AN EPQ MODEL OF EXPONENTIAL DETERIORATION WITH FUZZY DEMAND AND PRODUCTION WITH SHORTAGES

S.Sarkar and T.Chakrabarti

Abstract In the fundamental production inventory model, in order to solve the economic production quantity (EPQ) we always fix both the demand quantity and the production quantity per day. But, in the real situation, both of them probably will have little disturbances every day. Therefore, we should fuzzify both of them to solve the economic production quantity *(q*)* per cycle. Using α-cut for defuzzification the total variable cost per unit time is derived. Therefore the problem is reduced to crisp annual costs. The multi-objective model is solved by Global Criteria Method with the help of GRG (Generalized Reduced Gradient) Technique. In this model shortages are permitted and fully backordered. The purpose of this paper is to investigate a computing schema for the EPQ in the fuzzy sense. We find that, after defuzzification, the total cost in fuzzy model is less than in the crisp model. So it permits better use of the EPQ model in the fuzzy sense arising with little disturbances in the production, and demand.

1. Introduction

For solving the EPQ for each cycle, we always fix both the demand quantity and production quantity per day in the crisp model. But, in the real situation, both of them probably will have some little disturbances per day. In recent years, many researchers have studied inventory models for deteriorating items such as electronic components, food items, drugs and fashion goods. Deterioration is defined as decay, change or spoilage that prevent the items from being used for its original purpose. There are many items in which appreciable deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the model. Therefore, many authors have considered Economic order quantity models for deteriorating items.

Acting as the driving force of the whole inventory system, demand is a key factor that should be taken into consideration in an inventory study. There are mainly two categories demands in the present studies, one is deterministic demand and the other is stochastic demand. Some noteworthy work on deterministic demand are : Chung and Lin [1,2] have considered constant demand, Giri and Chakrabarty[3] and Teng and Chang[4] have considered time-dependent demand where as Giri and Chaudhuri[6], Bhattacharya[7] and Wu.et.al[8] have worked on inventory level-dependent demand and Wee and Law [9] have considered price-dependent demand . Among them, ramp type demand is a special type of time-dependent demand. Hill [10] was the first to introduce the ramp type demand to the inventory study. Then Mandal and Pal [11] introduced the ramp type demand to the inventory study of the deteriorating items. Deng.et.al [12] and Shah and Jaiswal[13] have extensively studied this type of demand. Stochastic demand includes two types of demands: the first type characterized by a known demand distribution and the second type characterized by arbitrary demand distribution i.e demand is fuzzy in nature.

In the classical inventory model depletion of inventory is caused by a constant demand rate alone. But subsequently, it was noticed that depletion of inventory may take place due to deterioration also. In the early stage of the study, most of the deteriorating rates in the models are constant, Padmanabhana and Vratb [14], and Bhunia and Maiti [15] worked on constant deterioration rate. In recent research, more and more studies have begun to consider the relationship between time and deteriorating rate.Wee[18], and Mahapatra[19], considered deterioration rate as linear increasing function of time. Chakrabarty.et.al[20] have considered three-parameter Weibull distribution. In this connection, studies of many researchers like Ghare and Schrader [21], Goyel et al. [22] are very important. Misra [23] developed a two parameter Weibull distribution deterioration for an inventory model. This investigation was followed by Shah and Jaiswal [24], Aggarwal [25], Dave and Patel [26], Datta and Pal [27], Jalan, Giri and Chaudhuri[28], Dixit and Shah [29], Giri and Goyel [30], Shah and Shah[31] etc.

The assumption of constant demand rate is not always appropriate for many inventory items. The works done by Donaldson [32], Silver [33], Ritchie [34], Pal and Mandal [35] are to be mentioned regarding time dependent demand rates.

In the present paper, efforts have been made to analyze an EPQ model that deteriorates exponentially assuming demand rate to be exponential. Here production is demand dependent. To make the model more realistic demand has been fuzzified. This paper investigates a computing schema for the economic production quantities in fuzzy sense. This fuzzy parameters are then represented in terms of interval numbers using triangular fuzzy number.

