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Change-Point Estimation of High-Yield Processes in the Presence of Serial Correlation

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ABSTRACT

Change-point estimation is as an effective method for identifying the time of a change in production and service processes. In most of the statistical quality control literature, it is usually assumed that the quality characteristic of interest is independently and identically distributed over time. It is obvious that this assumption could be easily violated in practice. In this paper, we use maximum likelihood estimation method to estimate when a step change occurs in a high-yield process by allowing a serial correlation between observations. Monte Carlo simulation is used as a vehicle to evaluate the performance of the proposed method. Results indicate satisfactory performance for the proposed method.

1. Introduction

The problem of change-point estimation of high-yield processes was studied first by Noorossana et al. (2009) where a maximum likelihood estimator (MLE) for a step-change point in the process fraction non-conforming was propose. Zandi et al. (2011) introduced a model for change-point estimation of the process fraction non-conforming with a linear trend and applied a maximum likelihood estimator when a linear trend disturbance was present. Then, Monte Carlo simulation was applied in order to evaluate the accuracy and precision of the proposed change-point estimator. Next, the proposed estimator was compared to the maximum likelihood estimator of the change-point of process fraction non-conforming derived under simple-step and monotonic changes following signals from a Shewhart *np* control chart. Noorossana et al. (2012) compared the performance of the maximum likelihood estimator and those of built-in change-point estimators of cumulative sum (CUSUM) and exponential weighted moving average (EWMA) control charts. They proposed a confidence set for the change point. The results of a Monte-Carlo simulation show the superiority of the MLE in identifying the real time of the change. Niaki and Khedmati (2013) proposed a maximum likelihood estimator for the change point in a high-yield process when a linear trend disturbance occurs in the proportion of process nonconformity. The performance of the proposed change-point estimator in terms of both accuracy and precision is compared to that of the MLE of the change-point designed for step changes. Niaki and Khedmati (2014) proposed a maximum likelihood estimator of a change point in high-yield processes while assuming that the change belonged to a family of monotonic changes. Following a signal from the cumulative count of conforming (CCC) control chart, the performance

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of the proposed monotonic change-point estimator was evaluated by comparing its performance to the ones designed for step-changes and linear-trend disturbances through extensive simulation experiments involving different single step-changes, linear-trend disturbances, and multiple-step changes.

In Section 2, the effect of correlation in high-yield process is discussed. Section 3 describes change-point estimation of high-yield process with single-step change. Section 4 provides a maximum likelihood estimation method when a step change occurs. Some numerical results are presented in Section 5. The proposed model's performance is evaluated in Section 6, and conclusions are provided in the final section.

2. Effect of Correlation on The High-Yield Process

Madsen (1993) considered several generalized binomial models, one of which is referred to as the correlation model. The key feature of this model is the allowance of a serial correlation for Bernoulli trials. For low values of p, i.e., the fraction nonconforming of the process, the generalized binomial distribution is approximated by a generalized Poisson distribution known as a modified Poisson distribution. Expression for $P(Z_n = k)$, where Z_n is the sum of n Bernoulli random variables defined as follows:

$$P(Z_n = k) = \begin{cases} \rho(1-p) + (1-\rho)(1-p)^n & k = 0 \\ (1-\rho)\binom{n}{k} p^k (1-p)^{n-k} & 0 < k \leq n-1 \\ \rho p + (1-\rho)p^n & k = n \end{cases} \quad (1)$$

where $\rho \geq 0$ is the correlation coefficient between any pair of observations

$$\rho = Corr(X_i, X_j), \quad i \neq j \quad (2)$$

Madsen (1993) also showed that the methods of moment estimator for ρ are as follows:

$$\hat{\rho} = \frac{[S^2 - np(1-p)]}{[n(n-1)p(1-p)]} \quad (3)$$

where S^2 denotes the sample variance of the observed data.

Consider a manufacturing process that produces individual items, each of which is inspected in the order of production. Let Y denote the total number of trials to obtain the first non-conforming item, and Y is conditioned on $X_0 = 1$. So, we have:

$$P(X_0 = 1, X_1 = 0, X_2 = 0, \ldots, X_{i-1} = 0, X_i = 1) = (1-\rho)p^2(1-p)^{i-1} \quad (4)$$

Now, the probability function of Y is as follows:

$$\begin{aligned} P(Y = i) &= P(X_1 = 0, X_2 = 0, \ldots, X_{i-1} = 0, X_i = 1 | X_0 = 1) \\ &= P(X_0 = 1, X_1 = 0, X_2 = 0, \ldots, X_{i-1} = 0, X_i = 1)/P(X_0 = 1) \\ &= (1-\rho)p(1-p)^{i-1} \quad i \geq 2 \end{aligned} \quad (5)$$

