International Journal of Industrial Engineering & Production Research

December 2014, Volume 25, Number 4
pp. 317-331

pISSN: 2008-4889

http://IJIEPR.iust.ac.ir/

A Robust Optimization Approach for a *p*-Hub Covering Problem with Production Facilities, Time Horizons and Transporter

Ali Ghodratnama, Reza Tavakkoli-Moghaddam, Armand Baboli, BehnamVahdani

Ali Ghodratnama *is an Assistant Professor in Department of Industrial Engineering, Faculty of Engineering, Kharazmi University, Karaj, Iran. E-mail:* ghodratn@ut.ac.ir

Reza Tavakkoli-Moghaddam *is a Pofessor in School of Industrial Engineering, College of Engineering, University of Tehran, Iran. E-mail:* tavakoli@ut.ac.ir

Armand Baboli *is an Associate Professor in DISP Laboratory, INSA-Lyon, F-69621, France. E-mail:* *armand.baboli@insa-lyon.fr*

Behnam Vahdani *is an Assistant Professor in Department of Industrial Engineering, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran. E-mail:* b.vahdani@ut.ac.ir

KEYWORDS

Hub covering problem,
Facilities location,
Opening and reopening modes,
Transporter vehicle,
Robust approach.

ABSTRACT

Hub location-allocation problems are currently a subject of keen interest in the research community. However, when this issue is considered in practice, significant difficulties such as traffic, commodity transportation and telecommunication tend to be overlooked. In this paper, a novel robust mathematical model for a p-hub covering problem, which tackles the inherent uncertainty of some parameters, is investigated. The main aim of the mathematical model is to minimize costs involving: 1) the covering cost; 2) the sum of the transportation costs; 3) the sum of the opening cost of facilities in the hubs; 4) the sum of the reopening cost of facilities in hubs; 5) the sum of the activating cost facilities in hubs; and 6) the sum of the transporters' purchasing cost. To solve this model, the new extensions of the robust optimization theory are used. To evaluate the robustness of the solutions obtained by the novel robust optimization approach, they are compared to those generated by the deterministic mixed-integer linear programming (MILP) model for a number of different test problems. Finally, the conclusions are presented.

1. Introduction

A number of helpful strategies (e.g., alliances and coalitions) are currently being either formed or investigated because of the significant increase in transportation, logistics and telecommunications. Many industrial studies are dedicated to these areas, in which hub-and-spoke structures have elicited a lot of interest. They present the possibilities of well-organized capacity allocation and fleet management on different legs of transportation routes.

In a hub-and-spoke network, the hub nodes are linked to common nodes by routes called spokes. Relations between

Corresponding author: *Reza Tavakkoli-Moghaddam*

Email *tavakoli@ut.ac.ir*

Paper first received Jun. 30, 2013, and in accepted form July, 05, 2014.

the hubs themselves account for most of the transport capacities, economies of scale and pollution decrease. The former two advantages are the main reasons for establishing such a transportation system. A hub has three main functions: to collect flows arriving from any node of the network; to redistribute flows toward each destination point; and to send the aggregated flow to another hub for further redistribution. The location of hubs in networks is a non-deterministic polynomial time complete (NP-complete) combinatorial optimization problem with a two-part solution which determines the network's vertices that should function as hubs, and attributes the origin/destination nodes of each flow to their respective hubs. The solution aims to present the lowest total network cost for routing the flows between all O–D (origin–destination) pairs. The capacitated single-allocation hub location problem (CSHLP) is the specific case that incorporates flow capacity limits, either in the net links or at the vertices. Its single-allocation feature restricts each node to send or receive flows by only one hub.

As a whole, the hub location-allocation problem is divided into three sub-problems, namely *p*-hub median, *p*-hub center and *p*-hub covering. In the *p*-hub median problem, the aim is to minimize the total shipping cost; however, the *p*-hub center problem minimizes the maximum cost or time in the communication lines.

In the *p*-hub covering problem, the main objective is to minimize the maximum covering radius in such a way that all of the customer nodes fall into the dedicated covering radius of potential hub centers. Even though hub location problems have been considered quite analytically in the last decade, only a few researchers have considered the hub covering problems. In general, the maximal covering location problem (MCLP) is a location problem that has been roughly considered in the literature. The main objective of the MCLP is to choose the location of facilities in order to maximize the population that has a facility within a maximum travel distance (or time). Thus, a population is covered if it is inside a predefined service distance (or time) from at least one of the existing facilities. The MCLP presents services to the maximum number of individuals in the population, bearing in mind the available sources. In respect of its formulation, various extensions of the MCLP have been devised to increase its applicability, in both the public and private sectors. Some applications exist in the literature; for example, Marianov and Serra [18] worked on congested systems.

In this paper, a different type of single-assignment hub covering problem is examined, with regard to production facilities and transporter vehicles. These production facilities are established only in hub centers considered as serving centers. Time horizons, through which facilities can be opened and reopened repeatedly, are considered. A variety of transporter vehicle types to ship the produced commodities are also considered.

Regarding the above-mentioned problems, the considered mathematical model has one objective, to minimize total cost involving: 1) the covering cost, 2) the sum of the transportation costs, 3) the sum of the opening costs of facilities in hubs, 4) the sum of the reopening costs of facilities in hubs, 5) the sum of the activating costs of facilities in hubs, and 6) the sum of the vehicle use costs. Minimizing these costs in such a way that customer demands are satisfied is the main aim. In addition, to come close to reality, some parameters are regarded as uncertain. The robust counterpart of the proposed MILP model is also developed to cope with the uncertainty. The numerical tests show the power of the proposed robust model in handling uncertainty in parameters and generating robust optimal solutions.

The remainder of this paper is organized as follows. After systematically reviewing the literature in Section 2, the problem is defined and an efficient mix-integer linear programming (MILP) model is developed in Section 3. The solution methodology for the proposed model is developed in Section 4, and the computational results are reported in Section 5. Finally, Section 6 concludes this paper and presents directions for further research.

2. Motivation

In this research, we take into account the model devised by Ghodratnama et al. [35]. They exploited from the unique characteristic of a hub and spoke network to install plants into the optimum points regarded as hub nodes. The plants installed into these nodes serve the client nodes or spoke nodes. In this respect, vehicles convey the commodity manufactured by these plants to customer or client node. However, in this research, to come close to reality, we consider uncertain environment by the name of robust. The robust approach used to handle this environment is the recent one devised by Ben-Tal et al [2,3]. The results show as a whole by solving the deterministic problem, in which the degree of uncertainty is zero the value of objective function is less than the robust one. Nevertheless, an increasing trend is reported as we increase the uncertain parameters of all mathematical model parameters. The sensitivity analysis shows that for some uncertain parameters there exists no feasible solution. Additionally, the constant trend for some uncertain parameter is observed. It means that as the uncertain parameters grow, the objective function value is constant. Interestingly, after specified uncertain parameter, the objective function value decreases and following the mentioned value the infeasibility is reported.

3. Literature Review

In network hub location allocation problems, a given network with n nodes, consisting of the set of origins, destinations and potential hub locations, is considered. The flow between origin–destination pairs, an important characteristic of flows in the network (e.g., cost, time, distance) and the hub-to-hub shipping discount factor are identified. In this problem, locating hub services on a plane rather than on the nodes of a network is taken into account.

Hub location has different application areas in telecommunication network and transportation plans. Research on many of these is reported in the literature. Mu [21] devised a unified framework for site selection and business forecasting using ANP. Levy [16] considered a case for sustainable security systems engineering: integrating national, human, energy and environmental. Lin et al. [17] considered a backbone of technology evolution in the modern era automobile industry security.

In telecommunication network design, the concept of the hub location problem has also been explored. For a wide review on hub location in network design.

In this research, Klincewicz specifically investigated telecommunication and computer systems. The hub location problem in the network plan differs somewhat from the usual hub location literature, for example, in locating hub facilities and allocating nodes to them. The cost of installing the capacity on each edge, essential to routing the traffic on the edge itself, was investigated by Carello et al. [6]. A comparable problem, called the uncapacitated hub location problem, with modular arc capacities, was studied by Yaman [31]. While trying to minimize the costs of establishing hubs and capacity units, he calculated the integer amounts of capacity units on the arcs. Yaman and Carello [31] considered the capacitated version of this problem, in which the capacity of a hub is defined as the amount of traffic passing through it. Yao and Hsu [33] proposed a new spanning tree-based genetic algorithm (GA) for determining the optimal locations of the hubs and the optimal transportation routes in such a way that the total costs are minimized.

