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An Additive Weighted Fuzzy Programming for Supplier Selection Problem in a Supply Chain

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ABSTRACT

Supplier selection is one of the most important activities of purchasing departments. This importance is increased even more by new strategies in a supply chain, because of the key role suppliers perform in terms of quality, costs and services which affect the outcome in the buyer's company. Supplier selection is a multiple criteria decision making problem in which the objectives are not equally important. In practice, vagueness and imprecision of the goals, constraints and parameters in this problem make the decision making complicated. Simultaneously, in this model, vagueness of input data and varying importance of criteria are considered. In real cases, where Decision- Makers (DMs) face up to uncertain data and situations, the proposed model can help DMs to find out the appropriate ordering from each supplier, and allows purchasing manager(s) to manage supply chain performance on cost, quality, on time delivery, etc. An additive weighted model is presented for fuzzy multi objective supplier selection problem with fuzzy weights. The model is explained by an illustrative example.

1. Introduction

In most industries the cost of raw materials and component parts constitutes the main cost of a product, such that in some cases it can account for up to 70% [9]. Thus the purchasing department can play a key role in an organization's efficiency and effectiveness because of the key role of a supplier's performance on cost, quality, delivery and service in achieving the objectives of a supply chain. Supplier selection is a multiple criteria decision-making (MCDM) problem which is affected by several conflicting factors. Consequently a purchasing manager must analyze the trade off among the several criteria. Multiple criteria decision-making techniques support the decision makers in evaluating a set of alternatives. Depending upon the purchasing situations, criteria have varying importance and there is a need to weight criteria [7].

In a real situation for a supplier selection problem, many input information are not known precisely. At the time of making decisions, the value of many criteria and constraints are expressed in vague terms such as "very high in quality" or "low in price". Deterministic models cannot easily take this vagueness into account. In these cases the theory of fuzzy sets is one of the best tools for handling uncertainty.

Fuzzy set theories are employed due to the presence of vagueness and imprecision of information in the supplier selection problem. In this paper, for the first time, a fuzzy multi objective model has been developed for the supplier selection problem, in which different fuzzy weights can be considered for various objectives.

2. Literature Review

The literature in this area discusses either the criteria or the methods of supplier selection.

Dickson [6] firstly identified and analyzed the importance of 23 criteria for supplier selection based on a survey of purchasing managers. He showed that quality is the most important criterion followed by delivery and performance history.

Weber et al. [27] reviewed 74 articles discussing supplier selection criteria, and showed that net price is

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the most important criterion for supplier selection. They also concluded that supplier selection is a multi criteria problem and the priority of criteria depends on each purchasing situation. Rao and Kiser [21] and Bache et al [1] identified, respectively, 60 and 51 criteria for supplier selection.

Gaballa [8] is the first author who applied mathematical programming to supplier selection in a real case. He used mixed integer programming to minimize the total discounted price of allocated items to the suppliers. Bender et al. [3] formulated a single objective, mixed integer programming to minimize the sum of purchasing, transportation and inventory costs by considering multiple items, multiple time periods, vendors' quality, delivery and capacity. Pan [20] used a single objective linear programming model to choose the best suppliers. This model minimizes total cost, and quality and service are considered as constraints. Sharma et al. [23] proposed a non-linear, mixed integer, goal-programming model for supplier selection. They considered price, quality, delivery and service in their model, in which all criteria are considered as goals. Weber and Current [27] used a multi-objective approach to systematically analyse the trade-offs between conflicting criteria in supplier selection problems. Three objectives are formulated to minimize aggregate purchasing cost, number of late deliveries and rejected items.

Ghodsypour and O'Brien [10] developed a decision support system (DSS) for reducing the number of suppliers according to supply-based optimization strategy. They used an integrated analytical hierarchy process (AHP) with mixed-integer programming and considered suppliers' capacity constraint and the buyers' limitations on budget and quality etc. in their DSS. Ghodsypour and O'Brien [11] developed an integrated AHP and linear programming model to consider both qualitative and quantitative factors in purchasing activity.

Karpak et el. [14] used a goal programming model to minimize costs and maximize delivery reliability and quality in supplier selection when assigning the order quantities to each supplier. Degreave and Roodhooft [5] developed a total cost approach with mathematical programming to treat supplier selection using activity based cost information. Ghodsypour and O'Brien [12] developed a mixed-integer non-linear programming approach to minimize total cost of logistics, including net price, storage, ordering costs and transportation in supplier selection. .

