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RESEARCH PAPER

Economic Production Quantity Under Possible Substitution: A Scenario Analysis Approach

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ABSTRACT
This paper designs a Scenario analysis approach to determine the joint production policy for two products under possible substitution. The Scenario analysis is designed to improve decision-making by considering possible outcomes and their implications. The traditional multi-product production models assume that there is no possible substitution between products. However, in real-world cases, there are many substitutable products where substitution may occur in the event of a product stock-out. The proposed model optimizes production quantities for two products under substitution with the aim of minimizing the total cost of the inventory system, including setup and holding costs, subject to a resource constraint. To analyze the problem, four special Scenarios are derived and discussed in detail. Furthermore, the total cost functions are derived for each Scenario separately, and then a solution procedure is suggested based on the Scenarios developed. The numerical examples are implemented, and the results are discussed in detail. Finally, sensitivity analysis is performed to get more insights. It is observed that the presented model is highly sensitive to the demand rate of products.

KEYWORDS: *Scenario analysis; Production-inventory systems; Substitutable products; Joint production policy.*

1. Introduction

In real-world industrial systems, the appropriate design and control of inventories have a great role and impact on performance. The raw materials, goods in processes, spare parts, and finished items are various kinds of inventory. The important decision in an inventory system is to determine how much and when should order [1]. If inventories are not controlled appropriately, they might incur costly outcomes. Therefore, designing an appropriate inventory system is a vital task to create an acceptable performance. The numerous models of inventory systems have been presented in the literature yet. Among them, the economic order quantity (EOQ) is the first and basic one [2]. In traditional EOQ, the demand is deterministic and constant over the planning horizon, and the order is received instantaneously. The model aims to delineate the optimal order quantity for items so as to minimize the total costs, including holding and ordering costs. Since the holding and ordering costs behave inversely in basic EOQ, the total cost function is convex, and then an intermediate amount of order quantity is optimal. Many versions of the inventory model have been proposed by relaxing some basic assumptions or adding new ones into the traditional EOQ model. The economic production quantity (EPQ) is one of the earlier extensions of EOQ [3]. In basic EOQ, it is assumed that the order quantity is received at the moment with an infinite rate, while, in EPQ, orders are received with a finite rate over time. The EPQ model, also known as the economic manufacturing quantity (EMQ), aims to determine the optimal production quantity for a manufacturing facility. The objective of the EPQ is to minimize the total inventory and production costs.

As a recent research, Pan, et al. [4] proposed an EPQ model integrated with the process control and maintenance problems. Wee, et al. [5] considered an EPQ model with a renewal reward procedure for imperfect items. Moreover, Dash,

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et al. [6] designed an EPQ for deteriorating inventories with time value of money and price-dependent demand. An imperfect EPQ problem was suggested by Karimi-Nasab and Sabri-Laghaie [7] with reworkable and non-reworkable items and random defectives. Nasr, et al. [8] utilized differential equations for an EPQ model with deteriorating raw materials. In addition, Pacheco-Velázquez and Cárdenas-Barrón [9] considered an EPQ problem by considering the inventory costs of the raw materials and finished items separately. Additionally, Jawad, et al. [10] analyzed a sustainable EPQ using the laws of thermodynamics. In another work, a multi-item EPQ with fuzzy demand was proposed by Sadeghi, et al. [11]. Moreover, Al-Salamah [12] suggested an EPQ model with the quality control process where the items are subjected to destructive or non-destructive inspection. Mokhtari, et al. [13] proposed an EPQ model for perishable products with shortage and stock-dependent demand. Karmakar, et al. [14] proposed a pollution-sensitive fuzzy EPQ model with a time-dependent rate of production. Taleizadeh, et al. [15] proposed sustainable production-inventory models under different shortage scenarios. They utilized a direct accounting approach to formulate carbon emission. Mokhtari and Rezvan [16] studied an EPQ model for multi-buyers and multi-products under a partial backordering shortage. Multi-product constrained EPQ models for imperfect quality items with rework policy were developed by Mokhtari, et al. [1]. They solved these models using Lagrangian relaxation method. Fallahi, et al. [17] designed an EPQ model for defective items under a multiple shipments policy. This model considered the carbon emission of the system under direct accounting policy. Moreover, Asadkhani, et al. [18] discussed the role of learning in inspection errors for EPQ models with different types of imperfect items. In addition, we can find integrated production-inventory and supply chain models as extensions of EPQ in literature [19-22].

The substitution usually occurs for inherently similar products, such as different coffee, chocolate, or pastry brands. When a company supplies two substitutable products, customers of one product may switch to another when the first product is unavailable and vice versa. Therefore, the effect of demand substitution on the multi-products inventory control problems is an important issue. However, the academic literature has treated little attention to studying the classical inventory models like EOQ and EPQ under substitution. To the best of our knowledge, the notable researches on substitutability are presented under EOQ framework, and there is no academic research for substitution under EPQ setting. Drezner, et al. [23] derived the joint replenishment policy for two substitutable products under EOQ model with one-to-one and full substitution. Gurnani and Drezner [24] extended the research presented by Drezner, et al. [23] to multiple products, and they considered one-way substitution where customers are allowed to switch to higher quality products. Shin, et al. [25] provided a review of the literature on substitutable products planning. In addition, Salameh, et al. [26] extended the work of Drezner, et al. [23] for partial substitution, where just a fraction of customers are willing to substitute. In addition, Krommyda, et al. [27] studied two substitutable products under a two-way setting, partial substitution and stock-dependent demand on the EOQ structure. Maddah, et al. [28] presented an EOQ model for multiple substitutable products under partial substitution. Giri, et al. [29] proposed a substitution balancing strategy for inventory systems of substitutable items. In this work, the demand was considered as a function of time. Mokhtari [30] studied a new inventory model for complementary substitutable products under a two-way substitution policy. Edalatpour and Al-e-Hashem [31] extended [30] work by considering non-linear holding cost and pricing strategy in the inventory model. Chen, et al. [32] determined the optimal lot-sizing strategy for an inventory system of imperfect substitutable items with real-world constraints. Shah, et al. [33] addressed the inventory problem of substitutable products with time-dependent demand. The goal of model was to maximize the total profit. The models were solved using heuristic algorithms. Durga and Chandrasekaran [34] developed an EOQ model of substitutable products under discount policy. They also analyzed the role of quadratic demand in their model. Table 1 shows the characteristics of previous articles in the literature at a glance.

