98	13	30	140	1838.3	1400	6440	10500	147.7
9	C_r	-20	7.1	2.8	45	11	27	200.8	893.6	1400	7100	6750	82.88
10		-10	4.9	2.5	93	9	21	104.9	901.4	1400	5880	5250	132.81
11		10	2.1	2.3	184	5	15	33.5	1294.3	1600	3360	3750	313.56
12		20	1.3	2.1	226	3	11	16.3	1132.5	1600	2340	2750	510.58
13	C_{PM}	-100	1.1	2.1	235	3	9	11.9	668.1	1300	1540	2250	525.89
14		-50	2.3	2.3	165	6	15	36.5	938.6	1300	3220	3750	266
15		50	3.8	2.7	107	13	29	112.2	1293.2	1500	5320	7250	141.38
16		100	4	2.9	97	15	35	142	1018.5	1600	5600	8750	123.02
17	μ_C	-1	1.4	1.6	186	2	10	15	732.4	2000	1960	2500	512.83
18		-0.5	2.1	2.2	164	5	16	35.1	824.9	1600	2940	4000	281.05
19		0.5	3.5	2.8	110	8	28	100.5	1561.1	1600	4900	7000	154.84
20		1	4.9	3	101	19	40	199	1890.2	1400	6860	10000	103.77
21	δ	-0.2	4.3	3.1	202	14	30	131	2011.5	1400	6020	7500	133.06
22		-0.1	3.6	3	193	11	27	99.2	1983.1	1400	5040	6750	157.99
23		0.1	2.2	2.9	117	5	15	35	1000.2	1600	3080	3750	283.72
24		0.2	1.9	2.4	107	3	9	19.1	780.5	1600	2660	2250	407.88

- **Changes of the cost of a defective unit**

As the unit rejection cost increases, the model reacts in a way to reduce the proportion of non-conforming units, by reducing the sampling interval. Moreover, when k^* decreases, the control chart becomes tighter and potentially more points close to the lower bounds and the average total cost of non-conforming items will reduce. Moreover, by reducing the sampling interval and by rising accuracy of the chart, the average of sample numbers to detect an out-of-control state reduce. In addition, by increasing α^* and reducing in PM frequency, the PM operations rise that will cause the total cost of maintenance to increase. As a result, the average time to undertake CM actions decreases and it increases the buffer stock size in order to improve the protection of the system against shortages.

- **Changes of the cost of preventive maintenance operation**

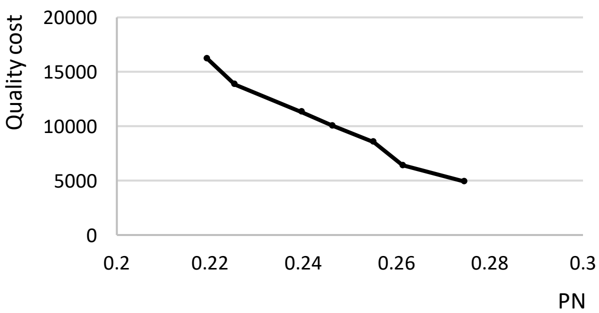
As the cost of a PM operation rises, the parameter α increases and reduce proportion $\frac{J}{\alpha}$ and in order to less preventative maintenance occurs and so, to reduce the average total cost of maintenance. However, by increasing in α^*, the sampling frequency rises and as a result, the average time to requiring a CM increases (increase of J^*), and the buffer stock size reduces. However, according to the impact of sampling interval and higher amount of J^*, the total average cost of inventory rises. In addition, the control limits become more distant and the accuracy of the control chart decreases, which increases the average cost of all non-conforming units. Finally, the total expected costs decreases considering all variations.

- **Changes of the average duration of corrective maintenance operation**

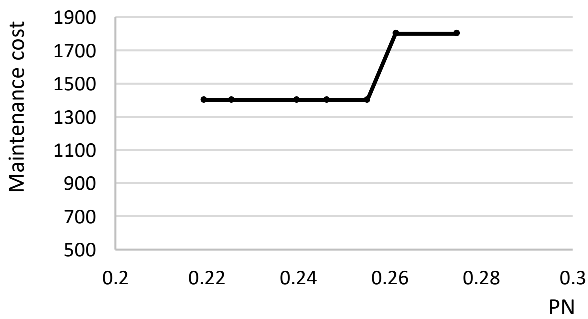
When the average duration of CM operation increases, the average period before the shift to the "out-of-control" state increases too (increase of the J) and less number of CM operation is needed. Moreover, the number of standard deviations between the center line and the control limits (k^*) increases. In this case, although the average duration of CM operation increases, due to notable decrease in the average time to perform CM operations, the buffer stock size decreases. Moreover, the parameter α increases and PM operations are performed less frequently. Finally, the total expected costs decrease considering all variations.

