MODAL ANALYSIS OF A TURBO-PUMP SHAFT: AN INNOVATIVE SUSPENDING METHOD TO IMPROVE THE RESULTS

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Abstract: *Modal parameter extraction of high speed shafts is of critical importance in mechanical design of turbo-pumps. Due to the complex geometry and peripheral components of turbo-pumps, difficulties can arise in determination of modal parameters. In this study, modal properties of a turbo-pump shaft, was studied by experimental modal analysis, and using different excitation techniques. An innovative suspending method is proposed to reduce noise-to-signal ratio, resulting from classic suspensions. Comparison of the experimental results obtained from the proposed suspension method and the traditional ones shows that the proposed approach was a promising method, when classic methods fall short of expectations in analysis of complex structures. To validate the experimental results, numerical solution was carried out using simplified geometric modeling combined with the Finite Element Method. The simplified modeling approach can be considered as a reliable theoretical method for numerical modal analysis of similar structures. Comparison of experimental and numerical results shows that there is a good conformity between the results of two approaches.*

Keywords*: Turbo-pump, shaft, experimental modal analysis, numerical modal analysis, suspension techniques*

1. Introduction

Design and manufacturing of turbo-pumps requires essential considerations, especially in high speed operating conditions. The high rotational speed of the shaft together with unavoidable tiny misalignment can be considered as one the most important sources of excitation, which may cause critical conditions due to the resonance. The extraction of modal parameters and the design of shaft bearings play important roles in design process [1]. Finite Element method has been widely used to determine natural frequencies and mode shapes of structures with complex geometry. However, due to the complex boundary condition the experimental modal analysis is an indispensable part of modal parameters extraction and determines the most realistic estimate of modal behavior of structures. The excitation method in experimental tests may be selected to be by shaker or by roving-hammer test,

according to the properties of structure and testing equipment [2].

Previous studies have used analytical and/or Experimental methods to analyze similar structures. Some studies have used extended experimental tests along with mathematical post processing of results [3-5]. Other studies have used Finite Element methods with various techniques such as sub-structuring [6, 7]. However very few of studies address the importance of suspension on the experimental results, to obtain a valid and error free signals. In this paper a new and simple method for suspension is introduced. The proposed method provides simple implementation and more accurate results, which facilitates the use of computational methods in complicated problems. This method was applied on several experiments, using different input sources; including both hammer and shaker excitation tests. Moreover, for better insight, modal parameters of the turbo-pump shaft assembly were determined, using a simplified model of the shaft and finite element analysis.

2. Model Description

The subject of study is the shaft of a turbo-pump, used for mixing of fluids. The shaft complex consists of main rod, the fins and helical attachments. (Figure 1)

Fig. 1. The Shaft and corresponding components of turbo-pump

The main rod and fins are made of stainless steel and the helical structures of aluminum. The nominal operating speed of shaft is in the range of 12000 to 18000 rpm. The details of structure dimensions are listed in table 1.

Tab. 1. Detailed dimensions of shaft components

Component	Dimension	Value
Main Rod		
	Maximum Diameter (mm)	455.0
	Minimum Diameter (mm)	30.0
	Length (mm)	758.5
Helical Components (227.5 & 630 mm from end of shaft)		
	Maximum Diameter (mm)	74.0
	Minimum Diameter (mm)	30.0
	Length (mm)	69.0
Other Attachments		
	Maximum Diameter (mm)	75.0
	Minimum Diameter (mm)	24.0

3. Theoretical Backgrounds
3-1. Experimental Modal Analysis

The experimental modal analysis to obtain the dynamic behavior of a system has been extendedly discussed in the literature [8, 9]. Assuming linear, time-invariant, observable system that complies with Maxwell's reciprocity principle, the basic formulation will be discussed as follows [2].

