# Mechanical Properties Characterization of Biax and Triax Composites Based on Limited Experimental Data 

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#### Abstract

The main goal of this research is to extract the full mechanical properties of stitch biax and triax composite materials which are necessary for finite element analysis, based on limited available experimental data and without performing full static characterization tests. Utilized experimental data are limited to elastic modulus of two $0^{\circ}$ and $45^{\circ}$ directions. Using presented technique and aforementioned data, mechanical properties of unidirectional fabrics of biax and triax are obtained and consequently mechanical properties of biax and triax composites are calculated. Evaluation of the results proved proper performance of the technique in this research.


Keywords: Multi axial composites, Mechanical properties, Characterization, Prepreg.

## 1. Introduction

Now days, bi-axial and tri-axial composite materials based on pre-preg technology are widely used in composite industries. For instance, all materials of horizontal axis wind turbine blade structure are prepreg. Raw composite materials of wind turbine blade are divided into three main groups as uni-directional, biaxial (biax) and tri-axial (triax) which are non-woven and stitched. Full experimental analyses in order to obtain complete mechanical properties which are necessary for any finite element analysis have to be carried out [1]. In this paper, presenting a theoretical method and merging it with a few experimental data, full mechanical properties of the biax abd triax composite materials will be characterized.
Prepregs have had a considerable impact on the evolution of composite industries in the late $20^{\text {th }}$ century. Used all aerospace programs worldwide, they are also enabling a new generation of high speed trains and fast ships and long wind turbine blades to become reality rather than designer's dream.
The position of prepreg technology in terms of performance and production volumes is compared below with other fabrication processes in figure 1.
A prepreg consists of a combination of a matrix (or resin) and fiber reinforcement. It is ready to use in the componenet manufacturing process. Infact, in this technology fiber is combined with resin before

[^0]manufacturing process and makes the manufacturing process easier and more economic. Furthermore, control of fiber volume fraction is handled better.


Fig. 1. Position of Prepreg in Fabrication Processes

Prepregs are available in [2] as:

- Uni-Directional (UD) form, one direction of reinforcement
- Fabric form, several directions of reinforcement These two groups are shown in figure 2.
The fabrics consist of at least two threads which are woven together and called the warp and weft or stitched together in non-woven form.
The weave style can be varied according to crimp and drapeability. Low crimp gives better mechanical performance because straighter fabrics carry greater loads. It has been proven that crimp will negatively affect fatigue life of structure. Geometric of woven fabric produces out of plane curvature in plies and
consequently stress concentration appears. Compressive strength of fibers in woven fabrics is approximately half of straight fibers in stitch form [3]. In order to overcome this problem using stitch form instead of woven fabrics is recommended [3].


Fig. 2. Different Forms of Prepreg

The main criteria influence the selection of prepregs for particular application are performance and cost and the main advantages of using them can be summarized as lower fabrication cost, reduced energy consumption, optimized weight and better mechanical properties under cyclic loading [4], tensile, stiffness and corrosion. Prepreg can be processed in different ways.
Vacuum bag and autoclave are the two common methods for the manufacture of components from prepreg. These days, prepreg materials based on Epoxy matrix are widely used in production of wind turbine blade as a full composite structure. They can be found in form of UD, Biax and Triax.

## 2. Available Mechanical Properties

Required properties for analyzing composites are divided into mechanical and strength properties. Mechanical properties are used for stress analysis and
strength properties are used for failure analysis. Material characterization means extracting aforementioned mechanical properties. Since composite materials are not isotropic materials, the required mechanical properties are [5]:

1-Longitudinal Elastic Modulus , $\mathrm{E}_{\mathrm{X}}$
2- Transverse Elastic Modulus, $\mathrm{E}_{\mathrm{Y}}$
3-Poisson's ratio, v
4- Shear Modulus, G
Also, strength properties are as follow [5]:
1-Longitudinal Tensile Strength
2- Transverse Tensile Strength
3- Longitudinal Compressive Strength
4- Transverse Compressive Strength
5-Shear Strentgh
Since, biax and triax composites are shaped based on overlaying unidirectional fibers in different directions, equivalent modulus of laminate will be expressed in two orthogonal directions.
According to information from supplier of these materials, available data for biax and triax is limited to two types of elastic modulus in 0 degree and 45 degree direction. These items are shown in figure 3.