Tab. 1. A review on the related problems in the literature

Article	Model type EOQ	Model type EPQ	Product substitution	Substitution way	Some other features
Drezner, et al. [23]	✓	×	✓	One-way	First EOQ model for substitutable products
Gurnani and Drezner [24]	✓	×	✓	One-way	Multi-product, quality aspects
Pan, et al. [4]	×	✓	×	×	Statistical process control, maintenance
Wee, et al. [5]	×	✓	×	×	Imperfect items, screening constraint
Salameh, et al. [26]	✓	×	✓	Two-way	Partial substitution
Dash, et al. [6]	×	✓	×	×	Deteriorating items, time value of money
Krommyda, et al. [27]	✓	×	✓	Two-way	Partial substitution, stock-dependent demand
Maddah, et al. [28]	✓	×	✓	Two-way	Multi-product, partial substitution
Al-Salamah [12]	×	✓	×	×	Destructive and non-destructive inspection
Mokhtari, et al. [13]	×	✓	×	×	Perishable product, stock-dependent demand, greed search heuristic
Karmakar, et al. [14]	×	✓	×	×	Carbon emission, time-dependent production rate
Mokhtari [30]	✓	×	✓	Two-way	Complementary substitutable products
Taleizadeh, et al. [15]	×	✓	×	×	Carbon emission, partial backordering
Shah, et al. [33]	✓	×	×	Two-way	Partial substitution, time-dependent demand
Edalatpour and Al-e-Hashem [31]	✓	×	✓	Two-way	Non-linear holding cost, pricing decisions
Mokhtari and Rezvan [16]	×	✓	×	×	Multi-buyer, multi-product, vendor managed inventory
Fallahi, et al. [17]	×	✓	×	×	Preventive maintenance, multiple shipments
Durga and Chandrasekaran [34]	✓	×	✓	Two-way	Complementary substitutable products, discount
Asadkhani, et al. [18]	×	✓	×	×	Inspection errors, different types of imperfect items
Current paper	×	✓	✓	Two-way	First EPQ model for substitutable products, Scenario analysis

As seen from the above review, all of the previous research on substitution is presented under the EOQ framework. Hence, in this paper, we study a production-inventory control model of two substitutable products in EPQ setting, where two-way substitution is possible with full and one-to-one substitution. To our knowledge, this problem has not been treated in literature yet. The details of the model will be discussed in subsequent sections.

The rest of the paper is arranged as follows. In the next section, notations and assumptions are presented. Section 3 discusses the problem definition and modeling and develops the possible Scenarios. Then, Section 4 presents the solution algorithm, and Section 5 presents numerical examples. Finally, Section 6 concludes the paper.

2. Notations and Assumptions

Before formulating the proposed model, the notations used throughout the paper are introduced below.

D_1	The demand rate of product 1
D_2	The demand rate of product 2
P_1	The production rate of product 1
P_2	The production rate of product 2
Q_1	The production quantity of product 1 per cycle
Q_2	The production quantity of product 2 per cycle
A_1	The fixed setup cost of product 1
A_2	The fixed setup cost of product 2

h_1	The holding cost of product 1 per unit time
h_2	The holding cost of product 2 per unit time
f_1	The amount of resources is required for one unit of product 1
f_2	The amount of resources is required for one unit of product 2
t_1^p	The production cycle of product 1 without substitution
t_1^d	The consumption cycle of product 1 without substitution
t_2^p	The production cycle of product 2 without substitution
t_2^d	The consumption cycle of product 2 without substitution
T_1	The inventory cycle of product 1 without substitution ($T_1 = t_1^p + t_1^d$)
T_2	The inventory cycle of product 2 without substitution ($T_2 = t_2^p + t_2^d$)
I_1	The initial inventory level of one product when another product runs out of stock (beginning of substitution period)
I_2	The maximum inventory level of one product when another product runs out of stock (during of substitution period)
t	The time interval in which maximum inventory of product within substitution period is consumed completely.
F	The total amount of resource that is available
TC_{1i}	The total cost per cycle of product 1 in Scenario i ($i = 1, 2, 3, 4$)
TC_{2i}	The total cost per cycle of product 1 in Scenario i ($i = 1, 2, 3, 4$)
TCU_{1i}	The total cost per unit time of product 1 in Scenario i ($i = 1, 2, 3, 4$)
TCU_{2i}	The total cost per unit time of product 1 in Scenario i ($i = 1, 2, 3, 4$)

3. Problem Definition and Modeling

There is a production-inventory system in a manufacturing plant with two products, working under EPQ setting. The manufacturer faces external demand for products, D_1 and D_2 Where demands are assumed to be deterministic and constant over the planning horizon. The manufacturer produces two products via finite production rates, P_1 and P_2 to meet the demands received from customers. The production-inventory system follows the basic EPQ model where the shortage is not allowed. Moreover, there is a finite amount of resources which products can use. Figure 1 shows the inventory level corresponding to a single product under EPQ framework. At every inventory cycle T, the production is processed until the inventory reaches the maximum level I_{max} during production cycle t^p, and then the stored inventory is consumed with the demand rate D until reaches reach to zero during the consumption cycle t^d. The setup process of production incurs a fixed cost denoted by A, and the produced inventory can be stored with a holding cost per unit time denoted by h. The aim is to find the economic production quantity Q so that the total cost of the inventory system involving setup and holding costs is minimized.

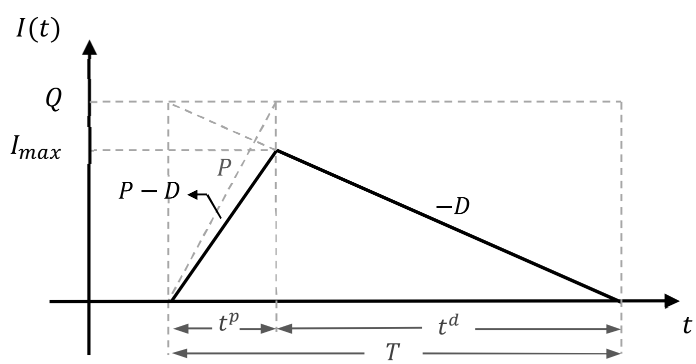

Fig. 1. The inventory level for a single product under EPQ framework

By considering the characteristics of the basic EPQ model (shown by Figure 1), the parameters of the model are obtained as $T = Q/D$, $t^p = Q/P$, $t^d = Q/D - Q/P$, and $I_{max} = Q(1 - D/P)$. Moreover, the total cost of product involving setup and holding costs is calculated as $TC = A + hQ^2/2D(1 - D/P)$ and, hence, the optimal production policy and the total cost is derived as $Q^* = [2AD/(h(1 - D/P))]^{1/2}$ and $TC(Q^*) = [2ADh(1 - D/P)]^{1/2}$, respectively. In our EPQ model with possible substitution, the products can be fully substituted when one product runs out of stock. That means if product 1 is depleted, then the customers will buy product 2, and vice versa. Product substitution occurs within special product categories which are inherently similar such as laptops, mobile phones and etc. [25]. For example, a laptop manufacturer produces two laptop models in a similar price range with a few differences in technical features. If a laptop model is not available, customers can shift to another one and vice versa. Product substitution has various benefits for the inventory system. It enhances the availability of the products and results in rapid response to the changes in customer demands [30]. In addition, the causes of lost sale shortage can be controlled by demand substitution in the inventory systems. It is assumed that the production of two products is started jointly in every inventory cycle, with the production quantities $\boldsymbol{Q_1}$ and Q_2. To analyze the problem, a Scenario analysis approach is utilized as a soft computing method, which is conventional in evaluating engineering problems [35-38]. To obtain optimal production quantities, Q_1^* and Q_2^*, four Scenarios are possible in terms of situations occur in relationship between t_1^p, t_1^d, t_2^p and t_2^d. That is, when $t_1^p + t_1^d \le t_2^p$ (Scenario I), when $t_2^p \le t_1^p + t_1^d \le t_2^p + t_2^d$ (Scenario II), when $t_1^p \le t_2^p + t_2^d \le t_1^p + t_1^d$ (Scenario III), and when $t_2^p + t_2^d \le t_1^p$ (Scenario IV). To ensure feasibility, we assume that $P_1 > D_1 + D_2$ and $P_2 > D_1 + D_2$, in the proposed model. Moreover, the proposed model is constructed based on the following assumptions:

- The production-inventory system involves two products
- The demand rate is deterministic and constant
- The production rate is finite and constant
- The lead time is assumed to be zero
- The shortage is not allowed
- The setup cost is fixed and incurred per cycle
- The holding cost is applied to the units of products
- The substitution is one-to-one between products
- The two-way substitution is possible between products
- The demand of one product can be fully substituted by another product

3.1. Scenario I

In the first Scenario (when $t_1^p + t_1^d \le t_2^p$ as depicted by Figure 2), product 1 is totally consumed within the production cycle of product 2. At this moment, substitution occurs for product 1 by product 2. Indeed, the demand of product 1, after depletion, is fulfilled from leftover inventory of product 2, at the rate $\boldsymbol{D_1}$. This case often occurred in manufacturing settings.

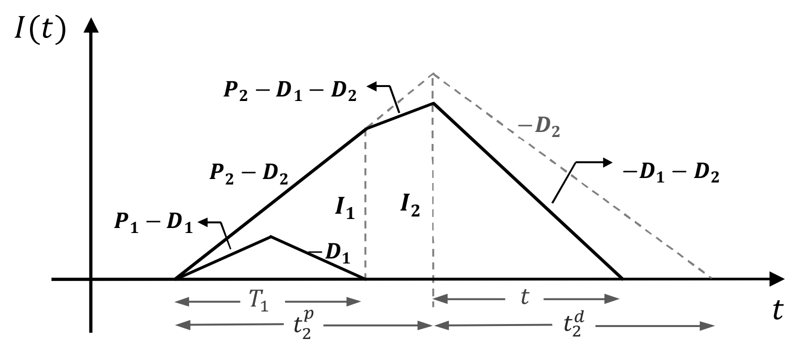

Fig. 2. The inventory level for Scenario I ($\boldsymbol{T_1 = t_1^p + t_1^d \le t_2^p}$)

In this Scenario, the total cost of product 1 per inventory cycle $T_1 = t_1^p + t_1^d$ is sum of setup and holding costs. As shown by Figure 2, the total cost of product 1 per cycle in Scenario I $(TC_{11}(Q_1, Q_2))$, is similar to the total cost of basic EPQ model for product 1, as follows.

$$TC_{11}(Q_1,Q_2) = A_1 + h_1\frac{Q_1^2}{2D_1}\left(1-\frac{D_1}{P_1}\right) \quad (1)$$

Moreover, the cost of product 2 includes the setup and holding costs. Before calculating the total cost of product 2 per cycle in Scenario I ($TC_{21}(Q_1,Q_2)$), note that the inventory level of product 2 when product 1 runs out of stock is $I_1 = (P_2 - D_2)T_1$, the maximum inventory level of product 2 within the substitution period is $I_2 = I_1 + (t_2^p - T_1)(P_2 - D_1 - D_2)$ and the time interval in which maximum inventory of product 2 is depleted is $t = I_2/(D_1 + D_2)$. By substituting the parameters $T_1 = Q_1/D_1$ and $t_2^p = Q_2/P_2$ into I_1, I_2 and t, it yields:

$$I_1 = (P_2 - D_2)Q_1/D_1 \quad (2)$$

$$I_2 = Q_2\left(1-\frac{D_1+D_2}{P_2}\right) + Q_1 \quad (3)$$

$$t = \left\{\left(\frac{Q_2}{P_2}-\frac{Q_1}{D_1}\right)(P_2 - D_1 - D_2) - \frac{Q_1(D_2-P_2)}{D_1}\right\}/(D_1 + D_2) \quad (4)$$

The fixed setup cost of product 2 is A_2, and the inventory holding cost is obtained by calculating the area under inventory level of product 2 in Figure 2, as follows.

$$h_2\left\{\frac{I_1T_1}{2} + \frac{(I_1+I_2)(t_2^p - T_1)}{2} + \frac{I_2t}{2}\right\} \quad (5)$$

By substituting the parameters I_1, I_2, t and t_2^p into the above holding cost and simplifying the results, the total cost $TC_{21}(Q_1,Q_2)$, as the sum of setup and holding costs, is written as follows.

$$TC_{21}(Q_1,Q_2) = A_2 - h_2\left\{\left(\frac{E}{2} - \frac{Q_1(D_2-P_2)}{2D_1}\right)\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right) - \frac{E^2}{2(D_1+D_2)} + \frac{Q_1^2(D_2-P_2)}{2D_1^2}\right\} \quad (6)$$

where $E = (Q_2/P_2 - Q_1/D_1)(P_2 - D_1 - D_2) - Q_1/D_1(D_2 - P_2)$. The total cost of products 1 and 2 per cycle in Scenario 1 is the sum of the total costs of products 1 and 2 per cycle, i.e., $TC_1(Q_1,Q_2) = TC_{11} + TC_{21}$. Finally, the total cost per unit time in Scenario 1, $TCU_1(Q_1,Q_2)$, is obtained by dividing $TC_1(Q_1,Q_2)$ by the inventory cycle $t_2^p + t$, as follows:

$$TCU_1(Q_1,Q_2) = TC_1(Q_1,Q_2)/(t_2^p + t) \quad (7)$$

which yields:

$$TCU_1(Q_1,Q_2) = \left\{A_1 + A_2 - h_2\left\{\left(\frac{E}{2} - \frac{Q_1(D_2-P_2)}{2D_1}\right)\left(\frac{Q_1}{D_1} - \frac{Q_2}{P_2}\right) - \frac{E^2}{2(D_1+D_2)} + \frac{Q_1^2(D_2-P_2)}{2D_1^2}\right\} + h_1\frac{Q_1^2}{2D_1}\left(1-\frac{D_1}{P_1}\right)\right\} / \left(\frac{Q_2}{P_2} + \frac{E}{D_1+D_2}\right) \quad (8)$$

3.2. Scenario II

In the second Scenario (when $t_2^p \le t_1^p + t_1^d \le t_2^p + t_2^d$ as depicted by Figure 3), product 1 is consumed within the consumption cycle of product 2. At this moment, substitution occurs for product 1 by product 2, and the demand of product 1 is fulfilled from the leftover inventory of product 2.

