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RESEARCH PAPER

Routing Flexibility for Unequal-Area Stochastic Dynamic Facility Layout Problem in Flexible Manufacturing Systems

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ABSTRACT

Any manufacturing system must be consistent with potential changes such as fluctuations in demand. Uncertainty also makes the above issue more essential. Routing Flexibility (RF) is one of the necessities to any modern manufacturing system such as Flexible Manufacturing System (FMS). This paper proposed three mixed integer nonlinear programming models for the Unequal-Area Stochastic Dynamic Facility Layout Problems (UA–SDFLPs) by considering the Routing Flexibility. The models were proposed when the independent demands followed the random variable with Poisson, Exponential, and Normal distributions. For the validation of the proposed models, many small-sized test problems derived from a real case in the literature were solved. The large-sized test problems were solved by Genetic Algorithm (GA) in a reasonable amount of computational time. The obtained results indicated that the discussed models for the UA–SDFLPs were valid and the managers could take these models to the manufacturing floor to adapt to the potential changes in today's competitive market.

KEYWORDS: *Uncertainty; Routing flexibility; Flexible manufacturing system; Unequal–area stochastic dynamic facility layout problems; Genetic algorithm.*

1. Introduction

A Facility Layout Problem (FLP) is related to managing facilities that contain the Material Handling Cost (MHC). A facility can be a physical item such as a manufacturing cell or a work center, etc. [1]. The facilities have different area requirements in ordinary situations. The MHC is induced by 20% –50% of the Total Manufacturing Cost (TMC) and a fit facility layout can diminish the cost from 10% to 30% [2]. The physical facility layout can affect operational performances such as throughput rate, manufacturing lead time, and Work-In-Process (WIP) [3]. It takes a substantial cost when a facility layout is installed. An inefficient facility layout can add as much as 36% to the MHC [4]. Because of these reasons, designing a suitable facility layout is obligatory.

Regarding the area of the facility, the FLP can be categorized into two parts: an Equal Area-FLP (EA-FLP) and an Unequal Area-FLP (UA-FLP). In the past, it was assumed that all of the facilities had a similar area. In real world, facilities of equal size are rarely possible. Thus, in recent cases, each of these facilities may have different areas. Moreover, the solution of the UA-FLP can be applied to some cases like macrocell placement [5] and very large-scale integration design [6]. Thus, it is highly justified that UA-FLP is studied. A schematic illustration of FLP types is depicted in Fig1. The upper half of the figure shows the EA-FLP and the lower half illustrates the UA-FLP.

Regarding changes in demand, the FLPs can be divided into three main categories. These categories are Static Facility Layout Problems (SFLPs), Dynamic Facility Layout Problems (DFLPs), and Stochastic Facility Layout Problems (STFLPs) [7]. In the FLP, it is regularly assumed that the material flow among facilities occurs through the centroid of two facilities. If the flow of materials does not change in the planning horizon, then the SFLPs can be used. When demand changes, the flow of materials among facilities alters consequently. At this time,

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the facility layout needs to be rearranged to make it efficient. Minimizing both of the MHCs and Rearrangement Cost (RC) is known as a DFLP [8].

The static facility layout problem (single period) with uncertainty in demand is turned into Stochastic Static Facility Layout Problem (SSFLP). Moreover, the dynamic facility layout problem (multi-period) is called Stochastic Dynamic Facility Layout Problem (SDFLP) when uncertainty exists in demand. In SSFLP, similar to FLP, an optimal facility layout is designed in an entire planning horizon by minimizing the MHC. One of the purposes of the SDFLP is to obtain an optimum layout for each period so that the MHC and the RC can be minimized simultaneously [9]. The demands are specified as independent chance variables with a known Probability Density Function (PDF), which alters from one period to another, randomly. In this type of model, a decision-maker that considers a level of uncertainty in demands by Confidence Level (α). As is clear, the RF is one of the principles of Flexible Manufacturing Systems (FMS). Flexible facility design is defined as the ability to adjust the changes without affecting performance considerably. The RF is determined as a measure of the average number of choices of the machine that an individual product can choose. However, the RF in a layout is its ability to produce a part by alternative routes through the system [10]. Flexibility is precious because it can sometimes facilitate productivity. When the performance of a facility or an infrastructure system is evaluated, flexibility should be considered. In other words, the definition of performance should connect to the value of flexibility [11].

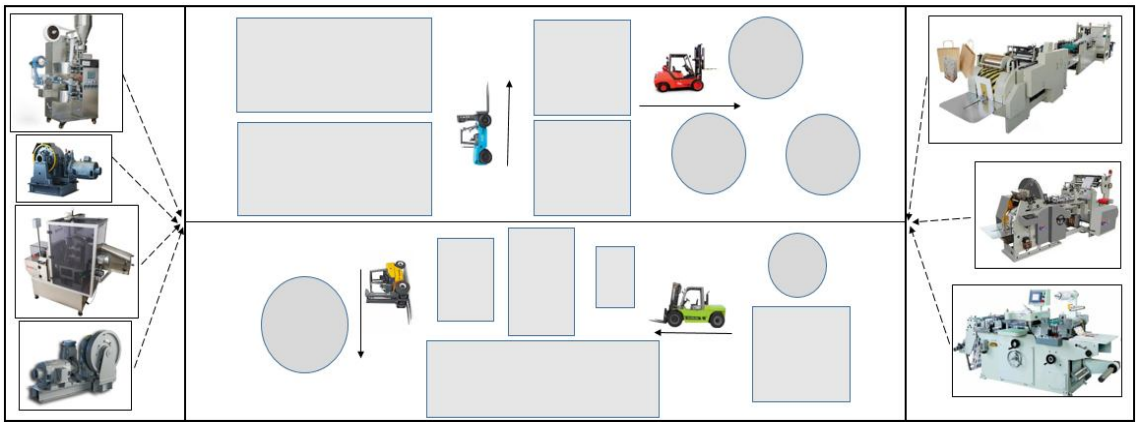

Fig. 1. Schematic illustration of types FLP (according to the area).

In fact, this paper is an extension of the referenced papers [7,9]. For real-world simulation, the proposed models are presented in UA-SDFLPs with three distribution random variables including Poisson, Exponential, and Normal. Moreover, the time value of money and RF is added to the SDFLPs, when each of the facility is located in a rectangular shape and fixed dimensions. Section 2 briefly surveys the literature related to the proposed models. In Section 3, the proposed models in UA-SDFLP are introduced. The independent demands are stochastic with known variance and expected value. In addition to the Normal distribution function, the UA-SDFLPs are modeled using the Poisson and Exponential distributions functions.