For $i = 1$, we have

$$\begin{aligned} P(Y = 1) &= P(X_1 = 1 | X_0 = 1) \\ &= \frac{P(X_0 = 1, X_0 = 1)}{P(X_0 = 1)} \\ &= \rho + (1-\rho)P \end{aligned} \quad (6)$$

If $\rho = 1$, equation (1) tells us that sequence $\{X_i\}$ is completely dependent; therefore, $P(X_1 = 1 | X_0 = 1) = 1$. Hence, equation (5) does make sense. Combining equations (4) and (5), we have

$$P(Y = x) = \begin{cases} \rho + (1-\rho)p & x = 1 \\ (1-\rho)p(1-p)^{x-1} & x \geq 2 \end{cases} \quad (7)$$

It is clear that, when $\rho = 0$, equation (1) reduces to the geometric distribution.

3. Change-Point Identification in Correlated High-Yield Processes with A Single-Step Change

Following an unknown point in the time, the fraction non-conformity level changes and the process gets out of control. It is assumed that control chart gives a signal at the time of T, meaning that observation X_T gets out of control limits. Given that our goal is to identify change-point with step change, observations $X_1, X_2, \ldots, X_\tau$ belong to the in-control process with fraction non-conforming p_0, while observations $X_{\tau+1}, X_{\tau+2}, \ldots, X_T$ come from the out-of-control process with fraction non-conformity level p_1. The control chart used for monitoring the process is geometric chart or g chart proposed by Benneyan (1991) and Kaminsky et al. (1992). Control limits of this chart are defined as follows:

$$UCL = \frac{1-p}{p} + k\sqrt{\frac{1-p}{p^2}} \quad (8)$$

$$CL = \frac{1-p}{p} \quad (9)$$

$$LCL = \frac{1-p}{p} - k\sqrt{\frac{1-p}{p^2}} \quad (10)$$

4. Maximum Likelihood Estimation for Correlated Model with A Single-Step Change

The proposed estimator for the period when a step change occurs in nonconformity level is obtained using the maximum likelihood estimation (MLE) method. When g control chart signals an out-of control condition, the proposed method can be applied to determine the period when the step change occurs in the process parameter, p_0 . When value of $i(0 \leq i < T)$ maximizes the likelihood function, the step change occurs in this period. Using the following probability distribution function, we can derive the estimator for the change point:

$$P(Y=x)=\begin{cases}\rho+(1-\rho)p & x=1\\(1-\rho)p(1-p)^{x-1} & x\geq 2\end{cases} \quad (11)$$

$$L(\tau,p_1|X)=\prod_{j=2}^{\tau}(1-\rho)p_0(1-p_0)^{x_j-1}\prod_{j=\tau+1}^{T}(1-\rho)p_1(1-p_1)^{x_j-1}$$

$$L(\tau,p_1|X)=(1-\rho)^{\tau-1}{p_0}^{\tau-1}(1-p_0)^{\sum_{j=2}^{\tau}x_j-\tau+1}(1-\rho)^{T-\tau}{p_1}^{T-\tau}(1-p_1)^{\sum_{j=\tau+1}^{T}x_j-t+\tau} \quad (12)$$

The logarithm of the likelihood function is as follows:

$$LnL(\tau,p_1|X)=(\tau-1)Ln(1-\rho)+(\tau-1)Ln(p_0)+\left(\sum_{j=2}^{\tau}x_j-\tau+1\right)Ln(1-p_0)+(T-\tau)Ln(p_1)+(T-\tau)Ln(1-\rho)+\left(\sum_{j=\tau+1}^{T}x_j-T+\tau\right)Ln(1-p_1) \quad (13)$$

The value ofp_1 that maximizes the likelihood function is $\widehat{p_{1,\tau}}=T-\tau/\sum_{j=\tau+1}^{T}x_j$