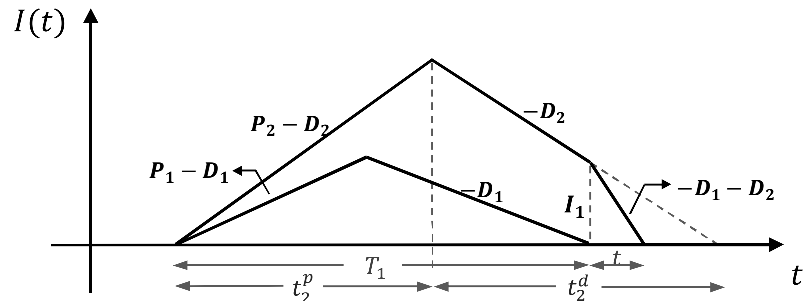

Fig. 3. The inventory level for Scenario II ($t_2^p \leq T_1 = t_1^p + t_1^d \leq t_2^p + t_2^d$)

In this Scenario, the total cost of product 1 per inventory cycle is similar to the first Scenario, as follows.

$$TC_{12}(Q_1, Q_2) = A_1 + h_1 \frac{Q_1^2}{2D_1}\left(1 - \frac{D_1}{P_1}\right) \tag{9}$$

Moreover, the cost of product 2 includes the setup and holding costs. First, note that the inventory level of product 2 when product 1 runs out of stock is $I_1 = Q_2(1 - D_2/P_2) - D_2(Q_1/D_1 - Q_2/P_2)$, and the time interval in which I_1 is depleted is $t = I_1/(D_1 + D_2)$.

The fixed setup cost of product 2 is A_2, and the inventory holding cost is obtained by calculating the area under inventory level of product 2 in Figure 3, as follows.

$$h_2\left\{\frac{Q_2^2}{2D_2}\left(1 - \frac{D_2}{P_2}\right) - \frac{(T_2 - T_1 - t)I_1}{2}\right\} \tag{10}$$

By substituting the parameters I_1, t, T_1 and T_2 into the above holding cost and simplifying the results, the total cost of product 2 is calculated as follows.

$$\begin{aligned} TC_{22}(Q_1, Q_2) = A_2 & \\ + h_2\Bigg\{\frac{F}{2}\Bigg(&\frac{F}{D_1 + D_2} - \frac{Q_1}{D_1} \\ &+ \frac{Q_2}{D_2}\Bigg) + \frac{Q_2^2}{2D_2}\left(1 - \frac{D_2}{P_2}\right)\Bigg\} \end{aligned} \tag{11}$$

where $F = D_2(Q_1/D_1 - Q_2/P_2) - Q_2(1 - D_2/P_2)$. The total cost of products 1 and 2 per cycle in Scenario 2 is the sum of the total costs of products 1 and 2 per cycle, i.e., $TC_2(Q_1, Q_2) = TC_{12} + TC_{22}$. Finally, the total cost per unit time in Scenario 2, $TCU_2(Q_1, Q_2)$, is achieved by dividing $TC_2(Q_1, Q_2)$ by the inventory cycle $T_1 + t$, as follows:

$$\begin{aligned} TCU_2(Q_1, Q_2) = \Bigg\{ & A_1 + A_2 \\ & + h_2\Bigg\{\frac{F}{2}\Bigg(\frac{F}{D_1 + D_2} \\ & - \frac{Q_1}{D_1} + \frac{Q_2}{D_2}\Bigg) \\ & + \frac{Q_2^2}{2D_2}\left(1 - \frac{D_2}{P_2}\right)\Bigg\} \\ & + h_1\frac{Q_1^2}{2D_1}\left(1 - \frac{D_1}{P_1}\right)\Bigg\} \\ & /\left(\frac{Q_1}{D_1} - \frac{F}{D_1 + D_2}\right) \end{aligned} \tag{12}$$

3.3. Scenario III

As depicted by Figure 4, the third Scenario occurs when $t_2^p \leq t_1^p + t_1^d \leq t_2^p + t_2^d$, in which product 2 is consumed totally within the consumption cycle of product 1. At this moment, substitution occurs for product 2 by product 1, and the demand of product 2 is fulfilled from the leftover inventory of product 1.

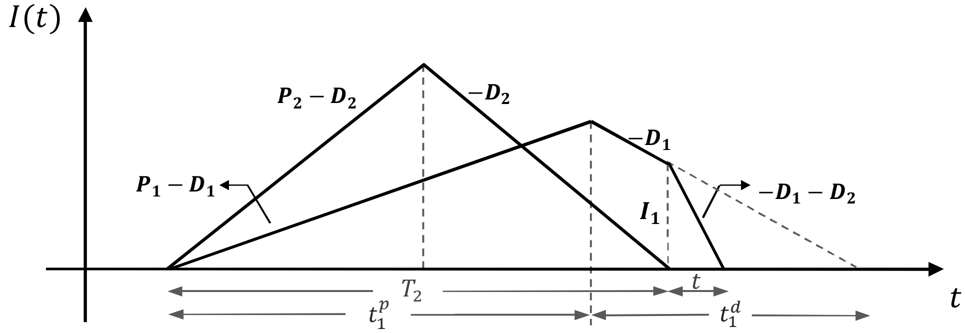

Fig. 4. The inventory level for Scenario III ($t_1^p \leq T_2 = t_2^p + t_2^d \leq t_1^p + t_1^d$)

In this Scenario, the total cost of product 2 per inventory cycle is similar to that of the basic EPQ model:

$$TC_{23}(Q_1, Q_2) = A_2 + h_2 \frac{Q_2^2}{2D_2}\left(1 - \frac{D_2}{P_2}\right) \quad (13)$$

Moreover, the cost of product 1 includes the setup and holding costs. First, note that the inventory level of product 1 when product 2 runs out of stock is $I_1 = Q_1(1 - D_1/P_1) - D_1(T_2 - t_1^p)$, and the time interval in which I_1 is depleted is $t = I_1/(D_1 + D_2)$.
The fixed setup cost of product 1 is A_1, and the inventory holding cost can be calculated by getting the area under inventory level of product 1 in Figure 4, as follows.

$$h_1 \left\{ \frac{Q_1^2}{2D_1}\left(1 - \frac{D_1}{P_1}\right) - \frac{(T_1 - T_2 - t)I_1}{2} \right\} \quad (14)$$

By substituting the parameters I_1, t, T_1 and T_2 into holding cost and simplifying the results, the total cost of product 1 is calculated as follows.

$$TC_{13}(Q_1, Q_2) = A_1 + h_1 \left\{ \frac{G}{2}\left(\frac{G}{D_1 + D_2} - \frac{Q_2}{D_2} + \frac{Q_1}{D_1}\right) + \frac{Q_1^2}{2D_1}\left(1 - \frac{D_1}{P_1}\right) \right\} \quad (15)$$

where $G = D_1(Q_2/D_2 - Q_1/P_1) - Q_1(1 - D_1/P_1)$. The total cost of products 1 and 2 per cycle in this Scenario is $TC_3(Q_1, Q_2) = TC_{13} + TC_{23}$. Finally, the total cost per unit time in Scenario 3, $TCU_3(Q_1, Q_2)$, is computed by dividing $TC_3(Q_1, Q_2)$ by the inventory cycle $T_2 + t$, as follows:

$$TCU_3(Q_1, Q_2) = \left\{ A_1 + A_2 + h_1 \left\{ \frac{G}{2}\left(\frac{G}{D_1 + D_2} - \frac{Q_2}{D_2} + \frac{Q_1}{D_1}\right) + \frac{Q_1^2}{2D_1}\left(1 - \frac{D_1}{P_1}\right) \right\} + h_2 \frac{Q_2^2}{2D_2}\left(1 - \frac{D_2}{P_2}\right) \right\} \Big/ \left(\frac{Q_2}{D_2} - \frac{G}{D_1 + D_2}\right) \quad (16)$$

3.4. Scenario IV

As depicted by Figure 5, in the fourth Scenario we have $t_2^p \leq t_1^p + t_1^d \leq t_2^p + t_2^d$. In this Scenario, product 2 is consumed totally within the production cycle of product 1. At this moment, substitution occurs for product 2 by product 1, and the demand of product 2 is fulfilled from inventory of product 1.