Fig. 3. Two Directions of Available Equivalent Elastic Modulus

In other word, equivalent longitudinal elastic modulus of biax composite is available for [+45/-45] configuration (direction of $0^{\circ}$ in figure 3) and [0/90] (direction of $45^{\circ}$ in figure 3).
Similarity, equivalent longitudinal elastic modulus of triax composite is available in [0/+45/-45] configuration (direction of $0^{\circ}$ in figure 3 ) and [0/90/-45] configuration (direction of $45^{\circ}$ in figure 3).
The overall picture of aforementioned available data has been summarized in table 1 [6]

Table. 1. Available Data from Experiments [6]

| Fabric Type | Configuration | Mechanical Properties |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{E}_{1}[\mathrm{GPa}]$ | $\mathrm{E}_{2}[\mathrm{GPa}]$ | $\mathrm{V}_{12}$ | $\mathrm{E}_{6}[\mathrm{GPa}]$ |
| Biax | $[ \pm 45]_{T}$ | 6.8 | 6.8 | N/A $^{*}$ | N/A |
| Biax | $[0 / 90]_{\mathrm{T}}$ | 16.7 | 16.7 | N/A | N/A |
| $\operatorname{Triax}$ | $[0 / \pm 45]_{T}$ | $20.7 \pm 3.1$ | N/A | N/A | N/A |
| Triax | $[0 / 90 /-45]_{\mathrm{T}}$ | $15.1 \pm 2.3$ | $15.1 \pm 2.3$ | N/A | N/A |

## *N/A: Not Available

It can be seen that in [45/-45], [0/90] and [0/90/-45] configurations, $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ are equal due to symmetry. In this paper, whenever " $x$ " or " $y$ " is used as subscript, it implies on lamina (one ply) and whenever a numerical form is used as subscript, it implies on laminate which can consist two or more lamina.
In order to derive unknown parameters which have been summarized in table 1 , two methods exist: direct and inverse. In the following these two methods are explained.

## 3. Direct Method

In direct method, equations of biax and triax are organized using mechanical properties of its unidirectional plies and macromechanical relations. This method needs full characterization of unidirectional ply and in this research complete mechanical properties of unidirectional are not available and this method will not be useful here.

## 4. Inverse Method

In inverse method, using equations of biax and triax based on limited experimental data and due to this fact that both biax and triax consist of same unidirectional fiber, we try to extract mechanical properties of unidirectional ply, inversely.
After that, mechanical properties of biax and triax will be characterized.
For this purpose, stiffness matrix of composites is shaped and is equalized by inverse of its compliance matrix.

## 4.1 . General Governing Equations of Inverse Method

Stiffness matrix of a ply in on-axis coordinate is expressed as follow [5]:

$$
\left(\begin{array}{ccc}
\mathrm{Q}_{\mathrm{XX}} & \mathrm{Q}_{\mathrm{XY}} & 0  \tag{1}\\
\mathrm{Q}_{\mathrm{XY}} & \mathrm{Q}_{\mathrm{YY}} & 0 \\
0 & 0 & \mathrm{Q}_{\mathrm{SS}}
\end{array}\right)
$$

Where;

$$
\begin{align*}
\mathrm{Q}_{\mathrm{XX}} & =\frac{\mathrm{E}_{\mathrm{X}}}{1-v_{\mathrm{XY}} v_{\mathrm{YX}}} \\
\mathrm{Q}_{\mathrm{YY}} & =\frac{\mathrm{E}_{\mathrm{Y}}}{1-v_{\mathrm{XY}} v_{\mathrm{YX}}}  \tag{2}\\
\mathrm{Q}_{\mathrm{XY}} & =\frac{v_{\mathrm{YX}} \mathrm{E}_{\mathrm{X}}}{1-v_{\mathrm{XY}} v_{\mathrm{YX}}} \\
\mathrm{Q}_{\mathrm{SS}} & =\mathrm{G}
\end{align*}
$$

Involved parameters in set of equations (2), are in the space of on-axis coordinate system and " $x$ " devotes to longitudinal fiber direction and " $y$ " devotes to transverse direction. $v_{\mathrm{xy}}$ is major Poisson's ratio and $V_{Y X}$ is minor Poisson's ratio.

If one needs stiffness matrix in any desired direction, transformation from on-axis coordinate system to offaxis has to be performed using following relation [5]:

$$
\left.\begin{array}{l}
\left(\begin{array}{l}
\mathrm{Q}_{11} \\
\mathrm{Q}_{22} \\
\mathrm{Q}_{12} \\
\mathrm{Q}_{66} \\
\mathrm{Q}_{16} \\
\mathrm{Q}_{26}
\end{array}\right)= \\
\left(\begin{array}{cccc}
\mathrm{m}^{4} & \mathrm{n}^{4} & 2 \mathrm{~m}^{2} \mathrm{n}^{2} & 4 \mathrm{~m}^{2} \mathrm{n}^{2} \\
\mathrm{n}^{4} & \mathrm{~m}^{4} & 2 \mathrm{~m}^{2} \mathrm{n}^{2} & 4 \mathrm{~m}^{2} \mathrm{n}^{2} \\
\mathrm{~m}^{2} \mathrm{n}^{2} & \mathrm{~m}^{2} \mathrm{n}^{2} & \mathrm{~m}^{4}+\mathrm{n}^{4} & -4 \mathrm{~m}^{2} \mathrm{n}^{2} \\
\mathrm{~m}^{2} \mathrm{n}^{2} & \mathrm{~m}^{2} \mathrm{n}^{2} & -2 \mathrm{~m}^{2} \mathrm{n}^{2} & \left(\mathrm{~m}^{2}-\mathrm{n}^{2}\right)^{2} \\
\mathrm{~m}^{3} \mathrm{n} & -\mathrm{mn}^{3} & \mathrm{mn}^{3}-\mathrm{m}^{3} \mathrm{n} & 2\left(\mathrm{mn}^{3}-\mathrm{m}^{3} \mathrm{n}\right) \\
\mathrm{mn}^{3} & -\mathrm{m}^{3} \mathrm{n} & \mathrm{mn}^{3}-\mathrm{m}^{3} \mathrm{n} & 2\left(\mathrm{mn}^{3}-\mathrm{m}^{3} \mathrm{n}\right)
\end{array}\right)\left(\begin{array}{l}
\mathrm{Q}_{\mathrm{XX}} \\
\mathrm{Q}_{\mathrm{XY}} \\
\mathrm{Q}_{\mathrm{XY}}
\end{array}\right)  \tag{3}\\
\mathrm{m}=\operatorname{Cos} \theta, \quad \mathrm{n}=\operatorname{Sin} \theta
\end{array}\right)
$$