In the literature, little attention has been paid to develop supplier selection models to deal with imprecise information and vagueness of the problem. In order to deal with incomplete and qualitative data, simple linear weighting models have been adapted to handle uncertainty in decision-making related to unstructured purchasing situations [24]. Narasimhan[18] and Nydick and Hill [19] proposed the use of AHP to deal with imprecision in supplier choice using liner weighting models for finding the supplier with the highest overall rating.

Morlacchi[16] developed a model that combines the use of fuzzy set theory (FST) with analytic hierarchy process(AHP) and implements it to evaluate small suppliers in the engineering and machine sectors.

In the literature, relatively scarce attention has been paid to develop effective supplier selection models for the supplier selection problem simultaneously trying to deal with: unstructured relevant information, quail/quantitative and/or absent/imprecise input data and the basic problem of weights (importance of evaluative criteria) assessment. Usually, these aspects have been analyzed one at a time by each model.

No authors discussed the fuzzy multiple criteria decision making methods to the problem of supplier selection with various important criteria, the current study addresses this research gap by providing a quantitative model. This fuzzy model enables the purchasing managers not only to consider the imprecision of information but also take the limitations of buyer and supplier into account to calculate the order quantity assigned to each supplier.

The paper is organized as follows: in section 3 the fuzzy multiobjective model and its crisp formulation for the supplier selection problem is presented in which the objectives are not equally important and have different weights. First, a general linear multi-objective formulation for this problem is considered and then some definitions and appropriate approach for solving this decision making problem are discussed. Section 4 presents the numerical example and explains the results. Finally, the concluding remarks are presented in section 5.

3. The Multi Objective Supplier Selection Model

A general multi objective model for the supplier selection problem can be stated as follows [27,12]:

$$Min\ Z_1, Z_2 \ldots Z_p \quad (1)$$

$$Max\ Zp_{+1}, Z_{k+2} \ldots Z_q \quad (2)$$

Subject to:

$$x \in X_d,\quad X_d = \{x / g(x) \le b_i,\ i = 1,2..m\} \quad (3)$$

in which the Z_1, Z_2, …Z_k are the negative objectives or criteria like cost, late delivery, etc. and Z_{k+1}, Z_{k+2},…Z_p are the positive objectives or criteria such as quality, on time delivery, after sale service and so on. X_d is the set of feasible solutions which satisfy the constraint such as buyer demand, supplier capacity, etc.

A typical linear model for supplier selection problems is as follows[20,27]:

$$\text{Min } Z_1 = \sum_{i=1}^{n} P_i X_i \quad (4)$$

$$\text{Max } Z_2 = \sum_{i=1}^{n} F_i X_i \quad (5)$$

$$\text{Max } Z_3 = \sum_{i=1}^{n} S_i X_i \quad (6)$$

Subject to:

$$\sum_{i=1}^{n} x_i \geq D \quad (7)$$

$$x_i \leq C_i \quad ,i=1, 2 \ldots n \quad (8)$$

$$x_i \geq 0 \quad ,i=1, 2 \ldots n \quad (9)$$

in which:
D = demand over period
x_i = the number of units purchased from the *i*th-supplier
P_i = per unit net purchase cost from supplier i
C_i = capacity of *i*th supplier
F_i =percentage of quality level of *i*th supplier
S_i = percentage of service level of *i*th supplier
n = number of suppliers

Three objective functions - net price (4), quality (5) and service (6) - are formulated to minimize total monetary cost, maximize total quality and service level of purchased items. Constraint (7) ensures that demand is satisfied. Constraint set (8) means that order quantity of each supplier should be equal or less than its capacity and constraint set (9) prohibits negative orders.
In a real case, decision makers do not have exact and complete information related to decision criteria and constraints. For supplier selection problems the collected data does not behave crisply and they are typically fuzzy in nature. A fuzzy multi objective model is developed to deal with the problem. Before presenting the fuzzy model, some definitions and notation should be discussed.

3.1. Definitions

Fuzzy set theory uses linguistic variables rather than quantitative variables to represent imprecise concepts. Linguistic variables analyse the vagueness of human language.

Fuzzy set: Let X be a universe of discourse, A is a fuzzy subset of X if for all $x \in X$, there is a number $\mu_A(x) \in [0,1]$ assigned to represent the membership of x to A, and $\mu_A(x)$ is called the membership function of A.