In facility covering problems, if customers are within a particular distance of a facility that can serve their demands, they are considered to be covered. Like the *p*-hub center problem, three coverage criteria for hubs were presented as follows:

The origin destination pair (i, j) is covered by hubs l and m only if:

Each of the origin-hub and hub-destination arcs meets distinct specific values.

The cost for each link in the path from i to j via l and m does not go beyond a definite value.

The cost from i to j via l and m does not surpass a definite value.

Locating hubs to cover up all the demands of customer nodes, such that the cost of opening hub facilities is minimized, is the main aim of the hub set-covering problem. On the other hand, the maximal hub-covering problem maximizes the demand covered up by means of a given number of hubs to locate. Kara and Tansel [14] produced the first effort improvement to the initial single assignment hub covering mathematical model. They considered the single-allocation hub set-covering problem and proved that it was NP-hardness. These authors plan and compare three different linearizations of the main quadratic model. In addition, they present a new linear model. The superiority of the performance of this new model is depicted to all of the other devised linear models.

New mathematical expressions for both single and multiple-allocation hub covering problems were designed by Wagner [27]. In his planned preprocessing methods, some hub assignments ruled out in this way required fewer variables and constraints than that of the mathematical model introduced by Kara and Tansel [14]. Wagner's mathematical model integrating certain constraints was thus an improvement. After that, a new mathematical model for the single-allocation hub set covering problem was devised by Ernst et al. [9] for the *p*-hub center problem considered by Ernst et al. [10]. They proposed a new idea in the hub covering problem area, namely the covering radius.

These authors strengthened the mathematical model proposed by Kara and Tansel [14] by replacing a constraint by its integrating form. They then compared this new model with the one presented by Kara and Tansel [14] found that it performed better in terms of the CPU time requirement. Ernst et al. [10] also considered the multiple allocation hub set-covering problems and devised two new mathematical models and an implicit enumerative method for this problem. Various mathematical models of the hub covering problem were subsequently compared by Hamacher and Meyer [13], who surveyed the feasibility polyhedron and recognized some facet-defining valid inequalities. They solved the hub set-covering problem for a specified cover radius b, and then iteratively decreased b to get the optimum solution of the p-hub center problem.

Calik et al. [5] studied the single assignment hub covering related to the incomplete hub network, and presented a mixed-integer mathematical model for the given problem. The main aim of their model is to find hub locations related to each other and customer nodes allocated to hub nodes in such a way that transportation times from origin nodes to destination nodes fall into the predefined interval. Weng et al. [29] studied a partial hub location allocation problem considering multiple assignments. Tan and Kara [26] presented a complete single-assignment hub covering model considering the commodity delivering centers. Berman et al. [4] meticulously surveyed the new developments in covering location models. Qu and Weng [23] proposed a path re-linking approach for a multiple hub maximal covering problem. Mohammadi et al. [34] considered a single assignment hub covering problem with capacities on hubs and solved their model by a hybrid algorithm based on genetic algorithms (GA) and simulated annealing (SA) using the random generated data.

Calik et al. [5] presented a single assignment hub covering model as a incomplete graph and proposed a heuristic approach to solve the model. Sahraeian and Korani [24] devised a hierarchical multiple assignment hub location-allocation problem considering partial covering with a predefined covering radius. Mohammadi et al. [20] presented a new model for the capacitated single allocation hub covering location problem and proposed the imperialist competitive algorithm to solve their multi-objective mathematical model. In addition, Karimi and Bashiri [15] investigated hub covering location problems with different coverage types. Dias et al. [8] presented a capacitated dynamic location mathematical model considering opening, closure and reopening of facilities. However, recently numerous research works have been accomplished in a hub location-allocation area. For further information, the reader can refer to the literature [36-41].

To the best of our knowledge, in uncertain environments (e.g., fuzzy, stochastic and robust ones), there is no research that has directly investigated hub covering problems. Some researchers have however recently investigated the stochastic hub location problem. Associated to this, a stochastic mathematical model for an air freight hub location and flight routes planning problem was developed by Yang [32], who investigated a real-case study related to good transportation. Conteras et al. [7] surveyed a stochastic uncapacitated hub location problem, in which uncertainty was related to demands and transportation costs. Sim et al. [25] considered the stochastic p-hub center problem with chance constraints with the service-level guarantees.

Menou et al. [19] considered a decision support problem for centralizing cargo at the Moroccan airport hub, using a stochastic multi-criteria acceptability analysis. They presented significant uncertainty in both the criteria measurements and the preferences. In addition, regarding fuzzy environments, Taghipourian et al. [11] developed a fuzzy programming approach for a dynamic virtual hub location problem. Table 1 summarizes the characteristics of relevant performed studies on hub covering problems, so far.

Tab. 1. Review of the existing literature

Authors	Year Published	Model Name	Type of Covering		Type of Model		Number of hubs	Cost of hub installation	Cost of hubs connecting	Cost of connecting hubs to customers
			Complete	Partial	Linear	Nonlinear				
Kara and Tansel [14]	2003	HC	√			√	√			
		HC-Lin	√		√		√			
Wagner [27]	2004	SAQI-W1	√		√		√			
		SAQI-W2	√		√		√			
		SAQD-W	√		√		√			
		MAQI-W	√		√		√			
Ernst *et al.* [10]	2005	USAHCOP-r	√		√		√			
		USAHCOP-3	√		√		√			
Weng *et al.* [29]	2006	MAHMCP		√	√					
Tan and Kara [26]	2007	--------	√		√		√			
Weng and Yang [29]	2006	MAHSCP	√		√		√	√		
Qu and Weng [23]	2009	MAHMCP		√	√					
		MAHSCP	√		√		√	√		
Calik *et al.* [5]	2009	--------	√		√		√	√	√	
Ghodsi *et al.* [12]	2010	CSAHCLP	√		√		√	√	√	√
Sahraeian and Korani [24]	2011	SA-TH-HMC		√	√					
Ghodratnama *et al.*[35]	2013	FBP-HC	√		√		√	√	√	√
Ghodratnama *et al.*	2014	RP-HC	√		√		√	√	√	√

Cont'd – Tab. 1. Review of the existing literature

Type	Assignment	Capacity	Solution Approach	Data Base									
Crisp	Robust	Fuzzy	Single	Multiple	Capacitated	Uncapacitated	Exact	Heuristic	Meta-heuristic	AP	CAB	Turkish	Random
√			√			√	√						√
√			√			√	√				√		
√			√			√	√						√
√			√			√	√			√	√		
√			√			√	√						√
√				√		√	√			√	√		
√			√			√	√			√	√		
√				√		√	√			√	√		
√				√		√	√			√			√

√			√			√	√				√	√	
√				√		√			√	√			
√				√		√		√		√			√
√				√		√	√						√
√			√			√		√			√	√	
√			√		√				√				√
√			√			√	√				√		
		√	√		√		√						√
	√		√		√		√						√

4. Mathematical model

In this section, the novel mathematical model for a p-hub covering location-allocation problem is considered. This model takes into account a number of facilities which produce goods and are active through time horizons. Each facility can be opened and reopened repeatedly over time. Depending on the demand operational constraint, activation and non-activation can take place. This model minimizes the total costs including covering, transporting, opening, reopening, activating and using vehicles. Some operational constraints relating to the covering problem, facility characteristics, demands and time horizons are also investigated. To come close to reality, some of the parameters are regarded as indefinite. Model assumptions, indices, parameters and decision variables are presented below.