So, in a ply that fibers have been oriented in direction of $0^{\circ}$, we have;

$$
\begin{align*}
& \mathrm{Q}_{11}^{0}=\mathrm{Q}_{\mathrm{xx}} \\
& \mathrm{Q}_{22}^{0}=\mathrm{Q}_{\mathrm{YY}} \\
& \mathrm{Q}_{12}^{0}=\mathrm{Q}_{21}^{0}=\mathrm{Q}_{\mathrm{xY}}  \tag{4}\\
& \mathrm{Q}_{66}^{0}=\mathrm{Q}_{\mathrm{SS}} \\
& \mathrm{Q}_{16}^{0}=\mathrm{Q}_{26}^{0}=0
\end{align*}
$$

Similarity, for 90 degree ply:

$$
\begin{align*}
& \mathrm{Q}_{11}^{90}=\mathrm{Q}_{22}^{0} \\
& \mathrm{Q}_{22}^{90}=\mathrm{Q}_{11}^{0} \\
& \mathrm{Q}_{12}^{90}=\mathrm{Q}_{21}^{90}=\mathrm{Q}_{12}^{0}  \tag{5}\\
& \mathrm{Q}_{66}^{90}=\mathrm{Q}_{66}^{0} \\
& \mathrm{Q}_{16}^{90}=\mathrm{Q}_{26}^{90}=\mathrm{Q}_{16}^{0}
\end{align*}
$$

For a $45^{\circ}$ ply and a $-45^{\circ}$ ply, we have:

$$
\begin{align*}
& \mathrm{Q}_{11}^{45}=\mathrm{Q}_{11}^{-45} \\
& \mathrm{Q}_{22}^{45}=\mathrm{Q}_{22}^{-45} \\
& \mathrm{Q}_{12}^{45}=\mathrm{Q}_{21}^{45}=\mathrm{Q}_{12}^{-45}=\mathrm{Q}_{21}^{-45}  \tag{6}\\
& \mathrm{Q}_{66}^{45}=\mathrm{Q}_{66}^{-45} \\
& \mathrm{Q}_{16}^{45}=\mathrm{Q}_{26}^{45}=-\mathrm{Q}_{16}^{-45}=-\mathrm{Q}_{26}^{-45}
\end{align*}
$$

Equivalent stiffness matrix of a composite consisting two plies, can be calculated using the following relation:

$$
\begin{equation*}
\frac{1}{\mathrm{~h}} \mathrm{~A}_{\mathrm{ij}}=\mathrm{Q}_{\mathrm{ij}}^{\alpha} \mathrm{p}^{\alpha}+\mathrm{Q}_{\mathrm{ij}}^{\beta} \mathrm{p}^{\beta} \tag{7}
\end{equation*}
$$

where, " $h$ " is thickness, $\alpha$ is angle of first ply in accordance with selected coordinate system, $\beta$ is angle of second ply and " p " is ply fraction.
Based on technical data from supplier of investigated biax material [6], both ply fractions are the same. Therefore, we have:

$$
\begin{equation*}
\mathrm{p}^{0}=\mathrm{p}^{90}=\mathrm{p}^{45}=\mathrm{p}^{-45}=.5 \tag{8}
\end{equation*}
$$

Using equations (7) and (8), we have:

$$
\begin{align*}
& \frac{1}{\mathrm{~h}} \mathrm{~A}_{\mathrm{ij}}^{[0 / 90]}=.5\left(\mathrm{Q}_{\mathrm{ij}}^{0}+\mathrm{Q}_{\mathrm{ij}}^{90}\right) \\
& \frac{1}{\mathrm{~h}} \mathrm{~A}_{\mathrm{ij}}^{[45 /-45]}=.5\left(\mathrm{Q}_{\mathrm{ij}}^{45}+\mathrm{Q}_{\mathrm{ij}}^{-45}\right) \tag{9}
\end{align*}
$$