These problems are one of the most practical and realistic problems among the FLPs and real-condition. To validate the proposed models, computational experiments are provided in Section 4. These are derived from literature. Also, Genetic Algorithm (GA) was applied to solve the proposed models for large-sized problems. The GA is an effective meta-heuristic method for many combinatorial optimization problems with large and complex search spaces such as Vehicle Routing (VR), Transportation Network Design Problem (TNDP), any problem with the continuous space, etc. GA starts with an initial remedy at each iteration. The solution may be obtained by a constructive-based algorithm or may be generated randomly. The meta-heuristic algorithm searches for the optimum solution among all possible solutions [12]. In Section 5, the sensitivity of the proposed models is examined. Finally, Section 6 shows the obtained results and suggests future work.

2. Literature Review

Nowadays, in the competitive market, equal-area facilities are hardly working. In most studies, the main objective of UA-FLP is to achieve minimal operating costs [13]. Generally, there are FLPs in the literature that deal with the arrangement of the rectangular facilities, e.g., planning of an airplane's dashboard, a city or a neighborhood, a layout of office buildings, and integrated circuits [14]. UA-SFLPs were considered with different areas and shapes using the center of each facility for determining the distances without counting specific directions for facilities [15]. Two problems of the UA-SFLPs were solved with a search algorithm called TOPOPT [16].

Two models introduced for the UA-SFLPs: single row layout and multi-row layout. Then, a heuristic algorithm was proposed for solving the problems [17]. A modified PSO was suggested for solving the UA-SFLPs and the UA DFLPs such that the shapes and areas of departments could not change throughout the whole-time horizon [18]. For the first, they applied the modified PSO to solve the UA SFLPs and UA DFLPs to minimize the sum of the MHC for static problems and minimize the sum of the MHC and the sum of the rearrangement costs for dynamic problems. The UA-DFLPs were examined so that the shapes and areas of facilities could be fixed throughout the planning horizon. They joined the Dynamic Programming (DP) and the GA for solving the problems [19]. The UA-DFLPs were studied where the shapes and areas of facilities were fixed and a two-stage algorithm was used to solve the problems. However, they did not report the RCs and only the sum of the MHC was published [20]. The UA-DFLPs were investigated where the shapes and areas of facilities could vary throughout the planning horizon. Flexible Bay Structure (FBS) was utilized to simplify the problems and the GA was used to solve them [21]. A model for the dynamic line layout problem was developed considering unequal size work centers, multiple types of material handling devices, and stochastic demand [22]. Moreover, the UA-DFLPs were modeled as Mixed Integer Programming (MIP) [23,24,25].

The SSFLPs were studied where the shapes and areas of facilities were fixed through the iteration of an algorithm [16,26]. The Equal-Area SSFLPs were formulated in which there were several scenarios for a material flow matrix with different probabilities. Dynamic programming was applied to solve the problems [27]. The Equal-Area SSFLPs were investigated in which there were several scenarios for a material flow matrix with different probabilities to minimize the expected material handling costs and the sum of the material handling costs, respectively [28,29]. The Equal-Area SSFLPs were formulated where the product demands were stochastic with known variance and expected value. They utilized a Simulated Annealing Algorithm (SAA) to solve the problems [30].

The Equal-Area SSFLPs were formulated where the product demands were stochastic with a known expected value and variance in Cellular Manufacturing Systems (CMS) [31]. The UA-SSFLPs were developed where the area of each facility was fixed, but the shape of each facility could change during the iteration of an algorithm. Moreover, the product demands were stochastic with known variance and expected value, and each product had several product routings. Kulturel-Konak et al. applied the FBS and Tabu Search Algorithm (TSA) to solve the problems [10]. To address this type of the UA-SSFLPs, the FBS and the GA were used, where each product only has a single routing [32].

The FLP is NP-complete. Optimization approaches are useful tools for obtaining an optimal solution to small-sized problems [33]. They stated that different metaheuristic techniques were proposed to solve FLPs. One of the best known and the most frequent of these methods is GA. These algorithms may be derived as optimal and suboptimal algorithms [34]. Moreover, they introduced an iterative construction procedure such that the whole time required was minimized by material handling systems to transport the part types among machines. Over the last decades, a large number of studies have suggested several meta-heuristic approaches to solve the UA-FLP by obtaining the approximate optimal solution in an acceptable amount of time [35,36]. Among them, the most popular and widely applied research is the subject of GA. For these reasons, GA is used. Of note, many other approaches have been suggested in the literature.

However, approaches based on the standard GAs are characterized by premature convergence, which means that a population for an optimization problem is converged too early to get an optimal solution. All individuals in the population should be selected, evaluated, crossed, and mutated in each generation [37]. The effectiveness of RF control was studied in FMS [38]. Moreover, the effects of different levels of RF were investigated on the performance of FMSs with and without the factor of machine breakdowns [39].

To the best of the knowledge, the application of SDFLP with Unequal-Area facilities remains unexplored. This paper is the first research that covers two related gaps and considers the RF and time value of money in UA-SDFLPs, simultaneously. Moreover, three integrated mathematical models are presented for the UA-SDFLPs where the independent demands follow the Poisson, Exponential, and Normal distributions random variables.

3. Problem Description

The uncertainty in DFLP creates SDFLP. The confidence level (α) determines uncertainty in demands so that the decision-maker can specify it. The purpose of the SDFLP is to obtain an optimum layout for each period so that the MHC and the RC can be minimized. The demands are considered as independent chance variables with a known Probability Density Function (PDF), which changes from one period to another period randomly. EA-SDFLP was first suggested by [9]. A mathematical model was proposed for the UA-DFLPs when the shapes and areas of departments remained unchanged throughout the time horizon [7]. Both of the previously referenced studies failed to consider the RF and the time value of money. In this section, three new mathematical models are formulated with the following assumptions, indexes, parameters, and decision variables. It should be noted that the proposed mathematical models are based on the assumptions and perspectives found in [7,9].

3.1. Assumptions

Assumptions of the proposed models are as follows:

1. The shapes and areas of machines cannot change throughout the time horizon.
2. The coordinate of the bottom left of the shop floor is *(0, 0)*.
3. The product demands whose parts are made in the manufacturing system are independent random variables with a known expected value and variance that change from period to another at random.
4. The parts flow in the batches among machines.
5. Both of the material handling and machine rearrangement costs are known.