4-1. Assumptions

- The number of customers is predefined.
- The number of hubs is predefined.
- The number of facilities satisfying the demands of customers is deterministic.
- The demands of customers are deterministic and predefined.
- Each facility has the minimum and maximum capacities.
- Each facility has the opening and reopening fixed costs.
- There is interchange between hubs by considering the discount factor in the related cost.
- Transportation costs between each facility and each customer are fixed.
- The planning horizon consists of multiple periods and is predefined.
- Facilities should be located in hub centers.
- Facilities can be opened only once.
- Facilities can be reopened numerously through time horizons.
- Capacities of facilities are predefined
- Capacities of vehicles are different
- Opening costs of facilities are predefined.
- Reopening costs of facilities are predefined.
- Using costs of vehicles are predefined.
- Each vehicle type has a unique capacity.

In addition, to illuminate the features of this problem and mathematical model, three figures have been considered.

Two hubs, namely k and m where facilities have been located, are taken into account, as shown in Figure. 1. They are represented as large rectangles. These facilities produce commodities to satisfy the customer node demands, represented as circles. For hubs k and m, the covering radii are r_k and r_m , which represent the distances to the farthest customer nodes served by them. The maximum cost is associated to the route i-k-m-j, in which the flow originating from customer node i destined to node j via hubs k and m. Customer nodes have been depicted as hexagons. To transmit commodities to customers, transporter vehicles are also used. These vehicles have been represented as small rectangles.

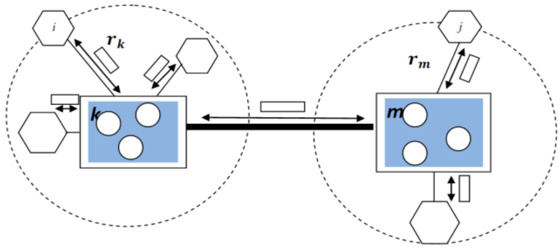

Fig. 1. Schema of hubs covering problem considering facilities located in hubs and transportation vehicles.

Figure. 2 depicts customer nodes as hexagons and their hub node as a big rectangle in which facilities serving customer nodes have been shown on a larger scale as circles. In this case, Facility 1 serves customer nodes one, seven and six, Facility 2 serves customer nodes two and three, and Facility 3 serves customer nodes four and five. In addition, vehicles serving the customer nodes, by shipping the commodities originating from the hub, have been represented as small rectangles. Figure. 3 shows that T time periods and n facilities are investigated. For instance, Facility 2 is opened in Period 1 and is active until the end of Period 2. It is inactive in Period 3. However, it is reopened at least once through the time horizons.

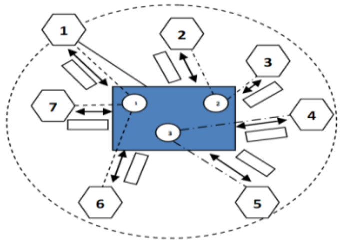

Fig. 2. Closer schema of one specific hub with its located facilities and transportation flows between its facilities and their customer via vehicles.

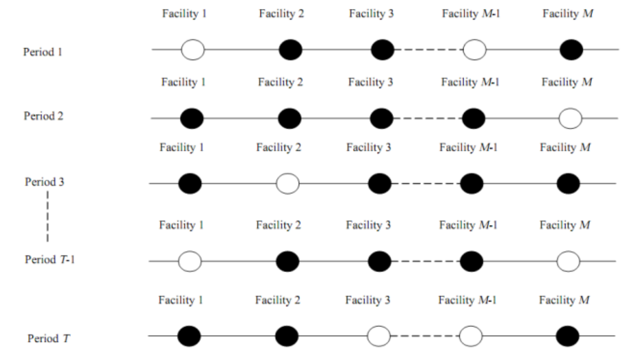

Fig. 3. Schema of *M* facilities on *T* period time horizon indicating the active facility in black.

4-2. Indices

$j, k, l, O \in \{1,..,N\}$	Node numbers
$i \in \{1,\dots, M\}$	Factory numbers
$t, t' \in \{1,\dots, T\}$	Period numbers
$v \in \{1,\dots,V\}$	Vehicle numbers

4-3. Parameters

NH	Number of hubs.
ct_{ikj}^t	Cost of transportation of product unit from factory *i* to customer *j* via hub *k* at period *t*.
cc_{lk}	Maximum cost considered between node *l* and node *k*.
ca_{ik}^t	Cost of being active, for factory *i* located in hub *k* at period *t*.
cv_v^t	Cost of using vehicle type *v* at period *t*.
d_j^t	Demand of customer *j* at period *t*.
A_{ik}^t	First time opening cost of factory *i* in hub *k* at period *t*.
R_{ik}^t	Reopening cost of factory *i* in hub *k* at period *t*.
Q_{ik}^{min}	Minimum production capacity of factory *i* in hub *k*.
Q_{ik}^{max}	Maximum production capacity of factory *i* in hub *k*.
cp_v	Capacity of vehicle *v*.
MM	A positive large number.

4-4. Decision variables

zr	Maximum cost of a path between any two nodes by using the respective hub radii.
rc_k	Maximum covering radius of hub *k*.
g_{lk}	1 if customer node *l* is allocated to hub *k*; 0, otherwise.
f_{ik}^t	1 if factory i located in hub *k* at period t is active; 0, otherwise.
a_{ik}^t	1 if factory i located in hub *k* at period *t* is opened for the first time; 0, otherwise.
r_{ik}^t	1 if factory *i* located in hub *k* at period *t* is reopened; 0, otherwise.
p_{ik}^t	Dedicated capacity to facility *i* located in hub *k* at period *t*.
x_{ikj}^t	Number of units produced by factory *i* located in hub *k* and transported to customer *j* at period *t*.
y_{vkj}^t	1 if vehicle *v* is selected for shipping the product to customer node *j* via hub *k* at period t; 0, otherwise.
NV_v^t	Number of vehicles type of *v* used at period *t*.

By using the above-mentioned notations, the presented mathematical programming model for the concerned *p*-hub covering location allocation problem is as follows:

$$\begin{aligned}\text{Min } Z_1 = zr &+ \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{N}\sum_{j=1}^{N} ct_{ikj}^t x_{ikj}^t \\ &+ \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{N} A_{ik}^t\, a_{ik}^t \\ &+ \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{N} R_{ik}^t\, r_{ik}^t \\ &+ \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{N} ca_{ik}^t\, f_{ik}^t \\ &+ \sum_{t=1}^{T}\sum_{v=1}^{V} cv_v^t NV_v^t\end{aligned} \tag{1}$$

s.t.

$$zr \geq rc_k + rc_o + \alpha cc_{ko} \qquad \forall k, o \tag{2}$$

$$rc_k \geq cc_{lk}.g_{lk} \qquad \forall k, l \tag{3}$$

$$\sum_{k=1}^{N} g_{kk} = NH \tag{4}$$

$$\sum_{k=1}^{N} g_{lk} = 1 \qquad \forall l \tag{5}$$

$$g_{lk} \leq g_{kk} \qquad \forall k, l \tag{6}$$

$$f_{ik}^t \leq g_{kk} \qquad \forall i, t, k \tag{7}$$

$$a_{ik}^t \leq g_{kk} \qquad \forall i, t, k \tag{8}$$

$$r_{ik}^t \leq g_{kk} \qquad \forall i, t, k \tag{9}$$

$$a_{ik}^t = f_{ik}^t \qquad \forall i, t, k \tag{10}$$

$$a_{ik}^t + r_{ik}^t \leq 1 - f_{ik}^{t-1} \qquad \forall i, t, k \tag{11}$$

$$\sum_{t=1}^{T} a_{ik}^t = g_{kk} \qquad \forall i, k \tag{12}$$

$$\sum_{t'=1}^{t-1} a_{ik}^{t'} \geq r_{ik}^t \qquad \forall i, k, t \tag{13}$$

$$\sum_{t'=1}^{t} a_{ik}^{t'} \geq f_{ik}^t \qquad \forall i, k, t \tag{14}$$

$$\sum_{j=1}^{n} x_{ikj}^t \leq p_{ik}^t \qquad \forall i, k, t \tag{15}$$

$$x_{ikj}^t \leq MM g_{jk} \qquad \forall i, j, k, t \tag{16}$$

$$\sum_{i=1}^{M}\sum_{k=1}^{N} x_{ikj}^{t} \geq d_{tj} \qquad \forall j,t \qquad (17)$$