By calculating inverse of "A" matrix, we have:

$$
\begin{equation*}
\left[\frac{1}{\mathrm{~h}}[\mathrm{~A}]\right]^{-1}=\mathrm{h}[\mathrm{~A}]^{-1}=\mathrm{h}[\mathrm{a}] \tag{10}
\end{equation*}
$$

Where, [a] is compliance matrix and its members are as follow [7]:

$$
[a]=h\left[\begin{array}{ccc}
1 / E_{1} & -v_{12} / \mathrm{E}_{2} & v_{16} / \mathrm{E}_{6}  \tag{11}\\
-v_{12} / \mathrm{E}_{2} & 1 / \mathrm{E}_{2} & v_{26} / \mathrm{E}_{6} \\
v_{16} / \mathrm{E}_{6} & v_{26} / \mathrm{E}_{6} & 1 / \mathrm{E}_{6}
\end{array}\right]
$$

It can be seen that in equation (11), all " $x$ " and " $y$ " subscripts have been changed to numerical subscripts which representing properties of a laminated composite instead of single ply.
Now, the same calculation has to be performed for other aforementioned configuration.

## 4.2 .Biax Composite with [45/-45] Configuration

Stiffness matrix of this composite is calculated using equations (3), (6) and (9):

$$
\begin{align*}
& \frac{1}{\mathrm{~h}}\left[\mathrm{~A}^{[ \pm 45]}\right]=\frac{1}{\mathrm{~h}}\left[\begin{array}{ccc}
\mathrm{A}_{11}^{[ \pm 45]} & \mathrm{A}_{11}^{[ \pm 45]} & 0 \\
\mathrm{~A}_{11}^{[ \pm 45]} & \mathrm{A}_{11}^{[ \pm 45]} & 0 \\
0 & 0 & \mathrm{~A}_{11}{ }^{[ \pm 45]}
\end{array}\right] \\
& {\left[\mathrm{A}_{11}^{[ \pm 45]}\right]=.25 \mathrm{Q}_{\mathrm{XX}}+.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}+\mathrm{Q}_{\mathrm{SS}}}  \tag{12}\\
& {\left[\mathrm{~A}_{22}{ }^{[ \pm 45]}\right]=.25 \mathrm{Q}_{\mathrm{XX}}+.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}+\mathrm{Q}_{\mathrm{SS}}} \\
& {\left[\mathrm{~A}_{12}^{[ \pm 4]]}\right]=\left[\mathrm{A}_{21}^{[ \pm 45]}\right]=.25 \mathrm{Q}_{\mathrm{XX}}+.5 \mathrm{Q}_{\mathrm{XY}}+} \\
& .25 \mathrm{Q}_{\mathrm{YY}}-\mathrm{Q}_{\mathrm{SS}} \\
& {\left[\mathrm{~A}_{66}^{[ \pm 45]}\right]=25 \mathrm{Q}_{\mathrm{XX}}-.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}}
\end{align*}
$$

Compliance matrix is calculated using equation (11) and by replacing known parameters from table 1 :

$$
[\mathrm{a}]=\mathrm{h}\left[\begin{array}{ccc}
1 / 6.8 & -\alpha / 6.8 & 0  \tag{13}\\
-\alpha / 6.8 & 1 / 6.8 & 0 \\
0 & 0 & 1 / \mathrm{G}_{[ \pm 45]}
\end{array}\right]
$$

Where, $\alpha$ is the major Poisson's ratio of composites. After inversing matrix of equation (13), equalizing them to their correspond members in equation (12) and omitting repeated equations, we have:

$$
\begin{align*}
& .25 \mathrm{Q}_{\mathrm{XX}}+.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}+\mathrm{Q}_{\mathrm{SS}}= \\
& \frac{.147059}{.0216263-.0216263 \alpha^{2}} \\
& .25 \mathrm{Q}_{\mathrm{XX}}+.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}-\mathrm{Q}_{\mathrm{SS}}=  \tag{14}\\
& \frac{.147059 \alpha}{.0216263-.0216263 \alpha^{2}} \\
& .25 \mathrm{Q}_{\mathrm{xX}}-.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}=\mathrm{G}_{[ \pm 45]}
\end{align*}
$$

### 4.3.Biax Composite with [0/90] Configuration

Stiffness matrix of this composite is derived using equations (3), (5) and (9):

$$
\begin{align*}
& .25 \mathrm{Q}_{\mathrm{XX}}+.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}+\mathrm{Q}_{\mathrm{SS}}= \\
& \frac{.147059}{.0216263-.0216263 \alpha^{2}} \\
& .25 \mathrm{Q}_{\mathrm{XX}}+.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}-\mathrm{Q}_{\mathrm{SS}}=  \tag{15}\\
& \frac{.147059 \alpha}{.0216263-.0216263 \alpha^{2}} \\
& .25 \mathrm{Q}_{\mathrm{XX}}-.5 \mathrm{Q}_{\mathrm{XY}}+.25 \mathrm{Q}_{\mathrm{YY}}=\mathrm{G}_{[ \pm 45]}
\end{align*}
$$

