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RESEARCH PAPER

Developing a Constrained Mathematical Model to Optimize the Expected Total Costs of Life Testing

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ABSTRACT
The subjects of reliability acceptance sampling plans and failure-censored life tests have usually been investigated from the viewpoint of statistical properties; indeed, few researchers have shed light on the economic aspects of these issues. In this research, a constrained mathematical model is developed to optimally design a reliability sampling plan under failure censoring life testing. Minimizing the expected total cost (ETC) involved in the sampling and life testing is considered as the objective function of the model. Ensuring the producer's and the consumer's risks is taken into consideration as the constraint of the model. To minimize the ETC, the model optimally determines three decision variables including the total number of the items put to the life test, the number of the failed items to terminate the test, and a criterion to make decisions about the acceptance or rejection of the lot. Examples are provided and analyses are conducted to gain some insight regarding the model performance.

KEYWORDS: *Reliability; Lifetime; Life testing; Acceptance sampling plan.*

Notation

ETC	Expected total cost of the experiment
ETT	Expected time to terminate the test
C_1	Operation cost of the test per time unit
C_2	The cost of each item placed on the test
C_3	The cost of each failed item during the test
n	Total number of the items put to the test (decision variable of the model)
r	Number of the failed items during the test (decision variable of the model)
k	Criterion to evaluate the quality of the lot (decision variable of the model)
α	Producer's risk
β	Consumer's risk
λ	Failure rate of the exponential distribution

1. Introduction

Acceptance sampling plans are classified under the traditional field of statistical quality control [1]. In a reliability acceptance sampling plan (RASP), the main parameter of quality is lifetime. The acquisition of data on lifetime, which is required in RASPs, usually involves conducting certain types of life testing. Different types of life testing schemes have been proposed including but not limited to failure censoring, time censoring, progressive censoring and hybrid censoring [2]. From another point of view, life testing schemes can be classified into right and left censoring.

Failure censoring schemes, which are also known as right censoring schemes type II, have been broadly employed by quality/reliability engineers and statisticians to evaluate lifetime. In the case of a failure censoring life test (FCLT), the test is terminated after a predetermined number of failures are observed. In particular, n items are randomly selected and put to the test simultaneously. The failure time of each item is recorded, and the test is terminated after $r (r \leq n)$ failures. Hence, there are two factors to impact the cost of the FCLT including the total number of the tested items (n) and the number of the failed items required to terminate the test (r). In an RASP, according to the results of the FCLT,

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an appropriate statistical operation is done, the obtained numerical data are compared to a predetermined criterion, and it is finally decided whether to accept the lot or reject it. Hence, in an RASP with lifetime tests, there exit three factors affecting the total cost of the procedure including the criterion applied to make decisions on the lot, the value of n, and the value of r.

The main costs incurred in the proposed RASP under FCLT are as follows:

a) Operation costs: It has a direct relationship with the length of the test, i.e., as the test duration increases, the cost increases too. The salary of the operators, the depreciation cost of the equipment and so on can be considered as the operation costs [3].

b) Sample cost: If n items are selected to be included in the test and the cost of each item is C_2, then the total sample cost is $C_2 n$.

c) The cost of failed items: If the cost of each failed item is C_3 and the test is terminated after r failures are observed, then the total cost of the failed items is $C_3 r$.

As discussed in the next sections, an increase in the number of the items put to the test decreases the expected time to terminate the test and, consequently, the operation costs. On the other hand, an increase of n makes the sample cost rise because it directly relates to the number of the tested items. The number of failed items and the criterion for the acceptance or rejection of the lot are mostly determined by the producer's and consumer's risks, the acceptable quality level (AQL) and the limiting quality level (LQL) of the plan. Accordingly, a trade-off is necessary between the operation costs and the sample costs.

In this paper, to optimally design an RASP under FCLT, a mathematical model is developed. The objective function of the model optimally determines the number of the items put to the test (n), the required number of failed items (r), and the criterion to evaluate the lot (k) in order to minimize the expected total cost (ETC) of the test. The producer's and consumer's risks are taken into consideration as the constraints of the model.

The rest of the paper is presented in several sections. In Section 2, a literature review is provided regarding the subject of the paper. In Section 3, the problem is stated, and the model is presented. Section 4 is dedicated to some numerical studies and analyses. Section 5 provides comparative analyses. Finally, the conclusion of the study and recommendations for future research end up the paper in Section 6.