The maximum likelihood estimate of the change point τ is:

$$\hat{\tau}=arg\max_{0\leq i<T}\{L_i\} \quad (14)$$

Therefore,

$$L_i=iLn\left(\frac{p_0(1-\hat{p}_{1,j})}{\hat{p}_{1,j}(1-p_0)}\right)-\sum_{j=\tau+1}^{T}x_j\,Ln\left(\frac{1-p_0}{1-\hat{p}_{1,j}}\right)+TLn\left(\frac{\hat{p}_{1,j}}{1-\hat{p}_{1,j}}\right)+Ln\left(\frac{1-p_0}{p_0}\right)+iLn\left(\frac{1-\rho}{1-\rho}\right)+TLn(1-\rho)-Ln(1-\rho) \quad (15)$$

$$\widehat{p_{1,\iota}}=\frac{T-i}{\sum_{j=i+1}^{T}x_j} \quad (16)$$

5. Numerical Example

Suppose that we have a process operating at 0.0005 non-conformity level; subsequently, the control limits for the chart can be calculated as follows:

$$UCL=\frac{1-0.0005}{0.0005}+3\sqrt{\frac{1-0.0005}{1-0.0005^2}}=7997.50$$

$$LCL=\frac{1-0.0005}{0.0005}-3\sqrt{\frac{1-0.0005}{1-0.0005^2}}=-3999.50$$

$$CL=\frac{1-0.0005}{0.0005}=1999$$

Given that the lower control limit is negative, it is rounded up to zero. Thus, if $X_i < 0$ or $X_i > 7997.50$, the chart signals an out-of-control condition, indicating a change in the process non-conformity level. The change-point estimator proposed herein for the period, when the step change occurs in the process nonconformity level, is based on the MLE method. When g control chart signals an out-of control condition, the proposed method can be applied to determine the period when the step change occurs in the process parameter $p_0 = 0.0005$. When the value of $i(0 \leq i < T)$ maximizes L_i, the step change occurs in that period. We consider the derivation of MLE for τ; the process change-point period using the MLE technique was proposed by Casella and Berger(1990). We consider MLE of change point τ as in $\hat{\tau}$ $(0 \leq \tau < T)$. The value of τ that maximizes the likelihood function is as follows:

$$L_i=iLn\left(\frac{p_0(1-\hat{p}_{1,j})}{\hat{p}_{1,j}(1-p_0)}\right)-\sum_{j=\tau+1}^{T}x_j\,Ln\left(\frac{1-p_0}{1-\hat{p}_{1,j}}\right)+TLn\left(\frac{\hat{p}_{1,j}}{1-\hat{p}_{1,j}}\right)+Ln\left(\frac{1-p_0}{p_0}\right)+iLn\left(\frac{1-\rho}{1-\rho}\right)+TLn(1-\rho)-Ln(1-\rho)$$

$\widehat{p_{1,\iota}}=\frac{T-i}{SX_{i,T}}$ is the estimate of process fraction nonconformity level, and $SX_{i,T}=\sum_{j=i+1}^{T}X_j$ is the sum of inspected units in the periods i+1, i+2, . . . ,T .The value of i that maximizes L_i is the estimate of the last period from the in-control process, and $\hat{p}_{1,i}$ is its corresponding estimate of the changed fraction nonconformity level.

Now, using a simple numerical example can point to the effectiveness of this method. Table (1) shows the number of inspected items, X_{i+1}, the values of $\hat{p}_{1,i}$, and $SX_{i,T}$ for $i = 0,1,2,\dots,29$.

According to this table, g control chart signals a change in the process non-conformity level at period T =30. To determine the period when the step change occurs, we have to check the above table for the largest value of L_i . The largest value of L_i is equal to period 26, showing that the change in the nonconformity level has most likely occurred at this step. It means that estimate of τ, the period when the step change occurs in the process nonconformity level, would be $\hat{\tau} = 26$. Therefore, we have to check the records of the process assigned to a cause that exists around period 26.

Tab. 1. Change-point estimation

Period	i	X_i	$SX_{i,T} = \sum_{j=i+1}^{T} X_j$	$\widehat{p_{1,i}} = T - i/SX_{i,T}$	L_i
1	0	4449	53062	0.000565	-220.940
2	1	802	48613	0.000597	-219.985
3	2	1059	47811	0.000586	-220.079
4	3	284	46752	0.000578	-220.144
5	4	1256	46468	0.000560	-220.253
6	5	1827	45212	0.000553	-220.289
7	6	5988	43385	0.000553	-220.293
8	7	886	37397	0.000615	-219.951
9	8	1507	36511	0.000603	-220.051
10	9	1491	35004	0.000600	-220.083
11	10	1316	33513	0.000597	-220.116
12	11	1159	32197	0.000590	-220.164
13	12	1194	31038	0.000580	-220.223
14	13	1658	29844	0.000570	-220.273
15	14	972	28186	0.000568	-220.288
16	15	751	27214	0.000551	-220.343
17	16	913	26463	0.000529	-220.390
18	17	2293	25550	0.000509	-220.410
19	18	3192	23257	0.000516	-220.406
20	19	494	20065	0.000548	-220.366
21	20	3063	19571	0.000511	-220.409
22	21	3541	16508	0.000545	-220.379
23	22	2321	12967	0.000617	-220.247
24	23	951	10646	0.000658	-220.171
25	24	3411	9695	0.000619	-200.284
26	25	99	6284	0.000796	-219.947
27	26	2482	6185	0.000647	-220.290
28	27	1156	3703	0.000810	-220.112
29	28	693	2547	0.000785	-220.235
30	29	1854	8856	0.000539	-220.409