Compliance matrix is calculated using equation (11) and by replacing known parameters from table 1 :

$$
[\mathrm{a}]=\mathrm{h}\left[\begin{array}{ccc}
1 / 16.7 & -\beta / 16.7 & 0  \tag{16}\\
-\beta / 16.7 & 1 / 16.7 & 0 \\
0 & 0 & 1 / \mathrm{G}_{[0 / 90]}
\end{array}\right]
$$

Where, $\beta$ is major Poisson's ratio of composites. After inversing matrix of equation (16), equalizing them to their corresponding members in equation (15) and omitting repeated equations, we have:

$$
\begin{align*}
& .5 \mathrm{Q}_{\mathrm{XX}}+.5 \mathrm{Q}_{\mathrm{YY}}=\frac{.0598802}{.00358564-.00358564 \beta^{2}} \\
& \mathrm{Q}_{\mathrm{XY}}=\frac{.0598802 \beta}{.00358564-.00358564 \beta^{2}}, \mathrm{Q}_{\mathrm{SS}}=\mathrm{G}_{[0 / 90]} \tag{17}
\end{align*}
$$

### 4.4.Triax Composite with [0/45/-45] Configuration

In order to calculate related matrixes for triax composites, the same method of biax composite is employed.
The only difference is referred to ply fractions. According to information from supplier [6], amount of fiber in direction of $0^{\circ}$ is equal to $254 \mathrm{~kg} / \mathrm{m}^{2}$ and amount of fiber in $45^{\circ}$ direction is equal to $-45^{\circ}$ direction and is $230 \mathrm{~kg} / \mathrm{m}^{2}$. Therefore, we have:

$$
\begin{align*}
& \mathrm{p}^{0}=\frac{425}{425+230+230}=0.48  \tag{18}\\
& \mathrm{p}^{45}=\mathrm{p}^{-45}=\frac{230}{425+230+230}=.26
\end{align*}
$$

Stiffness matrix of this configuration is in the following form using equation (4), (6), (7) and (18):

$$
\begin{align*}
& \frac{1}{\mathrm{~h}}\left[\mathrm{~A}^{[0 / \pm 45]}\right]= \\
& \frac{1}{\mathrm{~h}}\left[\begin{array}{ccc}
.61 \mathrm{Q}_{\mathrm{xx}}+.26 \mathrm{Q}_{\mathrm{xY}} & .13 \mathrm{Q}_{\mathrm{xx}}+.74 \mathrm{Q}_{\mathrm{xY}} & 0 \\
+.13 \mathrm{Q}_{\mathrm{YY}}+.52 \mathrm{Q}_{\mathrm{ss}} & +.13 \mathrm{Q}_{\mathrm{YY}}-.52 \mathrm{Q}_{\mathrm{ss}} & 0 \\
.13 \mathrm{Q}_{\mathrm{xx}}+.74 \mathrm{Q}_{\mathrm{xY}} & .13 \mathrm{Q}_{\mathrm{xx}}+.26 \mathrm{Q}_{\mathrm{xY}} & 0 \\
+.13 \mathrm{Q}_{\mathrm{YY}}-.52 \mathrm{Q}_{\mathrm{ss}} & +.61 \mathrm{Q}_{\mathrm{YY}}+.52 \mathrm{Q}_{\mathrm{SS}} & \\
0 & 0 & .13 \mathrm{Q}_{\mathrm{xx}}-.26 \mathrm{Q}_{\mathrm{xY}} \\
0 & & +.13 \mathrm{Q}_{\mathrm{YY}}+.48 \mathrm{Q}_{\mathrm{ss}}
\end{array}\right] \tag{19}
\end{align*}
$$

Compliance matrix is calculated using equation (11) and by replacing known parameters from table 1 :

$$
[\mathrm{a}]=\mathrm{h}\left[\begin{array}{ccc}
1 /(20.7 \pm 3.1) & -\varphi /(20.7 \pm 3.1) & 0  \tag{20}\\
-\varphi /(20.7 \pm 3.1) & 1 / \mathrm{E}_{2} & 0 \\
0 & 0 & 1 / \mathrm{G}_{[0 / \pm 45]}
\end{array}\right]
$$

Where, $\varphi$ is the major Poisson's ratio of the triax composites with $[0 /+45 /-45]$ configuration. After inversing matrix of equation (20), equalizing them to their corresponding members in equation (19) and omitting repeated equations, we have equation (21).