2. Assumptions and Notations.

2.1 Notations:
a) Replenishment rate is finite and it is demand dependent.
b) Lead time is zero.
c) T the cycle time.
d) I(t) inventory level at time t.
e) C_1 is the holding cost per unit time.
f) C_2 is the shortage cost per unit time.
g) C_3 is the unit purchase cost.
h) C_4 is the fixed ordering cost of inventory.
i) θ deterioration rate of finished items.

2.2 Assumptions:
a) The demand is taken as exponential ,R (t)= ae^{bt}.
b) Rate of production varies with demand i.e. $K = \beta R(t)$ where β is constant.
c) Replenishment is instantaneous.
d) Lead-time (i.e. the length between making of a decision to replenish an item and its actual addition to stock) is assumed to be zero. The assumption is made so that the period of shortage is not affected.
e) The rate of deterioration at any time t>0 is dependent on time.
f) Shortages are allowed and are fully backlogged.

3. Mathematical Modeling and Analysis.

Here we assume production starts at t=0 at the rate K and the stock attains a level Q at t= t_1. The production stops at t= t_1 and the inventory gradually depletes to zero at t=t_2 mainly to meet the demands and partly for deterioration .Now shortages occur and accumulate to the level S at time t=t_3.The production starts again at a rate K at t=t_3 and the backlog is cleared at time t=T when the stock is again zero. The cycle then repeats itself after time T.

The model is represented by the following diagram:

Space for Figure :1

Let I(t) be the inventory level at any time
t($0\leq t \leq$ T) and demand rate R(t) is assumed to be deterministic and is increasing exponentially with time.
Further let R(t)= $a e^{bt}$, $0\leq b<1$, a>0.

The differential equations describing instantaneous state of I(t) in the interval[0,T] are

$$\frac{dI(t)}{dt} + \theta t I(t) = ae^{bt}(\beta - 1) \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -ae^{bt} \qquad (2)$$

$$\frac{dI(t)}{dt} = -ae^{bt} \qquad (3)$$

$$\frac{dI(t)}{dt} = -(\beta - 1)ae^{bt} \qquad (4)$$

with the initial conditions I(t_0)=0 ,I(t_1)=Q, I(t_2)=0,I(t_3)=S, I(T)=0.

Now solving the above differential equations we get

$$I(t) = a(\beta-1)\{t + \frac{bt^2}{2} + \frac{b^2 t^3}{3} - \frac{\theta t^3}{3} - \frac{\theta t^5 b^2}{6}\}\ 0 \le t \le t_1 (5)$$

$$I(t) = Q + a\{(t_1 - t) + \frac{b(t_1^2 - t^2)}{2} + \frac{b^2(t_1^3 - t^3)}{3}$$
$$+ \frac{b^2\theta(t^5 - t_1^5)}{6} + \frac{\theta(t^3 - t_1^3)}{3}\} \qquad t_1 \le t \le t_2 \qquad (6)$$

$$I(t) = \frac{a(e^{bt_2} - e^{bt})}{b} \qquad t_2 \le t \le t_3 \qquad (7)$$

$$I(t) = \frac{(\beta-1)a(e^{bt_3} - e^{bt})}{b} + S \qquad t_3 \le t \le T \qquad (8)$$

Holding Cost (H.C) over the period [0,T]

$$= C_1 a(\beta - 1)[\frac{t_1^2}{2} + \frac{bt_1^3}{6} + \frac{b^2 t_1^4}{12} - \frac{\theta t_1^4}{12} - \frac{\theta t_1^6 b^2}{36}]$$

$$+ C_1[Q(t_2 - t_1) + a(t_1 t_2 - \frac{t_2^2}{2}) - \frac{at_1^2}{2} + \frac{b}{2}(t_1^2 t_2 - \frac{t_2^3}{3})$$

$$- \frac{bt_1^3}{3} + \frac{b^2}{3}(t_1^3 t_2 - \frac{t_2^4}{4}) - \frac{b^3}{4}t_1^4 + \frac{b^2\theta}{6}(\frac{t_2^6}{6} - t_1^5 t_2) + \frac{5\theta b^2 t_1^6}{36}$$

$$+ \frac{\theta t_2^4}{12} - \frac{\theta t_1^3 t_2}{3} + \frac{\theta}{4}t_1^4] \qquad (10)$$