3.2. Models decision variables

The proposed models have two decision variables as follows:

(x_{ti}, y_{ti}) Center-coordinate of machine i in period t.

$$r_{ti} = \begin{cases} 1 & ; \text{If the length and width of machine } i \text{ exchange in period } t \text{ in comparison with original length and width of machine } i \text{ (i.e., if the orientation of machine } i \text{ changes in period } t \text{ in comparison with the original orientation of machine } i) \\ 0 & ; \text{Otherwise} \end{cases}$$

3.3. Models indexes and parameters

Indexes and parameters of the proposed models are shown in Table 1.

Tab. 1. Indexes and parameters of the proposed model.

Indexes	Parameters
i,j Indexes of the machine;	W Length of shop floor
$i \neq j$, *i,j=1,2,...,M*;	H Width of shop floor
	w_i Length of machine *i*
M Number of machines;	h_i Width of machine *i*
	a_i Cost of shifting machine *i*
t Indexes of the period, *t=1,2,...,T*;	a_{ti} Cost of shifting machine *i* in period *t*
	C_k Cost of movements for the part *k*
T Number of periods under consideration;	C_{tk} Cost of movements for part *k* in period *t*
	B_k Transfer batch size for the part *k*
k Indexes for part, *k=1,2,...,K*;	P_{kn} Probability of passing a part of route *n*
	D_{tk} Demand for the part *k* in period *t*
K Number of parts;	*E()* Expected value of a parameter
	Var() Variance of a parameter
n Indexes of route of production ;	$Z_{1-\alpha}$ Standard normal *Z* value at confidence level,

$n = 1,2,\ldots,N$;	$1-\alpha$
	I_r Interest rate
N Number of routes of production.	β_{tijkn} : $\begin{cases} 1 & \text{if machine } j \text{ appears immediately after machine } i \text{ in the route } n \text{ of the part } k \text{ in period } t \\ 0 & \text{Otherwise.} \end{cases}$

3.4. Determining objective function value (OFV)

The flow of materials for the part k between machines i and j in route production n in period t (f_{tijkn}) can be calculated as follows:

$$f_{tijkn} = \begin{cases} \frac{C_{tk}P_{kn}}{B_k} D_{tk} & ,if\ material\ flow\ for\ part\ k\ moves\ between\ machines \\ & \quad i\ and\ j\ in\ period\ t\ in\ route\ production\ n \\ 0 & ,\ Otherwise \end{cases} \tag{1}$$

As mentioned in the assumptions of the problem, D_{tk} is a random variable. Therefore, f_{tijkn} has the same random variable with the expected value and variance shown in Equations (2) and (3).
Moreover, β_{tijkn} is a binary variable that ensures two consecutive operations, which are done on the part k by machines i and j in the route production n in period t.

$$E(f_{tijkn}) = \frac{\beta_{tijkn}C_{tk}P_{kn}}{B_k} E(D_{tk}) \tag{2}$$

$$Var(f_{tijkn}) = \left(\frac{\beta_{tijkn}C_{tk}P_{kn}}{B_k}\right)^2 Var(D_{tk}) \tag{3}$$

The total flow for part k between machines i and j in period t resulting from all routes of production can be written in the following according to Equation (1):

$$f_{tijk} = \sum_n f_{tijkn} = \sum_n \frac{\beta_{tijkn}C_{tk}P_{kn}}{B_k} D_{tk} \tag{4}$$

where f_{tijkn} is a random variable. Therefore, f_{tijk} is the same random variable with the expected value and variance as follows:

$$E(f_{tijk}) = \sum_n E(f_{tijkn}) = \sum_n \frac{\beta_{tijkn}C_{tk}P_{kn}}{B_k} E(D_{tk}) \tag{5}$$

$$Var(f_{tijk}) = \sum_n Var(f_{tijkn}) = \sum_n \left(\frac{\beta_{tijkn}C_{tk}P_{kn}}{B_k}\right)^2 Var(D_{tk}) \tag{6}$$

The total flow between machines i and j in the planning horizon can be written as follows:

$$f_{tij} = \sum_k f_{tijk} = \sum_{k,n} \frac{\beta_{tijkn}C_{tk}P_{kn}}{B_k} D_{tk} \tag{7}$$

where f_{tijk} is a random variable. Therefore, f_{tij} is the same random variable with the following expected value and variance:

$$E(f_{tij}) = \sum_k E(f_{tijk}) = \sum_{k,n} \frac{\beta_{tijkn}C_{tk}P_{kn}}{B_k} E(D_{tk}) \tag{8}$$

$$Var(f_{tij}) = \sum_{k} Var(f_{tijk}) = \sum_{k,n} \left(\frac{\beta_{tijkn} C_{tk} P_{kn}}{B_k}\right)^2 Var(D_{tk}) \tag{9}$$

The MHC for the layout π in the period t (i.e., $C(\pi_t)$) in the unequal status is defined as follows:

$$C(\pi_t) = \sum_{t,i,j} f_{tij} d_{tij} = \sum_{t,i,j} f_{tij}\left(|x_{ti} - x_{tj}| + |y_{ti} - y_{tj}|\right) \tag{10}$$

According to Equation (7), the following can be done:

$$C(\pi_t) = \sum_{t,i,j,k,n} \frac{\beta_{tijkn} C_{tk} P_{kn}}{B_k} D_{tk}\left(|x_{ti} - x_{tj}| + |y_{ti} - y_{tj}|\right) \tag{11}$$

The $C(\pi_t)$ has the same distribution random variable since f_{tij} is a random variable. In Equations (8), (9), the expected value and variance of the $C(\pi_t)$ are determined as follows:

$$E\left(C(\pi_t)\right) = \sum_{t,i,j,k,n} \frac{\beta_{tijkn} C_{tk} P_{kn}}{B_k} E(D_{tk})\left(|x_{ti} - x_{tj}| + |y_{ti} - y_{tj}|\right) \tag{12}$$

$$Var\left(C(\pi_t)\right) = \sum_{t,i,j,k,n} \left(\frac{\beta_{tijkn} C_{tk} P_{kn}}{B_k}\right)^2 Var(D_{tk})\left(|x_{ti} - x_{tj}| + |y_{ti} - y_{tj}|\right)^2 \tag{13}$$