### 4.4.Triax Composite with[0/90/-45] Configuration

 Extraction method of stiffness matrix of this configuration is the same as employed method for triax composite with [0/+45/-45] configuration. The only difference can be found in ply fraction due to new distribution of fibers as follow:$$
\begin{align*}
& \mathrm{p}^{-45}=\frac{425}{425+230+230}=0.48  \tag{22}\\
& \mathrm{p}^{0}=\mathrm{p}^{90}=\frac{230}{425+230+230}=.26
\end{align*}
$$

Stiffness matrix of this configuration is in the equation (23) using equation (4), (5), (6), (7) and (22):

Compliance matrix is calculated using equation (11) and by replacing known parameters from table 1 :

$$
[\mathrm{a}]=\mathrm{h}\left[\begin{array}{cll}
1 /(15 \pm 2.3) & -\xi /(15 \pm 2.3) & \mu / \mathrm{G}_{[0 / 90 /-45]}  \tag{24}\\
-\xi /(15 \pm 2.3) & 1 /(15 \pm 2.3) & \mu / \mathrm{G}_{[0 / 90 /-45]} \\
\mu / \mathrm{G}_{[0 / 90 /-45]} & \mu / \mathrm{G}_{[0 / 90 /-45]} & 1 / \mathrm{G}_{[0 / 90 /-45]}
\end{array}\right]
$$

Where, $\xi$ is the minor Poisson's ratio of triax composite with [0/90/-45] configuration and $\mu$ is the coupling Poisson's ratio in " 16 " direction.
In general case, $\mathrm{Q}_{16}$ is not equal to $\mathrm{Q}_{26}$. But in this particular case, it can be seen that these two parameters are equal and it has to be considered in compliance matrix too.
After inversing matrix of equation (24), equalizing them to their corresponding members in equation (23) and omitting repeated equations, we have equation (25).
$.61 \mathrm{Q}_{\mathrm{XX}}+. .26 \mathrm{Q}_{\mathrm{XY}}+.13 \mathrm{Q}_{\mathrm{YY}}+.52 \mathrm{Q}_{\mathrm{SS}}=\frac{1}{.0483092-.00233378 \mathrm{E}_{2} \varphi^{2}}$
$.13 \mathrm{Q}_{\mathrm{XX}}+.26 \mathrm{Q}_{\mathrm{XY}}+.61 \mathrm{Q}_{\mathrm{YY}}+.52 \mathrm{Q}_{\mathrm{SS}}=\frac{.0483092 \mathrm{E}_{2}}{0483092-.00233378 \mathrm{E}_{2} \varphi^{2}}$
$.13 \mathrm{Q}_{\mathrm{XX}}+.74 \mathrm{Q}_{\mathrm{XY}}+.13 \mathrm{Q}_{\mathrm{YY}}-.52 \mathrm{Q}_{\mathrm{SS}}=\frac{.0483092 \mathrm{E}_{2} \varphi}{0483092-.00233378 \mathrm{E}_{2} \varphi^{2}}$
$.13 \mathrm{Q}_{\mathrm{XX}}-.26 \mathrm{Q}_{\mathrm{XY}}+.13 \mathrm{Q}_{\mathrm{YY}}+.48 \mathrm{Q}_{\mathrm{SS}}=\mathrm{G}_{[0 / \pm 45]}$
$\frac{1}{\mathrm{~h}}\left[\mathrm{~A}^{[0 / 90 /-45]}\right]=\frac{1}{\mathrm{~h}}\left[\begin{array}{lll}.38 \mathrm{Q}_{\mathrm{XX}}+.24 \mathrm{Q}_{\mathrm{XY}} & .12 \mathrm{Q}_{\mathrm{XX}}+.76 \mathrm{Q}_{\mathrm{XY}} & -.12 \mathrm{Q}_{\mathrm{XX}}+.12 \mathrm{Q}_{\mathrm{YY}} \\ +.38 \mathrm{Q}_{\mathrm{YY}}+.48 \mathrm{Q}_{\mathrm{SS}} & +.12 \mathrm{Q}_{\mathrm{YY}}-.48 \mathrm{Q}_{\mathrm{SS}} & \\ .12 \mathrm{Q}_{\mathrm{XX}}+.76 \mathrm{Q}_{\mathrm{XY}} & .38 \mathrm{Q}_{\mathrm{XX}}+.24 \mathrm{Q}_{\mathrm{XY}} & -.12 \mathrm{Q}_{\mathrm{XX}}+.12 \mathrm{Q}_{\mathrm{YY}} \\ +.12 \mathrm{Q}_{\mathrm{YY}}-.48 \mathrm{Q}_{\mathrm{SS}} & +.38 \mathrm{Q}_{\mathrm{YY}}+.48 \mathrm{Q}_{\mathrm{SS}} & \\ -.12 \mathrm{Q}_{\mathrm{xX}}+.12 \mathrm{Q}_{\mathrm{YY}} & -.12 \mathrm{Q}_{\mathrm{xX}}+.12 \mathrm{Q}_{\mathrm{YY}} & .12 \mathrm{Q}_{\mathrm{XX}}-.24 \mathrm{Q}_{\mathrm{XY}} \\ & +.12 \mathrm{Q}_{\mathrm{YY}}+.52 \mathrm{Q}_{\mathrm{SS}}\end{array}\right]$

