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Pricing Decisions for Complementary Products of Competitive Supply Chains

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KEYWORDS

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Multi-product,
Pricing, Stackelberg game,
Three-stage supply chain,
Elastic demand.

ABSTRACT

This study considers pricing, production and transportation decisions in a Stackelberg game between three-stage, multi-product, multi-source and single-period supply chains called leader and follower. These chains consist of manufacturers, distribution centers (DCs), and retailers. Competition type is horizontal and SC vs. SC. The retailers in two chains try to maximize their profit through pricing of products in different markets and with regard to the transportation and production costs. A bi-level nonlinear programming model is formulated in order to represent the Stackelberg game. Pricing decisions are based on discrimination pricing rules, where we can put different prices in different markets. After that, the model is reduced to a single-level nonlinear programming model by replacing Karush-Kuhn-Tucker conditions for the lower level (follower) problem. Finally, a numerical example is solved in order to analyze the sensitivity of effective parameters to price and profit, and some managerial insights are explained during this analysis.

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1. Introduction

Nowadays, markets in different regions are competitive and dynamic. Technology development, Globalization, urbanization and diversified customer expectations are changing the type of competitions from competitive independent firms to competitive supply chains (CSCs). [1] According to Deloitte Consulting report [2], experts of different large industries in America and Canada comprising automotive, high-tech, aerospace and consumer product industries believe that, instead of competition among individual firms, the whole SCs will compete against each other in the near future [1]. Taylor (2003) mentioned that "in the 21st century, being the best at producing or selling a superior product is no longer enough. Success now depends on assembling a team of companies that can rise above the win/loss negotiations of conventional trading relationships and work together to deliver the best products at the best price. Excellence in manufacturing is just the admission fee to be a player in the larger game of SC competition" [3].

In supply chain context, there are three kinds of competitions: competition among the firms of one tier of a supply chain (Horizontal competition in one SC) [4-7]; competition among the firms of different tiers of a supply chain (Vertical competition in one SC) [8-10];

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competition between rival supply chains (Horizontal competition between two SCs) [11-16]. There is not much analytic work in the literature that studies the interaction of multi-product supply chains and particularly, supply chain versus supply chain competition.

Most of the research works in the literature studied single-product problems which are far away from applicable real world problems. There are few who talk about multi-product problems in supply chain competition [13] [17-19]. Having an integrated insight into supply chain management was a great motivation to investigate multi-product competition. It is also important in multi-product supply chains to consider the correlations between the different product types of supply chains. These correlations could be substitutable or complementary. Most of the papers do not consider these two correlations simultaneously. Therefore, this is important to bring an integrated real insight into supply chain competition which considers correlations between products of multi-product competitive SCs.

There are many real-world applications for these types of supply chains such as computer industries, home appliances, etc. For example, in computer industry, we can consider desktop and laptop computers alongside with their printers, scanners and external hard drives. As seen, there is a simultaneous complementary and substitutable relationship between the products of supply chains.

In this paper, supply chain pricing problem is formulated, and the later competition with the existing rival chain is anticipated in markets. Following that type of the competition, we have designated a Stackelberg game between two chains in duopolistic markets. One of them is the leader of the market, and the other is the follower of him. Both chains can affect each other's demand by pricing decisions of different products in different markets. Leader and Follower are manufacturing similar and multiple products. In fact, we are trying to determine what is the best strategy for the market leader in order to maximize his profit considering the follower's optimal pricing decision encounters with him, and also his entire multi-product's demand in different markets. On the other hand, there is a complementary relationship between the products at each chain that can affect SC's success in different markets.

This paper is organized as follows: In Section 2, the literature of our problem is reviewed and the research gaps alongside with contributions are explained. In Section 3, the problem scope is described and the mathematical model is proposed. In Section 4, we represent the KKT description and formulation. In Section 5, a numerical example is presented with some sensitivity analyses. We conclude the paper in Section 6.

2. Literature Review

Since there is not so much literature on multi-product competitive supply chain [13] [17-19], we focus on similar research works.