$$Q_{ik}^{min} f_{ik}^{t} \leq p_{ik}^{t} \qquad \forall i,k,t \qquad (18)$$

$$Q_{ik}^{max} f_{ik}^{t} \geq p_{ik}^{t} \qquad \forall i,k,t \qquad (19)$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{N} x_{ikj}^{t} \leq \sum_{v=1}^{V} cp_v \, NV_v^t \qquad \forall t \qquad (20)$$

$$\sum_{k=1}^{N}\sum_{j=1}^{N} y_{vkj}^{t} = NV_v^t \qquad \forall t,v \qquad (21)$$

$$\sum_{v=1}^{V} y_{vkj}^{t} = g_{jk} \qquad \forall j,k,t \qquad (22)$$

$$a_{ik}^{t} \in \{0,1\} \qquad \forall i,k,t \qquad (23)$$

$$r_{ik}^{t} \in \{0,1\} \qquad \forall i,k,t \qquad (24)$$

$$f_{ik}^{t} \in \{0,1\} \qquad \forall i,k,t \qquad (25)$$

$$y_{vkj}^{t} \in \{0,1\} \qquad \forall v,k,j,t \qquad (26)$$

$$p_{ik}^{t} \geq 0 \qquad \forall i,k,t \qquad (27)$$

$$x_{ikj}^{t} \geq 0 \qquad \forall i,j,k,t \qquad (28)$$

$$NV_v^t \geq 0, \text{integer} \qquad \forall v,t \qquad (29)$$

Objective function (1) tries to minimize: (i) the maximum value of the sum of the radius of hubs *k* and *o* as well as the maximum cost between hubs *k* and *o* multiplied by discount factorα; (ii) the transportation costs between each facility and customer; (iii) the opening costs of facilities in all hubs locations; (iv) the reopening costs of facilities in all hubs locations; (v) the using costs of vehicles serving to ship goods between customers and hubs. In Constraint (2), the maximum value of the sum of the radius of hubs *k* and *o*, as well as the maximum cost between hubs *k* and *o* multiplied by discount factor α, is computed for all hub centers. Note that *zr* is just one variable. Constraint (3) defines the maximum covering radius based on the considered cost providing the dedicated customer. Constraint (4) guarantees that exactly *NH* hubs are selected, while Constraint (5) ensures that every node is assigned to exactly one hub. Constraint (6) guarantees that customer *l* is dedicated to hub *k* providing that hub *k* is preformed. Constraint (7) ensures that facility *i* in hub *k* and period *t* is active if hub *k* is formed. Constraint (8) implies that facility *i* in hub *k* and period *t* is opened if hub *k* is formed. Constraint (9) implies that facility *i* in hub *k* and period *t* is reopened if hub *k* is formed.

Constraint (10) implies that when the facility *i* in hub *k* and period *t* is opened then facility *i* in hub *k* and period *t* will be active as well. Constraint (11) guarantees that facility *i* can be opened or reopened provided that in the previous period facility *i*is inactive. Constraint (12) makes sure that, over time, facility *i* can be opened only once. Constraint (13) implies that facility *i* can be reopened if in the previous periods it has been opened once. Constraint (14) ensures that if the facility is active, in the previous periods it has been opened. Constraint (15) ensures that the total transported goods to customers in each period is equal to the dedicated capacity of facility *i* in period *t*. Constraint (16) guarantees that transportation is implemented from facility *i* in hub *k* at period *t* to customer *j*, provided that hub *k* is also preformed. Constraint (17) guarantees that the total amount of transported goods to customer *j* is greater than the related demand.

Constraints (18) and (19) make sure that the dedicated capacity of facility *i* falls in the range of the minimum and maximum defined capacities, respectively. Constraint (20) implies that the total amount of commodities transported from facilities does not exceed the total vehicle capacity in period *t*. Constraint (21) computes the total number of type *v* vehicles used in period *t*. Constraint (22) implies that for each path between hub *k* and customer *j* at period *t* one vehicle type *v* is used. Constraints (23) to (26) relate to the binary restriction. Constraints (27) to (29) define the lower limits of considered variables. Note that for the last constraint an integer value is required.

5. Robust Optimization Model

To apply the robust optimization (RO) model the transportation costs, maximum costs between nodes, costs of factories being active , costs of using vehicles, demands of customers, first time opening costs of factories, reopening costs of factories, minimum production capacities of factories, maximum production capacities of factories, and capacities of vehicles are treated as uncertain parameters. In this section we briefly describe the principles of RO. For further details, readers may refer to [2]. Consider the following linear program (LP):

$$\min \quad cx \qquad (30)$$

s.t.:

$$Ax \geq b \qquad (31)$$

where$x \in \Re^n$ is the vector of decision variables,$b \in \Re^m$ is the right-hand side parameter vector, $c \in \Re^n$ is the vector of objective function coefficients, and $A \in \Re^{m\times n}$ with elements a_{ij} is the constraint coefficient matrix.

In a typical problem (e.g., LP), c,A and b are assumed to be deterministic, and by solving this problem an optimal solution is obtained. Some of the data parameters are considered as uncertain in the RO approach, yet they lie within a set that expresses limits on the uncertainty. The foregoing uncertainty set subsequently defines the limits on uncertainty, in which a solution will be immunized against. That is, the solution *x* deals with any possible uncertainty lying within the set. In the RO approach, the LP model is transformed into a robust counterpart by placing each constraint that has uncertain coefficients with a constraint reflecting the incorporation of the uncertainty set. In the following description, we focus on uncertainty in the objective function coefficients and constraint coefficient matrix. Let$\tilde{c}_j$ and $\tilde{a}_{ij}$denote an uncertain entry in the objective function coefficients and constraint coefficient, respectively. In the proposed model, each of the uncertain parameters is assumed to vary in a specified closed bounded box [3,22,42].

5-1. Box uncertainty

The general form of this box can be specified as follows:

$$u_{Box} = \{\xi \in \Re^n : |\xi_t - \bar{\xi}_t| \le \rho G_t, \quad t = 1,2,\ldots,n\} \quad (32)$$

where $\bar{\xi}_t$ is the normal value of the ξ_t as the *t*-th parameter of vector ξ, the positive number G_t represents the "uncertainty scale", and $\rho > 0$ is the "uncertainty level". A particular case of interest is $G_t = \bar{\xi}_t$, which corresponds to a simple case where the box contains ξ_t whose relative deviation from the nominal data can be as wide as ρ.

According to the above description, the robust counterpart of the LP model expressions (33) to (36) can be stated as expressions (36) to (40):

Mathematical model:

$$\text{Min} \quad \tilde{c}x + dy \quad (33)$$

s.t.

$$\tilde{a}x \ge b \quad (34)$$

$$ey \ge \tilde{f} \quad (35)$$

$$x, y \in \{0,1\} \quad (36)$$

Robust counterpart mathematical model:

$$\text{Min} \quad z \quad (37)$$

s.t.

$$\tilde{c}_j x_j + dy \le z \qquad \forall \tilde{c}_j \in u^c_{Box} \quad (38)$$

$$\tilde{a}_{ij} x_j \ge b_i \qquad \forall \tilde{a}_{ij} \in u^a_{Box} \quad (39)$$

$$e_i y \ge \tilde{f}_i \qquad \forall \tilde{f}_i \in u^f_{Box} \quad (40)$$

$$h_j x = \tilde{r}_j \qquad \forall \tilde{r}_j \in u^r_{Box} \quad (41)$$

Constraint (36).