In the following, some supplementary analyses are presented to highlights the influences of three main parameters on the total expected cost by considering a constant value for the other factors.

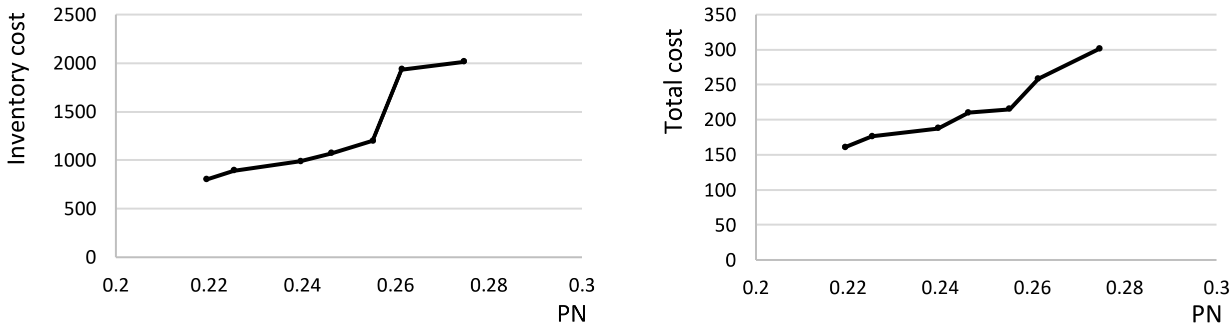
Figure 12 demonstrates the sensitivity of four mentioned cost factors to changes of P_N. As the figure shows, the total expected cost increases directly by increasing the average defective ratio of process in "out-of-control" state (see part d of the Figure 12). As can be seen in the figure, by increasing the defective ratio, the sampling distance decreases, and the control limits also move closer together to increase the accuracy of the control chart. These changes lead to a decrease in the cost of rejecting non-compliant units and the overall cost of quality (see part a of the Figure 12). In addition, by increasing in defective ratio, we need more preventive maintenance operations whenever the ratio of preventive maintenance (γ) shift down to reduce failures (see part b of the Figure 12). Finally, due to the result and by considering the effect of parameter δ as a part of the function p_N, the average total inventory cost is increased in proportion to p_N increasing (see part c of the Figure 12).



a: Changes in quality cost



b: Changes in maintenance cost

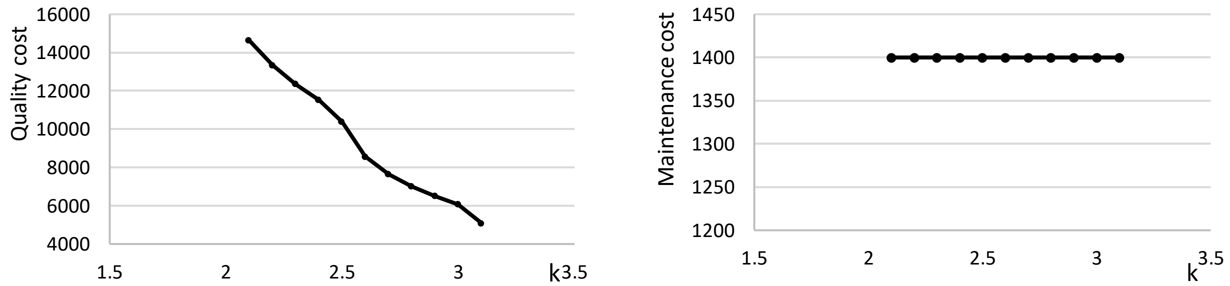


c: Changes in inventory cost

d: Changes in the total cost

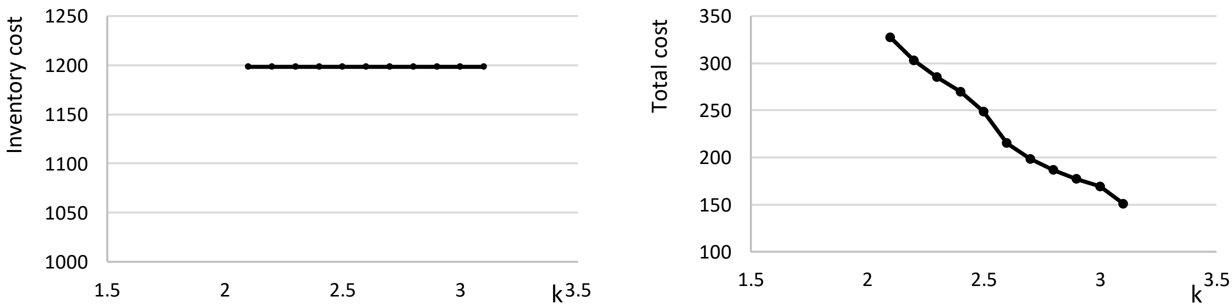
Fig. 12. Sensitivity analysis on the defective ratio (P_N) changes.