The overall dynamic behavior is described by a set of linear mass–damper spring systems, which can be described by linear second order differential equations of motion [10]:

$$\underset{\sim}{M}\ddot{x} + \underset{\sim}{C}\dot{x} + \underset{\sim}{K} x = f(t) \tag{1}$$

where x, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the time dependent displacement vector, the mass or inertia matrix, the viscous damping matrix, and the stiffness matrix, respectively. Also $\mathbf{f(t)}$ is the external applied force vector. The Laplace Transform of equation 1 yields:

$$[\underset{\sim}{M}s^2 + \underset{\sim}{C}s + \underset{\sim}{K}] X(s)$$
$$= F(s) + (\underset{\sim}{M} s + \underset{\sim}{C}) x(0) + \underset{\sim}{M} \dot{x}(0) \tag{2}$$

The modal model is fully described by a matrix of frequency response functions $\underset{\sim}{\mathbf{H}}_{pq}(\omega)$, whose elements may be obtained by calculating the ratio of the response signal at DOF p, $\mathbf{X}_p(\omega)$ to the input signal at DOF q, $\mathbf{F}_q(\omega)$ in the frequency domain [11]:

$$\underset{\sim}{H}_{pq} = \frac{X_p}{F_q} \tag{3}$$

Assuming zero initial condition, equation.2 may be written (in the frequency domain) as:

$$\mathbf{X}(\omega) = \underset{\sim}{\mathbf{H}}(\omega).\mathbf{F}(\omega) \tag{4}$$

where:

$$\underset{\sim}{H}(\omega) = \left[\underset{\sim}{K} - \omega^2 \underset{\sim}{M} + i\,\omega \underset{\sim}{C} \right]^{-1} \tag{5}$$

The relationship between the natural frequencies, mode shapes and the elements of $\underset{\sim}{\mathbf{H}}(\omega)$ matrix, can be obtained by [12, 13]:

$$H_{ij}(\omega) = \frac{X_i}{F_j} = \sum_{r=1}^{N} \frac{\phi_{ir}\phi_{jr}}{\omega_r^2 - \omega^2 + i\eta_r\omega_r^2} \tag{6}$$

3-2. Finite Element Method

For finite element analysis of turbo-pump, the shaft geometry as well as its components was modeled. In order to achieve a feasible FEM solution, the fins and helical components of shaft were modeled as simple solid disks with equivalent mass and moment of Inertia. The effect of simplification is negligible due to the ratio of dimension of components to the dimensions of main rod. The equation of motion for an undamped system may be expressed in matrix form as:

$$\underset{\sim}{M}\ddot{u} + \underset{\sim}{K}u = o \tag{7}$$

where $\underline{M}$ and $\underline{K}$ are Mass and Stiffness matrices respectively. Determination of natural frequencies and mode shapes, leads to solution of the following Eigenvalue problem:

$$\underline{K}u = \omega \underline{M}u \qquad (8)$$

The Eigenvalue problem in equation 8 can be solved by a general purpose finite element packages by different methods [14].

After development of simplified geometrical model of shaft, the modeled structure was meshed, using 3D linear 8 node elements (Figure 2). A total of 21800 elements were used which provides 141700 DOF. According to characteristics of problem and element type, the block lanczos method was chosen as the modal solver for eigenvalue extraction.
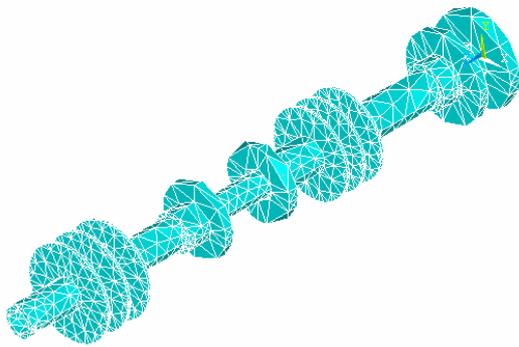

Fig. 2. Meshed model of the turbo-pump shaft, after simplification of model geometry.

4. Experiment Setup

4-1. Suspending the System

In order not to lose the generality of experiment and analysis, a free-free condition was chosen for modal analysis. To obtain minimal distortion of natural motion, the structure is usually sustained by sets of elastic strings [15]. The suitable elastic strings (Flaxes), are in general of very low stiffness while having high strength. The preliminary test results obtained by using available elastic strings, showed extremely noisy response. The data obtained by classic suspension method was so noisy and prone to error that could not be used for further processing (Figure 3a) Errors were probably due to the: specific characteristics of the structure, interaction between the supports and the structure, and the type of selected support itself. As a result, a different suspension setup, appropriate for our case and with new support materials and support method was needed. It is hypothesized that due to considerably limited excitation in the longitudinal direction of shaft, a partial distortion of natural motion along that direction, imposes only negligible error in the results. Considering the primary FEM results, showing limited excitation possibility along the longitudinal direction, this direction was chosen as less important direction. Therefore a small inhibition in motion along this direction was considered to generate very low percent of error in other directions. Consequently the shaft was