Bent-Tal *et al.* [3] demonstrated that in the closed bounded box, the robust counterpart problem can be converted to a tractable equivalent model where u_{Box} is replaced by a finite set u_{ext} consisting of the extreme points of u_{Box}. To represent the tractable form of the robust mathematical model, Expressions (37) to (40) should be converted to their equivalent tractable ones. For Constraint (38), we have:

$$\tilde{c}_j x \le z - dy,$$

$$\forall \tilde{c}_j \in u^c_{Box} | u^c_{Box} = \{\tilde{c}_j \in \Re^{n_c} : |\tilde{c}_j - \bar{c}_j| \le \rho_c G^c_j\} \quad (42)$$

$$\forall j = 1,2,\ldots,n_c$$

The left-hand side of expression (41) contains the vector of uncertain parameters, while all parameters of the right-hand side are certain. Thus, the tractable form of the above semi-infinite inequality could be written as follows:

$$\sum_j (\bar{c}_j x_j + \eta_j) \le z - dy, \quad (43)$$

$$\rho_c G^c_j x_j \le \eta_j \qquad \forall j \in \{1,2,\ldots,n_c\} \quad (44)$$

$$\rho_c G^c_j x_j \ge -\eta_j \qquad \forall j \in \{1,2,\ldots,n_c\} \quad (45)$$

For constraint $\sum_{j=1}^n \tilde{a}_{ij} x_j \ge b_i$, we only need to augment the left-hand side of the Constraint to reflect the uncertainty set in the formulation. Formally, in the augmented constraint we require the following expression for a given solution x [1,2]

$$\min_{\tilde{a} \in u^a_{Box}} \left\{\sum_{j=1}^n \tilde{a}_{ij} x_j\right\} \ge b_i \quad (46)$$

or

$$\min_{\tilde{a}_{ij} : |\tilde{a}_{ij} - \bar{a}_{ij}| \le \rho_a G^a_{ij}} \left\{\sum_{j=1}^n \tilde{a}_{ij} x_j\right\} \ge b_i \quad (47)$$

Given the structure of u_{Box}, the optimal solution of the optimization on the left-hand side is as follows:

$$\sum_{j=1}^n \bar{a}_{ij} x_j - \rho_a \sum_{j=1}^n G^a_{ij} |x_j| \ge b_i \quad (48)$$

It can be reformulated by:

$$\sum_{j=1}^n \bar{a}_{ij} x_j - \rho_a \sum_{j=1}^n G^a_{ij} l_j \ge b_i \quad (49)$$

$$-l_j \le x_j \le l_j \qquad \forall j (1,2,\ldots,n) \quad (50)$$

Similarly, for inequality (40), we have:

$$e_i y \ge \tilde{f}_i$$

$$\forall \tilde{f}_i \in u^f_{Box} | u^f_{Box} = \{\tilde{f}_i \in \Re^{n_f} : |\tilde{f}_i - \bar{f}_i| \le \rho_f G^f_i\} \quad (51)$$

$$\forall i \in \{1,2,\ldots,n_f\}$$

Thus, it can be rewritten as follows:

$$e_i y \ge \bar{f}_i + \rho_f G^f_i \quad (52)$$

$$\forall i \in \{1,2,\ldots,n_f\}$$

At last, in Equation (41), we have:

$$h_j x = \tilde{r}_j$$

$$\forall \tilde{r}_j \in u^r_{Box} | u^r_{Box} = \{\tilde{r}_j \in \Re^{n_r} : |\tilde{r}_j - \bar{r}_j| \le \rho_r G^r_j\} \quad (53)$$

$$\forall j \in \{1,2,\ldots,n_r\}$$

Therefore, it can be rewritten as follows:

$$h_j y \ge \tilde{r}_j - \rho_r G^r_j \quad (54)$$

$$\forall j \in \{1,2,\ldots,n_r\}$$

$$h_j y \le \tilde{r}_j + \rho_r G^r_j \quad (55)$$

$$\forall j \in \{1,2,\ldots,n_r\}$$

Regarding the above-mentioned descriptions, the structure of the robust counterpart p-hub covering mathematical model for the considered uncertain parameters is presented as follows:

$$\text{Min } Z_1 \tag{56}$$

$$\begin{aligned} zr &+ \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{N}\sum_{j=1}^{N}\left(\overline{ct}_{ikj}^{t} x_{ikj}^{t} + \eta_{ikj}^{t\ ct}\right) \\ &+ \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{N}(\bar{A}_{ik}^{t}\, a_{ik}^{t} + \eta_{ik}^{t\ A}) \\ &+ \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{N}(\bar{R}_{ik}^{t}\, r_{ik}^{t} + \eta_{ik}^{t\ R}) \\ &+ \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{N}(\overline{ca}_{ik}^{t}\, f_{ik}^{t} + \eta_{ik}^{t\ ca}) \\ &+ \sum_{t=1}^{T}\sum_{v=1}^{V}(\overline{cv}_{v}^{t} NV_{v}^{t} + \eta_{v}^{t\ cv}) \le Z_1 \end{aligned} \tag{57}$$

s.t.

$$\rho_{ct} G_{ikj}^{t\ ct} x_{ikj}^{t} \le \eta_{ikj}^{t\ ct} \qquad \forall i,k,j,t \tag{58}$$

$$\rho_{ct} G_{ikj}^{t\ ct} x_{ikj}^{t} \ge -\eta_{ikj}^{t\ ct} \qquad \forall i,k,j,t \tag{59}$$

$$\rho_{A} G_{ik}^{t\ A} a_{ik}^{t} \le \eta_{ik}^{t\ A} \qquad \forall i,k,t \tag{60}$$

$$\rho_{A} G_{ik}^{t\ A} a_{ik}^{t} \ge -\eta_{ik}^{t\ A} \qquad \forall i,k,t \tag{61}$$

$$\rho_{R} G_{ik}^{t\ R} r_{ik}^{t} \le \eta_{ik}^{t\ R} \qquad \forall i,k,t \tag{62}$$

$$\rho_{R} G_{ik}^{t\ R} r_{ik}^{t} \ge -\eta_{ik}^{t\ R} \qquad \forall i,k,t \tag{63}$$

$$\rho_{ca} G_{ik}^{t\ ca} f_{ik}^{t} \le \eta_{ik}^{t\ ca} \qquad \forall i,k,t \tag{64}$$

$$\rho_{ca} G_{ik}^{t\ ca} f_{ik}^{t} \ge -\eta_{ik}^{t\ ca} \qquad \forall i,k,t \tag{65}$$

$$\rho_{cv} G_{v}^{t\ cv} NV_{v}^{t} \le \eta_{v}^{t\ cv} \qquad \forall v,t \tag{66}$$

$$\rho_{cv} G_{v}^{t\ cv} NV_{v}^{t} \ge -\eta_{v}^{t\ cv} \qquad \forall v,t \tag{67}$$

$$zr \ge$$

$$rc_k + rc_o + \alpha \overline{cc}_{ko}(1+\rho_{cc}) \qquad \forall k, o \tag{68}$$

$$rc_k \ge \overline{cc}_{lk}(1+\rho_{cc}).\, g_{lk} \qquad \forall k,l \tag{69}$$

$$\sum_{i=1}^{M}\sum_{k=1}^{N} x_{ikj}^{t} \ge \bar{d}_{tj} + \rho_d G_{tj}^{d} \qquad \forall j,t \tag{70}$$

$$\bar{Q}_{ik}^{min}(1+\rho_{Q^{min}}) f_{ik}^{t} \le p_{ik}^{t} \qquad \forall i,k,t \tag{71}$$

$$\bar{Q}_{ik}^{max}(1-\rho_{Q^{max}}) f_{ik}^{t} \ge p_{ik}^{t} \qquad \forall i,k,t \tag{72}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{N} x_{ikj}^{t} \le \sum_{v=1}^{V} \overline{cp}_{v}\,(1-\rho_{cv}) NV_{v}^{t} \qquad \forall t \tag{73}$$

$$\eta_{ikj}^{t\ ct} \ge 0 \qquad \forall i,k,j,t \tag{74}$$

$$\eta_{ik}^{t\ A} \ge 0 \qquad \forall i,k,t \tag{75}$$

$$\eta_{ik}^{t\ R} \ge 0 \qquad \forall i,k,t \tag{76}$$

$$\eta_{ik}^{t\ ca} \ge 0 \qquad \forall i,k,t \tag{77}$$

$$\eta_{v}^{t\ cv} \ge 0 \qquad \forall v,t \tag{78}$$

Constraints (4) to (16) and (21) to (29).

6. Computational Results

To appraise the performance of the proposed model, four test problems are considered. The size of these problems is shown in Table 2.