2. Literature Review

There exists a huge body of literature which discusses life testing and RASPs from statistical points of view. For example, in references [4], [5], [6] and [7] , maximum likelihood estimators are provided for some parameters associated with life testing. In some other studies, RASPs are designed to minimize the expected number of failed items. For example, in reference [8], considering the lifetime performance index, a mathematical model is provided to minimize the number of the items failed in a failure censoring life test. Reference [9] provides another mathematical model to design an RASP under failure censoring while minimizing the number of failed items is considered as the objective function. In some studies such as [10] and [11], RASPs and life tests have been designed such that the producer's and consumer's risks are considered as constraints. In Reference [12], RASPs under failure censoring are investigated from the perspective of conditional value-at-risk. To this end, a risk-embedded model is developed, and lifetime is assumed to follow Weibull distribution. Minimizing the average sample number or the average failure number is another prevalent criterion investigated by some researchers. For example, reference [2] provides three mathematical models to minimize the average failure number under failure censoring. The producer's and consumer's risks are taken into consideration too. In reference [13], based on the process capability index, a mathematical model is developed to minimize the average sample number. Considering quick switching sampling systems, two mathematical models are developed in reference [14]. The models minimize the average sample number, and the risks facing the producer and the consumer are included as the constraints of the model.

To minimize the average number of failures under a quick switching reliability sampling plan, a mathematical model is proposed in [15]. The authors derive equations to compute the OC curve of the sampling plans. It is assumed that the lifetime of items is based on exponential distribution, so a lifetime performance index is employed. An accelerated life test has been designed under type-II censoring so as to extend the exponential distribution [16]. Designing multiple accelerated life tests is also discussed in [17] for log-normal distributed lifetime. In this regard, a real case of light-emitting device (LED) is presented to show the application of the plans. In another study [18], considering inspection errors, several economic single-sampling plans are proposed under different probability distributions. Using the Bayesian approach, three mathematical models are developed, and inspection errors are taken into account.

RASPs and life testing schemes have widely been designed to optimize such criteria as average sample number and average failure number or to meet the producer's and consumer's risks, but few researchers have shed light on the costs of conducting RASPs or life testing. In reference [19], using the expected warranty cost, a cost function approach is developed for products with Weibull distribution. The proposed RASP is studied under type-I generalized hybrid censoring scheme. Reference [3] introduces a model to minimize the total cost of an RASP under progressive type-I interval censoring. The model determines the sample size, inspection interval and number of inspections so that the expected total cost can be minimized. Reference [20] proposes a design for type-II interval censoring life test and seeks to minimize the asymptotic maximum likelihood estimation and the total costs of the experiments. The components of the cost include the set-up cost, inspection process cost and the cost of failed item during the experiment. Finding the best inspection interval is the only decision variable of the model. Reference [21] investigates the inspection plans for interval censored data as well as a constraint regarding the budget of the experiment.

In reference [22], a bi-level programming model is developed to optimize the costs of time censoring life testing while the lifetime follows Burr type XII. Reference [7] discussed the optimal design of an accelerated life test. The data of the lifetime are obtained using type-II censoring scheme, and Weibull distribution is applied for those data. An optimal acceptance sampling model is also proposed based on linear and Arrhenius stress life relations. Using the Bayesian approach, a modified chain sampling plan is proposed by [23]. The data on lifetime are dealt with through Weibull distribution, and equations are derived to obtain the OC curve.

In the present study, a mathematical model is developed to optimally design an RASP under FCLT. The objective function of the model optimally determines the number of the items put to the test (n), the required number of the failed items (r), and the criterion to evaluate the lot (k) in order to minimize the expected total cost (ETC) of the test. The model also addresses the producer's and consumer's risks by considering them as constraints. Thus, the main novelty of the presented model is optimizing the expected total costs of RASPs under failure censoring tests while the requirements for handling the producer's and consumer's risks are taken into consideration as the constraints of the model. To the best of the authors' knowledge, no study has provided an explicit mathematical model so that the expected total cost of RASPs can be optimized under the assumption of the risks facing consumers and producers.