Holding Cost (H.C) over the period [0,T]

$$= C_1 a(\beta - 1)[\frac{t_1^2}{2} + \frac{bt_1^3}{6} + \frac{b^2 t_1^4}{12} - \frac{\theta t_1^4}{12} - \frac{\theta t_1^6 b^2}{36}]$$

$$+ C_1[Q(t_2 - t_1) + a(t_1 t_2 - \frac{t_2^2}{2}) - \frac{at_1^2}{2} + \frac{b}{2}(t_1^2 t_2 - \frac{t_2^3}{3})$$

$$- \frac{bt_1^3}{3} + \frac{b^2}{3}(t_1^3 t_2 - \frac{t_2^4}{4}) - \frac{b^3}{4}t_1^4 + \frac{b^2\theta}{6}(\frac{t_2^6}{6} - t_1^5 t_2) + \frac{5\theta b^2 t_1^6}{36}$$

$$+ \frac{\theta t_2^4}{12} - \frac{\theta t_1^3 t_2}{3} + \frac{\theta}{4}t_1^4] \qquad (10)$$

Shortage Cost(S.C) over the period [0,T]

$$= C_2 \int_{t_2}^{T} I(t)dt$$

$$= C_2 \int_{t_2}^{t_3} I(t)dt + C_2 \int_{t_3}^{T} I(t)dt$$

$$= C_2 \frac{a}{b^2}(e^{bt_2} - e^{bt_3}) + \frac{C_2 a}{b}e^{bt_2}(t_3 - t_2)$$

$$+ \frac{C_2(\beta-1)ae^{bt_3}(T - t_3)}{b} - \frac{C_2(\beta-1)a(e^{bT} - e^{bt_3})}{b^2}$$

$$+ C_2 S(T - t_3) \qquad (11)$$

$$\text{Deteriorating Cost} = C_3 \int_0^{t_1} (P\text{-}D)dt - C_3 \int_{t_1}^{t_2} D\,dt$$

$$= aC_3 \frac{(\beta-1)(e^{bt_1}-1)}{b} - \frac{aC_3(e^{bt_2}-e^{bt_1})}{b} \quad (12)$$

$$\text{Set up Cost} = C_4 \qquad\qquad (13)$$

Total average Cost (TVC) = (Holding Cost+Deteoration Cost-Shortage Cost+Setup Cost)/T   (14)

$$TVC = C_1 a(\beta-1)[\frac{t_1^2}{2} + \frac{bt_1^3}{6} + \frac{b^2 t_1^4}{12} - \frac{\theta t_1^4}{12} - \frac{\theta t_1^6 b^2}{36}]$$

$$+C_1[Q(t_2-t_1) + a(t_1 t_2 - \frac{t_2^2}{2}) - \frac{at_1^2}{2} + \frac{b}{2}(t_1^2 t_2 - \frac{t_2^3}{3}) - \frac{bt_1^3}{3}$$

$$+\frac{b^2}{3}(t_1^3 t_2 - \frac{t_2^4}{4}) - \frac{b^3}{4}t_1^4 + \frac{b^2\theta}{6}(\frac{t_2^6}{6} - t_1^5 t_2) + \frac{50b^2 t_1^6}{36} + \frac{\theta t_2^4}{12}$$

$$-\frac{\theta t_1^3 t_2}{3} + \frac{\theta}{4}t_1^4] + aC_3 \frac{(\beta-1)(e^{bt_1}-1)}{b} - \frac{aC_3(e^{bt_2}-e^{bt_1})}{b}$$

$$-C_2 \frac{a}{b^2}(e^{bt_2} - e^{bt_3}) - \frac{C_2 a}{b}e^{bt_2}(t_3-t_2) - \frac{C_2(\beta-1)ae^{bt_3}(T-t_3)}{b}$$

$$+\frac{C_2(\beta-1)a(e^{bT}-e^{bt_3})}{b^2} - C_2 S(T-t_3) + aC_3 \frac{(\beta-1)(e^{bt_1}-1)}{b}$$

$$-\frac{aC_3(e^{bt_2}-e^{bt_1})}{b} \qquad\qquad (15)$$

Using the initial conditions the total average cost becomes function of t_1 and T. Hence we find the global optimal solution of total average cost by using LINGO 12.And the minimum cost in the deterministic model is compared with the minimum cost in fuzzy model.