In multi-product competitions: Zhang proposed a network economic model for SC vs. SC competition and comprises a heterogeneous multi-product supply chain competing for multiple markets. Then, he presented a variational inequality of the problem competing for multiple markets [13]. Mokhlesian and Zegordi considered an in-channel supply chain competition with two echelons: one manufacturer and several retailers. They investigated the coordination of inventory and pricing decisions in a competitive multi-product supply chain with different market powers. They proposed a multi-divisional nonlinear bi-level programming model considering the manufacturer as the upper level, and suggested a solution procedure based on GA [17]. Naimi Sadigh et al. proposed a Stackelberg game framework to a multi-product manufacturer-retailer supply chain where the demand of each product is jointly influenced by price and advertising expenditure. Several solution procedures, including an Imperialist Competitive Algorithm (ICA) and Evolution Strategy (ES) methods, have been proposed to solve Stackelberg games [18]. He et al. investigated an agent-based retail model (ARM), grounded in complex adaptive systems, which comprises two products and three types of agents: suppliers, retailers, and consumers. They derived the agent's optimal behaviors in response to competition by evaluating the evolutionary behavior of the ARM using optimization methods and genetic algorithm. They are seeking optimal pricing and inventory control policy with price-sensitive customers [19]. In ***single-product competitions***, Qian considered two competitive parallel distribution channels (PDCs), where the retailer plays as a leader and moves first, and the manufacturer is a follower with PDC one moving first, and PDC two moving next. She shows that the second-mover PDC has the advantage [20]. Baron et al. [21] studied the Nash equilibrium of two supply chains, each being composed of one manufacturer and one retailer by extending the seminal work of McGuire and Staelin [22]. They showed that both the traditional Manufacturer's Stackelberg (MS) and the Vertical Integration (VI) strategies are special cases of Nash bargaining on the wholesale price. Anderson and Bao investigated price competition having a linear demand function with deterministic parameters. They assumed that there are supply chains competing in the market with substitutable products, each having one manufacturer and one retailer. They studied the effect of varying the level of price competition on profits of the industry participants [23]. Boyaci and Gallego investigated a cross-channel competition between two SCs with one manufacturer and retailer. They modeled customer service competition using game theoretical concepts [24]. Khojasteh et al. developed a price competition model for a new supply chain that competes in a market with some rivals. This supply chain has a risk-neutral manufacturer as a leader and one risk-averse retailer as the follower. They finally obtained the optimal wholesale and retail prices in a real-world case [25]. Rezapour and Zanjirani Farahani proposed a bi-level model for strategic design of competing in centralized single-product supply chain networks with deterministic demands. They derived equilibrium conditions with the establishment of finite-dimensional inequality formulation and solved it using a modified projection method [26]. Rezapour et al. investigated a sequential game between two rival supply chains with single-product SC vs. SC competition and deterministic demand, considering that the new entrant supply chain has location decisions. Two strategies were represented: Stackelberg and minimum regret. The linear binary bi-level model was solved by combinatorial meta-heuristic [11]. Rezapour et al. proposed a bi-level model in order to represent the competition between two single-product SCs with probabilistic demand and distance as competition criterions. They solved the bi-level model using Nash equilibrium and an exact meta-heuristic algorithm [12]. Azari Khojasteh et al. investigated a price competition between two leader-follower single-product supply chains with one manufacturer and one retailer. They derived the optimal conditions in a Stackelberg game, and found that the follower has an advantage when the products are highly substitutable [27]. Esmaeili et. al. considered pricing and advertising decisions simultaneously for a three-level supply chain. The amount of market demand is influenced by pricing and advertising decisions. In this paper, three well-known approaches in game theory, i.e., Nash, Stackelberg, and cooperative games, are exploited to study the effects of pricing and advertising decisions [28]. Jafari et al. proposed a dual channel supply chain including one manufacturer and two retailers. A game theory approach is developed to analyze pricing decisions under centralized and decentralized scenarios. Finally, the equilibrium decisions are discussed and some managerial insights are revealed [29].

Research gaps:

As can be seen from the above reviewed literature, there exist some research gaps.

- The first one exists in multi-product competitive supply chain context. There is no concern of horizontal competition between multi-product supply chains.
- The second gap is not using the complementary and substitutable relationship simultaneously in a multi-product competition between SCs.

According to these explanations, this paper contributes to competitive supply chain in two ways:

- First, in today's fierce markets, there should be an integrated view of competition between big brands in world's market. So, all of the products that have been produced by one brand can be very helpful for understanding the competition complexity in today's market.
- Second, this paper tries to represent the complementary relationship between the products of one specific chain alongside with substitutable products of another chain that could have a noticeable impact on chain's demands.

3. Problem Description

In this section, the echelons of the centralized supply chain network structure, competition type, demand function, and mathematical model will be introduced. Our chain includes three stages: producers, distribution centers (DCs) and retailers. In this network structure, markets (customers) are demand points as shown in Figure 1. Our model includes decision-making for tactical and operational levels. Determination of the amounts of products transported to customers makes our tactical decisions. After that, pricing of products in different markets sets the decisions at the operational level.