Tab. 2. Size of test problems

Problem no.	No. of customers (N)	No. of hubs (NH)	No. of facilities (M)	No. of time periods (T)	No. of vehicles (V)
1	3	1	3	3	3
2	3	1	4	4	4
3	4	2	5	4	5
4	5	3	6	5	6
5	6	4	7	5	7
6	7	5	8	6	8

Table 3 shows the interval of the generated input parameters; namely demands, capacities and costs related to each test problem. Note that the parameters are generated uniformly between the related lower and upper bounds. Table 4 illustrates the number of variables, including binary, positive and free ones, and the number of formed constraints for each test problem. In this table, for each test problem, two deterministic and robust types are taken into account. All the instances are carried out with a branch-and-bound (B&B) method by using the CPLEX solver of the GAMS commercial software to analyze the mixed-integer linear model, which is executed on a computer with characteristics of Intel(R), Core (TM) 2 Duo CPU P8400 @ 2.26 GHz, 3.00 GB of RAM. To do a sensitivity analysis, for uncertainty levels of all parameters, the analysis has first been investigated. The impact of the following four important and efficient factors is then evaluated separately.

- Uncertainly level of the demand.
- Uncertainly level of the minimum capacity.
- Uncertainly level of the maximum capacity.
- Uncertainly level of the vehicle capacity.

For the first phase, four test problems with different uncertainty levels are taken into account. For the second phase, test problem 4 has also been chosen to execute the sensitivity analysis. Note that, for the first phase, all uncertainty levels have increased simultaneously and for the second phase all uncertainty levels are fixed at zero values. Then, for considered parameters, it is increased step wisely with the 0.1 value.

6-1. Analysis based wholly on the uncertainty level of all parameters

In this section the impact of the uncertainty level of all parameters is investigated. The results show that when we pass through the higher uncertainty level the more objective function is yielded. Thus, we obviously have an ascending pattern. Note that for a bigger uncertainty level the infeasibility mode also occurs. In addition, because we increase the uncertainty level simultaneously, the impacts of other parameters are not analyzed individually. Table 5 and Figure 4 illustrate and depict our results quantitatively.

6-.2. Analysis based on the uncertainty level of the demand

In this respect, and due to importance of the uncertainty of demand, only an ascending pattern is reported. Note that only some operational constraints are included for this feature. Table 6 presents our results numerically, and Figure 5 elucidates the associated results graphically.

Tab. 3. Demands, capacities and costs for the four test problems

Problem no.	Demands	Capacities			Costs					
	$d_{tj\theta}$	$Q_{ik\theta}^{min}$	$Q_{ik\theta}^{max}$	cp_v	$cc_{lk\theta}$	$ct_{ikj\theta}^t$	$ca_{ik\theta}^t$	cv_v^t	$A_{ik\theta}^t$	$R_{ik\theta}^t$
1	(50,180)	(500,1000)	(5000,10000)	(400,800)	(1000,2000)	(1500,2500)	(2000,3000)	(2500,3500)	(5000,8000)	(4000,7000)
2	(130,400)	(800,1300)	(8000,13000)	(1000,1800)	(3500,4500)	(4000,5000)	(4500,5500)	(5000,6000)	(7500,10500)	(6500,9500)
3	(210,650)	(1100,1600)	(11000,16000)	(1600,2800)	(6000,7000)	(6500,7500)	(7000,8000)	(7500,8500)	(10000,13000)	(9000,12000)
4	(300,900)	(1400,1900)	(14000,19000)	(2200,3800)	(8500,9500)	(9000,10000)	(9500,10500)	(10000,11000)	(12500,15500)	(11500,14500)
5	(390,1150)	(1700,2200)	(17000,22000)	(2800,4800)	(11000,12000)	(11500,12500)	(12000,13000)	(12500,13500)	(15000,18000)	(14000,17000)
6	(480,1400)	(2000,2500)	(20000,25000)	(3400,5800)	(13500,14500)	(14000,15000)	(14500,15500)	(15000,16000)	(17500,20500)	(16500,19500)

Tab. 4. Number of variables and constraints of the test problems

Problem size $N \times M \times T \times V$	Number of variables						Number of constraints	
	Deterministic			Robust			Deterministic	Robust
	Binary	Positive	Free	Binary	Positive	Free		
3×3×3×3	171	121	1	171	292	1	439	763
3×4×4×4	297	212	1	297	516	1	735	1343
4×5×4×5	576	425	1	576	1005	1	1297	2457
5×6×5×6	1225	936	1	1225	2166	1	2546	5006
6×7×5×7	1836	1122	1	1836	2777	1	3767	7617
7×8×6×8	3283	2114	1	3283	5102	1	6313	13129

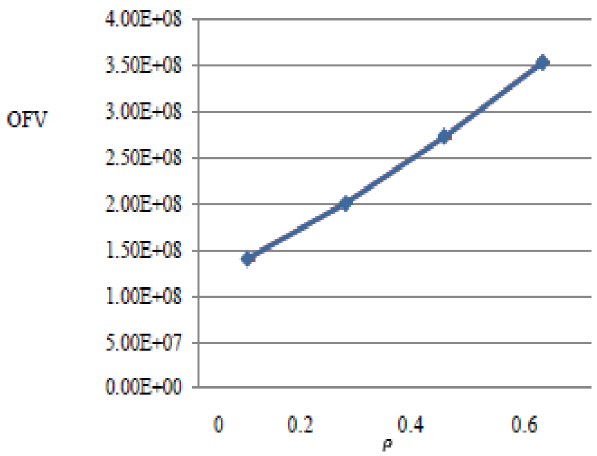

Fig. 4. Robust approach considering the impact of an indefinite level of all parameters

Tab. 5. Comparison of outputs based on the uncertainty level of all parameters in terms of the time and OFV

Problem size $N \times M \times T \times V$	Uncertainty level (ρ)	OFV		Time (Sec.)	
		Deterministic	Robust	Deterministic	Robust
3×3×3×3	0	1105416.5	1105416.5	0.45	0.172
	0.2		1578727.8		1.45
	0.4		2136115.1		0.344
	0.6		2777578.4		0.188
	0.8		Infeasible		Infeasible
	1		Infeasible		Infeasible
3×4×4×4	0	14897373	14897373	0.266	0.25
	0.2		21420396.24		0.265
	0.4		29124602.36		0.329
	0.6		38010090.56		0.234
	0.8		Infeasible		Infeasible
	1		Infeasible		Infeasible
4×5×4×5	0	49619727	49619727	0.281	0.282
	0.2		71600810.88		0.36
	0.4		97378895.32		0.391
	0.6		120717046.4		0.422
	0.8		Infeasible		Infeasible
	1		Infeasible		Infeasible
5×6×5×6	0	141220459	141220459.5	0.516	0.453
	0.2		201341236.9		0.5
	0.4		273764817.3		0.5
	0.6		353952903.5		0.719
	0.8		Infeasible		Infeasible
	1		Infeasible		Infeasible
6×7×5×7	0	267378490	267378490	1.492	0.877
	0.2		382860194.64		0.804
	0.4		523111198.96		0.735
	0.6		666625573.76		1.38
	0.8		Infeasible		Infeasible
	1		Infeasible		Infeasible
7×8×6×8	0	567406943	567406943	1.061	1.11
	0.2		820356005.04		1.189
	0.4		1111371285.92		1.208
	0.6		1459054996.16		1.195
	0.8		Infeasible		Infeasible
	1		Infeasible		Infeasible

Tab. 6. Comparison of outputs based on the uncertainty level of the demand parameter in terms of the time and OFV

Problem size $N \times M \times T \times V$	Uncertainty level (ρ_d)	OFV		Time (Sec.)	
		Deterministic	Robust	Deterministic	Robust
5×6×5×6	0	141220459	141220460	0.516	0.516
	0.1		153560700		0.531
	0.2		167784761		0.484
	0.3		181348863		0.515
	0.4		195546885		0.515
	0.5		209136671		0.485
	0.6		223378460		0.515
	0.7		236923593		0.531
	0.8		251208900		0.562
	0.9		266405522		0.5
	1		281706859		0.516