3. Problem Statement, Model Development and Operating Characteristic Curve

Let's assume T is a random variable denoting the lifetime of an item. In this study, it is assumed that T follows an exponential distribution with the following probability density function (p.d.f):

$$f(t;\lambda) = \lambda e^{-\lambda t} \tag{1}$$

In this equation, λ is the unknown parameter of the distribution, which is inversely related to the mean of the lifetime, i.e., $\mu = \frac{1}{\lambda}$. It is also assumed that a batch or lot of these items is available; it is desirable to evaluate the quality of product, i.e., their lifetime. In particular, it is of significance to implement the following hypothesis test:

$$\begin{cases} H_0: \lambda = \lambda_0 \\ H_1: \lambda = \lambda_1 > \lambda_0 \end{cases} \tag{2}$$

Let's denote the probability of type I and type II errors in the hypothesis test with α and β respectively. These two parameters are defined in the context of reliability acceptance sampling plans. From the standpoint of the producer, a batch at the quality level of $\lambda = \lambda_0$ should be accepted with at least the probability of $1-\alpha$. Furthermore, the consumer's desire is that a batch with $\lambda = \lambda_1$ should be accepted with at most the probability of β. In the terminology of acceptance sampling plans, the values of λ_0 and λ_1 are usually referred to as AQL and LQL or the lot tolerance present defective (LTPD) respectively. Also, α and β are usually referred as the producer's and consumer's risks respectively.

In order to assess the quality of the batch of items, a failure censoring life test is conducted. For this purpose, at first, n items are randomly selected from the lot and readily put to the test. The test continues until $r (r \le n)$ failures are observed. According to the common notations in the order statistics, let $t_{(i)}$ denote the failure time of the i_{th} item during the test. The failure time of each item is recorded to form the order statistic $t_{(1)}, t_{(2)}, \dots, t_{(r)}$. As suggested by some researchers [8, 15, 24], the recorded failure times are used to define the following statistic:

$$W = \sum_{j=1}^{r} t_{(j)} + (n-r)t_{(r)} \tag{3}$$

where $t_{(r)}$ is the failure time of the last item in the test, and W is the value of the statistic. It has been proved that $2\lambda W$ follows a chi-square distribution with $2r$ degrees of freedom, which is denoted as follows [8]:

$$2\lambda W \sim \chi^2_{2r} \tag{4}$$

It has also been proved that $\frac{r}{W}$ is the maximum likelihood estimator (MLE) of λ [8]. Hence, the following rule is applied to make a decision regarding the lot:
If $\frac{r}{W} \leq k$, the lot is accepted; otherwise, the lot is rejected. In this rule, k is a critical value determined by the model. Given the values of n and r, the expected time to terminate the test (*ETT*) is computed as follows:

$$ETT(r,n) = \frac{1}{\lambda}\sum_{j=0}^{r-1}\frac{1}{n-j} \tag{5}$$

Also, given the values of n, r and k, the *ETC* of the test can be computed as follows:

$$ETC(n,r,k) = \frac{C_1}{\lambda}\sum_{j=0}^{r-1}\frac{1}{n-j} + nC_2 + rC_3 \tag{6}$$

According to Equation 5, *ETT* and, consequently, the first term of Equation 6 depends on λ, while the actual value of λ is unknown. Different objective functions can be thus considered for the model. This study uses a similar approach proposed in [25] to evaluate the objective functions for three different values of λ including the values of λ at AQL, i.e., λ_0, the value of λ at LQL, i.e., λ_1 , and the value of λ at $0.5(\lambda_0 + \lambda_1)$. As stated before, from the viewpoint of the producer, a lot with $\lambda = \lambda_0$ should be accepted with at least the probability of $1-\alpha$. This statement yields the following constraint for the model:

$$\alpha \geq P(\frac{r}{W} \geq k | \lambda = \lambda_0) = P(\frac{W}{r} \leq \frac{1}{k} | \lambda = \lambda_0) = P(W \leq \frac{r}{k} | \lambda = \lambda_0) = P(2\lambda W \leq \frac{2\lambda r}{k} | \lambda = \lambda_0) = P(\chi^2_{2r} \leq \frac{2\lambda_0 r}{k}) \tag{7}$$