Similarly, Figure 13 represents the sensitivity of the four abovementioned cost factors to changes of coefficient values in calculating control limits (k). By increasing the parameter k (assuming the other values remain constant), the average total expected cost is reduced (see part d of Figure 13). Increase the amount of parameter k causes to the control chart gets wider. As a result, the accuracy of the chart decreases and it increases the cost of rejecting non-compliant units and, consequently, the cost of quality (as seen in part a of Figure 13). It should be noted that, changes in coefficient of control limits don’t affect maintenance costs or inventory costs (parts b and c of Figure 13). Therefore, as can see in part d of the figure, as the value of k increases, the total expected cost increases proportionally increase the cost of quality (see part d of Figure 13).



a: Changes in quality cost

b: Changes in maintenance cost

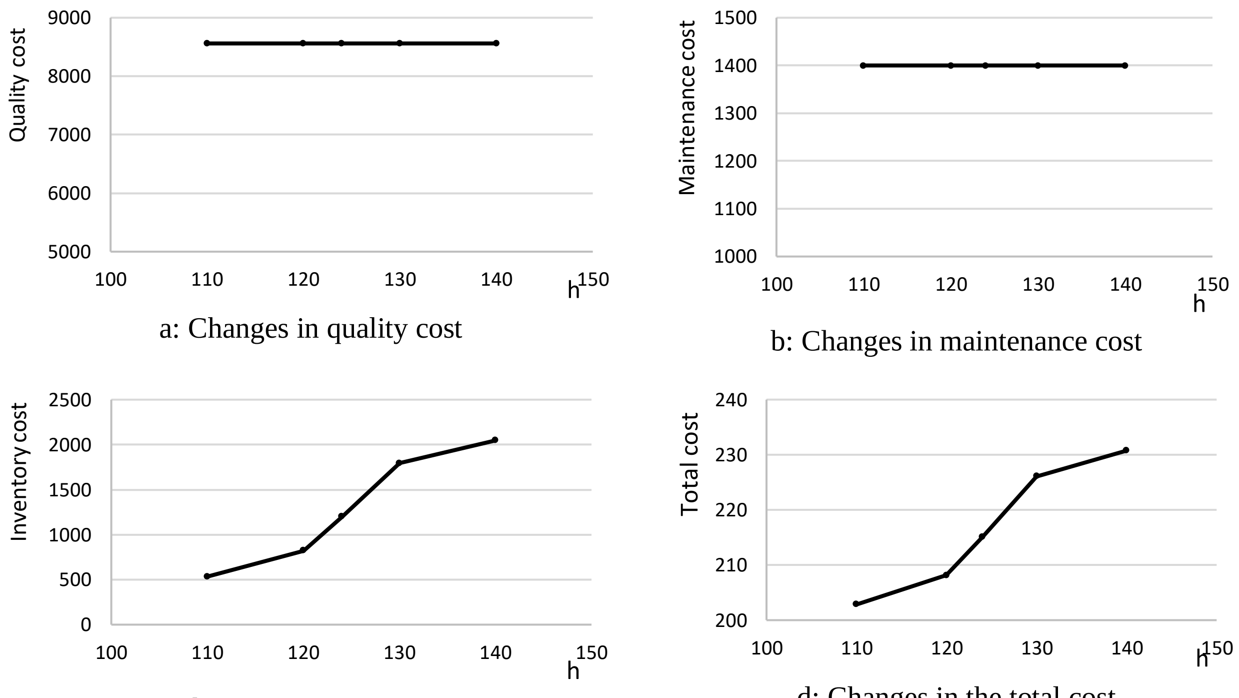


c: Changes in inventory cost

d: Changes in the total cost

Fig. 13. Sensitivity analysis on the coefficient of control limits (k).