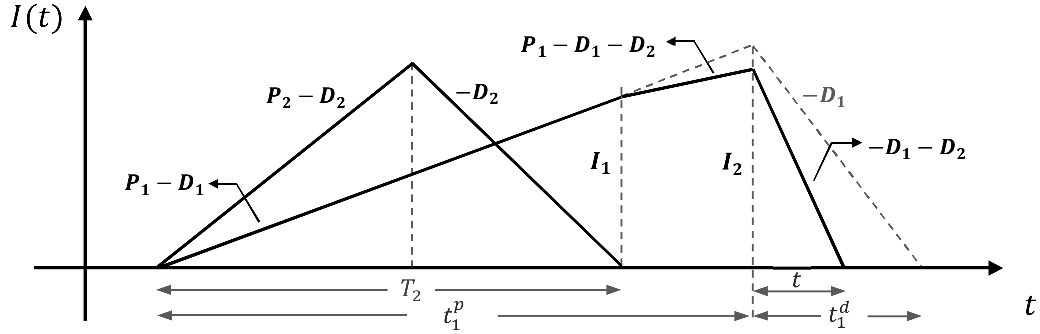

Fig. 5. The inventory level for Scenario IV ($T_2 = t_2^p + t_2^d \le t_1^p$)

In this Scenario, the total cost of product 2 per inventory cycle is similar to Scenario III, as follows.

$$TC_{24}(Q_1,Q_2) = A_2 + h_2\frac{Q_2^2}{2D_2}\left(1-\frac{D_2}{P_2}\right) \qquad (17)$$

To calculate cost of product 1, first, note that the inventory level of product 1 when product 2 runs out of stock is $I_1=(P_1-D_1)T_2$, the maximum inventory level of product 1 is $I_2=I_1+\left(t_1^p-T_2\right)(P_1-D_1-D_2)$ and the time interval in which maximum inventory of product 2, i.e. I_2, is depleted is $t=I_2/(D_1+D_2)$. By substituting the parameters $T_2=Q_2/D_2$ and $t_1^p=Q_1/P_1$ into I_1, I_2 and t, we have:

$$I_1=(P_1-D_1)Q_2/D_2 \qquad (18)$$

$$I_2=Q_1\left(1-\frac{D_1+D_2}{P_1}\right)+Q_2 \qquad (19)$$

$$t=\left\{\left(\frac{Q_1}{P_1}-\frac{Q_2}{D_2}\right)(P_1-D_1-D_2) - \frac{Q_2(D_1-P_1)}{D_2}\right\}/(D_1+D_2) \qquad (20)$$

The fixed setup cost of product 1 is A_1, and the inventory holding cost is obtained by calculating the area under inventory level in Figure 5, as follows.

$$h_1\left\{\frac{I_1T_2}{2}+\frac{(I_1+I_2)(t_1^p-T_2)}{2}+\frac{I_2t}{2}\right\} \qquad (21)$$

By substituting the parameters I_1, I_2, t and t_2^p into holding cost and simplifying the results, the total cost $TC_{14}(Q_1,Q_2)$ is computed as follows.

$$TC_{14}(Q_1,Q_2)=A_1 - h_1\left\{\left(\frac{H}{2}-\frac{Q_1(D_2-P_2)}{2D_1}\right)\left(\frac{Q_1}{D_1}-\frac{Q_2}{P_2}\right)-\frac{H^2}{2(D_1+D_2)}+\frac{Q_1^2(D_2-P_2)}{2D_1^2}\right\} \qquad (22)$$

where $H=(Q_1/P_1-Q_2/D_2)(P_1-D_1-D_2)-Q_2(D_1-P_1)/D_2$. The total cost of products 1 and 2 per cycle in Scenario 1 is $TC_4(Q_1,Q_2)=TC_{14}+TC_{24}$. Finally, the total cost per unit time, $TCU_4(Q_1,Q_2)$, is gained by dividing $TC_4(Q_1,Q_2)$ by the inventory cycle t_1^p+t, as follows:

$$TCU_4(Q_1,Q_2)=\left\{A_1+A_2 - h_1\left\{\left(\frac{H}{2}-\frac{Q_2(D_1-P_1)}{2D_2}\right)\left(\frac{Q_2}{D_2}-\frac{Q_1}{P_1}\right)-\frac{H^2}{2(D_1+D_2)}+\frac{Q_2^2(D_1-P_1)}{2D_2^2}\right\}+h_2\frac{Q_2^2}{2D_2}\left(1-\frac{D_2}{P_2}\right)\right\}/\left(\frac{Q_1}{P_1}+\frac{H}{D_1+D_2}\right) \qquad (23)$$

4. Solution Algorithm

In this section, we are going to find the optimal production policy, including the economic production quantities, the optimal inventory intervals, and the optimized total cost, by considering all Scenarios discussed earlier. The derived total costs per unit time will be used to determine the solution. To this end, we first derive the conditions of Scenarios in terms of decision variables, i.e., (Q_1, Q_2), as linear constraints in optimization models, as follows.

Scenario I: $t_1^p + t_1^d \leq t_2^p \sim Q_1/D_1 \leq Q_2/P_2$
Scenario II: $t_2^p \leq t_1^p + t_1^d \leq t_2^p + t_2^d \sim (Q_2/P_2 \leq Q_1/D_1)$ & $(Q_1/D_1 \leq Q_2/D_2)$
Scenario III: $t_1^p \leq t_2^p + t_2^d \leq t_1^p + t_1^d \sim (Q_1/P_1 \leq Q_2/D_2)$ & $(Q_2/D_2 \leq Q_1/D_1)$
Scenario IV: $t_2^p + t_2^d \leq t_1^p \sim Q_2/D_2 \leq Q_1/P_1$

Moreover, the following constraint ensures the required amount of resources does not violate the total amount of available resources.

$$f_1Q_1 + f_2Q_2 \leq F \tag{24}$$

So, the optimal production policy can be found by the following algorithm.