hanged along its symmetric axes (longitudinal direction) from a frame by using a hook, attached to shaft by industrial glue (Figure 4). It should be noted that the low stiffness of the applied glue in suspension provides appropriate freedom for natural motion. In this case of suspension, the signal to noise ration improved considerably in comparison with the former classic suspending method (Figure 3b). It should be emphasized that no type of averaging was used in estimation of the FRF's shown in the figures.

4-2. Excitation Methods

To ensure that more mode shapes are properly excited, both hammer test and shaker test was used to provide the required excitation for the structure. In hammer test (Type 8202, B & K Inc., Denmark, with a force transducer with a sensitivity of 0.94 pc/N, mass of hammer=21 gram), we applied the force to four points respectively (Points A-D in Figure 5). The coordinates of excitation points are listed in Table 2a. In shaker test, an eccentric excitation point (to excite rotational DOF) is considered for input (Shaker Type 4809, B & K Inc. Denmark, with a force transducer with a sensitivity of 0.94 pc/N) (Point E in Figure 5). The coordinates of excitation point are listed in Table 2a. Random signal was chosen as input to achieve desired excitation.
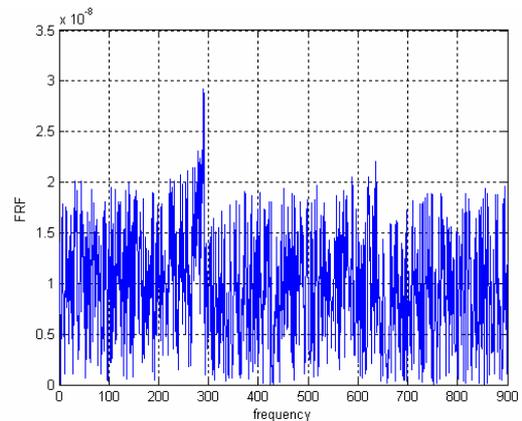

Fig. 3a Linear amplitude of Frequency Response Function, obtained with classic method of suspension
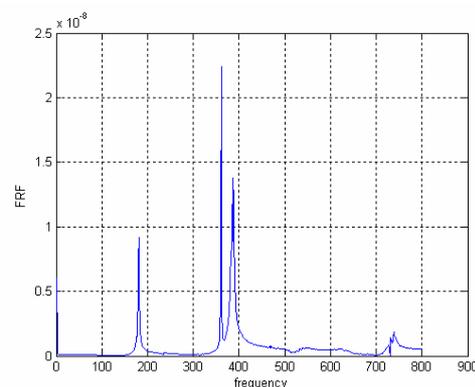

Fig. 3b. Linear amplitude of Frequency Response Function, obtained with the new innovative suspending method.

Tab. 2a. Coordinates of excitation points in Hammer and Shaker tests. R, Theta and Z are the radius, angle and height in cylindrical coordinate system, respectively. (see also Figure 5a)

E excitation Point	R (mm)	θ (degree)	Z (mm)
A	227.5	90	8
B	47	90	205
C	19	90	290
D	40	90	460
E	17	270	350

4-3. Measurement

A 3-axial accelerometer (Type 4504, B & K Inc.Denmark, with mass of 14 gram) was used to measure the output of system (Figure 6).

Fig. 4. The Turbo-pump shaft, dangled from the support frame by a glued hook

Tab. 2b. Coordinates of measurement points in Hammer and Shaker tests. R, Theta and Z are the radius, angle and height in cylindrical coordinate system, respectively. (see also Figure 5b)

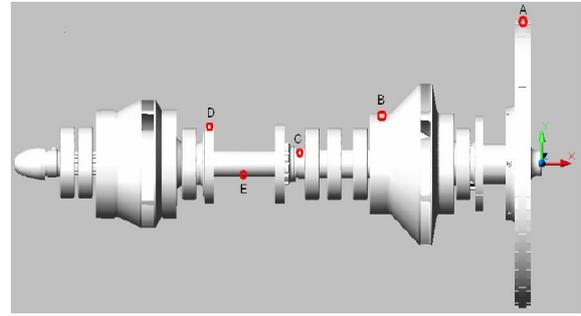| Measurement Point	R(mm)	θ (degree)	Z(mm)
1	227.5	0	10
2	227.5	120	10
3	227.5	240	10
4	47	0	205
5	47	120	205
6	47	240	205
7	19	0	290
8	19	120	290
9	19	240	290
10	17	0	350
11	17	180	350
12	47	0	490
13	47	120	490
14	47	240	490
15	17	270	350