On the other hand, from the viewpoint of the consumer, a lot at the quality level of $\lambda = \lambda_1$ should be accepted with at most the probability of β. This leads to another constraint for the model presented as follows:

$$\beta \geq P(\frac{r}{W} \leq k | \lambda = \lambda_1) = P(\frac{W}{r} \geq \frac{1}{k} | \lambda = \lambda_1) = P(W \geq \frac{r}{k} | \lambda = \lambda_1) = P(2\lambda W \geq \frac{2\lambda r}{k} | \lambda = \lambda_1) = P(\chi^2_{2r} \geq \frac{2\lambda_1 r}{k}) \tag{8}$$

Finally, the following mathematical model is presented to minimize *ETC*:

$$Minimize\ ETC(n,r,k) = \frac{C_1}{\lambda}\sum_{j=0}^{r-1}\frac{1}{n-j} + nC_2 + rC_3 \tag{9}$$

$$\textbf{Subject to}: \quad P(\chi^2_{2r} \leq \frac{2\lambda_0 r}{k}) \leq \alpha$$

$$P(\chi^2_{2r} \geq \frac{2\lambda_1 r}{k}) \leq \beta$$

$$n \geq r, k > 0$$

The model optimally determines the values of n, r and k in order to minimize *ETC*. The constraints of the model guarantee the producer's and consumer's risks. As stated before, in the objective function, the true value of λ is unknown. Thus, for the analyses in the next section, three cases are studied including λ_0 , λ_1 and $0.5(\lambda_0 + \lambda_1)$.
As in the following, this study proceeds to derive an equation to compute the operating characteristic (OC) curve of the proposed RASP. An OC curve shows the acceptance probability of a lot for different quality levels. Given that the

quality of a lot is λ, the following equation can be derived to obtain the OC curve of the plan:

$$\pi_a(\lambda) = P\left(\frac{r}{W} \le k\right) = P\left(\frac{W}{r} \ge \frac{1}{k}\right) = P\left(W \ge \frac{r}{k}\right) = P\left(2\lambda W \ge \frac{2\lambda r}{k}\right) = P(\chi^2_{2r} \ge \frac{2\lambda r}{k}) \quad (10)$$

As Equation 10 implies, the probability of accepting a lot depends on λ, k and r and is not affected by n.

4. Numerical Examples and Analyses

In this section, numerical examples and sensitivity analyses are provided regarding the proposed model. The data of an example are presented in Table 1.

Tab. 1. The data of the example

C_1	C_2	C_3	α	β	λ_0	λ_1
10	150	1	0.01	0.05	0.001	0.002

A grid search algorithm is applied to optimize the model. For the three alternatives of the objective function, which correspond to λ_0 , λ_1 and $0.5(\lambda_0 + \lambda_1)$, the result of the model optimization is provided in Table 2.

Tab. 2. The result of optimizing the example

Decision variables Value of λ in the objective function	r	n	k	ETC
λ_0	36	70	0.0016	17682
λ_1	36	57	0.0016	13504
$0.5(\lambda_0 + \lambda_1)$	36	61	0.0016	15055

For the case in which the objective function is optimized with λ_0 considered, the results indicate that, a sample with the size of 70 should be put to the test, and the test continues until 36 failed items are observed. Using the data of the experiment, the value of $\frac{r}{W}$ is computed and compared to $k = 0.0016$. The lot is accepted, which means the acceptance of H_0, if $\frac{r}{W} < 0.006$; otherwise it is rejected. The results of the example can be interpreted, and the objective function is optimized assuming λ_1 or $0.5(\lambda_0 + \lambda_1)$. As the results of Table 2 show, the use of λ_1 or $0.5(\lambda_0 + \lambda_1)$ instead of λ_0 in the objective function of Equation 9 does not affect the values of r and k, while this change impacts the values of ETC and n. More precisely, optimizing the objective function for the larger values of λ leads to smaller n. Figure 1 shows the OC curve of this plan. According to the figure, for example, the probability of accepting a lot with $\lambda = 0.0017$ is 0.336. As the OC curve suggests, with an increase in λ, i.e., deterioration of the quality, the acceptance probability of the lot decreases. Figure 2 shows the impact of the sample size (n) on the different components of the cost. As discussed before and according to this figure, an increase of n leads to a decrease in the operation costs, which, in turn, increases the sampling cost. Hence, with an increase of n from 40, the expected total cost first decreases and then increases. In this case, ETC minimizes at $n = 70$. The figure also shows that, around the optimal value of n, the curve of ETC is relatively smooth. This means that the slight changes of n do not have a significant effect on ETC.