Then, we want to generate this case for $\rho = 0.04$.

Tab. 2. Change-point estimation of $\rho = 0.04$

Period	i	X_i	$SX_{i,T} = \sum_{j=i+1}^{T} X_j$	$\widehat{p_{1,i}} = T - i/SX_{i,T}$	L_i
1	0	6404	64164	0.000468	-221.526
2	1	1013	57760	0.000502	-221.595
3	2	1299	56747	0.000493	-221.593
4	3	448	55448	0.000487	-221.586
5	4	1522	55000	0.000473	-221.554
6	5	2187	53478	0.000467	-221.538
7	6	5013	51291	0.000468	-221.542
8	7	1106	46278	0.000497	-221.595
9	8	1810	45172	0.000487	-221.588
10	9	1792	43362	0.000484	-221.585
11	10	1590	41570	0.000481	-221.580
12	11	1412	39980	0.000475	-221.571
13	12	1451	38568	0.000467	-221.552
14	13	1987	37117	0.000458	-221.528
15	14	1202	35130	0.000455	-221.524
16	15	957	33928	0.000442	-221.477
17	16	1136	32971	0.000425	-221.398
18	**17**	**2760**	**31835**	**0.000408**	**-221.310**
19	18	3989	29075	0.000413	-221.360
20	19	676	25086	0.000438	-221.496
21	20	3798	24410	0.000410	-221.383
22	21	4538	20612	0.000437	-221.509
23	22	2795	16074	0.000498	-221.595
24	23	1178	13279	0.000527	-221.586
25	24	4327	12101	0.000496	-221.595
26	25	250	7774	0.000643	-221.449
27	26	3002	7524	0.000532	-221.588
28	27	1409	4522	0.000663	-221.486
29	28	893	3113	0.000642	-221.538
30	29	2220	2220	0.000450	-221.590

In table(2), to determine the period when the step change has occurred, we have to check the above table for the largest value of L_i that is equal to period 18, showing that the change in the nonconformity level has most likely occurred at this step. It means that estimate of τ, the period when the step change occurs in the process nonconformity level, would be $\hat{\tau} = 18$. Therefore, we have to check records in the process for an assignable cause that exists around period 18. It is noticeable that a correlation is applied in this case.

6. Performance Evaluation Model

Performance of the proposed estimator for the period when the step change has occurred in the nonconformity level is investigated using Monte Carlo simulation while a correlation exists throughout the process. Using a geometric distribution, 100 observations from an in-control process with $p = p_0$ are generated first. If any of the observations exceeds the control limits, it is assumed to be a false alarm. We simply replace this observation with an in-control one. This procedure is repeated until all 100 observations are placed between the two control limits. At the beginning 101 period, a shock is induced to the process and nonconformity level changes from p_0 to $p_1 = \delta p_0$. Then, observations are generated until an out-of-control signal is detected. The point estimate of the period when a change has occurred, τ, which should be close to 100, is computed. To estimate the expected period, when the first alarm is given by g control chart, $\hat{E}(T)$, the period number whose signal is determined has to be recorded. This process is repeated 10,000 times for $p_0 = 0.0005$ and different values of δ. The mean and standard error of 10000 the estimates of the periods that the step change has occurred, $\bar{\hat{\tau}}$ and $Se(\bar{\hat{\tau}})$, along with the estimate of the expected period when the first alarm is given, $\hat{E}(T) = ARL + \tau$, for different values of increases (δ> 1) and decreases (δ <1) in the fraction nonconforming is shown in the following tables. The average run length (ARL) in the above expression for $\hat{E}(T)$ refers to the number of points plotted on the control chart prior to observing any signal, and it can easily be obtained by τ, subtracted from $\hat{E}(T)$.