4.Fuzzy Model and solution procedure.

T The instantaneous states of the inventory level I(t) at time t t(0≤t≤T) can be described by the following equations:

$$\frac{d\tilde{I}(t)}{dt} + \theta t\tilde{I}(t) = \tilde{a}e^{bt}(\beta-1) \qquad 0 \le t \le t_1 \qquad (16)$$

$$\frac{d\tilde{I}(t)}{dt} + \theta t\tilde{I}(t) = -\tilde{a}e^{bt} \qquad t_1 \le t \le t_2 \qquad (17)$$

$$\frac{d\tilde{I}(t)}{dt} = -\tilde{a}e^{bt} \qquad t_2 \le t \le t_3 \qquad (18)$$

$$\frac{d\tilde{I}(t)}{dt} = -(\beta-1)\tilde{a}e^{bt} \qquad t_3 \le t \le T \qquad (19)$$

With the initial conditions I(t_0)=0 ,I(t_1)=Q, I(t_2)=0,I(t_3)=S, I(T)=0.

The differential equations (1) to (8) are fuzzy differential equations. To solve this differential equation at first we take the α-cut then the differential equations reduces to

$$\frac{dI_1^+(t)}{dt} + \theta t I_1^+(t) = a^+ e^{bt}(\beta-1) \qquad 0 \le t \le t_1 \quad (20)$$

$$\frac{dI_1^-(t)}{dt} + \theta t I_1^-(t) = a^- e^{bt}(\beta-1) \qquad 0 \le t \le t_1 \quad (21)$$

$$\frac{dI_2^+(t)}{dt} + \theta t I_2^+(t) = -a^- e^{bt} \qquad t_1 \le t \le t_2 \quad (22)$$

$$\frac{dI_2^-(t)}{dt} + \theta t I_2^-(t) = -a^+ e^{bt} \qquad t_1 \le t \le t_2 \quad (23)$$

$$\frac{dI_3^+(t)}{dt} = -a^- e^{bt} \qquad t_2 \le t \le t_3 \qquad (24)$$

$$\frac{dI_3^-(t)}{dt} = -a^+ e^{bt} \qquad t_2 \le t \le t_3 \qquad (25)$$

$$\frac{dI_4^+(t)}{dt} = -(\beta-1)a^- e^{bt} \qquad t_3 \le t \le T \qquad (27)$$

$$\frac{dI_4^-(t)}{dt} = -(\beta-1)a^+ e^{bt} \qquad t_3 \le t \le T \qquad (28)$$

where

$$I_i^+ = \sup\{x \in R : \mu_{\tilde{I_i}}(x) \ge \alpha\}, i = 1,2,3,4,5.$$

$$I_i^- = \inf\{x \in R : \mu_{\tilde{I_i}}(x) \ge \alpha\}, i = 1,2,3,4,5.$$

Similarly $a^+(\alpha), a^-(\alpha)$ have usual meaning

Now solving the above differential equations we get

$$I_1^+(t) = \{a_3 - \alpha(a_3-a_2)\}(\beta-1)\{t + \frac{bt^2}{2} + \frac{b^2 t^3}{3} - \frac{\theta t^3}{3} - \frac{\theta t^5 b^2}{6}\} \quad 0 \le t \le t_1 \quad (29)$$

$$I_1^-(t) = \{a_1 + \alpha(a_2-a_1)\}(\beta-1)\{t + \frac{bt^2}{2} + \frac{b^2 t^3}{3} - \frac{\theta t^3}{3} - \frac{\theta t^5 b^2}{6}\} \quad 0 \le t \le t_1 \quad (30)$$