In the rest of this section, some analyses are conducted to provide an insight concerning the performance of the model and the proposed RASP. It is worth noting that the forthcoming analyses assume λ_0 to optimize the objective function of Equation 9.

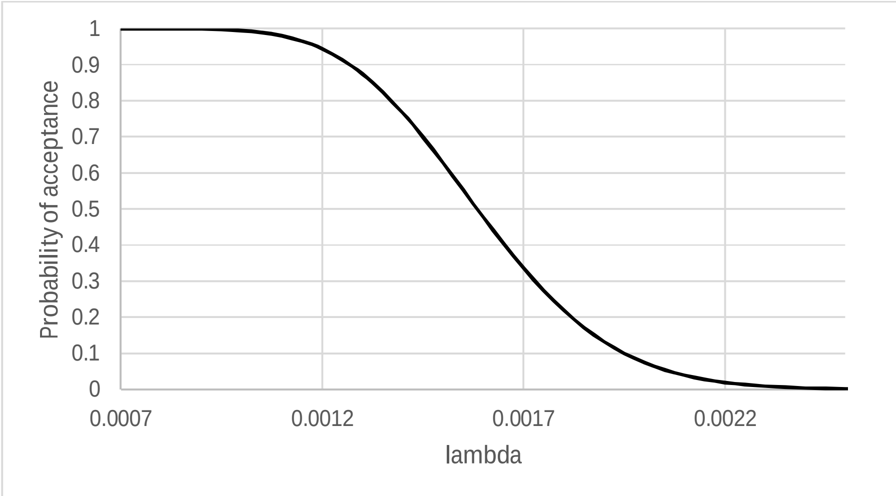

Fig. 1. Operating characteristic curve of the plan

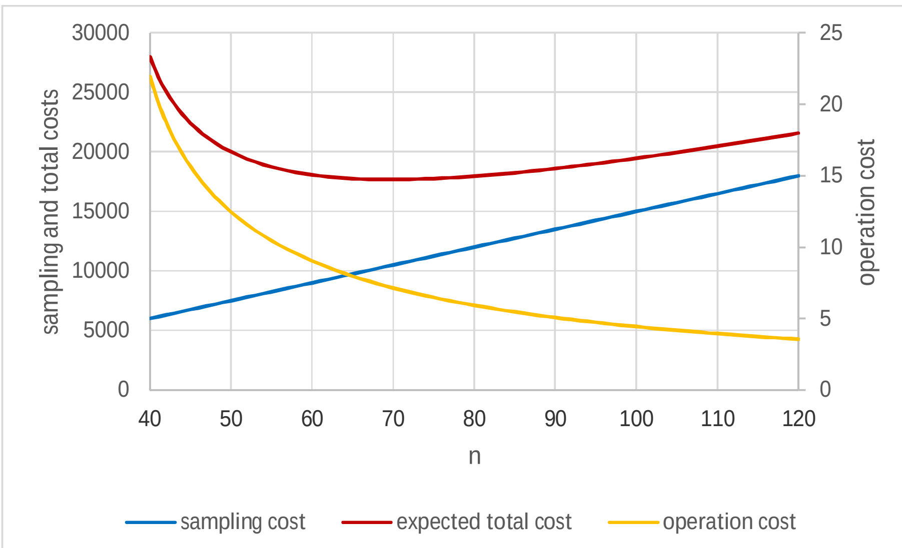

Fig. 2. Changes of the different components of costs involved in the RASP versus the sample size

The impacts of λ_0 and λ_1 are illustrated in Table 3. Generally, the change of these two parameters gives the insight that, as the difference between λ_0 and λ_1 becomes wider, the values of n, r and *ETC* decrease. It implies that, the same level of producer's and consumer's risks can be guaranteed for the cases with a larger difference between λ_0 and λ_1 and a smaller sample size. Also, due to the decrease of n and r, the value of *ETC* decreases too.