Fuzzy number: A fuzzy number A is a normal and convex subset of X. Normality implies

$$\exists x \in R \vee m_A(x) = 1.$$

Convexity implies

$$\forall x_1 \in X, x_2 \in X, \forall a \in [0,1]$$

$$\mu_A(\alpha x_1 + (1-\alpha) x_2) \geq \min \mu_A(x_1), \min \mu_A(x_2).$$

Fuzzy decision: A fuzzy decision is defined in an analogy to non-fuzzy environments "as the selection of activities which simultaneously satisfy objective functions and constraints" In fuzzy set theory the intersection of sets normally corresponds to the logical "and". The "decision" in a fuzzy environment can therefore be viewed as the intersection of fuzzy constraints and fuzzy objective functions [29].
Constructing fuzzy decision model depends upon the selection of operators. For fuzzy decision making, the selection of appropriate operators is very important. Zimmermann [30] classified eight important criteria that may be helpful for selecting the appropriate operators in fuzzy decisions. In the next section, the appropriate operator related to the fuzzy supplier selection problem is discussed.

3.2. The Fuzzy Supplier Selection Model

In this section, first the general multi objective model for supplier selection is presented and then appropriate operators for this decision making problem are discussed.
A general linear multi objective model can be presented as:
Find a vector x written in the transformed form $x^T = [x_1, x_2 \ldots x_n]$ which minimizes objective functions Z_k and maximizes objective function Z_l:

$$Min\ Z_k\ (c_k, x) = \sum_{j=1}^{n} c_{kj} x_{j,}\ ,\ k=1, 2 \ldots p \quad (10)$$

$$Max\ Z_l\ (c_l, x) = \sum_{j=1}^{n} c_{lj} x_{j,}\ ,\ l=p+1, p+2 \ldots q \quad (11)$$

with constraints:

$$x \in X_d,\ X_d = \{x / g(x) = \sum_{j=1}^{n} a_{ij} x_j \leq b_i,\ \ i=1,2,\ldots m,\ \ x \geq 0\} \quad (12)$$

where c_{kj}, c_{lj}, a_{ij} and b_i are crisp or fuzzy values.
Zimmermann [29] has solved the problem (10), (11) and (12) by using fuzzy linear programming. He formulated the fuzzy linear program by separating every objective function Z_j into its maximum Z_j^+ and minimum Z_i^- value by solving:

$$Z_k^+ = Max\ Z_k(x),\ x \in X_a,\ Z_k^- = Min\ Z_k(x),\ x \in X_d \quad (13)$$

$$Z_l^+ = Max\ Z_l(x),\ x \in X_d,\ Z_l^- = Min\ Z_l(x),\ x \in X_a \quad (14)$$

Z_k^-, Z_l^+ are obtained through solving the multi-objective problem as a single objective using, each time, only one objective and $x \in X_d$ means that solutions must satisfy constraints while X_a is the set of all optimal solutions through solving as single objective.

Since for every objective function Z_j, its value changes linearly from Z_j^- to Z_j^+, it may be considered as a fuzzy number with the linear membership function $\mu_j(z_j)$ as shown in Figure 1. It was shown that a linear programming problem (10), (11) and (12) with fuzzy goal and fuzzy constraints may be presented as follows:

Find a vector x to satisfy:

$$\tilde{Z}_k(x) = \sum_{j=1}^{n} c_{kj} x_j \lesssim Z_k^o, \quad k=1, 2... p \tag{15}$$

$$\tilde{Z}_l(x) = \sum_{j=1}^{n} c_{l_j} x_j \gtrsim Z_l^o, \quad l=p+1, p+2... q \tag{16}$$

subject to:

$$\tilde{g}_i(x) = \sum_{j=1}^{n} a_{ij} x_j \lesssim b_i, \quad i = 1,2...h \tag{17}$$

$$g_p(x) = \sum_{j=1}^{n} a_{pj} x_j \le b_p, \quad p = h+1,...m \tag{18}$$

$$x \ge 0 \tag{19}$$

In this model, the sing ~ indicates the fuzzy environment. The symbol $\lesssim$ in the constraints set denotes the fuzzified version of $\le$ and has linguistic interpretation "essentially smaller than or equal to" and Z_k^o is the upper bound of minimizing goal Z_k and Z_l^o is the lower bound of maximizing goal l (aspiration levels that the decision maker wants to reach). Assuming that membership functions, based on preference or satisfaction are linear, the linear membership for minimization goals (Z_k) and maximization goals (Z_l) are given as follows:

$$\mu_{zk}(x) = \begin{cases} 1 & \text{for } Z_k \le Z_k^- \quad (k=1, 2 .. p) \\ (Z_k^+ - Z_k(x))/(Z_k^+ - Z_k^-) & \text{for } Z_k^- \le Z_k(x) \le Z_k^+ \\ 0 & \text{for } Z_k \ge Z_k^+ \end{cases} \tag{20}$$

$$\mu_{zl}(x) = \begin{cases} 1 & \text{for } Z_l \ge Z_l^+ \quad (l=p+1,p+2...q) \\ (Z_l(x) - Z_l^-)/(Z_l^+ - Z_l^-) & \text{for } Z_l^- \le Z_l(x) \le Z_l^+ \\ 0 & \text{for } Z_l \le Z_l^- \end{cases} \tag{21}$$

The linear membership function for the fuzzy constraints is given as:

$$\mu_{gi}(x) = \begin{cases} 1 & \text{for } g_i(x) \le b_i, (i=1, 2..., h \\ 1 - (g_i(x) - b_i)/d_i & \text{for } b_i \le gi(x) \le b_i + d_i \\ 0 & \text{for } g_i(x) \ge b_i + d_i \end{cases} \tag{22}$$

The d_i is subjectively chosen constants expressing the limit of the admissible violation of the *i*th inequalities constraints (tolerance interval). In the next section some important fuzzy decision making operators will be presented.

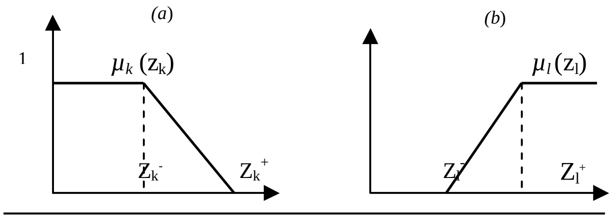

Fig. 1.Objective function as fuzzy number:a) min Z_k and b) max Z_l

3.3. Decision Making Operators

First, the max-min operator is discussed, which was used by Zimmermann [29,30] and for fuzzy multi objective problems. Then, the convex (weighted additive) operator is stated that enables the decision makers (DMs) to assign different weights to various criteria.

In fuzzy programming modeling, using Zimmermann's approach, a fuzzy solution is given by the intersection of all the fuzzy sets representing either fuzzy objective or fuzzy constraints. The fuzzy solution for all fuzzy objectives and h fuzzy constraints may be given as:

$$\mu_D(x) = \left\{ \{ \bigcap_{j=1}^{q} \mu_{z_j}(x) \} \cap \{ \bigcap_{i=1}^{h} \mu_{g_i}(x) \} \right\} \tag{23}$$

The optimal solution(x^*) is given by:

$$\mu_D(x^*) = \max_{x \in X_d} \mu_D(x) = \max_{x \in X_d} \min \left[\min_{j=1,..,q} \mu_j(x), \min_{i=1,..h} \mu_i(x) \right] \tag{24}$$

In this solution the relationship between constraints and objective functions in a fuzzy environment is fully symmetric [29]. In other words, in this definition of the fuzzy decision, there is no difference between the fuzzy goals and fuzzy constraints. Therefore, depending on the supplier selection problem, situations in which fuzzy goals and fuzzy constraints have unequal importance to DMs and other patterns, as the confluence of objectives and constraints, should be considered.

The weighted additive model can handle this problem which is described as follows. The weighted additive model is widely used in vector objective optimization problems; the basic concept is to use a single utility function to express the overall preference of DMs to draw out the relative importance of criteria [15].

In this case, a linear weighted utility function is obtained by multiplying each membership function of fuzzy goals by their corresponding weights and then adding the results together.

The convex fuzzy model proposed by Bellman and Zadeh [2], Sakawa [22] and the weighted additive model, Tiwari et al. [25] is:

$$m_D(\mathrm{X}) = \sum_{j=1}^{q} a_j m_{z_j}(\mathrm{X}) + \sum_{i=1}^{h} b_i m_{g_i}(\mathrm{X}) \quad (25)$$

$$\sum_{j=1}^{q} a_j + \sum_{i=1}^{h} b_i = 1, \quad a_j, b_i \geq 0 \quad (26)$$

where α_j and β_i are the weighting coefficients that present the relative importance among the fuzzy goals and fuzzy constraints. The following crisp single objective programming is equivalent to the above fuzzy model:

$$\text{Max} \sum_{j=1}^{q} a_j l_j + \sum_{i=1}^{h} b_i g_i \quad (27)$$

Subject to:

$$l_j \pounds m_{z_j} \quad , J = 1, \& q \quad (28)$$

$$g_i \leq m_{g_i}(x), \quad i = 1,2, \dots \mathrm{h} \quad (29)$$

$$g_p(x) \leq b_p, \quad p = h+1, \dots m \quad (30)$$

$$l_j, g_i \in [0,1] \quad (31)$$

$$\sum_{j=1}^{q} a_j + \sum_{i=1}^{h} b_i = 1, \quad a_j, b_i \geq 0 \quad (32)$$

$$x \geq 0 \quad (33)$$

To elicit the relative importance, weight or priority among goals/objectives from DMs is a very important initial process to solve this model. However, the DMs may provide either crisp weights or (vaguely) linguistic of weights. In the next part, vague weights is discussed.

3.3.1. Fuzzy Weights

To specify weights, there are some good approaches in the literature [13,17,28]. Here, Zenely's maximin approach is discussed. Suppose that weights are described by linguistic variables which are given to describe membership grades for a set of all possible weight values. Then, the set of weights always adding up to one is obtained such that the minimal membership value is maximized.

$$\text{Max } Min_k \mu_k(\alpha_k) \quad \text{Where } \sum_k \alpha_k = 1 \quad (34)$$

For example, if there are three goals with associated weights α_1, α_2 and α_3 the optimal weight is obtained by solving the following problem:

$$\text{Max } [Min[\mu_1(\alpha_1), \mu_2(\alpha_2), \mu_3(\alpha_3)] \quad (35)$$

In the next part, we are going to present the proposed fuzzy multi objective supplier selection problem by numerical example.

4. Numerical Example

For supplying a new product to a market, assume that three suppliers should be managed. The purchasing criteria are net price, quality and service. The capacity constraints of suppliers are also considered.

It is assumed that the input data from suppliers' performance on these criteria are not known precisely. The de-fuzzified values of their cost, quality and service level and constraints of suppliers are presented in Table 1. The demand is a fuzzy number and is predicted to be about 1,000, as shown in Table 2.

Tab. 1. Suppliers quantitative information

	Cost	Quality	Service	Capacity
Supplier 1	3	%85	%75	500
Supplier 2	2	%80	%90	600
Supplier 3	5	%95	%85	550

The fuzzified formulation of the numerical example is presented as:

$$\tilde{Z}_1 = 3x_1 + 2x_2 + 5x_3 \lesssim Z_1^0$$
$$\tilde{Z}_2 = 0.85x_1 + 0.8x_2 + 0.95x_3 \gtrsim Z_2^0$$
$$\tilde{Z}_3 = 0.75x_1 + 0.9x_2 + 0.85x_3 \gtrsim Z_3^0$$

Subject to:

$$x_1 + x_2 + x_3 \cong 1000$$
$$x_1 \leq 500$$
$$x_2 \leq 600$$
$$x_3 \leq 550$$
$$x_i > 0, \quad i = 1,..,3$$

The linear membership function is used for fuzzifying the objective functions and demand constraint for the above problem according to (20), (21) and (22). The data set for the values of the lower bounds and upper bounds of the objective functions and a fuzzy number for the demand are given in Table 2.

Tab. 2. The data set for membership functions

	$m = 0$	$m = 1$	$m = 0$
Z_1(net cost)	-	2400	4100
Z_2(quality level)	820	905	-
Z_3(service level)	805	880	-
Demand	950	1000	1100

In Appendix A, the membership functions for three objectives and the demand constraint are provided by which to minimize the total monetary cost and maximize the total quality and service level of the purchased items. The fuzzy multi objective formulation for the example problem is as:

Find X to satisfy:

$$\tilde{Z}_1 = 3x_1 + 2x_2 + 5x_3 \tilde{\leq} 2400$$
$$\tilde{Z}_2 = 0.85x_1 + 0.8x_2 + 0.95x_3 \tilde{\geq} 905$$
$$\tilde{Z}_3 = 0.75x_1 + 0.9x_2 + 0.85x_3 \tilde{\geq} 880$$

subject to:

$x_1 + x_2 + x_3 \cong 1000$
$x_1 \leq 500$
$x_2 \leq 600$
$x_3 \leq 550$
$x_i > 0,\ i=1,..,3$

a_j (j=1, 2, 3) and b_1 are the weights associated with the jth objective and demand constraint. In this example the DMs relative importance or weights of the fuzzy goals are given vaguely as:
important for cost objective, rather important for quality and service, and rather unimportant for demand constraint.
The following information is also provided by DMs:

Tab. 3. The value of weight membership functions

α , β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mu(\alpha_1)$	0	0	0.1	0.5	0.8	0.9	1	1	1	1
$\mu(\alpha_2)$	0.1	0.7	0.9	0.5	0.2	0	0	0	0	0
$\mu(\alpha_3)$	0.1	0.7	0.9	0.5	0.2	0	0	0	0	0
$\mu(\beta_1)$	0.9	0.8	0.5	0.3	0	0	0	0	0	0

Then, we can use Equation (37) to get optimal weights. The result is $a_1 = 0.5,\ a_2 = 0.2,\ a_3 = 0.2$, and the weight of the fuzzy constraint is $b_1 = 0.1$ with the membership grade= 0.7= min (0.8, 0.7, 0.7,0.9)
Based on the convex fuzzy decision making (35)-(41) and the weights which are given by DMs, the crisp single objective formulation for the numerical example is as follows:

Max $0.5l_1 + 0.2l_2 + 0.2l_3 + 0.1g_1$

subject to:

$$l_1 \leq \frac{4100 - (3x_1 + 2x_2 + 5x_3)}{1700}$$
$$l_2 \leq \frac{(0.85x_1 + 0.8x_2 + 0.95x_3) - 820}{85}$$
$$l_3 \leq \frac{(0.75x_1 + 0.9x_2 + 0.85x_3) - 805}{75}$$
$$g_1 \leq \frac{1100 - (x_1 + x_2 + x_3)}{100}$$
$$g_1 \leq \frac{(x_1 + x_2 + x_3) - 950}{50}$$

$X_1 \leq 500$
$X_2 \leq 600$
$X_3 \leq 550$
$X_1, X_2, X_3 \geq 0$

The linear programming software LINDO/LINGO is used to solve this problem. The optimal solution for the above formulation is obtained as follows:
X_1=400, X2=600 and X_3=0

$$m_{z_1}(x) = l_1 = 1,\ m_{z_2}(x) = l_2 = 0.0,$$
$$m_{z_3}(x) = l_3 = 0.466 \text{ and } g_1 = 1$$

Corresponding to DMs preferences (0.2, 0.5, 0.2, 0.1), in this solution, 600(maximum capacity) items is assigned to be purchased from supplier 2, because of the lowest price and the remaining items are ordered to supplier 1.
If DMs relative importance or weights of cost criterion changes to rather important like as service and priority of quality changes to important, Table4 presents this variation of DMs in cost and quality priority. Then, the order quantities vary as follows:
X_1=77, X_2=392 and X_3=550 and the membership function values are obtained as follows:

$$m_{z_1}(x) = l_1 = 0.195,\ m_{z_2}(x) = l_2 = 0.969,$$
$$m_{z_3}(x) = l_3 = 1 \text{ and } g_1 = 0.0$$

Corresponding to DMs preferences (0.2, 0.5, 0.2, 0.1), in this solution, 550(maximum capacity) items is assigned to be purchased from supplier 3, because of the highest quality level of supplier 3 performance on the quality criterion. The remaining items are split between supplier 2 and supplier1. In this case that variation in priority of criteria will cause variation in order quantities.

Tab. 4. The value of weight membership functions according to variation of DMs in cost and quality priority

α , β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mu(\alpha_1)$	0.1	0.7	0.9	0.5	0.2	0	0	0	0	0
$\mu(\alpha_2)$	0	0	0.1	0.5	0.8	0.9	1	1	1	1
$\mu(\alpha_3)$	0.1	0.7	0.9	0.5	0.2	0	0	0	0	0
$\mu(\beta_1)$	0.9	0.8	0.5	0.3	0	0	0	0	0	0

5. Summary and Conclusions

Supplier selection is one of the most important activities of purchasing departments. This importance is increased even more by new strategies in a supply chain, because of the key role suppliers perform in terms of quality, costs and services which affect the outcome in the buyer's company. Supplier selection is a multiple criteria decision making problem in which the objectives are not equally important. In real cases, many input data are not known precisely for decision making. For the first time a fuzzy multi objective model is developed for supplier selection in order to assign different fuzzy weights to various criteria. Simultaneously, in this model, vagueness of input data and varying importance of criteria are considered. In real cases, where DMs face up to uncertain data and situations, the proposed model can help DMs to find out the appropriate ordering from each supplier, and allows purchasing manager(s) to manage supply chain performance on cost, quality, on time delivery, etc. Moreover, the fuzzy multi objective supplier selection problem transforms into a convex (weighted additive) fuzzy programming model and its equivalent crisp single objective LP programming. This transformation reduces the dimension of the system, giving less computational complexity, and makes the application of fuzzy methodology more understandable.