Step 1: Solve the constrained optimization problem for Scenario I as follows:
$Min\ TCU_1(Q_1, Q_2)$
Subject to:
$Q_1/D_1 \leq Q_2/P_2$
$f_1Q_1 + f_2Q_2 \leq F$
$Q_1, Q_2 \geq 0$

And set the optimal solution of this problem as (Q_{11}^*, Q_{21}^*).
Step 2: Solve the constrained optimization problem for Scenario II as follows:
$Min\ TCU_2(Q_1, Q_2)$
Subject to:
$Q_2/P_2 \leq Q_1/D_1$
$Q_1/D_1 \leq Q_2/D_2$
$f_1Q_1 + f_2Q_2 \leq F$
$Q_1, Q_2 \geq 0$

And set the optimal solution of this problem as (Q_{12}^*, Q_{22}^*).
Step 3: Solve the constrained optimization problem for Scenario III as follows:
$Min\ TCU_3(Q_1, Q_2)$
Subject to:
$Q_1/P_1 \leq Q_2/D_2$
$Q_2/D_2 \leq Q_1/D_1$
$f_1Q_1 + f_2Q_2 \leq F$
$Q_1, Q_2 \geq 0$

And set the optimal solution of this problem as (Q_{13}^*, Q_{23}^*).
Step 4: Solve the constrained optimization problem for Scenario IV as follows:
$Min\ TCU_4(Q_1, Q_2)$
Subject to:
$Q_2/D_2 \leq Q_1/P_1$
$f_1Q_1 + f_2Q_2 \leq F$
$Q_1, Q_2 \geq 0$

And set the optimal solution of this problem as (Q_{14}^*, Q_{24}^*).
Step 5: Find the minimum total cost obtained in Steps 1-4, $\min \left\{ \begin{matrix} TCU_1(Q_{11}^*, Q_{21}^*), TCU_2(Q_{12}^*, Q_{22}^*), TCU_3 \\ (Q_{13}^*, Q_{23}^*), TCU_4(Q_{14}^*, Q_{24}^*) \end{matrix} \right\}$, and introduce the corresponding solution as the optimal solution of the problem (Q_1^*, Q_2^*).

5. Computational Results

In this section, two numerical examples are solved to illustrate the performance of models. In addition, to investigate the inventory system's behavior, sensitivity analysis is performed for the first numerical example. Finally, some managerial insights are discussed to provide better insights for decision-makers.

5.1. Numerical examples

In order to illustrate the application and performance of the proposed model, we present and discuss two numerical examples in this section. In the first example, consider a manufacturer which produces two products with independent external demands. Two-way substitution is possible between products. Moreover, the demand for one product can be fully substituted by another product. One unit of a product is substituted with one unit of another product when shortage is occurred (one-to-one substitution). There is a finite amount of resource, e.g., space, money and labor, $F = 400$, which can be used by products. The characteristics of these products, including the production and demand rates, the production setup costs, and the inventory holding costs, are presented in Table 2.

Tab. 2. The characteristics of first numerical example

Parameters	Product 1	Product 2
Demand rate	150	250
Production rate	450	550
Setup cost	20	15
Holding cost	2	4
Resource usage	1	2

To solve this example, we implement the solution algorithm presented in the previous section. First, we solve all constrained optimization problems in Steps 1-4 of the algorithm. To this end, we utilized commercial solver Lingo. The following results are achieved:
Scenario I results: $Q_{11}^* = 0.00,\ Q_{21}^* = 160.21,\ TCU_1 = 174.7726$
Scenario II results: $Q_{12}^* = 45.97,\ Q_{22}^* = 76.61,\ TCU_2 = 228.4334$
Scenario III results: $Q_{13}^* = 45.97,\ Q_{23}^* = 76.61,\ TCU_3 = 228.4334$
Scenario IV results: $Q_{14}^* = 354.96,\ Q_{24}^* = 0.00,\ TCU_4 = 78.8811$

According to the above results, the minimum total cost is $TCU^* = \min\{174.7726, 228.4334, 228.4334, 78.8811\} = 78.8811$, which is related to the Scenario IV whose optimal production quantities are $Q_1^* = 354.96,\ Q_2^* = 0.00$. That means the demand of both products is satisfied by the inventory of product 1. As can be seen, the second and third Scenarios yield the same results. This is a general observation which is due to the similar structure of these Scenarios. Corresponding to this optimal quantities, the optimal inventory intervals are obtained as $t_1^p = Q_1^*/P_1 = 0.789$, $t_2^p = Q_2^*/P_2 = 00.00$, $T_1 = Q_1^*/D_1 = 2.366$, $T_2 = Q_2^*/D_2 = 00.00$.

In order to investigate the superiority of the proposed problem under substitution against the basic model, we compare the results under substitution with the results of the production problem without substitution. For this purpose, the formula of the basic EPQ is employed, which lead to $Q_1 = [2A_1D_1/(h_1(1-D_1/P_1))]^{1/2} = 67.08$, and $Q_2 = [2A_2D_2/(h_2(1-D_2/P_2))]^{1/2} = 58.63$. This is a feasible solution, since $f_1Q_1 + f_2Q_2 = 1*67.08 + 2*58.63 = 185.34 \nleq 400$. This solution incurs the total cost $TCU(Q_1, Q_2) = A_1Q_1/D_1 + h_1Q_1(1-D_1/P_1)/2 + A_2Q_2/D_2 + h_2Q_2(1-D_2/P_2)/2 = 259.8794$. As it is obvious, by using the substitution policy, total cost reduces from 259.8794 to 151.1111, which shows 71.98% improvement.
In the second example, we consider a manufacturer with two substitutable products where substitution is assumed to be two-way, one-to-one, and fully possible between products. There is a finite amount of resource $F = 200$. The characteristics of the products of this example is presented in Table 3. To solve this example, we solve all constrained optimization problems in Steps 1-4 of the algorithm. The following results are obtained.

Tab. 3. The characteristics of second numerical example

Parameters	Product 1	Product 2
Demand rate	400	200
Production rate	800	1000
Setup cost	40	60
Holding cost	10	15
Resource usage	5	3

Scenario I results: $Q_{11}^* = 0.00,\ Q_{21}^* = 66.67,\ TCU_1 = 1100.000$

Scenario II results: $Q_{12}^* = 30.77,\ Q_{22}^* = 15.38,\ TCU_2 = 1469.231$

Scenario III results: $Q_{13}^* = 30.77$, $Q_{23}^* = 15.38$, $TCU_3 = 1469.231$
Scenario IV results: $Q_{14}^* = 34.78$, $Q_{24}^* = 8.70$, $TCU_4 = 1498.261$