Tab. 3. Average change-point estimates $\bar{\hat{\tau}}$ and standard error $p_0 = 0.0005$, $\tau = 100$, $p_1 > p_0$

	p_1	0.0006	0.0007	0.0008	0.0009
	$\bar{\hat{\tau}}$	153	124.5	108	101.5
ρ	$Se(\bar{\hat{\tau}})$	4.75	4.555	4.252	4.12
$= 0$	$\hat{E}(T)$	505.9	481.7	446.5	432.2

Tab. 4. Average change-point estimates $\bar{\hat{\tau}}$ and standard error $p_0 = 0.0005$, $\tau = 100$, $p_1 < p_0$

	p_1	0.0004	0.0003	0.0002	0.0001
	$\bar{\hat{\tau}}$	111.5	104.5	103	102.5
	$Se(\bar{\hat{\tau}})$	1.417	0.629	0.791	0.745
ρ $= 0$	$\hat{E}(T)$	234.2	157.2	113.9	102.2

Table (3) reveals that a 40% increase in the fraction nonconformity level $p_1 = 0.0007$ and $\rho = 0$ would be detected by *g* control chart on average of 381.7 periods after the change has actually occurred in the process. However, the *MLE* provides an average estimate of 124.5 for the period when the step change occurred in fraction nonconforming level that is very close to real change point. The standard error of the estimates is *4.555*, which is small. The results in Table (3) indicate that the estimates of the period when the step change has occurred get closer to the true value as the size of the shift in the process nonconforming level increases, which is reasonable enough.

According to Table (4), *g* control chart signals 57.2 periods on average after the process fraction nonconforming level drops by 40%, $p_1 = 0.0003$ .The change-point estimator performs relatively well by yielding 104.5 as the average estimate for the period when the step change has occurred. The results in Table (4) reveal that the performance of the *MLE* improves as the value of the fraction non-conforming decreases. In other words, as the deviation from the in-control value of fraction nonconformity level increases, the standard error of estimates decreases.

Tab. 5. Average change-point estimate of $\bar{\hat{\tau}}$ and standard error for $p_0 = 0.0005$, $\tau = 100$, $p_1 > p_0$, and $\rho = 0.1$

	p_1	0.0006	0.0007	0.0008	0.0009
	$\bar{\hat{\tau}}$	142	134	107.5	106.5
ρ $= 0.1$	$Se(\bar{\hat{\tau}})$	2.662	2.466	1.160	1.087
	$\hat{E}(T)$	301.3	288.1	225.4	223.2

Tab. 6. Average change-point estimate of $\bar{\hat{\tau}}$ and standard error for $p_0 = 0.0005$, $\tau = 100$, $p_1 < p_0$, and $\rho = 0.1$

	p_1	0.0004	0.0003	0.0002
	$\bar{\hat{\tau}}$	103	102	102
$\rho = 0.1$	$Se(\bar{\hat{\tau}})$	0.791	0.707	0.7071
	$\hat{E}(T)$	203.4	212.2	214.4

Tab. 7. Average change-point estimates of $\bar{\hat{\tau}}$ and standard error for $p_0 = 0.0005$, $\tau = 100$, $p_1 > p_0$, and $\rho = 0.5$

	p_1	0.0006	0.0007	0.0008	0.0009
	$\bar{\hat{\tau}}$	106	105	103.5	102
ρ	$Se(\bar{\hat{\tau}})$	1.0488	0.9682	0.8366	0.7070
$= 0.5$	$\hat{E}(T)$	221	219	215.5	212.2

Tab. 8. Average change point estimate of $\bar{\hat{\tau}}$ and standard error for $p_0 = 0.0005$, $\tau = 100$, $p_1 < p_0$, and $\rho = 0.5$

	p_1	0.0004	0.0003	0.0002
	$\bar{\hat{\tau}}$	102	101	101
$\rho = 0.5$	$Se(\bar{\hat{\tau}})$	0.7072	0.7071	0.707
	$\hat{E}(T)$	208.9	211.1	211.1

Tab. 9. Average change point estimate of $\bar{\hat{\tau}}$ and standard error for $p_0 = 0.0005$, $\tau = 100$, $p_1 > p_0$, and $\rho = 0.8$

	p_1	0.0006	0.0007	0.0008	0.0009
	$\bar{\hat{\tau}}$	100.5	100.5	100.4	100.2
ρ	$Se(\bar{\hat{\tau}})$	0.7071	0.7071	0.707	0.706
$= 0.8$	$\hat{E}(T)$	210	210	211.1	210