In a real situation, the proposed model can be implemented as a vector optimization problem; the basic concept is to use a single utility function to express the preference of DMs, in which the values of criteria and constraints are expressed in vague terms and are not equally important

References

[1] Bache, J., Carr, R., Parnaby, J., Tobias, A.M., *Supplier Development Systems*. International Journal of Technology Management 2(2), 1987, pp. 219-228.

[2] Bellman, R.G., Zadeh, L.A., *Decision making a fuzzy environment*, Management Sciences 17, 1970, B141-B164.

[3] Bender, P.S., Brown, R.W., Isaac, H., Shapiro, J.F., "*Improving Purchasing Productivity at IBM with a Normative Decision Support System*", Interfaces 15 (3), 1985, pp. 106-115.

[4] Bufa, F.P., Jackson, W.M., *"A Goal Programming Model for Purchase Planning"*, Journal of Purchasing and Materials Management 19 (3), 1983, pp. 27-34.

[5] Degraeve, Z., Roodhooft, F., "*A Mathematical Programming Approach for Procurement Using Activity Based Costing*". Journal of business finance and accounting 27 (1-2), 2000, pp. 69-98.

[6] Dickson, G.W., "*An Analysis of Vendor Selection Systems and Decisions*", Journal of Purchasing 2 (1), 1966, pp. 5-17.

[7] Dulmin, R., Mininno, V., "*Supplier Selection using a multi-criteria decision aid method*", Journal of purchasing & supply management 9, 2003, pp. 177-187

[8] Gaballa, A.A., "*Minimum Cost Allocation of Tenders*", Operational Research Quarterly 25 (3), 1974, pp. 389-398.

[9] Ghobadian, A., Stainer, A., Kiss, T., "*A Computerized Vendor Rating System, Proceedings of the First International Symposium on Logistics*", The University of Nottingham, Nottingham, UK, July 1993, pp. 321-328.

[10] Ghodsypour, S.H., O'Brien, C., *A Decision Support System for Reducing the Number of Suppliers and Managing the Supplier Partnership in a JIT/TQM Environment*, Proceedings of the Third International Symposium on Logistics, University of Padua, Padua, Italy, 1997.

[11] Ghodsypour, S.H., O'Brien, C., *A Decision Support System for Supplier Selection Using an Integrated Analytical Hierarchy Process and Linear Programming*. International Journal of production Economics 56-57, 1998, pp. 199-212.

[12] Ghodsypour, S.H., O'Brien, C., *The Total Cost of Logistic in Supplier Selection, Under Conditions of Multiple Sourcing, Multiple Criteria and Capacity Constraint*, International journal of production economics 73, 2001, pp. 15-27.

[13] Hwang, C.L., Yoon, K., *Multiple Attribute Decision Making:* Methods and Applications (Springer- Verlag, Heidelberg), 1987.

[14] Karpak, B., Kumcu, E., Kasuganti, R., *An Application of Visual Interactive Goal Programming: a Case in Vendor Selection Decisions*. Journal of Multi-Criteria decision analysis 8, 1999, pp. 93-105.

[15] Lai, Y.J., Hwang, C.L., *Fuzzy Multiple Objective Decision Making*, Methods and Applications. Springer-Verlag, Berlin, 1994.

[16] Morlacchi, P., *Small and Medium Enterprises in Supply Chain: a Supplier Evaluation Model and Some Empirical Results*. Proceedings IFPMM Summer School, August, Saltzburg, 1997.

[17] Narasimhan, R., *A Geometric Averaging Procedure for Constructing Super Transitive Approximation of Binary Comparison Matrices*, Fuzzy Sets and Systems, 1982, pp. 53-61.

[18] Narasimhan, R., *An Analytic Approach to Supplier Selection*. Journal of Purchasing and Supply Management 1, 1983, pp. 27-32.

[19] Nydick, R.L., Hill, R.P., *Using the Analytic Hierarchy Process to Structure the Supplier Selection Procedure*. International Journal of Purchasing and Materials Management 28 (2), 1992, pp. 31-36.