According to the obtained results, the minimum total cost is $TCU^* = \min\{1100.000, 1469.231, 1469.231, 1498.261\} = 1100.000$ which is related to the Scenario I, whose optimal production quantities are $Q_1^* = 0.00$, $Q_2^* = 66.67$. That means the demand of both products is satisfied by the inventory of product 2. The optimal inventory intervals are also obtained as $t_1^p = Q_1^*/P_1 = 0.00$, $t_2^p = Q_2^*/P_2 = 0.08$, $T_1 = Q_1^*/D_1 = 0.00$, $T_2 = Q_2^*/D_2 = 0.33$.
In addition, we compare the results under substitution with the results of the production problem without substitution. The basic EPQ leads to $Q_1 = [2A_1D_1/(h_1(1 - D_1/P_1))]^{1/2} = 80.00$, and $Q_2 = [2A_2D_2/(h_2(1 - D_2/P_2))]^{1/2} = 44.72$. However, this is not a feasible solution, since $f_1Q_1 + f_2Q_2 = 5 * 80.00 + 3 * 44.72 = 534.16 \nleq 200$. To ensure feasibility, we use Lagrangian relaxation as a conventional approach in such cases. To do so, the Lagrangian total cost is obtained by adding resource constraint $f_1Q_1 + f_2Q_2 - F$ with Lagrange multiplier θ into the original total cost as follows.

$$LR(Q_1, Q_2, \theta) = \frac{A_1Q_1}{D_1} + \frac{h_1Q_1}{2}\left(1 - \frac{D_1}{P_1}\right) + \frac{A_2Q_2}{D_2} + \frac{h_2Q_2}{2}\left(1 - \frac{D_2}{P_2}\right) + \theta(f_1Q_1 + f_2Q_2 - F)$$

By setting derivatives of $LR(Q_1, Q_2, \theta)$ with respect to Q_1, Q_2 and θ, to zero, the following equations are achieved.

$$Q_1 = \left[\frac{2A_1D_1}{(h_1 + 2\theta f_1)(1 - D_1/P_1)}\right]^{\frac{1}{2}}, \quad Q_2 = \left[\frac{2A_2D_2}{(h_2 + 2\theta f_2)(1 - D_2/P_2)}\right]^{\frac{1}{2}}$$

And

$$f_1\left[\frac{2A_1D_1}{(h_1 + 2\theta f_1)(1 - D_1/P_1)}\right]^{\frac{1}{2}} + f_2\left[\frac{2A_2D_2}{(h_2 + 2\theta f_2)(1 - D_2/P_2)}\right]^{\frac{1}{2}} = F$$

By solving the above system of equations, the feasible solution is achieved as $Q_1^* = 23.72$ and $Q_2^* = 27.14$. This solution incurs the total cost $TCU(Q_1, Q_2) = A_1Q_1/D_1 + h_1Q_1(1 - D_1/P_1)/2 + A_2Q_2/D_2 + h_2Q_2(1 - D_2/P_2)/2 = 1338.928$. Obviously, using the substitution policy, total cost reduces from 1338.928 to 1100.000, which shows a relatively high cost-saving value (21.72% improvement).

5.2. Sensitivity analysis

In the real-world situation, the changes in inventory systems' input parameters are inevitable, and the parameters fluctuate. These changes in the input parameters can significantly impact the decision variables and the objective function of the problem. Sensitivity analysis is a systematic approach to studying the impact of parameter fluctuation on the optimal decision of models. In this section, for the first numerical example, we analyze the impact of changes in total demand D_1 and D_2, setup cost A_1 and A_2, and inventory holding cost h_1 and h_2 on economic production quantity and the system's total cost. The results are provided in Table 4-6. In addition, Figure 6-8 shows the sensitivity of the objective function to the parameters schematically.
As reported in Table 3 and Figure 6, if the demand for products D_1 and D_2 is increased, the total cost of the system is decreased, and the increasment in parameters have a positive impact on TCU^*.

Tab. 4. The sensitivity of TCU^* due to change in demand rates

%Change in D_1	Q_1^*	Q_2^*	TCU^*	%Change in D_2	Q_1^*	Q_2^*	TCU^*
−30	242.60	0.00	102.431	−30	202.361	0.00	112.422
−20	269.89	0.00	95.962	−20	234.787	0.00	104.349
−10	305.43	0.00	88.235	−10	280.624	0.00	93.541
0	355.00	0.00	78.8811	0	354.96	0.00	78.8811
+10	400.00	0.00	67.423	+10	400.00	0.00	59.409
+20	400.00	0.00	55.402	+20	400.00	0.00	39.375
+30	400.00	0.00	43.381	+30	400.00	0.00	19.340

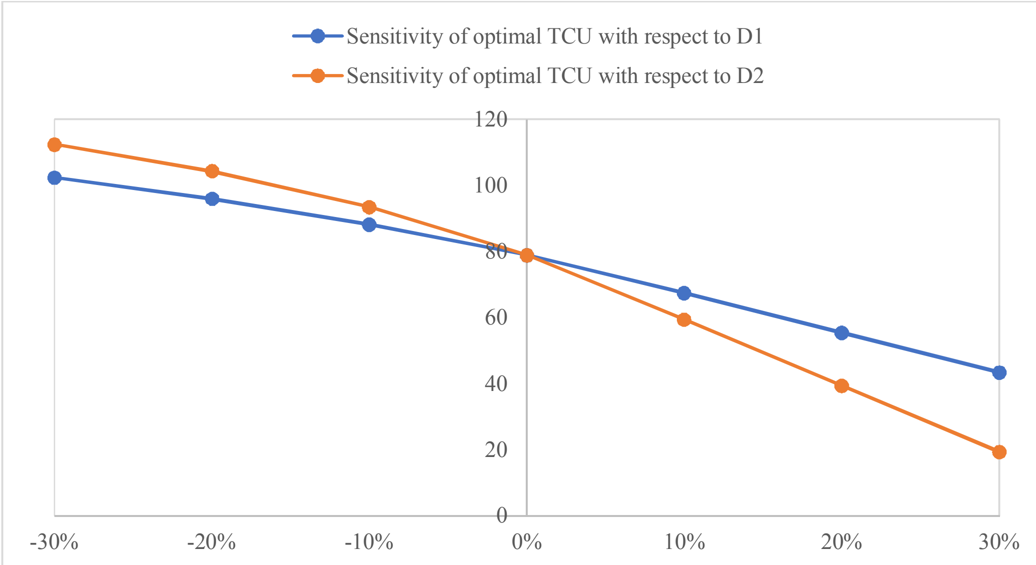

Fig. 6. The impact of D_1 and D_2 on TCU^*

Moreover, the provided results in Table 5 and Figure 7 confirm that the increase in setup cost A_1 and A_2 affect the total cost negatively, where A_1 is more impactful than A_2.