If the decision-maker considers $U(\pi_t, \alpha)$ as the maximum value (upper bound) of $C(\pi_t)$ with the confidence level α, $U(\pi_t, \alpha)$ can be minimized instead of $C(\pi_t)$. Therefore, we have

$$P\left(C(\pi_t) \le U(\pi_t, \alpha)\right) = 1 - \alpha \tag{14}$$

$$\rightarrow \quad P\left(\frac{C(\pi_t) - E\left(C(\pi_t)\right)}{\sqrt{Var\left(C(\pi_t)\right)}} \le \frac{U(\pi_t, \alpha) - E\left(C(\pi_t)\right)}{\sqrt{Var\left(C(\pi_t)\right)}}\right) = 1 - \alpha \tag{15}$$

Equation (15) can be standardized as follows:

$$P\left(Z \le \frac{U(\pi_t, \alpha) - E\left(C(\pi_t)\right)}{\sqrt{Var\left(C(\pi_t)\right)}}\right) = 1 - \alpha \tag{16}$$

Thus, $Z \sim N(0,1)$, i.e., Z is a variable with a Standard Normal distribution. $F(Z)$ is assumed as the Cumulative Distribution Function (CDF) of the random variable Z. Therefore, we have:

$$F\left(\frac{U(\pi_t, \alpha) - E\left(C(\pi_t)\right)}{\sqrt{Var\left(C(\pi_t)\right)}}\right) = 1 - \alpha \tag{17}$$

$$\rightarrow \quad F^{-1}(1 - \alpha) = \frac{U(\pi_t, \alpha) - E\left(C(\pi_t)\right)}{\sqrt{Var\left(C(\pi_t)\right)}} \tag{18}$$

If F^{-1} is considered as the inverse function for F, then Equation (18) can be rewritten as follows:

$$F(Z_{1-\alpha}) = 1 - \alpha \tag{19}$$

Since the $Z_{1-\alpha}$ is a standard normal Z value for percentile α, Equation (21) can be written as follows:

$$F^{-1}(1-\alpha) = Z_{1-\alpha} \tag{20}$$

According to Equation (18), Equation (21) can be written as follows:

$$\frac{U(\pi_t,\alpha) - E\big(C(\pi_t)\big)}{\sqrt{Var\big(C(\pi_t)\big)}} = Z_{1-\alpha} \tag{21}$$

$$\rightarrow \quad U(\pi_t,\alpha) = E\big(C(\pi_t)\big) + Z_{1-\alpha}\sqrt{Var\big(C(\pi_t)\big)} \tag{22}$$

Therefore, the upper bound of $C(\pi_t)$ equals that of Equation (22). The OFV_t is subject to the Rearrangement Cost (RC). The sum of the MHC and the RC generated for the layout (permutation) π in period t (OFV_t) is as follows:

$$OFV_t = C(\pi_t) + RC \tag{23}$$

The RC was formulated in [9]. $U(\pi_t,\alpha)$ can be minimized instead of $C(\pi_t)$ as follows:

$$OFV = U(\pi_t,\alpha) + \sum_{t=2}^{T}\sum_{i=1}^{M} a_{ti}\, r_{ti} \tag{24}$$

Therefore, if the time value of money is considered, then:

$$C_{tk} = C_k(1+I_r)^t \qquad ;\ \forall k \tag{25}$$
$$a_{ti} = a_i(1+I_r)^t \qquad ;\ \forall i \tag{26}$$

According to Equations (12), (13), (22), (25), and (26), Equation (24) can be rearranged as follows:

$$\begin{aligned} OFV = \sum_{t,i,j,k,n} & \frac{\beta_{tijkn} C_k (1+I_r)^t P_{kn}}{B_k} E(D_{tk})\big(|x_{ti}-x_{tj}| + |y_{ti}-y_{tj}|\big) \\ & + Z_{1-\alpha}\left(\sum_{t,i,j,k,n}\left(\frac{\beta_{tijkn} C_k (1+I_r)^t P_{kn}}{B_k}\right)^2 Var(D_{tk})\big(|x_{ti}-x_{tj}| \right. \\ & \left. + |y_{ti}-y_{tj}|\big)^2\right) + \sum_{t=2}^{T}\sum_{i=1}^{M} a_i(1+I_r)^t\, r_{ti} \qquad ;\ i \neq j \end{aligned} \tag{27}$$

In the next section, Equation (27) is rewritten according to the type of D_{tk} distribution and its $E(D_{tk})$ and $Var(D_{tk})$.

3.5. Mathematical models

In this section, Equation (27) is formulated according to the type of D_{tk} distribution and its $E(D_{tk})$ and $Var(D_{tk})$. At first, in Subsection 3.5.1, the independent demand follows the Poisson distribution. Moreover, the independent demands follow the Exponential and Normal distributions in Subsections 3.5.2 and 3.5.3, respectively. Each of the above distributions has its own known mean and variance.

3.5.1. Modeling under the poisson distribution condition

D_{tk} can be a random variable with a Poisson distribution in any time period ($D_{tk} \sim p(\lambda_t)$). The average number of events at an interval is designated λ_t at time t ($\lambda_t > 0$). The expected value and variance of a Poisson distribution are equal together. Therefore, we have:

$$E(D_{tk}) = Var(D_{tk}) = \lambda_t \tag{28}$$

When independent random variables are summed, their properly normalized sum leads to a normal distribution even if the main variables are not normally distributed (Central Limit Theorem (CLT)) [40]. Thus,

$$\lim_{k\to\infty} \sum_{k=1}^{K} D_{tk} \sim Normal\ (K\lambda_t, K\lambda_t) \qquad ;\ \forall\ t \tag{29}$$

If the average number of events at an interval (λ_t) is equal to each other for all parts at time t ($D_{tk} \sim i.i.d\ P(\lambda_t)$) and $K \geq 10$, then the CLT will be established. Thus, a new model of the UA-SDFLP is as follows by inserting Equation (29) into (27):

$$\begin{aligned} &Minimize\ OFV \\ &= K\left(\sum_{t,i,j,k,n} \frac{\beta_{tijkn} C_k (1+I_r)^t P_{kn} \lambda_t}{B_k} \left(|x_{ti} - x_{tj}| + |y_{ti} - y_{tj}|\right)\right) \\ &+ Z_{1-\alpha}\sqrt{K}\left(\sqrt{\sum_{t,i,j,k,n} \left(\frac{\beta_{tijkn} C_k (1+I_r)^t P_{kn}}{B_k}\right)^2 \lambda_t \left(|x_{ti} - x_{tj}| + |y_{ti} - y_{tj}|\right)^2}\right) \\ &+ \sum_{t=2}^{T}\sum_{i=1}^{M} a_i (1+I_r)^t r_{ti} \qquad ;\ i \neq j \end{aligned} \tag{30}$$