[20] Pan, A.C., *Allocation of order quantity among suppliers*, Journal of Purchasing and Materials Management 25 (3), 1989, pp. 36-39.

[21] Roa, C.P., Kiser, G.E., *Educational Buyers' Perceptions Of Vendor Attributes*. Journal of Purchasing and Materials Management. 16, 1980, pp. 25-30.

[22] Sakawa, M., *Fuzzy Sets and Interactive Multi Objective Optimization*. Plenum Press, New York, 1993.

[23] Sharma, D., Benton, W.C., Srivastava, R., *Competitive Strategy and Purchasing Decision*, Proceedings of the 1989 Annual Conference of the Decision Sciences Institute, 1989, pp. 1088-1090.

[24] Soukup, W.R., *Supplier selection strategies*. Journal of Purchasing and Materials Management 23 (3), 1987, pp. 7-12.

[25] Tiwari, R.N., Dharmahr, S., Rao, J.R., *Fuzzy Goal Programming-an Additive Model*. Fuzzy sets and systems 24, 1987, pp. 27-34.

[26] Weber, C.A., Current, J.R., Benton, W.C., *Vendor Selection Criteria and Methods,* European Journal of Operational Research 50, 1991, pp. 2-18.

[27] Weber, C.A., Current, J.R., *A Multi Objective Approach to Vendor Selection,* European Journal of Operational Research 68, 1993, pp. 173-184.

[28] ZenelyM, M., *Compromise Programming*, Multiple Criteria Decision Making (University of South Carolina, Columbia), 1973.

[29] Zimmermann, H.J., *Fuzzy Programming and Linear Programming with Several Objective Functions,* Fuzzy sets and systems 1, 1978, pp. 45-55.

[30] Zimmermann, H.J., *Fuzzy Set Theory and its Applications*. Kluwer Academic Publishers, Boston, 1993.

Appendix A:

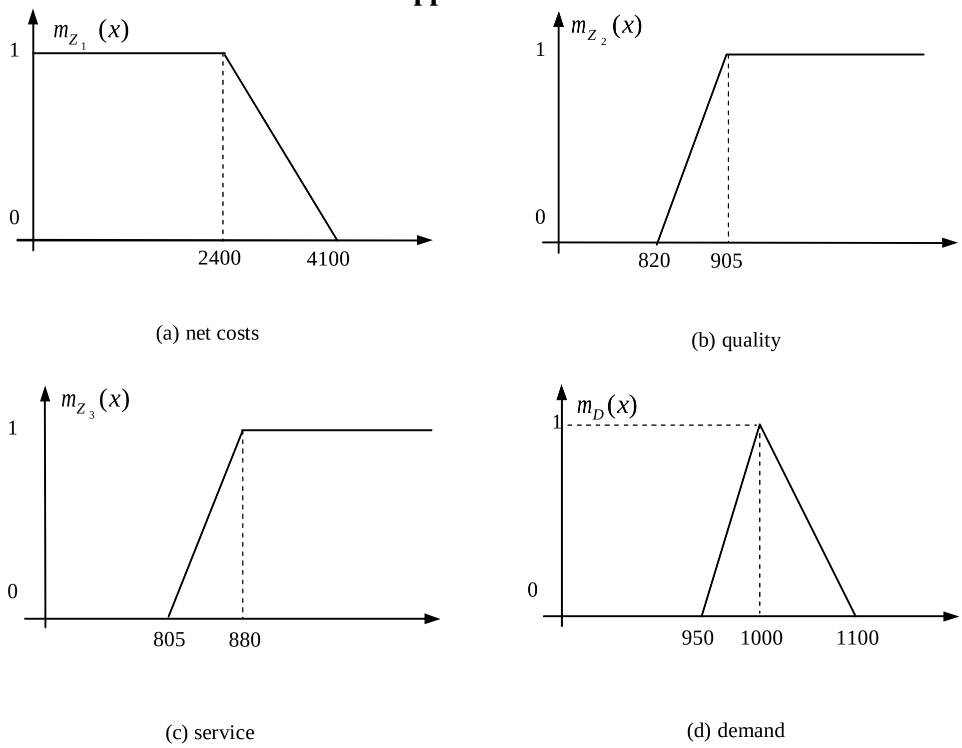

Fig. 1. Membership functions: a) net costs (Z_1) objective function, b) quality (Z_2) objective function, c) service Z_3) objective function, d) demand constraint (Z_D)