$$I_2^+(t) = Q + \{a_1 + \alpha(a_2-a_1)\}\{(t_1-t) + \frac{b(t_1^2-t^2)}{2} + \frac{b^2(t_1^3-t^3)}{3} + \frac{b^2\theta(t^5-t_1^5)}{6}$$
$$+ \frac{\theta(t^3-t_1^3)}{3}\} \qquad\qquad t_1 \le t \le t_2 \quad (31)$$

$$I_2^-(t) = Q + \{a_3 - \alpha(a_3-a_2)\}\{(t_1-t) + \frac{b(t_1^2-t^2)}{2} + \frac{b^2(t_1^3-t^3)}{3} + \frac{b^2\theta(t^5-t_1^5)}{6}$$
$$+ \frac{\theta(t^3-t_1^3)}{3}\} \qquad\qquad t_1 \le t \le t_2 \quad (32)$$

$$I_3^+(t) = \frac{\{a_1 + \alpha(a_2 - a_1)\}(e^{bt_2} - e^{bt})}{b}, \qquad t_2 \le t \le t_3 \quad (33)$$

$$I_3^-(t) = \frac{\{a_3 - \alpha(a_3 - a_2)\}(e^{bt_2} - e^{bt})}{b}, \qquad t_2 \le t \le t_3 \quad (34)$$

$$I_4^+(t) = \frac{(\beta-1)\{a_1 + \alpha(a_2 - a_1)\}(e^{bt_3} - e^{bt})}{b} + S \quad t_3 \le t \le T \quad (35)$$

$$I_4^-(t) = \frac{(\beta-1)\{a_3 - \alpha(a_3 - a_2)\}(e^{bt_3} - e^{bt})}{b} + S \quad t_3 \le t \le T \quad (36)$$

Therefore the upper α-cut of fuzzy stockholding cost $(HC^+) = C_1 \int_0^{t_2} \tilde{I}(t)dt$

$$= C_1 \int_0^{t_1} I_1^+(t)dt + C_1 \int_{t_1}^{t_2} I_2^+(t)dt$$

$$= \{a_3 - \alpha(a_3 - a_2)\}C_1(\beta-1)\left[\frac{t_1^2}{2} + \frac{bt_1^3}{6} + \frac{b^2 t_1^4}{12} - \frac{\theta t_1^4}{12} - \frac{\theta t_1^6 b^2}{36}\right]$$

$$+ C_1\left[Q(t_2 - t_1) + a\left(t_1 t_2 - \frac{t_2^2}{2}\right) - \frac{at_1^2}{2} + \frac{b}{2}\left(t_1^2 t_2 - \frac{t_2^3}{3}\right) - \frac{bt_1^3}{3}\right.$$

$$+ \frac{b^2}{3}\left(t_1^3 t_2 - \frac{t_2^4}{4}\right) - \frac{b^3}{4}t_1^4 + \frac{b^2\theta}{6}\left(\frac{t_2^6}{6} - t_1^5 t_2\right) + \frac{5\theta b^2 t_1^6}{36} + \frac{\theta t_2^4}{12}$$

$$\left. - \frac{\theta t_1^3 t_2}{3} + \frac{\theta}{4}t_1^4\right] \quad (37)$$

And the lower α-cut of fuzzy stockholding cost $(HC^-) = C_1 \int_0^{t_2} \tilde{I}(t)dt$

$$= C_1 \int_0^{t_1} I_1^-(t)dt + C_1 \int_{t_1}^{t_2} I_2^-(t)dt$$

$$= \{a_1 + \alpha(a_2 - a_1)\}C_1(\beta-1)\left[\frac{t_1^2}{2} + \frac{bt_1^3}{6} + \frac{b^2 t_1^4}{12} - \frac{\theta t_1^4}{12} - \frac{\theta t_1^6 b^2}{36}\right]$$

$$+ C_1\left[Q(t_2 - t_1) + a\left(t_1 t_2 - \frac{t_2^2}{2}\right) - \frac{at_1^2}{2} + \frac{b}{2}\left(t_1^2 t_2 - \frac{t_2^3}{3}\right) - \frac{bt_1^3}{3}\right.$$