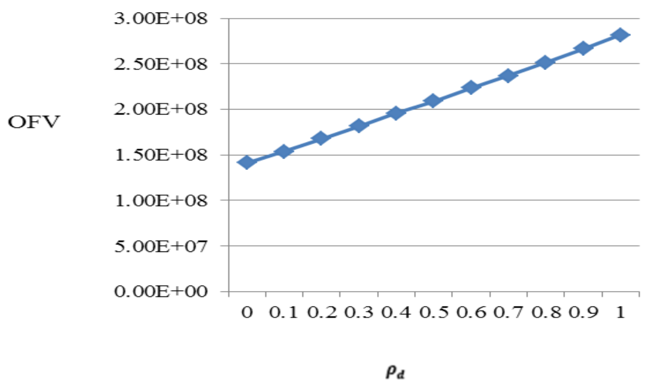

Fig. 5. Robust approach considering the impact of an indefinite level of the demand parameter

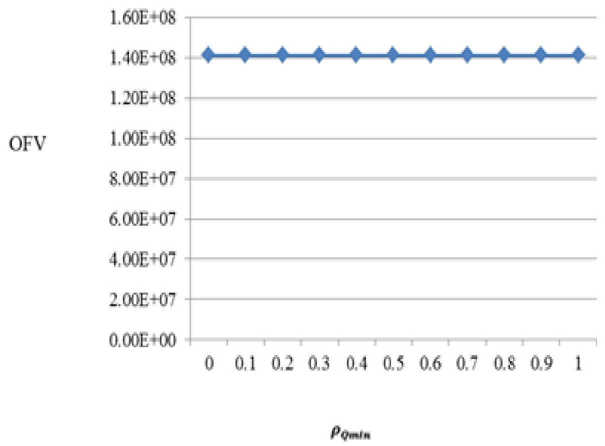

Fig. 6. Robust approach considering the impact of an indefinite level of the minimum capacity of the facility parameter

6-.3. Analysis based on the uncertainty level of the minimum capacity

In this respect there is no change in the objective function values through various uncertainty levels. It seems that the minimum capacity has no significant impact on the feasible solution area. Table 7 reports our results numerically, and Figure 6 clarifies them graphically.

Tab. 7. Comparison of outputs based on the uncertainty level of the minimum capacity of the facility parameter in terms of the time and OFV

Problem size $N \times M \times T \times V$	Uncertainty level (ρ_{Qmin})	OFV		Time (Sec.)	
		Deterministic	Robust	Deterministic	Robust
5×6×5×6	0	141220459	141220460	0.516	0.5
	0.1		141220460		0.515
	0.2		141220460		0.484
	0.3		141220460		0.515
	0.4		141220460		0.406
	0.5		141220460		0.515
	0.6		141220460		0.453
	0.7		141220460		0.515
	0.8		141220460		0.64
	0.9		141220460		0.516
	1		141220460		0.516

6-4. Analysis based on the uncertainty level of the maximum capacity

In this respect, there is no change in the objective functions up to the definite value of the uncertainty level, that is, 0.7. However, after this point a sudden decrease takes place and after that there is no feasible solution. To be critical, this parameter is the main reason for this pattern. In other words, this parameter is linked to satisfy the demands, and for some uncertainty levels this infeasibility occurs. Table 8 reports the results numerically, and Figure 7 clarifies them graphically.

6-5. Analysis based on the uncertainty level of the vehicle capacity

In this respect, as for maximum capacity, the same behavior is depicted. As the vehicle capacity is linked to the number of vehicles and to the amount of shipment, the importance of this parameter is recognized as well. Up to the definite point of the uncertainty level, there is no significant change; however, after this point a sudden decrease in the objective function takes place and after that there are no feasible solutions. Table 9 and Figure 8 illustrate and depict our results numerically and graphically.

Tab. 8. Comparison of outputs based on the uncertainty level of the maximum capacity of the facility parameter in terms of the time and OFV

Problem size $N{\times}M{\times}T{\times}V$	Uncertainty level (ρ_{Qmax})	OFV		Time (Sec.)	
		Deterministic	Robust	Deterministic	Robust
5×6×5×6	0	141220459	141220460	0.516	0.5
	0.1		141220460		0.516
	0.2		141220460		0.5
	0.3		141220460		0.484
	0.4		141220460		0.469
	0.5		141220460		0.515
	0.6		141220460		0.547
	0.7		141220460		0.516
	0.8		139176809		0.516
	0.9		Infeasible		Infeasible
	1		Infeasible		Infeasible

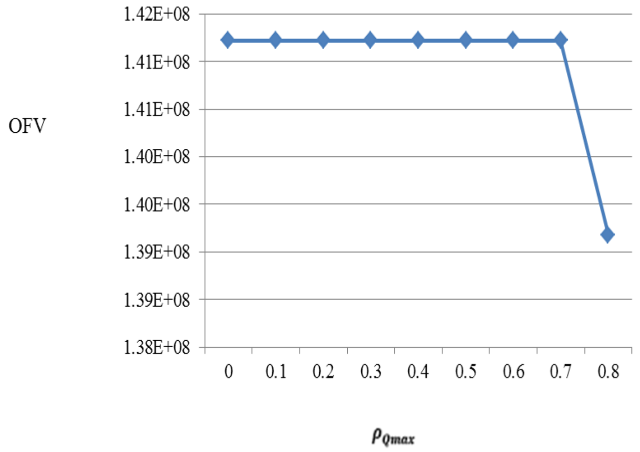

Fig. 7. Robust approach considering the impact of an indefinite level of the maximum capacity of the facility parameter

Tab. 9. Comparison of outputs based on the uncertainty level of the vehicle capacity parameter in terms of the time and OFV

Problem size $N{\times}M{\times}T{\times}V$	Uncertainty level (ρ_{cp})	OFV		Time (Sec.)	
		Deterministic	Robust	Deterministic	Robust
5×6×5×6	0	141220459	141220460	0.516	0.5
	0.1		141220460		0.5
	0.2		141220460		0.485
	0.3		141220460		0.484
	0.4		140748749		0.5
	0.5		141327484		0.5
	0.6		141220460		0.5
	0.7		141220460		0.5
	0.8		138752386		0.656
	0.9		Infeasible		Infeasible
	1		Infeasible		Infeasible

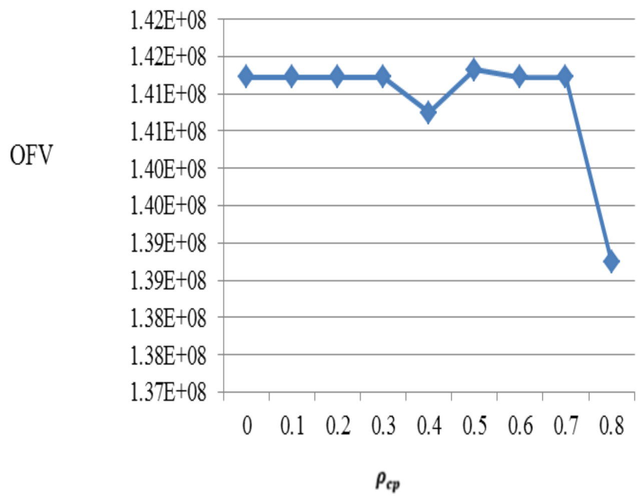

Fig. 8. Robust approach considering the impact of indefinite level of the vehicle capacity parameter

7. Conclusion

In this paper, a robust environment with a novel *p*-hub covering problem has been investigated. The presented model takes into account new features, such as production facilities located in hubs and vehicle transporters that ship commodities. These facilities are opened once through time horizons and can be reopened several times. An objective function involving covering, transportation, opening, reopening, activation and using vehicles has been taken into account. Because of satisfying demands and other hypotheses, some operational constraints have also been considered. To come close to reality, some strategic parameters are regarded as indefinite. A recent extension of robust theory is used to investigate and tackle these kinds of parameters. The computational results have explained the capability of this method for dealing with this uncertain environment. A wide sensitivity analysis has also been executed to recognize the performance of the model through various indefinite levels. The results have shown that, as a whole, an ascending pattern is observed; however, for some critical parameters some oscillations are depicted. The main reasons can be the trade-off between other costs and the impact of their operational constraints. In addition, for further research on solving the proposed mathematical model on a large scale, the use of heuristic and meta-heuristic algorithms is also proposed. Extending the related mathematical model and involving other practical aspects such as queuing theory and scheduling in it may also be relevant.