Tab. 10. Average change point estimate of $\bar{\hat{\tau}}$ and standard error for $p_0 = 0.0005$, $\tau = 100$, $p_1 < p_0$, and $\rho = 0.8$

	p_1	0.0004	0.0003	0.0002
	$\bar{\hat{\tau}}$	100.5	100.5	100.4
$\rho = 0.8$	$Se(\bar{\hat{\tau}})$	0.707	0.707	0.7071
	$\hat{E}(T)$	210	211.1	210

Table (5) shows that an increase in the fraction nonconformity level $p_1 = 0.0008$ and $\rho = 0.1$ would be detected by *g* control chart of 125.4 periods on average after the change has occurred in the process. The *MLE* provides an average estimate of 107.5 for the period when the step change has occurred in fraction nonconformity level that is very close to real change point. The standard error of the estimates is 1.160 that is very small. Table (5) indicates that the estimate for the change-point period in the case of a correlation is closer to the true value.

Table (6) indicates that *g* control chart signals 103.4 periods on average after the process fraction nonconformity level decreases by 40% to $p_1 = 0.0003$ .The change-point estimator performs relatively well by yielding 103 as the average estimate for the period when the step change has occurred. The results in Table (6) reveal that the performance of the *MLE* improves as the value of the fraction nonconforming decreases and correlation increases.

Table (7) shows that the increase in the fraction nonconformity level $\boldsymbol{p_1 = 0.0007}$ and $\boldsymbol{\rho = 0.5}$ would be detected by g control chart of 119 periods on average after the change has occurred in the process. The *MLE* provides an average estimate of 105 for the period when the step change has occurred in fraction nonconformity level that is very close to real change point. The standard error of the estimates is *0.9682* that is very small. Table (7) indicates that the estimate for period when the correlation exists in process is closer to the true value.

Table (8) indicates that g control chart signals 108.9 periods on average after a decrease in the process fraction nonconformity level to $\boldsymbol{p_1 = 0.0004}$. The change-point estimator performs relatively well by yielding 102 as the average estimate for the period when the step change has occurred. The results in Table (8) reveal that the performance of the *MLE* improves as the value of the fraction nonconforming decreases and correlation increases.

Table (9) shows that an increase in the fraction nonconformity level to $\boldsymbol{p_1 = 0.0006}$ and $\boldsymbol{\rho = 0.8}$ would be detected by g control chart 110 periods on average after the change has occurred in the process. The *MLE* provides an average estimate of 100.5 for the period when the step change has occurred in fraction nonconformity level that is very close to real change point. The standard error of the estimates is 0.7071 which is very small. Table (9) indicates that the estimates for the period with the correlation in process when the step change occurred gets closer to the true value.

We now consider the frequency with which the change point estimate is within m periods of the true change point for m=1, 2, 3, 4, 5, 10, 15, 20, 25, 30, and 40. The results derived from the same simulation study for the increase and decrease in the process fraction nonconformity levels are given in Tables (11) and (12).

Tab. 11. Precision of the estimator: $\boldsymbol{p_0 = 0.0005, \tau = 100, p_1 > p_0}$

	p_1	0.0006	0.0007	0.0008	0.0009
$\rho = 0$	$\hat{P}(\hat{\tau} = \tau)$	0.0110	0.04136	0.062369	0.014587
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 1)$	0.011069	0.062836	0.064681	0.016435
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 2)$	0.022139	0.085671	0.067362	0.032871
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 3)$	0.033208	0.148507	0.069043	0.049306
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 4)$	0.044278	0.151343	0.071724	0.065741
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 5)$	0.055347	0.164178	0.073405	0.082177
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 10)$	0.110694	0.178358	0.146812	0.164355
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 15)$	0.166043	0.192538	0.22022	0.246537
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 20)$	0.221393	0.256722	0.293633	0.328724
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 25)$	0.276745	0.320908	0.36705	0.410917
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 30)$	0.332099	0.385098	0.440472	0.493119
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 40)$	0.442818	0.513493	0.58734	0.657554

Tab. 12. Precision of the estimator: $\boldsymbol{p_0 = 0.0005, \tau = 100, p_1 < p_0}$

	p_1	0.0004	0.0003	0.0002
$\rho = 0$	$\hat{P}(\hat{\tau} = \tau)$	0.0106	0.08236	0.147565
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 1)$	0.02653	0.158157	0.156753
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 2)$	0.03305	0.116313	0.195941
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 3)$	0.04958	0.17447	0.232627
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 4)$	0.05061	0.290783	0.391883
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 5)$	0.07263	0.36519	0.498623
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 10)$	0.1238	0.37896	0.581567
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 15)$	0.2042	0.41582	0.587825
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 20)$	0.3304	0.56231	0.783767
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 25)$	0.4700	0/65478	0.872352
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 30)$	0.5055	0.78952	0.979711
	$\hat{P}(\|\hat{\tau} - \tau\| \leq 40)$	0.5379	0.89552	0.986325