The size of the buffer stock is another important factor that can affect the solution result and total expected cost. As expected and parts a and b of Figure 14 confirm, quality costs and maintenance costs are independent of the changes in the buffer stock (h). However, inventory costs are directly related to this factor and part c of the figure demonstrate this dependence (see part c of figure 14). Consequently, by changing the size of the buffer stock, the total expected cost will directly change similar to the inventory cost.



a: Changes in quality cost

b: Changes in maintenance cost

c: Changes in inventory cost

d: Changes in the total cost

Fig. 14. Sensitivity analysis on the buffer stock (h).

7. Conclusion

In this study, an integrated approach was proposed to optimize shop floor activities, considering three main functions including production plan, maintenance policy, and quality aspects based on a P-control chart. Optimizing these three functions simultaneously results in significant savings in operational costs and improved efficiencies for any production system. The proposed method was adopted for a single-machine production system where a constant demand has to be satisfied. We developed mathematical relations and an iterative numerical procedure to simultaneously determine the optimal values of the sampling interval, the coefficient in the calculation of the control limits, the preventive maintenance ratio, and the buffer inventory. The considered objective function was to minimize the total cost including inventory cost, maintenance cost, and quality cost. However, the considered metric for optimization was the average total cost per time unit.

A numerical instance of a real case study was solved using the proposed solution approach to investigate the performance of the iterative procedure. Moreover, sensitivity analyses based on several numerical experiments have been performed to show the effect of the main parameters and the robustness of the proposed integrated model. According to the result, reduction in k^* (coefficient value in calculating control limits) makes the control chart tighter and the accuracy of the chart increases. Therefore, the average total cost of nonconforming items also decreases. In addition, due to the constant value of the sample size and the inspection cost of one unit, the sampling cost decreases. However, while overall costs are expected to decrease, an increase in overall costs due to a reduction in the inspection cycle has been highlighted. Furthermore, as the defective ratio increases, the sampling distance decreases, and the control limits also move closer together to increase the accuracy of the control chart. These changes lead to a decrease in the cost of rejecting non-compliant units and the overall cost of quality. However, by increasing in defective ratio, we need more preventive maintenance operations whenever the ratio of preventive maintenance shift down to reduce failures.

It would be worth investigating situations where there are some parallel machines instead of one machine. Another possible extensions of this work include conducting the problem under uncertainty, especially stochastic demand. Another interesting suggestion for future related studies is to consider operational constraints such as a capacity limit to close the problem to real-world condition.

Compliance with Ethical Standards:

Ethical approval: We confirm that all the research meets the ethical guidelines, including adherence to the legal requirements of the study country.

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Conflict of Interest: The authors declare that they do not have any conflict of interest of other works.

References

[1] Aazami, A. and Saidi-Mehrabad, M. Benders decomposition algorithm for robust aggregate production planning considering pricing decisions in competitive environment: A case study. Scientia Iranica, Vol. 26, No. 5, (2019), pp. 3007-3031.

[2] Ayvaz, S. and Alpay, K., Predictive maintenance system for production lines in manufacturing: A machine learning approach using IoT data in real-time. Expert Systems with Applications, Vol. 173, (2021), p. 114598.

[3] Babaeimorad, S., Fatthi, P. and Fazlollahtabar, H., A joint optimization model for production scheduling and preventive maintenance interval. International Journal of Engineering, Vol. 34, No. 11, (2021), pp. 2508-2516.

[4] Bahria, Nadia, Anis Chelbi, Hanen Bouchriha, and Imen Harbaoui Dridi. “Integrated Production, Statistical Process Control, and Maintenance Policy for Unreliable Manufacturing Systems.” *International Journal of Production Research* Vol. 57, No. 8, (2019), pp. 2548-70.

[5] Bahria, Nadia, Imen Harbaoui Dridi, Anis Chelbi, and Hanen Bouchriha. “Joint Design of Control Chart, Production and Maintenance Policy for Unreliable Manufacturing Systems.” *Journal of Quality in Maintenance Engineering*, (2020).

[6] Ben-Daya, M., and M. A. Rahim. “Integrated Production, Quality & Maintenance Models: An Overview.” In *Integrated Models in Production Planning, Inventory, Quality, and Maintenance*, Springer US, (2001), pp. 3-28.

[7] Bouslah, B., A. Gharbi, and R. Pellerin. “Integrated Production, Sampling Quality Control and Maintenance of Deteriorating Production Systems with AOQL Constraint.” *Omega (United Kingdom)* Vol. 61, (2016), pp. 110-26.

[8] Bouslah, Bassem, Ali Gharbi, and Robert Pellerin. “Joint Production, Quality and Maintenance Control of a Two-Machine Line Subject to Operation-Dependent and Quality-Dependent Failures.” *International Journal of Production Economics* Vol. 195, (2018), pp. 210-26

[9] Chiu, Singa Wang, Shan Ling Wang, and Yuan Shyi Peter Chiu. “Determining the Optimal Run Time for EPQ Model with Scrap, Rework, and Stochastic Breakdowns.” *European Journal of Operational Research* Vol. 180, No. 2, (2007), pp. 664-76.