Subject to: (31)

$$\frac{w_i}{2}(1-r_{ti}) + \frac{h_i}{2} r_{ti} \leq x_{ti} \leq W - \left(\frac{w_i}{2}(1-r_{ti}) + \frac{h_i}{2} r_{ti}\right) \qquad ;\ \forall\ t, i$$

$$\frac{h_i}{2}(1-r_{ti}) + \frac{w_i}{2} r_{ti} \leq y_{ti} \leq H - \left(\frac{h_i}{2}(1-r_{ti}) + \frac{w_i}{2} r_{ti}\right) \qquad ;\ \forall\ t, i \tag{32}$$

$$\begin{aligned} &|x_{ti} - x_{tj}| + |y_{ti} - y_{tj}| \\ &\geq \left(\frac{w_i}{2}(1-r_{ti}) + \frac{h_i}{2} r_{ti} + \frac{w_j}{2}(1-r_{tj}) + \frac{h_j}{2} r_{tj}\right) \\ &+ \left(\frac{h_i}{2}(1-r_{ti}) + \frac{w_i}{2} r_{ti} + \frac{h_j}{2}(1-r_{tj}) + \frac{w_j}{2} r_{tj}\right) \qquad ;\ \forall\ t, i, j (i \neq j) \end{aligned} \tag{33}$$

$$x_{ti}, x_{tj}, y_{ti}, y_{tj} \geq 0, \qquad r_{ti}, r_{tj} \in \{0,1\}, \qquad \forall\ t, i, j \tag{34}$$

Equation (30) displays the Objective Function Value (OFV) of unequal-area stochastic dynamic facility layout problems that should be minimized. In this OFV, D_{tk} follows a random variable with a Poisson distribution.

Based on Equations (31) and (32), machines must be located in the workspace along the x-axis and y-axis, respectively. In other words, each machine is restricted to the shop floor based on Equations (31) and (32). Equation (33) prevents the overlap between each pair of machines. In other words, Equation (33) ensures that there is no interference or overlap among the machines. Equation (34) shows all decision variables.

3.5.2. Modeling under the exponential distribution condition

For real-world simulation, D_{tk} (demand for the part k at time t) is followed by a random variable with an Exponential distribution. If λ'_t ($\lambda'_t > 0$) be the parameter of an Exponential distribution for all parts at time t, then $D_{tk} \sim Exp(\lambda'_t)$. Its expected value and variance will be as follows:

$$E(D_{tk}) = \frac{1}{\lambda'_t} \tag{35}$$

$$Var(D_{tk}) = \frac{1}{{\lambda'_t}^2} \tag{36}$$

According to the CLT, we have:

$$\lim_{k\to\infty}\sum_{k=1}^{K} D_{tk} \sim Normal\left(\frac{K}{\lambda'_t}, \frac{K}{{\lambda'_t}^2}\right) \qquad ; \forall t \tag{37}$$

If $D_{tk} \sim i.i.d\ Exp(\lambda'_t)$ and $K \geq 20$, then the CLT will be established [40]. A new model of the UA-SDFLP is given below by inserting Equation (37) into (27):

$$\begin{aligned} &Minimize\ \text{OFV} \\ &= K\left(\sum_{t,i,j,k,n} \frac{\beta_{tijkn} C_k (1+I_r)^t P_{kn}}{\lambda'_t B_k}\left(|x_{ti}-x_{tj}| + |y_{ti}-y_{tj}|\right)\right) \\ &+ Z_{1-\alpha}\sqrt{K}\left(\sqrt{\sum_{t,i,j,k,n}\left(\frac{\beta_{tijkn} C_k (1+I_r)^t P_{kn}}{\lambda'_t B_k}\right)^2 \left(|x_{ti}-x_{tj}| + |y_{ti}-y_{tj}|\right)^2}\right) \\ &+ \sum_{t=2}^{T}\sum_{i=1}^{M} a_i (1+I_r)^t r_{ti} \qquad ; \quad i \neq j \end{aligned} \tag{38}$$

Equation (38) demonstrates the Objective Function Value (OFV) of unequal-area stochastic dynamic facility layout problems that should be minimized. In this OFV, independent demands (D_{tk}) have a random variable with an Exponential distribution. Its constraints are one with Subsection 3.5.1.

3.5.3. Modeling under the normal distribution condition

If D_{tk} be a random variable with the Normal distribution, then its expected value and variance will be as follows:

$$E(D_{tk}) = \mu_{tk} \tag{39}$$

$$Var(D_{tk}) = {\sigma^2}_{tk} \tag{40}$$

D_{tk} follows a normal distribution; therefore, D_{tk} does not need k to tend to ∞. In this case, regardless of k, the distribution of the summation will continue to be normal [40]. By inserting Equations (39), (40) into (27), a new model of the UA-SDFLP is given on the next page.

Equation (41) shows the OFV of unequal-area stochastic dynamic facility layout problems that should be minimized. Here, the independent demand (D_{tk}) follows a Normal distribution random variable. Its constraints are one with Subsection 3.5.1.

$$\begin{aligned} \text{Minimize OFV} = &\sum_{t,i,j,k,n} \frac{\beta_{tijkn} C_k (1+I_r)^t P_{kn}}{B_k} \mu_{tk}\left(|x_{ti}-x_{tj}| + |y_{ti}-y_{tj}|\right) \\ &+ Z_{1-\alpha}\left(\sqrt{\sum_{t,i,j,k,n}\left(\frac{\beta_{tijkn} C_k (1+I_r)^t P_{kn} \sigma_{\text{tk}}}{B_k}\right)^2 \left(|x_{ti}-x_{tj}| + |y_{ti}-y_{tj}|\right)^2}\right) \\ &+ \sum_{t=2}^{T}\sum_{i=1}^{M} a_i (1+I_r)^t r_{ti} \qquad ; \quad i \neq j \end{aligned} \tag{41}$$

4. Computational Experiments

In this section, the performance of the proposed models is evaluated by many test problems. In Subsection 4.1, the proposed models are evaluated when the independent demand follows the Normal distribution function.