$$+ \frac{b^2}{3}\left(t_1^3 t_2 - \frac{t_2^4}{4}\right) - \frac{b^3}{4}t_1^4 + \frac{b^2\theta}{6}\left(\frac{t_2^6}{6} - t_1^5 t_2\right) + \frac{5\theta b^2 t_1^6}{36}$$

$$\left. + \frac{\theta t_2^4}{12} - \frac{\theta t_1^3 t_2}{3} + \frac{\theta}{4}t_1^4\right] \quad (38)$$

As demand is fuzzy in nature shortage cost is also fuzzy in nature.

Therefore the upper α-cut of shortage cost $(SC^+) = C_2 \int_{t_2}^{T} \tilde{I}(t)dt$

$$= C_2 \int_{t_2}^{t_3} I_3^+(t)dt + C_2 \int_{t_3}^{T} I_4^+(t)dt$$

$$= C_2 \frac{\{a_1 + \alpha(a_2 - a_1)\}}{b^2}(e^{bt_2} - e^{bt_3}) + \frac{C_2\{a_1 + \alpha(a_2 - a_1)\}}{b}e^{bt_2}(t_3 - t_2)$$

$$+ \frac{C_2(\beta-1)\{a_1 + \alpha(a_2 - a_1)\}e^{bt_3}(T - t_3)}{b}$$

$$- \frac{C_2(\beta-1)\{a_1 + \alpha(a_2 - a_1)\}(e^{bT} - e^{bt_3})}{b^2} + C_2 S(T - t_3) \quad (39)$$

Also the lower α-cut of shortage cost $(SC^-) = C_2 \int_{t_2}^{T} \tilde{I}(t)dt$

$$= C_2 \int_{t_2}^{t_3} I_3^-(t)dt + C_2 \int_{t_3}^{T} I_4^-(t)dt$$

$$= C_2 \frac{\{a_3 - \alpha(a_3 - a_2)\}}{b^2}(e^{bt_2} - e^{bt_3}) + \frac{C_2\{a_3 - \alpha(a_3 - a_2)\}}{b}e^{bt_2}(t_3 - t_2)$$

$$+ \frac{C_2(\beta-1)\{a_3 - \alpha(a_3 - a_2)\}e^{bt_3}(T - t_3)}{b} - \frac{C_2(\beta-1)\{a_3 - \alpha(a_3 - a_2)\}(e^{bT} - e^{bt_3})}{b^2}$$

$$+ C_2 S(T - t_3) \quad (40)$$

Since demand is fuzzy in nature and production is dependent in demand so production is also fuzzy. So deterioration cost is also fuzzy in nature.

Deteriorating Cost $= C_3 \int_0^{t_1} (P-D)dt - C_3 \int_{t_1}^{t_2} D dt$

$$= C_3 \int_0^{t_1} \tilde{a}(\beta-1)e^{bt}dt - C_3 \int_{t_1}^{t_2} \tilde{a}e^{bt}dt$$

Upper α-cut of deterioration Cost $(DC^+) = C_3(\beta-1)\left[\frac{a_3}{b}(e^{bt_1} - 1)(1-\alpha) + \frac{\alpha a_2(e^{bt_1} - 1)}{b}\right]$

$$- C_3\left[\frac{a_1(e^{bt_2} - e^{bt_1})(1-\alpha)}{b} + \frac{\alpha a_2(e^{bt_2} - e^{bt_1})}{b}\right] \quad (41)$$

Lower α-cut of deterioration Cost $(DC^-) = C_3(\beta-1)\left[\frac{a_1}{b}(e^{bt_1} - 1)(1-\alpha) + \frac{\alpha a_2(e^{bt_1} - 1)}{b}\right]$

$$- C_3\left[\frac{a_3(e^{bt_2} - e^{bt_1})(1-\alpha)}{b} + \frac{\alpha a_2(e^{bt_2} - e^{bt_1})}{b}\right] \quad (42)$$