Fig. 5. Model of turbo-pump shaft. Excitation points in Hammer and Shaker tests are indicated by numbered circles

It should be noted that by using 3-axial accelerometer the measurement of rotational DOF's as well as longitudinal ones can be achieved. In shaker test, 14 measurement points were considered for output. The coordinates of measurement points are listed in Table 2b. In hammer test, the system output in one point (Point 15) is measured. The coordinates of measurement points are listed in Table 2b.

Fig. 6. Experiment setup for modal test of turbo-pump shaft in hammer test. Hammer, Accelerometer, and Data Acquisition unit are shown.

4-4. Data Acquisition

The data acquisition process is controlled using a four channel Dynamic Signal Analyzer Pulse System (Type 2827, B & K Inc. Denmark). The signal of the force sensor is windowed by a rectangular window. An exponential decay window in time domain was used to the response signal, in the hammer test and a Hanning window in the shaker test.

The algorithm requires that the number of time samples (N) be a power of two [16]. The frequency of measured data ranges from 0 to 1 kHz. In order to ensure good-quality measurements, a coherence of more than 90% is required to accept the measurement [13]. The FRF matrix is used to estimate the modal parameters by the Pulse system PC software (Type 7754-J, B & K Inc., Denmark).

5. Results

5-1. Experimental Results

The FRF diagram of the shaker test corresponding to measurement from Point 8 is shown in Figure 7. The peaks of the FRF function indicate the excited natural frequencies. The Natural Frequencies and Mode shapes obtained from hammer tests and shaker tests are listed in Table 3. It may be seen that a total of 4 bending mode shapes, one longitudinal, and one torsion mode shape, are excited. Two of mode shapes were not excited in hammer test.
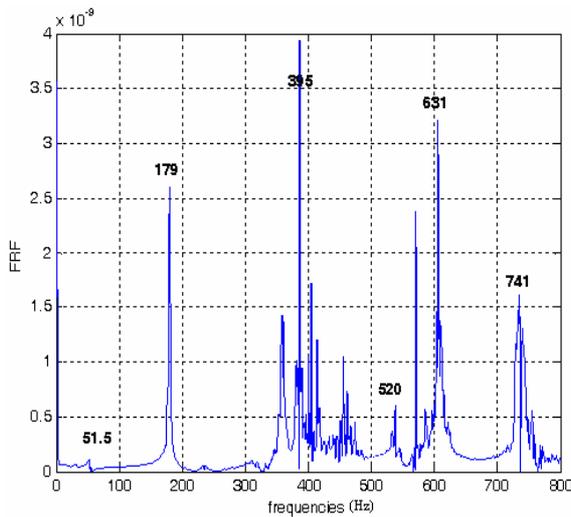

Fig. 7. Linear amplitude of Frequency Response Function, corresponding to measurements from point 8 in shaker test

(See Fig. 5b).

Tab. 3. Natural Frequencies and Mode Shapes from hammer tests and shaker tests

Case	Shaker Test	Hammer Test	Mode Shape
1	51.5	48.5	1st Bending Mode
2	179	182	1st Torsion Mode
3	395	401	2nd Bending Mode
4	520	Not excited	3rd Bending Mode
5	622	Not excited	1st Longitudinal Mode
6	741	742	4th Bending Mode

5-2. FEM Results

The result of Finite Element solution, including natural frequencies and mode shapes are listed in table 4. The results include natural frequencies in range of 0 to 1.1 kHz. The analysis results are in the range of the operating condition of the shaft (1-300Hz). There are 7 Bending mode shapes, 2 torsion mode shapes, 1 longitudinal mode shape and 3 complex mode shapes with no distinct motion along the defined DOF's. The first bending mode shape is shown in figure 8.