We will continue to explain pricing decisions within the introduction of demand function.

3-1. Competition type

In general, our competition structure is competition with foresight. This type of competition helps industries in anticipating later reaction of existing or new rivals to keep their market shares and incomes. It might be interesting for the industries not to needlessly invest in a region in which investment will be lost to the followers, anyway. The situation becomes quite different when a market becomes aware of other rivals entering it soon afterwards. It will then be necessary to make decisions with foresight about this competition, which itself will enter a market where competition is already present. These games have potential applications for network routing and pricing in transportation systems, competitive designing with foresight, and many others [11].

Considering duopoly in our markets, we are trying to design a sequential Stachkelberg game between two competitors, called Leader and Follower. Competition structure in our model is a horizontal competition between two SCs. In fact, pricing decisions of each chain in different markets and for different products can affect our competition. Because of our competition type, we are going to model the competition between two SCs in the form of bi-level programming.

3-2. Demand function

It is assumed that the demand function is linear and price-dependent. Self-price, competitor's price, and complement prices affect the demand of each product in different markets. In multi-product case, the linear function for leader's chain is shown as follows: [17]

$$D^L_{mr} = a^L_{mr} - b^L_{mrr}P^L_{mr} + d^F_{mr}P^F_{mr} - \sum_{l\neq r} b^L_{mlr}P^L_{ml} \qquad (1)$$

where a^L_{mr} is the potential demand matrix for market $m = 1..M$ and product $r = 1..R$ of the leader's chain. Also, b^L_{mlr} is the matrix of price sensitivity coefficients with $b^L_{mrr} < 0$, which means that increasing the price of each product leads to the decrease in its demand. The sign of $b^L_{mlr}, l \neq r$ for the complement products is $b^L_{mlr} < 0$ due to the negativity in cross-price elasticity for complementary products. It shows that an increase in the price of each product will decrease the demand of its complements, if $b^L_{mlr} \neq 0$. After that, d^F_{mr} is the matrix of follower's coefficients which affects leader's demand for product *r* in market *m*. In fact, this is the cross-price elasticity for substitutable products. Also, P_{mr} denotes the price of product *r* for market *m* in both leader (P^L_{mr}) and follower's (P^F_{mr}) demand. Therefore, as the leader, we can define the follower's demand function as follows:

$$D^F_{mr} = a^F_{mr} - b^F_{mrr}P^F_{mr} + d^L_{mr}P^L_{mr} - \sum_{l\neq r} b^F_{mlr}P^F_{ml} \qquad (2)$$

As explained, the price of each product has different quantities in different markets by the same provider. Pricing strategy of this paper is the third-degree discriminatory pricing, where each chain can charge different prices to different groups of customers. Since the model's markets are demand points, obviously different groups of customers can be defined.

3-3. Mathematical model

In this study, we consider two supply chains, called leader and follower, with three levels. At the first level, we have many producers that transport their products to distribution centers (DCs) in the second level. Then, we have retailers in the third level that receive products from DCs and transport them to the customers in different markets. There exists a horizontal competition between two supply chains in retailer's level. However, it is worth noting that there is no competition between the retailers of one specific supply chain. According to the Stackelberg game, we formulate the problem as a bi-level model. The upper level is the leader of the market and tries to maximize his profit considering the prices of his own products (both self-price and complement-price effects) and the price of the follower's products. Similarly, we have the lower level which contains the maximization of follower's profit considering competitor's prices and the prices of his own.

3-3-1. Assumptions

Here are the assumptions used for the development of the mathematical model:

- Demand of each product in each market for both upper and lower levels follows a linear function of self-price, competitor's price, and the complement-prices of their own chain.
- For both chains, the model is incapacitated.
- For both chins, shortage is not permitted.
- Probabilistic behavior of customers (according to price and transportation cost) causes different demands for the retailers of two chains.
- Each customer is free to purchase his favorite products from different retailers in any volume.

- For both Chains, the transportation cost in the third-stage is meant to be the distance of each retailer from different markets (customers).