4.1. Evaluation of the proposed model under normal distribution

In this section, to evaluate the performance of the proposed model when the demand follows a Normal distribution, two numerical examples are studied. The real case is STFLP-RC with seven

departments (i.e., Cutting machine, Guillotine machine type *I*, Guillotine machine type *II*, Drilling or Punching machine, Wire welding machine, CO_2 welding machine, and Powder seam welding machine) and six products such that the machines should be arranged on the shop floor in which its length and width are equal to *60 (W*H=60*60)* [7]. In this paper, the production routes are added to the STFLP-RC.

In the first problem *(I)*, a part of the real case (STFLP-RC) is derived from literature studies as a small-sized problem. Three machines (i.e., Cutting machine, Guillotine machine type, and Powder seam welding machine), three parts, and six production routes (*M=3*, *K=3*, and *N=6*) are considered for the problem *(I)*. The GAMS 24.1.3 software and the GA encoded in MATLAB have solved both problems at *T=3* and *T=5* and at three confidence levels of $\alpha=0.75$, $\alpha=0.85$, and $\alpha=0.95$ in a system with the RAM specifications of 8GB, 2.4 GHz CPU, and Corei7.

The GAMS that used BARON solver for the problem *(I)* is compared with the result of the GA encoded in MATLAB. Table 2 presents the Objective Function Value (OFV) of these comparisons. Figure. 2 exhibits the OFV of the Normal distribution by GAMS and MATLAB for the problem *(I)*. Figure 3 displays a graphical exhibition of the best layout for the problem (*I*), where *T=3* and $\alpha=0.75$.

Tab. 2. The OFV of the Normal distribution function for the problem *(I)*.

Period	Confidence level (α)	GAMS	MATLAB (Mean value)	Difference (%)
T=3	0.75	6043.42	6156.32	1.83%
	0.85	6163.93	6222.56	0.94%
	0.95	6368.13	6448.874	1.25%
T=5	0.75	13362.88	14396.367	7.18%
	0.85	13559.67	14594.147	7.09%
	0.95	13893.11	14381.334	3.39%

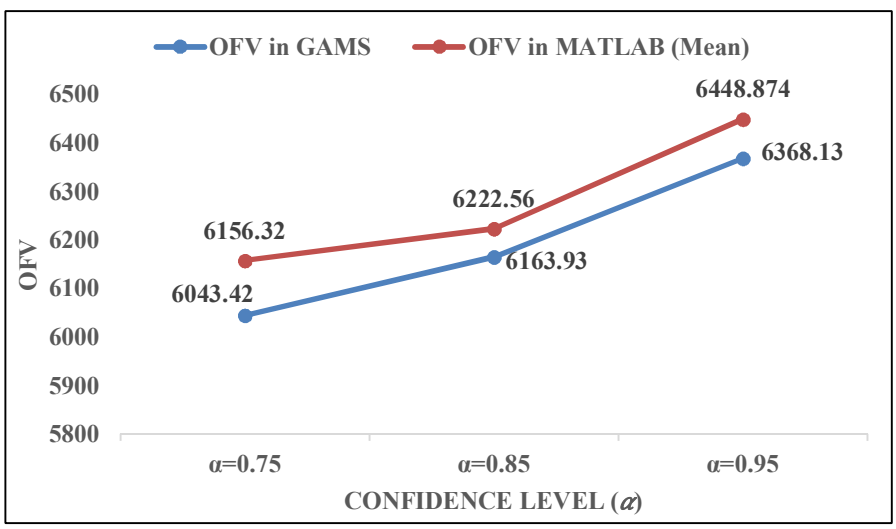

Fig. 2. The OFV of the Normal distribution in GAMS and MATLAB for the problem *(I)*.

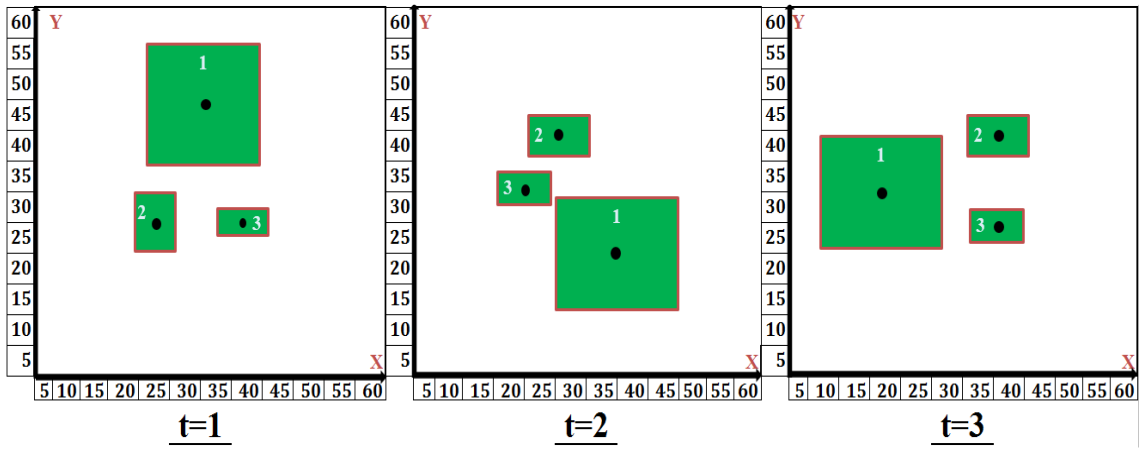

Fig. 3. The best layout for the problem *(I)* in *T=3* and *α=0.75* condition.

Table 3 shows a statistical evaluation when the problem *(I)* is solved by the GA encoded in MATLAB.

Tab. 3. Statistical evaluation of the problem *(I)* (10 trials).

Period	Confidence level (α)	Objective Function Value (OFV)			
		Worst	Mean	Best	Std. Dev.
T=3	0.75	6748.76	6156.32	5750.89	338.95
	0.85	6579.77	6222.56	5611.11	253.94
	0.95	6923.31	6448.87	6037.22	283.98
T=5	0.75	15810.77	14396.36	13019.33	892.15
	0.85	15310.48	14594.14	13878.38	533.62
	0.95	15827.35	14381.33	13230.75	796.57

According to Tables 2, 3 and Figure 2, it can be claimed that the proposed models are valid. In the problem *(II)*, the STFLP-RC is considered as a large-sized problem where *M=7*, *K=6*, and *N=14*. Table 4 presents statistical evaluation when the problem *(II)* is solved by the GA encoded in MATLAB. The primal values of parameters for Problems (I), (II) are presented in Table 5. In Tables 6 and 7, the routes of the parts and their probability are given for Problems (I), (II), respectively.