Tab. 5. The sensitivity of TCU^* due to setup costs

%Change in A_1	Q_1^*	Q_2^*	TCU^*	%Change in A_2	Q_1^*	Q_2^*	TCU^*
-30	323.110	0.00	71.802	-30	331.361	0.00	73.636
-20	334.066	0.00	74.236	-20	339.411	0.00	75.425
-10	344.674	0.00	76.594	-10	347.275	0.00	77.172
0	355.00	0.00	78.881	0	354.96	0.00	78.881
$+10$	364.966	0.00	81.103	$+10$	362.491	0.00	80.554
$+20$	374.700	0.00	83.267	$+20$	369.865	0.00	82.192
$+30$	384.187	0.00	85.374	$+30$	377.094	0.00	83.799

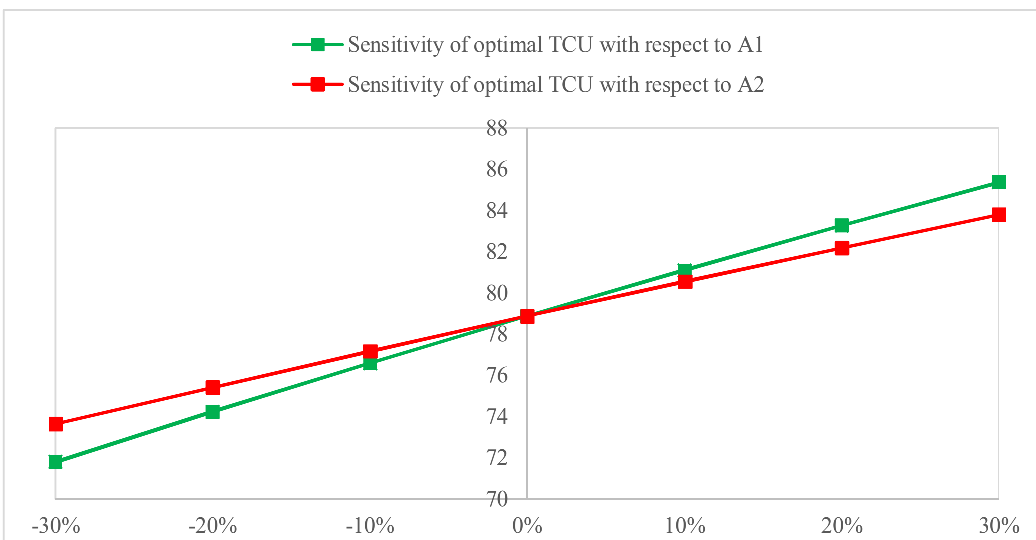

Fig. 7. The impact of A_1 and A_2 on TCU^*

Finally, the inventory holding cost h_1 and h_2 is addressed in the sensitivity analysis. Similar to the setup cost, increasing the holding cost of product 1 imposes more costs on the inventory system. Interestingly, the holding cost of product 2 has no impact on TCU^*. Table 6 And Figure 8 illustrate these results better.

Tab. 6. The sensitivity of TCU^* due to holding cost

%Change in h_1	Q_1^*	Q_2^*	TCU^*	%Change in h_2	Q_1^*	Q_2^*	TCU^*
−30	400.00	0.00	66.111	−30	354.96	0.00	78.881
−20	396.86	0.00	70.553	−20	354.96	0.00	78.881
−10	374.17	0.00	74.833	−10	354.96	0.00	78.881
0	355.00	0.00	78.881	0	354.96	0.00	78.881
+10	338.45	0.00	82.731	+10	354.96	0.00	78.881
+20	324.04	0.00	86.410	+20	354.96	0.00	78.881
+30	311.32	0.00	89.938	+30	354.96	0.00	78.881

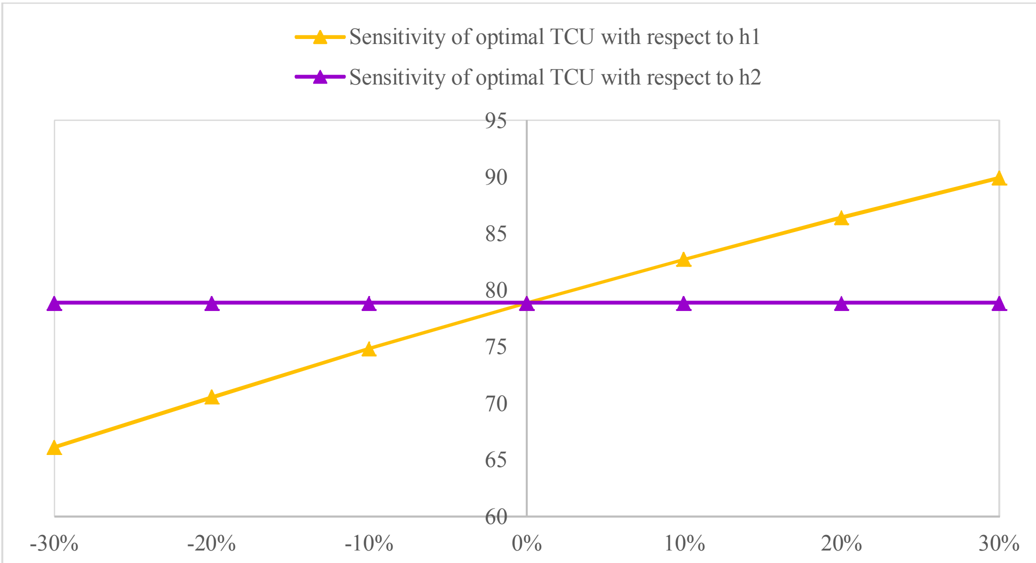

Fig. 8. The impact of h_1 and h_2 on TCU^*

5.3. Managerial insights

Although several extensions of the traditional EPQ model are discussed in the literature, the current work is a new research line for production-inventory researches. In many multi-product manufacturing systems, a joint production policy is established for different types of items. However, this approach may not be cost-effective for substitutable products. The currently developed model helps the managers to create flexibility for the inventory cycle of two-substitutable items and enhance the classic inventory models. The utilized Scenario analysis approach can investigate the possible outcomes and provide a framework for optimal decision making. The comparison of the new inventory model with traditional EPQ shows significant cost improvements in two numerical examples. Based on the sensitivity analysis, the optimal total cost of the proposed inventory system is highly sensitive to the demand rate of products, especially the second product. Hence, it is highly recommended to predict and monitor the parameter efficiently. Moreover, the impact of product 1 holding cost is more than the setup cost of both products.

6. Conclusions

This paper proposed a production-inventory system where a manufacturing plant with two products is working under EPQ setting with possible substitution. To satisfy the external demand for products, the manufacturer produces the products via finite production rates. The shortage is not allowed, and there is a finite amount of resources that products can use. The production setup process incurs a fixed cost, and the produced inventory can be stored with a holding cost per unit time. The aim is to find the economic production quantities to minimize the total cost of the inventory system involving setup and holding costs. To analyze the problem, the Scenario analysis as a process of analyzing possible outcomes was proposed. To this end, four special Scenarios were derived, and then a solution procedure was suggested based on the Scenarios developed. Two numerical examples were presented and solved via analyzing developed Scenarios. The results were compared with the results obtained by basic EPQ (without substitution). To this end, the Lagrangian relaxation was employed to handle the resource constraint. The comparisons show that the production model under substitution can significantly save costs as opposed to the traditional model. The sensitivity analysis is also performed for some parameters, and the results revealed that the optimal solution is highly sensitive to the demand of products. Developing the current model in a multi-echelon supply chain framework is an interesting suggestion for the feature researches. In addition, a sustainable extension of the model can be provided by considering some carbon emission regulations such as carbon tax and cap-and-trade.

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