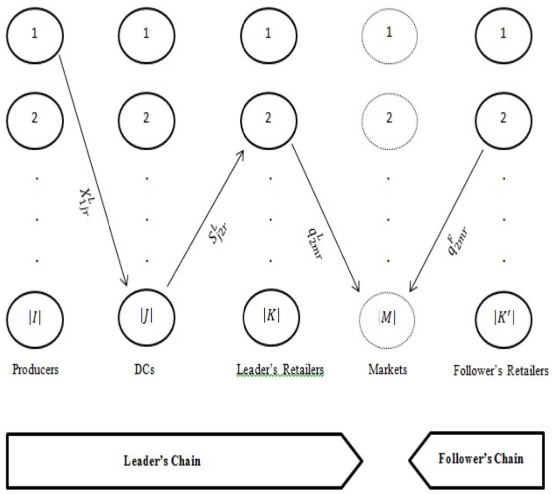

Fig.1. SC structure for both Leader and Follower

- Competition is horizontal between retailers of two chains.

3-3-2. Notations

Parameters and variables of the upper level (Leader):

i Index of manufacturers
j Index of DCs
k Index of retailers
m Index of demand points (markets)
r Index of products
PC^L_{ir} Production cost for producer i and product r
$TC1^L_{ijr}$ Transportation cost of product r from producer i to DC j
$TC2^L_{jkr}$ Transportation cost of product r from DC j to retailer k
$TC3^L_{kmr}$ Transportation cost of product r from retailer k to customer (market) m
X^L_{ijr} Decision variable: Transportation variable for amounts of product r transported from producer i to DC j
S^L_{jkr} Decision variable: Transportation variable for amounts of product r transported from DC j to retailer k
P^L_{mr} Decision variable: retail price of product r to be charged on market m by leader's chain.
$q^L{}_{kmr}$ Decision variable: quantity of product r transported to customer (market) m by retailer k in leader's chain.

Parameters and variables of the lower level (Follower):

i' Index of manufacturers
j' Index of DCs
k' Index of retailers
$PC^F_{i'r}$ Production Cost for producer i' and product r
$TC1^F_{i'j'r}$ Transportation Cost of product r from producer i' to DC j'
$TC2^F_{j'k'r}$ Transportation Cost of product r from DC to retailer k'
$TC3^F_{k'mr}$ Transportation Cost of product r from retailer k' to customer (market) m
$X^F_{i'j'r}$ Decision variable: Transportation variable for amounts of product r transported from producer i' to DC j'
$S^F_{j'k'r}$ Decision variable: Transportation variable for amounts of product r transported from DC j' to retailer k'
P^F_{mr} Decision variable: retail price of product r to be charged on market m by follower's chain.
$q^F{}_{k'mr}$ Decision variable: quantity of product r transported to customer (market) m by retailer k' in leader's chain.

3-3-3. Bi-level model

According to the assumption and notations described, the bi-level model is presented as follows:

Leader's model (upper level):

$$Max\ f_L = \sum_m \sum_r P^L_{mr} D^L{}_{mr} - \sum_i \sum_j \sum_r PC^L_{ir} X^L_{ijr} - \sum_i \sum_j \sum_r TC1^L_{ijr} X^L_{ijr} - \sum_j \sum_k \sum_r TC2^L_{jkr} S^L_{jkr} - \sum_k \sum_m \sum_r TC3^L{}_{kmr} q^L{}_{kmr} \qquad (3)$$

St:

$$D^L_{mr} = a^L_{mr} - b^L_{mrr} P^L_{mr} + d^F_{mr} P^F_{mr} - \sum_{l \neq r} b^L_{mlr} P^L_{ml} \qquad \forall\ m, r \qquad (4)$$

$$\sum_m q^L_{kmr} \leq \sum_j S^L_{jkr} \qquad \forall\ r\ , k \qquad (5)$$

$$\sum_k S_{jkr}^L \leq \sum_i X_{ijr}^L \qquad \forall j, r \tag{6}$$

$$\sum_k q_{kmr}^L = D_{mr}^L \qquad \forall m, r \tag{7}$$

$$X_{ijr}^L, S_{jkr}^L, P_{mr}^L, q_{kmr}^L, D_{mr}^L \geq 0 \qquad \forall i, j, k, m, r \tag{8}$$

Follower's model (lower level):

$$Max\ f_F = \sum_m \sum_r P_{mr}^F D_{mr}^F - \sum_{i'} \sum_{j'} \sum_r PC_{i'r}^F X_{i'j'r}^F - \sum_{i'} \sum_{j'} \sum_r TC1_{i'j'r}^F X_{i'j'r}^F - \sum_{j'} \sum_{k'} \sum_r TC2_{j'k'r}^F S_{j'k'r}^F - \sum_{k'} \sum_m \sum_r TC3^F{}_{k'mr} q_{k'mr}^F \tag{9}$$