Tab. 3. Analyses of the effects of λ_0 and λ_1
$\alpha = 0.01$; $\beta = 0.05$; $C_1 = 10$; $C_2 = 150$; $C_3 = 1$

λ_0	λ_1	n	r	k	*ETC*
0.001	0.002	70	36	0.0016	17682
	0.0025	49	21	0.0018	12891
	0.003	39	15	0.0021	10641
0.002	0.003	126	100	0.0026	26815
	0.0035	76	53	0.0028	17354
	0.004	61	40	0.003	14445
0.003	0.005	80	63	0.0041	17149
	0.006	50	35	0.0046	11472

The effects of the type-I and type-II errors, i.e., the producer's and consumer's risks, are shown in Table 4. The main finding is that the increment of α or β values has a decreasing effect on the values of n, r and ETC. This is in line with a basic concept of the sampling theory of statistics. According to it, in case the decision maker can endure more risks, a smaller sample size can be selected. For example, as the table shows, when $\alpha = 0.01$ and $\beta = 0.01$, the sample size is 84 and the test terminates after the failure of 47 items. This leads to the minimum expected total cost of 20771. According to one of the results of the experiment gained through Equation 3, if $\frac{W}{r} = \frac{W}{47} < 0.0015$, the lot is accepted; otherwise, it is rejected.

Tab. 4. Analyses of the effects of the producer's and consumer's risks
$\lambda_0 = 0.001$; $\lambda_1 = 0.002$; $C_1 = 10$; $C_2 = 150$; $C_3 = 1$

α	β	n	r	k	ETC
0.01	0.01	84	47	0.0015	20771
	0.05	70	36	0.0016	17682
	0.1	62	30	0.0016	15869
0.05	0.01	66	33	0.0014	16789
	0.05	53	24	0.0015	13927
	0.1	46	19	0.0015	12171
0.1	0.01	56	26	0.0013	14591
	0.05	44	18	0.0014	11801
	0.1	39	15	0.0015	10641

Finally, the cost parameters C_1, C_2 and C_3 are analyzed in Table 5. For the fixed values of C_2 and C_3, the increment of the operating cost leads to an increase in the values of *ETC* and *n*. This can be interpreted in due terms. According to Equation 5, as the number of the items put to the test (*n*) increases, the expected time to terminate the test decreases. Hence, for the larger values of the operation cost per time unit, the value of *n* increases to mitigate the impact of increasing C_1. For the fixed values of C_1 and C_3, an increase of C_2 has a decreasing effect on n. Considering the impacts of C_1 and C_2 briefly, as the ratio of $\frac{C_1}{C_2}$ rises, more items should be put to the test. From Table 5, it seems that the cost of the failed items, C_3, does not have a significant effect on the decision variables.

Tab. 5. Analyses of the effects of the cost parameters
$\lambda_0 = 0.001$; $\lambda_1 = 0.002$; $\alpha = 0.01$; $\beta = 0.05$

C_1	C_2	C_3	n	r	k	ETC
5	100	1	64	36	0.0016	10520
		10	64	36	0.0016	10844
	150	1	57	36	0.0016	13504
		10	57	36	0.0016	13828
	200	1	52	36	0.0016	16223
		10	52	36	0.0016	16547
10	100	1	80	36	0.0016	13964
		10	80	36	0.0016	14288
	150	1	70	36	0.0016	17682
		10	70	36	0.0016	18006
	200	1	64	36	0.0016	21003
		10	64	36	0.0016	21327
15	100	1	93	36	0.0016	16629
		10	93	36	0.0016	16953
	150	1	80	36	0.0016	20927
		10	80	36	0.0016	21251
	200	1	72	36	0.0016	24730
		10	72	36	0.0016	25054

For the RASP under failure censoring, the average number of failures is optimized, and two real data sets are analyzed [8]. The first one belongs to the electrical insulating fluids, and the second concerns the endurance of deep-grove ball bearings. Unfortunately, since no data have been provided on the costs of these experiments, it is not possible to directly apply our model in these real data sets. Naturally, once the data are provided, the proposed model can be easily employed to optimize the parameters of the plans.