Tab. 4. The statistical evaluation of Problem *(II)* (10 trials).

Period	Confidence level (α)	Objective Function Value (OFV)			
		Worst	Mean	Best	Std. Dev.
T=3	0.75	6.17×10^{18}	4.64×10^{18}	3.68×10^{18}	0.70×10^{18}
	0.85	6.97×10^{18}	4.92×10^{18}	3.24×10^{18}	1.03×10^{18}
	0.95	6.33×10^{18}	5.91×10^{18}	4.98×10^{18}	0.45×10^{18}
T=5	0.75	9.87×10^{18}	9.02×10^{18}	8.11×10^{18}	0.59×10^{18}
	0.85	1.46×10^{19}	1.18×10^{19}	1.05×10^{19}	0.13×10^{18}
	0.95	1.63×10^{19}	1.34×10^{19}	1.02×10^{19}	0.21×10^{18}

Tab. 5. Primal values of parameters for Problems *(I)* and *(II)*.

Parameters	Primal value
Interest rate (I_r)	%20
Batch size (B_k)	50
Cost of movement for part k (C_k)	100
Cost of shifting machine i * (a_i)	1000

* a_i is the same for all machines in any time period

Tab. 6. Routes of parts and their probability for Problem *(I)*.

Parts (k)	Machine sequence	Probability of route (P_{kn})
1	2-3-1	0.5
	2-3	0.2
	2-1	0.3
2	3-1-2	1
3	1-2	0.7
	1-3	0.3

Tab. 7. Routes of parts and their probability for Problem *(II)*.

Parts (k)	Machine sequence	Probability of route (P_{kn})
1	4-2-6-3-1	0.5
	7-5-3	0.5
2	1-2-6-7	0.9
	3-6-7	0.05
	5-4-1-2-3	0.05
3	7-1-3	0.25
	1-2-6	0.75
4	5-3-1-2	1
5	7-6-5-1-3-4	0.3
	2-7-6-5	0.35
	1-2-3-5	0.2
6	4-6-1	0.15
	2-1-5-7	0.85
	3-4-6-1-5-2	0.15

For example, the machine sequence of the third part in Problem *(I)* is 1 →2 with a probability rate of 0.7 and *1→3* with a probability of *0.3*. It is implied that the first, second, and third operations on the third part with a probability rate of *0.7* are performed by Machines 1 and 2, respectively. The first, second, and third operations on the third part three are performed by Machines 1 and 3 with a probability rate of *0.3*. Table 8 shows the length and width of each machine for the STFLP–RC.

Tab. 8. Length and width of each machine for the STFLP-RC.

Machines	1	2	3	4	5	6	7
Original length (w_i)	20	10	8	10	14	14	14
Original width (h_i)	18	7	5	10	12	12	10

Table 9 indicates the means and variances of parts for all periods (*t=1* to *t=5*). The center coordinates of the facility are shown in Table 10 for Problem *(I)*.

Tab. 9. Means and variances of parts for periods *T=1* to *T=5*.

Parts	Period 1		Period 2		Period 3		Period 4		Period 5	
(k)	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
1	6.22	1.073	5.656	1.118	3.764	2.584	6.503	2.629	6.503	1.553
2	2.565	2.824	8.863	2.442	6.636	1.609	9.101	1.344	7.589	2.940
3	7.623	1.893	9.12	2.318	3.543	1.372	4.554	1.668	5.948	1.388
4	2.067	1.573	4.347	2.578	2.646	2.986	9.746	2.262	8.496	1.812
5	8.965	1.283	2.358	2.251	2.720	1.909	7.540	1.898	8.085	1.663
6	8.736	2.892	9.998	1.190	7.804	1.045	3.677	1.417	2.066	2.998

Tab. 10. Center coordinates of the facility for the best layout for Problem *(I)*, *α=0.75*.

Period	Machines	Center coordinate of machine along x-axis (x_i)	Center coordinate of machine along y-axis (y_i)	r_{ti}
t=1	1	30.915	44.01	1
	2	22.558	24.868	1
	3	37.415	25.01	0
t=2	1	34.701	20.117	0
	2	26.156	39.072	0
	3	19.656	30.572	0
t=3	1	17.263	30.455	0
	2	36.263	38.955	0
	3	36.419	24.111	0

5. Sensitivity Analysis

In this section, the sensitivity of the proposed models to four parameters is examined. The confidence level (α), the interest rate (I_r), the cost of movements for the part k (C_k), and the transfer batch size for the part k (B_k) are studied in Subsections 5.1, 5.2, 5.3, and 5.4, respectively.

5.1. Confidence level (α)

According to Figures 2, it is evident that as the confidence level (α) increases, the Objective Function Value increases, too. In other words, the confidence level (α) is directly related to the Objective Function Value.

5.2.Interest rate (I_r)

As the Interest rate (I_r) decreases, the Objective Function Value decreases, too. In other words, the interest rate (I_r) is directly related to the Objective Function Value. Figure 4 exhibits the result of solving the proposed UA-SDFLP by GAMS for Problem *(I)* with *T=3* and *α=0.75* when the demand follows a normal distribution function.

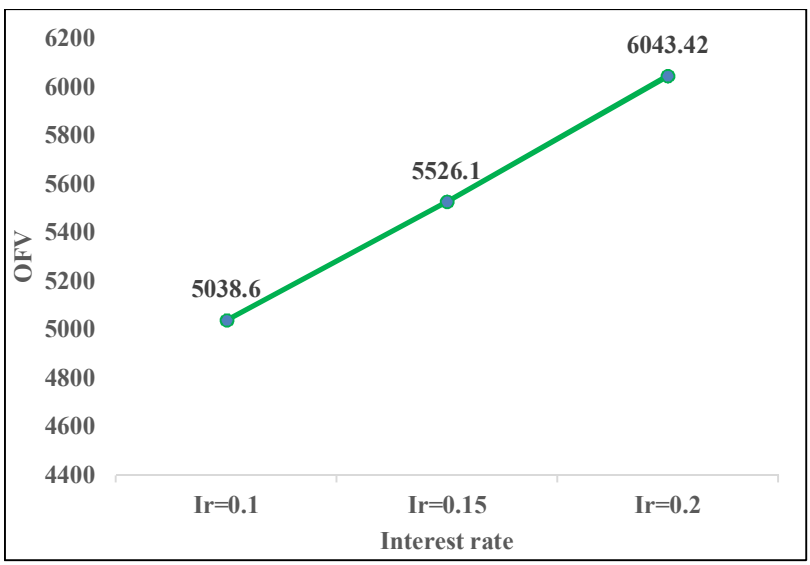

Fig. 4. Communication between the interest rate (I_r) and the OFV.