$$
\begin{align*}
& .38 \mathrm{Q}_{\mathrm{XX}}+.24 \mathrm{Q}_{\mathrm{XY}}+.38 \mathrm{Q}_{\mathrm{YY}}+.48 \mathrm{Q}_{\mathrm{SS}}=\frac{.066 \mathrm{G}_{[0 / 90 /-45]}-\eta^{2}}{\mathrm{G}_{[0 / 90 /-45]}\left(.004356-.004356 \xi^{2}\right)-.132(1+\xi) \eta^{2}} \\
& .12 \mathrm{Q}_{\mathrm{XX}}+.76 \mathrm{Q}_{\mathrm{XY}}+.12 \mathrm{Q}_{\mathrm{YY}}-.48 \mathrm{Q}_{\mathrm{SS}}=\frac{.066 \mathrm{G}_{[0 / 90 /-45]}+\eta^{2}}{\mathrm{G}_{[0 / 90 /-45]}\left(.004356-.004356 \xi^{2}\right)-.132(1+\xi) \eta^{2}} \\
& .13 \mathrm{Q}_{\mathrm{XX}}+.74 \mathrm{Q}_{\mathrm{XY}}+.13 \mathrm{Q}_{\mathrm{YY}}-.52 \mathrm{Q}_{\mathrm{SS}}=\frac{.066 \mathrm{G}_{[0 / 90 /-45]}(1+\xi) \eta}{\mathrm{G}_{[0 / 90 /-45]}\left(.004356-.004356 \xi^{2}\right)-.132(1+\xi) \eta^{2}}  \tag{25}\\
& .12 \mathrm{Q}_{\mathrm{XX}}-.24 \mathrm{Q}_{\mathrm{XY}}+.12 \mathrm{Q}_{\mathrm{YY}}+.52 \mathrm{Q}_{\mathrm{SS}}=\frac{\mathrm{G}_{[0 / 90 /-45]}^{2}\left(.004356-.004356 \xi^{2}\right)}{\mathrm{G}_{[0 / 90 /-45]}\left(.004356-.004356 \xi^{2}\right)-.132(1+\xi) \eta^{2}}
\end{align*}
$$

## 5. Extracting Mechanical Properties

Considering sets of equations (14), (17), (21) and (24), we have 14 equations with 14 unknown parameters which unknown parameters are unknown mechanical properties of unidirectional, biax and triax. At a glance, it can be seen that solving this set of equations due to its non-linear nature is not simple. Since some of the equations are highly dependent to another one, ordinary and classical solution is not possible. In order to simplify solution of them, knowing this fact that the major Poisson's ratio of a [0/90] composites has to have an amount between 0.05 and 0.1 , we choose this point as a start point of solution. Considering $\beta$ as a known parameter will reduce number of unknowns. Further more, in this way, there is no need to use complex equations of triax composite with [0/90/-45] configuration. The equations of [0/90/-45] will be used just to verify the results.According to table 1, elastic modulus of triax composite is located between the upper and lower bands. Using upper and middle bands as the first guess in inverse method will not lead to acceptable results, therefore lower band magnitude for elastic modulus is selected.

The obtained results in terms of different amounts of $\beta$, are inserted in table 2 .

Table. 2. Mechanical Properties of Unidirectional Fibers in Biax and Triax

| $\beta$ | $\mathbf{E}_{\mathbf{X}}[\mathbf{G P a}]$ | $\mathbf{E}_{\mathbf{Y}}[\mathbf{G P a}]$ | v | $\mathbf{G}[\mathbf{G P a}]$ |
| :--- | :--- | :--- | :--- | :--- |
| Lower Band: $E_{\text {Triax }}=20.7-3.1=17.6$ |  |  |  |  |
| 0.05 | 28.5491 | 4.5643 | .21 | 2.107 |
| 0.06 | 28.8925 | 4.36248 | .26 | 2.096 |
| 0.07 | 29.01 | 4.2215 | .32 | 2.09 |
| 0.08 | 29.127 | 4.10 | .37 | 2.085 |
| 0.09 | 29.1972 | 3.8243 | .4 | 2.08 |
| 0.1 | 29.234 | 3.69422 | .43 | 2.077 |

When Poisson's ratio of [0/90] is considered 0.06, the best result in table 2 is obtained. Now, using obtained results and available calculated equations, $E_{2}$ of triax composite with [0/45/-45] configuration and its Poisson's ratio can be calculated.Therefore, full mechanical properties of unidirectional, biax and triax composites are extracted. These values are inserted in table 3.