5. Comparative Studies

In many studies regarding RASPs, minimizing the average failure number (AFN) has been considered as an objective function, and the RASPs has been designed without considering the cost parameters. Herein, in order to perform a comparative study, another mathematical model is developed to optimally design RASPs under failure censoring and to minimize the value of *AFN* during the test as the objective function. The procedure of the failure censoring in this model is similar to the one proposed in Section 3. That is, n items are randomly selected and put to the test. Then, the test continues until observing r failures. Based on the data of the test, the proper statistics are computed according to Equation 3. If $\frac{r}{W} \le k$, the lot is accepted; otherwise, the lot is rejected. In this case, k is a critical value determined by the model. Accordingly, the following optimization model is proposed:

$$Minimize\ AFN(r,k) = r \quad (11)$$

$$\textbf{Subject to}:\quad P(\chi^2_{2r} \le \frac{2\lambda_0 r}{k}) \le \alpha$$

$$P(\chi^2_{2r} \ge \frac{2\lambda_1 r}{k}) \le \beta$$

$$k > 0, r \in Z^+$$

As the model shows, the objective is to minimize the *AFN*, and the parameters of cost are not taken into consideration. Once the optimal values of r and k are obtained, the tester can specify the value of n so as to have $n \ge r$. In other words, in this approach, the value of n is not specified exactly by the model; after the value of r is determined, any value of n which is larger than (or equal to) r is acceptable. To provide an insight, the results of the comparative studies are reported in Table 6. As the table shows, the model proposed in this paper not only optimizes the *ETC* of the test but also yields appropriate results for *AFN*. For example, according to the data in the first row of the table, the model delineated through Equation 9 specifies that 70 items should be put to the test and the test can be terminated after 36 items fail. However, the model in Equation 11 does not specify an exact number for the sample size; it can be any number larger than 35.

Tab. 6. Comparative studies

Parameters	The model of minimizing ECT (Equation 9)			The model of minimizing AFN (Equation 11)		
	n	r	k	n	r	k
$\alpha = 0.01, \beta = 0.05, \lambda_0 = 0.001, \lambda_1 = 0.002, c_1 = 10, c_2 = 150, c_3 = 1$	70	36	0.0016	$n > 35$	36	0.0015
$\alpha = 0.01, \beta = 0.01, \lambda_0 = 0.001, \lambda_1 = 0.002, c_1 = 10, c_2 = 150, c_3 = 1$	84	47	0.0015	$n > 46$	47	0.0015
$\alpha = 0.01, \beta = 0.05, \lambda_0 = 0.001, \lambda_1 = 0.003, c_1 = 10, c_2 = 150, c_3 = 1$	39	15	0.0021	$n > 15$	16	0.002

6. Conclusion

Three factors affect the total costs of a reliability sampling plan under failure censoring. They include b) the total number of the items put to the test, b) the observed number of failed items to terminate the test, and c) the value of the criterion employed to decide about the acceptance/rejection of the lot. In this study, a mathematical model is developed to optimally determine the values of these three factors. Minimizing the total expected cost of the experiment is considered as the objective function, and producer's and consumer's risks serve as the constraints of the model. An equation is also derived to compute the OC curve of the plan. Finally, numerical examples are provided and analyses are performed regarding the impact of the model parameters.

Most RASPs involve the conduction of life testing schemes which are intrinsically destructive. Hence, it is worth investigating how to minimize the expected total costs of experiments. In some studies, life testing schemes and RASPs are designed to minimize the average failure number without considering the cost parameters. In this paper, however, both approaches are studied and compared. Designing RASPs according to the mathematical models proposed in this study insures minimizing the expected total costs of the test while the risks of producers and consumers are guaranteed too. According to the analyses, increasing n leads to a decrease in the operation costs, and this increases the sampling cost. So, when n increases, the expected total cost first decreases and then increases. Also, *ETC* is minimized around a proper value, which is, indeed, the optimal value of n. Another important aspect of the proposed model concerns the *ETC* curve; around the optimal value of n, the curve is relatively smooth, implying that the slight changes of n do not have a significant effect on *ETC*.

The present study can be extended in several directions. For example, the mathematical model proposed here can also consider Weibull, gamma or log-normal distribution to deal with lifetime. Designing other sampling schemes, e.g., repetitive group sampling or sequential sampling, under failure censoring is another suggestion to optimize the expected total costs. Presenting mathematical models by considering multiple objective functions (e.g., minimizing the costs and the average number of failures) may also be of insight in future studies.

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