5.3. Cost of movements for the part k (C_k)

If the cost of movements for the part k (C_k) increases, then OFV will increase, too. In other words, the cost of movements for the part k (C_k) is directly related to the Objective Function Value. Figure 5 exhibits the result of solving the proposed UA-SDFLP by GAMS when D_{tk} follows an exponential distribution function for Problem *(I)* with *T=3* and *α=0.75*.

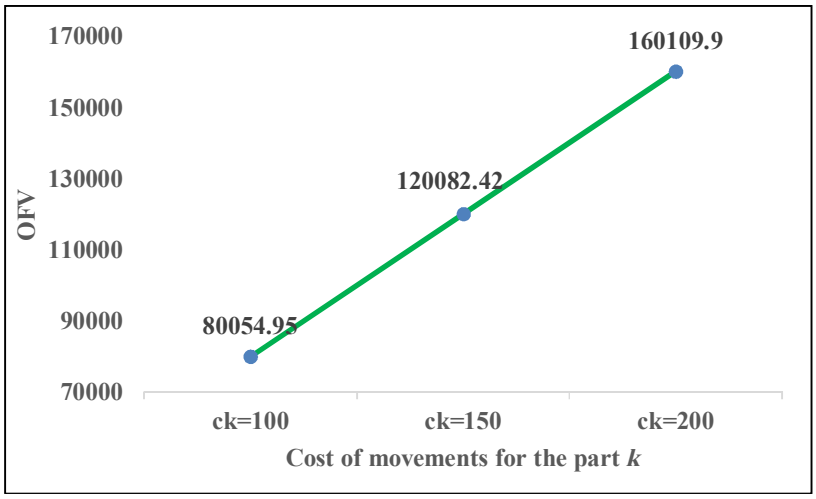

Fig. 5. Consistency between the cost of movements for the part *k* and the OFV.

5.4. Transfer batch size for the part k (B_k)

When the transfer batch size for the part k (B_k) increases, the OFV decreases. In other words, these values run in opposite directions. Figure 6 exhibits the result of solving the proposed UA-SDFLP by GAMS when the demand follows a normal distribution for Problem *(I)* with $T=3$ and $\alpha=0.75$.

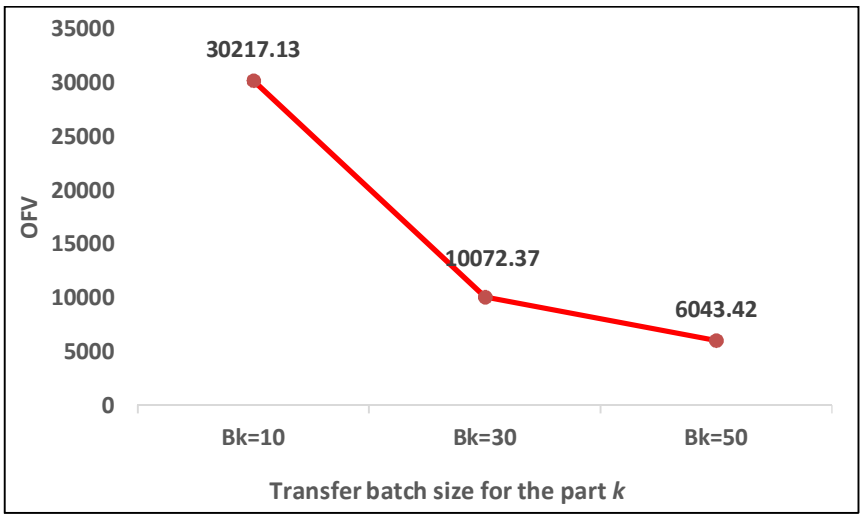

Fig. 6. Consistency between the transfer batch size for the part *k* and the OFV.

5. Conclusion and Future Work

This paper proposed three new mathematical models for designing a dynamic layout with the Unequal-Area facility in an uncertain environment. Two important research gaps were covered in Unequal–Area Stochastic Dynamic Facility Layout Problems. First, this study considered the RF and time value of money in the Unequal-Area Stochastic Dynamic Facility Layout Problems. These issues can be applied to any manufacturing system and provide practical insight for managers. Second, for real-world simulation, this paper modeled the Unequal-Area Stochastic Dynamic Facility Layout Problems, where the independent demands followed the Poisson, Exponential and Normal distributions random variables. Moreover, two test problems were generated randomly with three and six parts as well as three and seven machines, respectively. The discussed problems were derived from a real case study in literature. The test problems were solved by the GA approach at three different confidence levels ($\alpha=0.75$, $\alpha=0.85$, $\alpha=0.95$) and two different periods ($T=3$ and $T=5$) in a reasonable amount of computational time. The obtained results indicate that the proposed models for the Unequal-Area Stochastic Dynamic Facility Layout Problems are valid. Generally, significant contributions can be summarized as follows:

- Modeling the Stochastic Dynamic Facility Layout Problems in Unequal-Area facility status (Unequal–Area SDFLPs).
- Adding the Routing Flexibility to the SDFLPs with the Unequal-Area facility and considering the time value of money.
- Considering the Poisson, Exponential, and Normal distributions of random variables for independent demands, which vary from period to period for real-world simulation.
- Using the proposed models in a real case study in literature.

Finally, this work can be extended to the future researches as shown below:

- Design of a robust layout by other methods such as MULVEY.
- Further study of the stability of the output layout by considering RF.
- Development of a new hybrid meta-heuristic approach by combining GA with other methods such as Craft.
- Use of the proposed models in this paper for the concurrent design of inter-cell and intra-cell layout designs in the other modern system manufacturing.
- Considering the constraints such as closeness ratio, aisles, and budget constraint with the RF simultaneously.

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