St:

$$D_{mr}^F = a_{mr}^F - b_{mrr}^F P_{mr}^F + d_{mr}^L P_{mr}^L - \sum_{l \neq r} b_{mlr}^F P_{ml}^F \qquad \forall m, r \tag{10}$$

$$\sum_m q_{k'mr}^F \leq \sum_j S_{jk'r}^F \qquad \forall r, k' \tag{11}$$

$$\sum_{k'} S_{j'k'r}^F \leq \sum_i X_{i'j'r}^F \qquad \forall j', r \tag{12}$$

$$\sum_{k'} q_{k'mr}^F = D_{mr}^F \qquad \forall m, r \tag{13}$$

$$D_{mr}^F + D_{mr}^L \leq a_{mr}^L + a_{mr}^F \qquad \forall m, r \tag{14}$$

$$X_{i'j'r}^F, S_{j'k'r}^F, P_{mr}^F, q_{k'mr}, D_{mr}^F \geq 0 \qquad \forall i', j', k', m, r \tag{15}$$

Objective functions (3) and (9) maximize the annual profit of leader and follower's chain, respectively. Constraint sets (4) and (10) show the linear demand function of leader and follower, respectively. Constraint sets (5)-(6) and (11)-(12) consider network balancing for leader and follower. Constraint sets (7) and (13) show the demand assignment to the retailers of both chains. Constraint set (14) controls the demand of both chains, not to exceed the potential demand of each market for each product. Last constraint sets (8) and (15) represent the allowed sign of variables in both chains.

4. KKT conditions

In this section, we propose Karush-Kuhn-Tucker conditions for our bi-level model. According to KKT conditions for bi-level problems, we must have convexity conditions for follower's constraints. After that, we must consider the concavity of the follower's objective function. Bi-level programming is formulated as follows [30]:

$$\max_x f_L(x, y) \quad s.t \tag{16}$$

$$G(x, y) \leq 0 \tag{17}$$

$$H(x, y) = 0 \tag{18}$$

$$\max_y f_F(x, y) \quad s.t \tag{19}$$

$$g_i(x, y) \leq 0 \qquad i = 1, \dots, m \tag{20}$$

$$h_j(x, y) = 0 \qquad j = 1, \dots, l \tag{21}$$

$$x \geq 0\ ,\ y \geq 0 \tag{22}$$

Considering the presented bi-level model and the KKT conditions, the single-level model is as follows [17, 30]:

$$\max_x f_L(x, y)$$

$$s.t$$

$$G(x, y) \leq 0$$

$$H(x, y) = 0$$

$$\nabla f_{F_{y^*}}(x, y^*) = \sum_{i=1}^{m} u_i \nabla g_{i_{y^*}}(x, y^*) + \sum_{j=1}^{l} \lambda_j \nabla h_{j_{y^*}}(x, y^*) \tag{23}$$

$$u_i g_i(x, y^*) \leq 0 \quad i = 1, \dots, m \tag{24}$$

$$g_i(x, y) \leq 0 \qquad i = 1, \dots, m$$

$$h_j(x, y) = 0 \qquad j = 1, \dots, l$$

$$x \geq 0\ ,\ y \geq 0$$

$$u_i \geq 0 \qquad i = 1, \dots, m \tag{25}$$

where u_i and λ_j are the values of dual variables and Lagrangian multipliers; (x, y) are the decision variables. Equations (23) and (24) satisfy the optimality conditions for the lower level problem, called follower in our model.

Stackelberg equilibrium strategy for the upper level (leader) problem is achieved by the assumption that the lower level (follower) plays his optimal strategy. Achieving the KKT conditions for the lower level can convert our bi-level model into a single level. Thus, we can guarantee the globality of the optimum solution because of the convex constraints and concave objective functions in both levels.

Clearly, constrained convexity of leader and follower is proven because of the continuous variables and linear constraints.

Lemma. Objective functions $\mathrm{f_L}$ and $\mathrm{f_F}$ are concave functions.

Proof. Concavity of the objective functions can be proven using negative definite Hessian matrix of the objective functions.