Annual ordering Cost $= C_4 \quad (43)$

Therefore total variable cost per unit time is a fuzzy quantity and is defined by

$$\widetilde{TVC} = \begin{cases} TVC^+ \\ TVC^- \end{cases} \qquad (44)$$

where

$$TVC^+ = \sup\{x \in R : \mu_{T\widetilde{VC}}(x) \ge \alpha\},$$

$$TVC^- = \inf\{x \in R : \mu_{T\widetilde{VC}}(x) \ge \alpha\},$$

The upper α-cut of total variable cost per unit time is

$$TVC^+ = \{a_3 - \alpha(a_3 - a_2)\}C_1(\beta-1)\{[\frac{t_1^2}{2} + \frac{bt_1^3}{6} + \frac{b^2t_1^4}{12} - \frac{\theta t_1^4}{12} - \frac{\theta t_1^6 b^2}{36}]$$

$$+ C_1[Q(t_2-t_1) + a(t_1t_2 - \frac{t_2^2}{2}) - \frac{at_1^2}{2} + \frac{b}{2}(t_1^2 t_2 - \frac{t_2^3}{3}) - \frac{bt_1^3}{3}$$

$$+ \frac{b^2}{3}(t_1^3 t_2 - \frac{t_2^4}{4}) - \frac{b^3}{4}t_1^4 + \frac{b^2\theta}{6}(\frac{t_2^6}{6} - t_1^5 t_2) + \frac{50b^2 t_1^6}{36} + \frac{\theta t_2^4}{12}$$

$$- \frac{\theta t_1^3 t_2}{3} + \frac{\theta}{4}t_1^4] + C_3(\beta-1)[\frac{a_3}{b}(e^{bt_1}-1)(1-\alpha) + \frac{\alpha a_2(e^{bt_1}-1)}{b}]$$

$$- C_3[\frac{a_1(e^{bt_2}-e^{bt_1})(1-\alpha)}{b} + \frac{\alpha a_2(e^{bt_2}-e^{bt_1})}{b}]$$

$$- C_2\frac{\{a_1+\alpha(a_2-a_1)\}}{b^2}(e^{bt_2}-e^{bt_3}) - \frac{C_2\{a_1+\alpha(a_2-a_1)\}}{b}e^{bt_2}(t_3-t_2)$$

$$- \frac{C_2(\beta-1)\{a_1+\alpha(a_2-a_1)\}e^{bt_3}(T-t_3)}{b}$$

$$+ \frac{C_2(\beta-1)\{a_1+\alpha(a_2-a_1)\}(e^{bT}-e^{bt_3})}{b^2} - C_2S(T-t_3) + C_4\}/T \qquad (45)$$

The lower α-cut of total variable cost per unit time is

$$TVC^- = \{a_1 + \alpha(a_2-a_1)\}\{C_1(\beta-1)[\frac{t_1^2}{2} + \frac{bt_1^3}{6} + \frac{b^2t_1^4}{12} - \frac{\theta t_1^4}{12} - \frac{\theta t_1^6 b^2}{36}]$$

$$+ C_1[Q(t_2-t_1) + a(t_1t_2 - \frac{t_2^2}{2}) - \frac{at_1^2}{2} + \frac{b}{2}(t_1^2 t_2 - \frac{t_2^3}{3}) - \frac{bt_1^3}{3} + \frac{b^2}{3}(t_1^3 t_2 - \frac{t_2^4}{4})$$

$$- \frac{b^3}{4}t_1^4 + \frac{b^2\theta}{6}(\frac{t_2^6}{6} - t_1^5 t_2) + \frac{50b^2 t_1^6}{36} + \frac{\theta t_2^4}{12} - \frac{\theta t_1^3 t_2}{3} + \frac{\theta}{4}t_1^4]$$

$$+ C_3(\beta-1)[\frac{a_3}{b}(e^{bt_1}-1)(1-\alpha) + \frac{\alpha a_2(e^{bt_1}-1)}{b}] - C_3[\frac{a_1(e^{bt_2}-e^{bt_1})(1-\alpha)}{b}$$