Table. 3. Full Mechanical Properties of Unidirectional, Biax and Triax

| Material | Config. | Mechanical Properties |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} \mathrm{E}_{1} \\ {[\mathrm{GPa}]} \end{gathered}$ | $\begin{gathered} \mathrm{E}_{2} \\ {[\mathrm{GPa}]} \end{gathered}$ | $v_{\text {Major }}$ | $\begin{gathered} \mathrm{E}_{6} \\ {[\mathrm{GPa}]} \end{gathered}$ |
| UD* | - | 28.89 | 4.36 | 0.26 | 2.096 |
| Biax | [0/90] ${ }_{\text {T }}$ | 16.7 | 16.7 | 0.06 | 2.01 |
| Triax | $[0 / \pm 45]_{T}$ | 17.6 | 7.01 | 0.52 | 5.075 |

In order to verify the results, elastic modulus of triax composite with [0/90/-45] configuration is also calculated and compared with its original amount which has been obtained from experiment and reported by its supplier [6].
According to aforementioned calculation, amount of this elastic modulus is equal to 13.6 GPa and it is located in the reported domain $(15 \pm 2.3 \mathrm{GPa})$.

In order to double check the results, flexural modulus of triax composite is also reported by its supplier and we can compare calculate flexural modulus and compare it with its original amount [6].
The calculation of flexural modulus has been performed using PROMAL software based on CLT $^{2}$ theory [8]. Obtained results are inserted in table 4.

[^1]Table. 4. Comparison between Calculated and
Experimental Flexural Modulus

| Flexural <br> Modulus | Triax [0/45/-45] |  |
| :---: | :---: | :---: |
|  | Theoretical | Experimental |
|  | 17.85 | $16.7 \pm 2.5$ |
|  | Triax [0/90/-45] |  |
| Flexural | Theoretical | Experimental |
| Modulus | 15.23 | $15.1 \pm 2.3$ |

It can be seen that results has a good correlation with experiments and it shows the accuracy of obtained results.

## 6.Conclusion

In this research a new method has been presented to extract full mechanical properties of non-woven biax and triax composite materials which are widely used in industry.
Generally, mechanical properties extraction of composites needs full material characterization experiments [9].
In this research, limited experimental data have been used to extract full mechanical properties. Applied experimental data were limited to four elastic modulus: two elastic modulus of biax and triax in $0^{\circ}$ direction and two elastic modulus of biax and triax in $45^{\circ}$ direction.
Since these composites are not woven, using direct approach in macromechanical field would be a proper approach.
This approach needs full mechanical properties of constructed unidirectional fibers.
In the second approach which is called inverse method, stiffness and compliance matrixes of biax and triax composites are organized and inversely full mechanical properties of their constructed unidirectional fiber will be derived.
Therefore, full mechanical properties of biax and triax can be obtained employing direct method.
This technique has been applied to the Glass/Epoxy prepreg composites as a case study.
Obtained results showed the technique is applicable. In order to evaluate the results flexural modulus is also calculated and has been compared with its original amount which has been reported by its supplier. The results were in a good correlation with experimental data.

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## Appendix

## List of Symbols

| $\mathrm{E}_{\mathrm{X}}$ | Longitudinal Modulus of Lamina |
| :---: | :---: |
| $\mathrm{E}_{Y}$ | Transverse Modulus of Lamina |
| $\mathrm{E}_{\text {S }}$ | In-Plane Shear Modulus of Lamina |
| $\mathrm{v}_{\mathrm{XY}}$ | Major Poisson's Ratio of Lamina |
| $v_{12}$ | Minor Poisson's Ratio of Lamina |
| $\mathrm{E}_{1}$ | Longitudinal Modulus of Laminate |
| $\mathrm{E}_{2}$ | Transverse Modulus of Laminate |
| $\mathrm{E}_{6}$ | In-Plane Shear Modulus of Laminate |
| $v_{12}$ | Major Poisson's Ratio of Laminate |
| $\mathrm{Q}_{\mathrm{xx}}, \quad \mathrm{Q}_{\mathrm{xy}}$, | On-Axis Stiffness Matrix |
| $\mathrm{Q}_{\mathrm{yy}}$, $\mathrm{Q}_{\text {ss }}$ | Components of A Lamina |
| $\mathrm{Q}_{11}, \quad \mathrm{Q}_{22}$, | Off-Axis Stiffness Matrix |
| $\begin{aligned} & \mathrm{Q}_{12}, \quad \mathrm{Q}_{16}, \\ & \mathrm{Q}_{26}, \mathrm{Q}_{66} \end{aligned}$ | Components of A Lamina |
| h | Ply Thickness |
| p | Ply Fraction |
| $\mathrm{A}_{11}, \quad \mathrm{~A}_{22}$, | Equivalent Stiffness Matrix |
| $\begin{aligned} & \mathrm{A}_{12}, \quad \mathrm{~A}_{16}, \\ & \mathrm{~A}_{26}, \mathrm{~A}_{66} \end{aligned}$ | Components of A Laminate |
| [a] | Compliance Matrix of A Laminate |
| $\alpha$ | Major Poisson's Ratio of [+45/-45] |
| $\beta$ | Major Poisson's Ratio of [0/90] |
| $\varphi$ | Major Poisson's Ratio of [0/45/-45] |
| $\xi$ | Minor Poisson's Ratio of [0/90/-45] |


[^0]:    Received by the editor August, 21, 2004; final revised: April, 15, 2006.
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[^1]:    ${ }^{2}$ Classical Lamination Theory