We know that the summation of multiple concave functions will be a concave function. So, it is only necessary to prove the concavity of the first nonlinear statement of the objective functions. Thus, using the Hessian matrix for follower, if we put $(m = 3, r = 2)$, we will have the following statement for the first term of the objective function:

$$P_{11}(D_{11}) + P_{12}(D_{12}) + P_{21}(D_{21}) + P_{22}(D_{22}) + P_{31}(D_{31}) + P_{32}(D_{32}) \tag{26}$$

According to equations (4) and (10), we can replace the following statements with their equals. After that, we have the following Hessian matrix for $P_{11}, P_{12}, P_{21}, P_{22}, P_{31}, P_{32}$ according to Table 2 (price coefficients):

$$\begin{bmatrix} -2b_{111}^F & -b_{121}^F - b_{112}^F & 0 & 0 & 0 & 0 \\ -b_{121}^F - b_{112}^F & -2b_{122}^F & 0 & 0 & 0 & 0 \\ 0 & 0 & -2b_{211}^F & -b_{221}^F - b_{212}^F & 0 & 0 \\ 0 & 0 & -b_{221}^F - b_{212}^F & -2b_{222}^F & 0 & 0 \\ 0 & 0 & 0 & 0 & -2b_{311}^F & -b_{321}^F - b_{312}^F \\ 0 & 0 & 0 & 0 & -b_{321}^F - b_{312}^F & -2b_{322}^F \end{bmatrix}$$

Now, b_{mlr}^F values can be replaced in Hessian matrix; after that, the following eigenvalues can be driven by Matlab software:

$$eig\left(\begin{bmatrix} -3 & -0.8 & 0 & 0 & 0 & 0 \\ -0.8 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & -0.95 & 0 & 0 \\ 0 & 0 & -0.95 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & -0.94 \\ 0 & 0 & 0 & 0 & -0.94 & -3 \end{bmatrix}\right) = \begin{bmatrix} -3.95 \\ -3.94 \\ -3.8 \\ -2.2 \\ -2.06 \\ -2.05 \end{bmatrix}$$

Clearly, we can conclude that this symmetric matrix is negative definite due to the negative eigenvalues of Hessian matrix. Consequently, the follower's objective function is concave. The same applies to the leader's objective function.

$$\begin{aligned} Max\ f_L = & \sum_m \sum_r P_{mr}^L \left(a_{mr}^L - b_{mrr}^L P_{mr}^L + d_{mr}^F P_{mr}^F - \sum_{l \neq r} b_{mlr}^L P_{ml}^L\right) - \sum_i \sum_j \sum_r PC_{ir}^L X_{ijr}^L \\ & - \sum_i \sum_j \sum_r TC1_{ijr}^L X_{ijr}^L - \sum_j \sum_k \sum_r TC2_{jkr}^L S_{jkr}^L \\ & - \sum_k \sum_m \sum_r TC3^L{}_{kmr} q^L{}_{kmr} \end{aligned} \tag{27}$$

St:

$(4) - (7)$

According to these assumptions, the proposed converted KKT single-level model will be presented as follows:

$$a^L_{mr} - b^L_{mrr}P^L_{mr} + d^F_{mr}P^F_{mr} - \sum_{l\neq r} b^L_{mlr}P^L_{ml} \geq 0 \qquad \forall\, m, r \tag{28}$$

$$\sum_m \sum_r (\, a^L_{mr} - b^L_{mrr}P^L_{mr} + d^F_{mr}P^F_{mr} - \sum_{l\neq r} b^L_{mlr}P^L_{ml} - \sum_{l\neq r} b^L_{mrl}P^L_{ml} + \lambda_{mrr}b^F_{mrr} - \sum_{l\neq r} \lambda_{mrl}b^F_{mrl} - u3_{mrr}b^F_{mrr} + \sum_{l\neq r} u3_{mrl}b^F_{mrl} + u4_{mrr}b^F_{mrr} - \sum_{l\neq r} u4_{mrl}b^F_{mrl} - u4_{mrr}d^F_{mr}) = 0 \tag{29}$$

$$-|J'| * \sum_{i'} \sum_r PC^F_{i'r} - \sum_{i'} \sum_{j'} \sum_r TC1^F_{i'j'r} + |I'| * \sum_{j'} \sum_r u2_{j'r} = 0 \tag{30}$$

$$-\sum_{j'} \sum_{k'} \sum_r TC2^F_{j'k'r} - |J'| * \sum_{k'} \sum_r u2_{k'r} - |K'| * \sum_{j'} \sum_r u2_{j'r} = 0 \tag{31}$$

$$-\sum_{k'} \sum_m \sum_r TC3^F{}_{k'mr} + |K'| * \sum_m \sum_r \lambda_{mrr} - |M| * \sum_{k'} \sum_r u1_{k'r} = 0 \tag{32}$$

$$u1_{k'r}\left(\sum_m q^F_{k'mr} - \sum_j S^F_{j'k'r}\right) = 0 \qquad \forall\, k', r \tag{33}$$