A complete list of FEM and experimental results and the percentage of error between experimental and numerical results cab be found in table 5. The error shows the error between computational results and average hammer and shaker test results.

Tab. 4. Natural frequencies and mode shapes of shaft, from FEM solution. (Non-zero frequencies only)

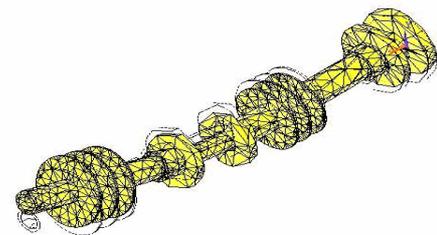| Case	Natural Frequency (Hz)	Mode shape
1	49.52	1st Bending Mode
2	174.7	1st Torsion Mode
3	329.3	2nd Bending Mode
4	419.3	2nd Torsion Mode
5	499.0	3rd Bending Mode
6	510.7	Complex mode shape
7	632.0	1st Longitudinal Mode
8	680.9	4th Bending mode shape
9	903.1	5th Bending Mode
10	966.2	Complex mode shape
11	1023.1	6th Bending Mode
12	1051.7	7th Bending Mode
13	1085.2	Complex mode shape

Fig. 8. Graphical representation of the first Bending Mode shape, obtained from Finite Element solution

Tab. 5. Natural Frequencies and Mode Shapes from hammer tests and shaker tests and FEM solutions.

Case	FEM (Hz)	Experiment (Shaker Test) (Hz)	Experiment (Hammer Test) (Hz)	Error (%)	Mode shape
1	49.52	51.5	48.5	0.96	1st Bending Mode
2	174.7	179	182	3.21	1st Torsion Mode
3	329.3	395	401	17.26	2nd Bending Mode
4	419.3	-	-		2nd Torsion Mode
5	499.0	520	Not excited	4.04	3rd Bending Mode
6	510.7	-	-	-	Complex undefined mode shape
7	632.0	622	Not excited	1.61	1st Longitudinal Mode
8	680.9	741	742	8.17	4th Bending mode shape
9	903.1	-	-	-	5th Bending Mode
10	966.2	-	-	-	Complex undefined mode shape
11	1023.1	-	-	-	6th Bending Mode
12	1051.7	-	-	-	7th Bending Mode
13	1085.2	-	-	-	Complex undefined mode shape

6. Conclusions

Comparison of computational and experimental results of table 5 reveals that some mode shapes are not excited in experiments. This is mainly originated from common limitations of experimental tests. Unexcited mode shapes include second torsion mode, high-frequency bending mode, and complex mode shapes. These results are expected, due to difficulty of excitation of these modes with the available setup [17-19].

The shaker test excited more mode shapes which can be addressed to its continuous random excitation force and the eccentric application of input force [20]. The low percentage of error between the FEM and experimental results (except for the second bending mode) implies that attained modal parameters are of desirable accuracy. The compatibility of the results validates the modeling and experimental processes.

The errors can be attributed to simplifications in assumptions, limitations in modeling, and other issues in experiments. The simplified modeling of shaft components, assumption of unitized body and neglecting the areas in contact, and meshing issues can be other sources of error. Common sources of error, including data acquisition, leakage, windowing, filtering and other computational sources of error exist and may affect the results[7]. Other experiment limitations such as variability of applied transient force in hammer tests also exist [21].

We implemented an innovative support condition for modal analysis. It may be discussed that dangling the shaft from a glued hook, attached to the support frame, distorts the natural motion of shaft, especially in longitudinal direction. It should be noted that the material used as glue is considerably low in stiffness and the total connection provides adequate freedom of motion. In addition, the results show only negligible error in longitudinal mode (1.61 %) which is an indication of undistorted natural motion.

The conclusion about the FEM modeling can be made as: the proposed simplified modeling may be used in similar cases, which eliminates the need of complex procedures such as sub-structuring. The numerical and experimental results were in good conformity with each other, which may be used in design and modification schemes.

Future works should focus on the effect of shaft bearings and fluids in contact with it during operation of shaft, on the response of system. The effect of mentioned factors may be considered as a modified boundary conditions and exerted loads on the shaft. Sensitivity Analysis of shaft bearings, detailed FEM analysis, extended experimental tests and post-processing of experimental data can shed light to unrevealed aspects of dynamic response of shaft.

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