Table (3) shows that, for a 40% increase in the fraction nonconformity level $\boldsymbol{p_1 = 0.0007}$, the control chart yields an *ARL* of 481.7. According to Table (11), the proposed *MLE* estimates the true change point 4.1% of the times correctly. The change point is estimated 16.41% of the times within five periods of the process change point. Similarly, for a 60% increase in the fraction nonconformity level $\boldsymbol{p_1 = 0.0008}$, the control chart yields an *ARL* of 346.5. For this step change, the proposed *MLE* estimates the process change point 8.2% of the times correctly. The change point is estimated 22% of the times within 15 periods of the process change point. The results in Table (11) indicate that the performance of the estimator improves as the magnitude of the change increases.

Table (4) indicates that for a 40% decrease in the process fraction nonconformity level $\boldsymbol{p_1 = 0.0003}$,the control chart *ARL* drops to 157.2. For a step change of this magnitude, according to Table (12), the true process change point is estimated 8.2% of the times correctly. For a 60% decrease $\boldsymbol{p_1 = 0.0002}$ in the fraction nonconformity level, the control chart yields an *ARL* of 13.9, and the true process change point is estimated 14.75% of the times correctly. Simulation results indicate that the change point is estimated 49.86% of the times within five periods of the true process change point. According to Tables (11) and (12), it can be shown that with an increase or decrease in the change magnitude, a closer estimate of the change point is expected.

We now consider the number of times a change point estimation is around *m* periods *m*= 1,2,3,4,5. The results of the simulations with different increasing and decreasing in the fraction nonconforming when correlation is present are shown in the following tables.

Tab. 13. Precision of the estimator: $\boldsymbol{p_0 = 0.0005, \tau = 100, p_1 > p_0, \rho = 0.1}$

	$\boldsymbol{p_1}$	0.0006	0.0007	0.0008	0.0009
	$\boldsymbol{\hat{P}(\hat{\tau} = \tau)}$	0.246	0.330	0.497	0.537
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 1)}$	0.273	0.339	0.509	0.556
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 2)}$	0.312	0.345	0.517	0.599
$\boldsymbol{\rho = 0.1}$	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 3)}$	0.358	0.486	0.529	0.641
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 4)}$	0.466	0.576	0.693	0.712
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 5)}$	0.518	0.674	0.788	0.879

Tab. 14. Precision of the estimator: $\boldsymbol{p_0 = 0.0005, \tau = 100, p_1 < p_0, \rho = 0.1}$

	$\boldsymbol{p_1}$	0.0004	0.0003	0.0002
	$\boldsymbol{\hat{P}(\hat{\tau} = \tau)}$	0.369	0.418	0.524
$\boldsymbol{\rho = 0.1}$	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 1)}$	0.389	0.433	0.569
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 2)}$	0.426	0.498	0.636
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 3)}$	0.445	0.541	0.754
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 4)}$	0.530	0.626	0.366
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 5)}$	0.632	0.799	350.8

Table (5) shows that for a 40% increase in $\boldsymbol{p_1 = 0.0007}$, the control chart yields an *ARL* of 113.1. Table (13) shows that the *MLE* estimates the true change point correctly 33% of the times. The change point is estimated 67.4% of the times within five periods of the process change point. Similarly, for a 60% increase $\boldsymbol{p_1 = 0.0008}$, the control chart yields an *ARL* of 112.2. The proposed *MLE* estimates the process change point correctly 49.7% of the times. The change point is estimated 78.8% of the times within 5 periods of the process. The results in Table (13) indicate that the performance of the estimator improves as the magnitude of the change and correlation increase.

Table (6) indicates that for $\boldsymbol{p_1 = 0.0003}$,the control chart *ARL* decreases to 112.2. According to Table (14), the true process change point is estimated correctly 41.8% of the times. For $\boldsymbol{p_1 = 0.0002}$, *ARL* is 114.4 and the true process change point is estimated correctly 52.4% of the times. Results indicate that the change point is estimated correctly 85.3% of the times within five periods of the true process change point.