References

[1] Baron, O., Milner, J. Facility Location: A Robust Optimization Approach. Production and Operations Management Society 10, 2010, 1059–1478, 1937 – 5956.

[2] Ben-Tal, A., El-Ghaoui, L., Nemirovsky, A. Robust Optimization. Princeton University Press, Princeton 2009, NJ.

[3] Ben-Tal, A., Golany, B., Nemirovsky, A., Vial, J-P. Retailer supplier flexible commitments contracts – a robust optimization approach. Manufacturing and Service Operations Management 7, 2005, 248–271.

[4] Berman, O., Drezner, Z., Krass, D. Generalized coverage: New developments in covering location models. Computers & Operational Research, 37, 2010, 1675–1687.

[5] Calik, H., Alumur, S.A., Kara, B.Y., Karasan, O.E. A tabu-search based heuristic for the hub covering problem over incomplete hub networks. Computers & Operations Research, 36, 2009, 3088–3096.

[6] Carello, G., Della Croce, F., Ghirardi, M., Tadei, R. Solving the hub location problem in telecommunication network design: A local search approach. Networks, 44 (2), 2004, 94–105.

[7] Conteras, I., Cordeau, J.F., Laporte, G. Stochastic uncapacitated hub location. European Journal of Operational Research, 212, 2011, 518–528.

[8] Diast, J., Captivo, M.E., Climaco, J. Capacitated dynamic location problems with opening, closure and reopening of facilities. Journal of Management Mathematics, 17, 2006, 317–348.

[9] Ernst, A., Hamacher, H., Jiang H., Krishnamoorthy, M., Woeginger, G. Uncapacitated single and multiple allocation p-hub center problems. Unpublished Report, CSIRO Mathematical and Information Sciences, Australia, 2002.

[10] Ernst, A.T., Jiang, H., Krishnamoorthy, M. Reformulations and computational results for uncapacitated single and multiple allocation hub covering problems. Unpublished Report, CSIRO Mathematical and Information Sciences, Australia, 2005.

[11] Taghipourian, F., Mahdavi, I., Mahdavi-Amiri, N.,Makui, A. A fuzzy programming approach for dynamic virtual hub location problem. Applied Mathematical Modelling, 36(7), 2011, 3257–3270.

[12] Ghodsi, R., Mohammadi, M., Rostami, H. Hub covering location problem under capacity constraints. 4th International Conference on Mathematical Modeling and Computer Simulation: 2010, 204–208.

[13] Hamacher, H.W., Meyer, T. Hub cover and hub center problems. Working paper, Department of Mathematics, University of Kaiserslautern, Kaiserslautern, Germany, 2006.

[14] Kara, B.Y., Tansel, B.C. The single-assignment hub covering problem: Models and linearizations. Journal of the Operational Research Society, 54, 2003, 59–64.

[15] Karimi, H. & Bashiri, M. Hub covering location problems with different coverage types. Scientia Iranica, 18(6), 2011, 1571–1578.

[16] Levy, J.K. A case for sustainable security systems engineering: Integrating national, human, energy and environmental security. Journal of systems science and systems engineering, 18(4), 2009, 385–402.

[17] Lin, Y., Chen, J., Chen, Y. Backbone of technology evolution in the modern era automobile industry: an analysis by the patents citation network. Journal of Systems Science and Systems Engineering, 20(4), 2011, 416–442.

[18] Marianov, V., Serra, D. Hierarchical location-allocation models for congested systems. European Journal of Operational Research, 135, 2001, 195–208.

[19] Menou, A., Benallou, A., Lahdelma, R., Salminen, P. Decision support for centralizing cargo at a Moroccan airport hub using stochastic multi-criteria acceptability analysis. European Journal of Operational Research, 204, 2010, 621–629.

[20] Mohammadi, M., Tavakkoli-Moghaddam, R., Rostami, H. A multi-objective imperialist competitive algorithm for a capacitated hub covering location problem. International Journal of Industrial Engineering Computations, 2, 2011, 671–688.

[21] Mu, E. A unified framework for site selection and business forecasting using ANP. Journal of systems science and systems engineering, 15(2), 2006, 178–188.

[22] Pishvaee, M.S., Rabbani, M., Torabi, S.A. A robust optimization approach to closed-loop supply chain network design under uncertainty. Applied Mathematical Modelling, 35, 2011, 637–649.

[23] Qu, B., Weng, K. Path relinking approach for multiple allocation hub maximal covering problems. Computers and Mathematics with Applications, 57, 2009, 1890–1894.

[24] Sahraeian, R., Korani, E. The hierarchical hub maximal covering problem with determinate cover radius. International Conference on Industrial Engineering and Engineering Management, 2010, 1329–1333.

[25] Sim, T., Lowe, T.J., Thomas, B.W. The stochastic p-hub center problem with service-level constraints. Computers & Operations Research, 36, 2009, 3166–3177.

[26] Tan, P.Z., Kara, B.Y.A hub covering model for cargo delivery systems, Networks, 49, 2007, 28–39.

[27] Wagner, B. Model formulations for hub covering problems. Working paper, Institute of Operations Research, Darmstadt University of Technology, Darmstadt, Germany, 2004.

[28] Wang, J., Shu, Y.F. A possibilistic decision model for new product supply chain design. European Journal of Operational Research, 177, 2007, 1044–1061.

[29] Weng, Kerui, Yang, C., Ma, Y.F. Two artificial intelligence heuristics in solving multiple allocation hub maximal covering problem. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 4113 LNCS-I: 2006, 737–744.

[30] Yaman, H., Carello, G. Solving the hub location problem with modular link capacities. Computers & Operation Researches, 32 (12), 2006, 3227–3245.

[31] Yaman, H. Polyhedral analysis for the uncapacitated hub location problem with modular arc capacities. SIAM Journal on Discrete Mathematics, 19 (2), 2005, 501– 522.

[32] Yang, T.H. Stochastic air freight hub location and flight routes planning. Applied Mathematical Modelling, 33, 2009, 4424–4430.

[33] Yao, M.J., Hsu, H.W. A new spanning tree-based genetic algorithm for the design of multi-stage supply chain networks with nonlinear transportation costs. Engineering and Optimization, 10, 2009, 219–237.

[34] Mohammadi, M., Tavakkoli-Moghaddam, R., Ghodratnama, A., Rostami, H. Genetic and improved shuffled frog leaping algorithms for a 2-stage model of a hub covering location network. International Journal of Industrial Engineering & Production Research, 22(3), 2011, 179-187.

[35] Ghodratnama, A., Tavakkoli-Moghaddam, R., Azaron, A. A fuzzy possibilistic bi-objective hub covering problem considering production facilities, time horizons and transporter vehicles. The International Journal of Advanced Manufacturing Technology, 66(1-4), 2013, 187-206.

[36] Peiró, J., Corberán, A., Martí, R. GRASP for the uncapacitated r-allocation p-hub median problem. Computers & Operations Research, 43, 2014, 50–60.

[37] An, Y., Zeng, B., Zhang, Y., Zhao, L. Reliable p-median facility location problem: two-stage robust models and algorithms. Transportation Research Part B: Methodological, 64, 2014, 54–72.

[38] Liang, H. The hardness and approximation of the star p-hub center problem. Operations Research Letters, 41(2), 2013, 138–141.

[39] Yang, K., Liu, Y., Yang, G. An improved hybrid particle swarm optimization algorithm for fuzzy p-hub center problem. Computers & Industrial Engineering, 64(1), 2013, 133–142.

[40] Davari, S., Fazel Zarandi, M.H., Turksen, I.B. The incomplete hub-covering location problem considering imprecise location of demands. Scientia Iranica 20(3), 2013, 983–991.

[41] Hwang, Y.H., Lee, Y.H. Uncapacitated single allocation p-hub maximal covering problem. Computers & Industrial Engineering 63(2), 2012, 382–389.

[42] Vahdani, B., Tavakkoli-Moghaddama, R., Modarres, M., Baboli, A. Reliable design of a forward/reverse logistics network under uncertainty: A robust-M/M/c queuing model. Transportation Research Part E 48, 2012, 1152–1168