$$u2_{j'r}\left(\sum_{k'} S^F_{j'k'r} - \sum_i X^F_{i'j'r}\right) = 0 \qquad \forall\, j', r \tag{34}$$

$$u3_{mrr}\left(-a^F_{mr} + b^F_{mrr}P^F_{mr} - d^F_{mr}P^L_{mr} + \sum_{l\neq r} b^F_{mlr}P^F_{ml}\right) = 0 \qquad \forall\, m, r \tag{35}$$

$$u4_{mrr}(\left(a^L_{mr} - b^L_{mrr}P^L_{mr} + d^L_{mr}P^F_{mr} - \sum_{l\neq r} b^L_{mlr}P^L_{ml}\right) + \left(a^F_{mr} - b^F_{mrr}P^F_{mr} + d^F_{mr}P^L_{mr} - \sum_{l\neq r} b^F_{mlr}P^F_{ml}\right) - (a^L_{mr} + a^F_{mr})) = 0 \qquad \forall\, m, r \tag{36}$$

$$a^F_{mr} - b^F_{mrr}P^F_{mr} + d^F_{mr}P^L_{mr} - \sum_{l\neq r} b^F_{mlr}P^F_{ml} \geq 0 \tag{37}$$

$(10) - (14)$

$$X^F_{i'j'r}, S^F_{j'k'r}, P^F_{mr}, q^F_{kmr}, X^L_{ijr}, S^L_{jkr}, P^L_{mr}, q^L_{kmr}, u1_{k'r}, u2_{j'r}, u3_{mrr}, u4_{mrr} \geq 0 \qquad \forall\, i, j, k, m, r, i', j', k' \tag{38}$$

Equation (27) is the leader's objective function subject to the mentioned leader's constraints (D_{mr}^{L} and D_{mr}^{F} are substituted by their equal values from constraints (4) and (12)). Constraint sets (28) and (37) prevent the demand violation. Constraints (29)-(36) satisfy the optimality conditions of follower's model. Finally, constraint (38) shows the allowed sign of the variables in both models.

5. Numerical Example

In this section, a numerical example is solved using the proposed final KKT transformed model. Solving procedure is performed by a personal computer with Intel ® core™ i3 CPU 2.27 GHz and 6GB RAM, and the software GAMS 24.1.2 with Baron solver was used to solve the proposed NLP problem. The execution time was about 8 seconds.

Inputting the indices $m = 3,\ r = 2,\ i = 2,\ j = 3\ , k = 2,\ i' = 2,\ j' = 3\ ,\ k' = 2$ for the number of markets, products, leader's producers, DCs, retailers, follower's producers, DCs, and retailers, gives us a small-sized problem, in turn.

The potential demand of each product in each market for both chains is shown in Table 1.

Tab. 1. Potential demand for both chains

a_{mr}	Product 1	Product 2
a_{1r}^{L}	1,500,000	1,000,000
a_{2r}^{L}	1,000,000	1,500,000
a_{3r}^{L}	1,200,000	1,500,000
a_{1r}^{F}	1,700,000	1,000,000
a_{2r}^{F}	1,300,000	1,100,000
a_{3r}^{F}	1,300,000	1,200,000

Also, price-sensitive coefficients (b_{mrr}^{L}, b_{mrr}^{F}) and (d_{mr}^{L}, d_{mr}^{F}) are randomly generated as summarized in Table 2 [17].

Tab. 2. Price-sensitive coefficients

Coefficient type	Coefficient range
b_{mrr}^{L}	U(0.5,1.5)
b_{mrr}^{F}	U(0.8,1.3)
d_{mr}^{L}	U(0.9,1.2)
d_{mr}^{F}	U(0.3,1.6)

Production and transportation costs are randomly generated as shown in Table 3 [17].

Tab. 3. Costs of both chains

Type of cost	Cost range
$TC1$	U(100,200)
$TC2$	U(150,500)
$TC3$	U(100,700)
PC	U(10,30)

5-1. Execution

In this subsection, the implementation results of GAMS run are shown in tables (4)-(6).

Tab. 4. Prices and profits of both chains for different products and markets

P_{mr}^{L}	$r = 1$	$r = 2$
$m = 1$	686,598	223,830
$m = 2$	359,582	538,127
$m = 3$	343,365	535,785
Leader's Profit	2,462,827,182,430	
P_{mr}^{F}	$r = 1$	$r = 2$
$m = 1$	737,657	219,244
$m = 2$	462,069	364,262
$m = 3$	375,011	479,479
Follower's Profit	2,414,480,188,480	

These tables consist of optimal price with profits, optimal demands, and optimal market assignments, respectively. Table 5 shows the demands of each market for each product taken by both chains. The differences between prices in various markets can be seen in this table. On the other hand, because of the follower's greater complementary impact of product 1 on product 2 in the first market (b_{112}^F), P_{12}^F cannot increase more than P_{12}^L, and the opposite analysis is taken for P_{11}^F. This price can increase more than P_{11}^L because of $b_{112}^F < b_{112}^L$.