[10] Dhouib, K., A. Gharbi, and M. N. Ben Aziza. “Joint Optimal Production Control/Preventive Maintenance Policy for Imperfect Process Manufacturing Cell.” *International Journal of Production Economics* Vol. 137, No. 1, (2012), pp. 126-36.

[11] Duffuaa, S., Kolus, A., Al-Turki, U. and El-Khalifa, A., An integrated model of production scheduling, maintenance and quality for a single machine. *Computers & Industrial Engineering*, Vol. 142, (2020), p. 106239.

[12] Jafarian-Namin, S., Fallahnezhad, M.S., Tavakkoli-Moghaddam, R., Salmasnia, A. and Fatemi Ghomi, S.M.T., An integrated quality, maintenance and production model based on the delayed monitoring under the ARMA control chart. *Journal of Statistical Computation and Simulation*, (2021), pp. 1-25.

[13] Khatab, Abdelhakim, Claver Diallo, El Houssaine Aghezzaf, and Uday Venkatadri. “Integrated Production Quality and Condition-Based Maintenance Optimisation for a Stochastically Deteriorating Manufacturing System.” *International Journal of Production Research* Vol. 57, No. 8, (2019), pp. 2480-

97.

[14] Lopes, R., Integrated model of quality inspection, preventive maintenance and buffer stock in an imperfect production system. *Computers & Industrial Engineering*, Vol. 126, (2018), pp.650-656.

[15] Mishra, A.K., Shrivastava, D., Tarasia, D. and Rahim, A., Joint optimization of production scheduling and group preventive maintenance planning in multi-machine systems. Annals of Operations Research, (2021), pp.1-44.

[16] *Montgomery, D. C. Introduction to Statistics Quality Control. 6th edu* & Sons Inc., (2009).

[17] Pandey, Divya, Makarand S. Kulkarni, and Prem Vrat. "Joint Consideration of Production Scheduling, Maintenance and Quality Policies: A Review and Conceptual Framework." *International Journal of Advanced Operations Management* Vol. 2, Nos. 1/2, (2010).

[18] Pandey, Divya, Makarand S. Kulkarni, and Prem Vrat. "A Methodology for Joint Optimization for Maintenance Planning, Process Quality and Production Scheduling." *Computers and Industrial Engineering* Vol. 61, No. 4, (2011), pp. 1098-1106. http://dx.doi.org/10.1016/j.cie.2011.06.023.

[19] Radhoui, M., N. Rezg, and A. Chelbi. "Integrated Model of Preventive Maintenance, Quality Control and Buffer Sizing for Unreliable and Imperfect Production Systems." In *International Journal of Production Research*, Taylor & Francis Group, (2009), pp. 389-402.

[20] Radhoui, M., N. Rezg, and A. Chelbi. "Integrated Maintenance and Control Policy Based on Quality Control." *Computers and Industrial Engineering* Vol. 58, No. 3, (2010), pp. 443-51. http://dx.doi.org/10.1016/j.cie.2009.11.002.

[21] Rahim, M. A., & Ben-Daya, M. A generalized economic model for joint determination of production run, inspection schedule and control chart design. *International Journal of Production Research*, Vol. 36, No. 1, (1998), pp. 277-289.

[22] Rivera-Gómez, Héctor et al. "Joint Optimization of Production and Maintenance Strategies Considering a Dynamic Sampling Strategy for a Deteriorating System." *Computers and Industrial Engineering* Vol. 140, (2020), p. 106273.

[23] Salmasnia, Ali, Farzaneh Soltani, Elham Heydari, and Samira Googoonani. "An Integrated Model for Joint Determination of Production Run Length, Adaptive Control Chart Parameters and Maintenance Policy." *Journal of Industrial and Production Engineering* Vol. 36, No. 6, (2019), pp. 401-17.

[24] Wang, L., Lu, Z. and Han, X., Joint optimisation of production, maintenance and quality for batch production system subject to varying operational conditions. *International Journal of Production Research*, Vol. 57, No. 24, (2019), pp. 7552-7566.

[25] Zhang, J., Zhao, X., Song, Y. and Qiu, Q., Joint optimization of condition-based maintenance and spares inventory for a series–parallel system with two failure modes. Computers & Industrial Engineering, Vol. 168, (2022), p.108094.

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