$$+ \frac{\alpha a_2(e^{bt_2}-e^{bt_1})}{b}] - C_2\frac{\{a_3-\alpha(a_3-a_2)\}}{b^2}(e^{bt_2}-e^{bt_3})$$

$$- \frac{C_2\{a_3-\alpha(a_3-a_2)\}}{b}e^{bt_2}(t_3-t_2)$$

$$- \frac{C_2(\beta-1)\{a_3-\alpha(a_3-a_2)\}e^{bt_3}(T-t_3)}{b}$$

$$+ \frac{C_2(\beta-1)\{a_3-\alpha(a_3-a_2)\}(e^{bT}-e^{bt_3})}{b^2} - C_2S(T-t_3) + C_4\}/T \qquad (46)$$

The objective in this paper is to find an optimal cycle time to minimize the total variable cost per unit time.
Therefore this model mathematically can be written as
Minimize {TVC$^+$,TVC$^-$}   (47)
Subject to $0 \le \alpha \le 1$
Therefore the problem is a multiobjective optimization problem.To convert it as a single objective optimization problem we use global criteria(GC) method.
Then the above problem reduces to

Minimize   GC   (48)

Subject to $0 \le \alpha \le 1$

5. Global criteria method

The model presented by (34) is a multi-objective model which is solved by Global Criteria (GC) Method with the help of Generalized Reduced Gradient Technique.
The Multi-Objective Non –linear Integer Programming(MONLIP) problems are solved by Global Criteria Method converting it to a single objective optimization problem .The solution procedure is as follows:
Step-1: Solve the multi-objective programming problem (34) as a single objective problem using only one objective at a time ignoring other.
Step-2: From the results of Step-1, determine the ideal objective vector, say (TVC^{+min},TVC^{-min}) and the corresponding values of (TVC^{+max},TVC^{-max}).Here, the ideal objective vector is used as a reference point.The problem is then to solve the following auxiliary problem:

$$Min(GC) = Minimize\{(\frac{TVC^+ - TVC^{+min}}{TVC^{+max} - TVC^{+min}})Q + (\frac{TVC^- - TVC^{-min}}{TVC^{-max} - TVC^{-min}})Q\}^{\frac{1}{Q}}$$

where $1 \le Q < \infty$.This method is also sometimes called Compromise Programming.

6.Numerical Example.

We now consider a numerical example showing the utility of the model from practical point of view. According to the developed solution procedure of the proposed inventory system , the optimal solution has been obtained with the help of well known generalized reduced gradient method(GRG).To illustrate the developed model, an example with the following data has been considered:
Deterministic Model:
C_1=\$8per unit, C_2=\$.9 per unit, C_3=\$.5 per unit, C_4=\$100, a=1.9, b=0.5, q=50, β=1.0, θ=.001, S=10.
Hence the optimal Solution is \$155.629.
Fuzzy Model:
Let a_1= 0.9unit/month ,a_2= 1.6unit/month, a_3=0.8unit/month, C_1=\$8per unit,
C_2=\$.9 per unit,C_3=\$.5 per unit, C_4=\$100 per order, b=.5, α= .25 , β=1,q =50, θ=.001, t_0=.006 T=10hrs.
Substituting above parameters, Global Criteria (GC) is obtained as
GC= 0.0044436
The compromise solutions are TVC$^+$ = \$ 25.52763, TVC$^-$ =\$ 25.90769

7. Conclusion.

In the present paper an EPQ model of time dependent deteriorating items has been studied and a methodology has been developed to determine the total average cost in fuzzy sense and to minimize the same The basic assumption of the model is production is demand dependent which is exponential in nature. To make the model more realistic demand has been fuzzified. This paper investigates a computing schema for the economic production quantities in fuzzy sense. This fuzzy parameters are then represented in terms of interval numbers. The original inventory model with interval coefficients is transformed into an equivalent multiobjective deterministic model. The multiobjective model is then solved by Global Criteria Method with the help of GRG (Generalized Reduced Gradient) technique. In future this model can be extended by taking deterioration parameter as fuzzy.

8.References

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