As seen in Table 5, the mentioned market has greater demand for the follower in product 1 and lower demand of product 2.

Demand assignment of each chain is shown by q^L and summarized in Table 6.

These variables represent the amount of products transported to markets according to the transportation costs (TC3). It is worth noting that the price of each product is set by each chain in different markets, and each retailer is forced to follow the chains' prices in different markets. Thus, distance criteria (TC3) can affect customers' choice of a retailer in one specific market.

Tab. 5. Demand share of chains for different products and markets

D_{mr}^L	$r = 1$	$r = 2$
$m = 1$	1,118,228	608,859
$m = 2$	726,745	967,644
$m = 3$	727,780	990,508
D_{mr}^F	$r = 1$	$r = 2$
$m = 1$	1,214,339	548,518
$m = 2$	801,625	845,488
$m = 3$	772,436	905,140

Tab. 6. Demand assignment to leader and follower (regarding the TC3 costs)

q_{kmr}^L	k = 1 (Retailer 1)		k = 2 (Retailer 2)	
	r=1	r=2	r=1	r=2
m=1	-	608,859	1,118,228	-
m=2	-	-	726,745	967,644
m=3	-	-	727,780	990,508
$q_{k'mr}^F$	$k' = 1$		$k' = 2$	
	r=1	r=2	r=1	r=2
m=1	-	548,518	1,214,339	-
m=2	-	845,488	801,625	-
m=3	-	-	772,436	905,140

5-2. Sensitivity analysis

In this subsection, the sensitivity of b_{mrr} and d_{mr} is investigated for both chains in order to represent the verification of the demand function. First, the self-price coefficient was multiplied by 0.5 and, also simultaneously, we have doubled the cross-price for substitutable products. Good results were obtained as shown in Table 7 on leader's analysis. These results show that the leader can defeat his rival through decreasing self-price sensitivity and increasing that for cross-price. We have 514% profit improvement for leader and 184% improvement for follower.

After that, the complementary coefficient for leader was investigated. Thus, b_{112}^L was multiplied by 5. It means that product 1 has significant impact on product 2 in leader's chain, and price increase of product 1 will cause a huge decrease in the demand of product 2.

Tab. 7. Prices and profits of both chains after changing the coefficients

P_{mr}^L	$r = 1$	$r = 2$
$m = 1$	2,500,000	-
$m = 2$	-	2,432,494
$m = 3$	-	2,329,132
Leader's Profit	15,121,860,608,200	
P_{mr}^F	$r = 1$	$r = 2$

$m = 1$	1,400,593	-
$m = 2$	124,415	976,344
$m = 3$	45,012	1,240,438
Follower's Profit	6,863,420,661,100	

The result shows that all price and demand changes are in the market due to the coefficient increase in market 1. Thus, the demand of product 2 in that market went down to zero due to the huge price effect of his complement (Product 1). It is worth noting that a huge complementary relationship between two products causes a decrease in supply chain's profit in comparison with a normal one, especially when we have lower self-price sensitive demands. Therefore, it is recommended for the managers not to charge the product prices with big difference when we have complementary products and lower self-price coefficients.

6. Conclusions

This paper developed a bi-level programming model for the Stackelberg game between leader and follower supply chains under a linear price-dependent demand. In this model, products of one SC can compete with the same products of another SC with a substitutable and complementary relationship between them. This study also considered production, transportation, and pricing decisions in multi-product competitive leader-follower supply chains. KKT approach was used in order to find the optimal conditions of lower level problem. Finally, a numerical example was solved in order to analyze some sensitive parameters of the model. The results show a notable profit growth, when we decrease self-price and increase the cross-price (replaceable) coefficients. The managers of supply chains might try to reduce self-price by developing brand loyalty for their products and employing marketing activities that lead to more frequent brand switching in order to increase the cross-price.

For the future research, this method can be used for designing the networks of reverse, close-loop, and open-loop SCs. One can extend this model to include more competitor SCs in the markets. In addition, there could be more criteria with respect to the demand function. Inventory decision and back-order products can be investigated in the future.

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