Tab. 15. Precision of the estimator: $\boldsymbol{p_0 = 0.0005, \tau = 100, p_1 > p_0, \rho = 0.5}$

	$\boldsymbol{p_1}$	0.0006	0.0007	0.0008	0.0009
	$\boldsymbol{\hat{P}(\hat{\tau} = \tau)}$	0.456	0.567	0.599	0.634
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 1)}$	0.483	0.578	0.631	0.694
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 2)}$	0.562	0.661	0.694	0.765
$\boldsymbol{\rho = 0.5}$	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 3)}$	0.678	0.789	0.799	0.879
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 4)}$	0.769	0.881	0.891	0.891
	$\boldsymbol{\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 5)}$	0.846	0.947	0.976	0.986

Tab. 16. Precision of the Estimator: $p_0 = 0.0005, \tau = 100, p_1 < p_0, \rho = 0.5$

p_1		0.0004	0.0003	0.0002
	$\hat{P}(\hat{\tau} = \tau)$	0.565	0.606	0.619
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 1)$	0.670	0.689	0.774
$\rho = 0.5$	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 2)$	0.742	0.787	0.827
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 3)$	0.797	0.847	0.870
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 4)$	0.838	0.899	0.885
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 5)$	0.897	0.982	0.956

Table (7) shows that for $p_1 = 0.0007$, the control chart yields an *ARL* of 219. Table (15) shows that the proposed *MLE* estimates the true change point correctly 56.7% of the times. The change point is estimated 94.7% within five periods of the process change point. For $p_1 = 0.0008$, *ARL* is 215.5. The proposed *MLE* estimates the process change point correctly 59.9% of the times. The change point is estimated 97.6% of the times within 5 periods of the process. Table (15) indicates that the performance of the estimator improves as the magnitude of the change and correlation increase. Table (8) indicates that for $p_1 = 0.0003$, the control chart *ARL* decreases to 211.1. According to Table (16), the true process change point is estimated correctly 60.6% of the times. Results indicate that the change point is estimated95.6% of the times within five periods of the true process change point.

Tab. 17. Precision of the estimator: $p_0 = 0.0005, \tau = 100, p_1 > p_0, \rho = 0.8$

p_1		0.0006	0.0007	0.0008	0.0009
	$\hat{P}(\hat{\tau} = \tau)$	0.971	0.988	0.988	0.988
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 1)$	0.985	0.989	0.988	0.988
$\rho = 0.8$	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 2)$	0.988	0.998	0.998	0.998
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 3)$	1	1	1	1
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 4)$	1	1	1	1

Tab. 18. Precision of the estimator: $p_0 = 0.0005, \tau = 100, p_1 < p_0, \rho = 0.8$

p_1		0.0004	0.0003	0.0002
	$\hat{P}(\hat{\tau} = \tau)$	0.987	0.988	0.988
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 1)$	0.988	0.988	0.998
$\rho = 0.8$	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 2)$	0.999	0.989	0.999
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 3)$	1	1	1
	$\hat{P}(\lvert\hat{\tau} - \tau\rvert \le 4)$	1	1	1

Tables (17) and (18) show that by increasing or decreasing the amount of change and correlation, the accurate probability of estimate of the change point increases. For example, for a 40% increase in $p_1 = 0.0007$, the proposed *MLE* estimates the true change point correctly 98.8% of the times, and 100% times estimated change point is within 4 periods of the true process change point. Similarly, for a 40 % reduction in the fraction nonconforming $p_1 = 0.0003$, according to Table (18) in 98.8% of the times, the change point is correctly estimated within one period and 100% of the times within 4 periods of the true change point.

7. Conclusion

Knowing the real-time change in the process not only helps process engineers to discover and eliminate sources of assignable causes effectively, but also increases production efficiency in industry. This paper proposed an estimator based on MLE method when correlation can be present to identify the period of step change for fraction nonconformity level in high-yield processes. The performance of the proposed estimator with different values of fraction nonconforming level and correlation was investigated. Results show that the estimator has a reasonable performance for different levels of nonconformity.

References

[1] Noorossana, R., Saghaei, A., Paynabar, K., & Abdi, S.Identifying the period of a step change in high-yield processes. Quality and Reliability Engineering International; Vol. 25; 2009, pp. 875–883.

[2] Zandi, F., Niaki, S. T. A., Nayeri, M. A., & Fathi, M. Change-point estimation of the process fraction non-conforming with a linear trend in statistical process control.

International Journal of Computer Integrated Manufacturing; Vol. 24, 2011, pp. 939–947.

[3] Noorossana, R., Paynabar, K., Moradimanesh M. Estimating Time of a Simple Step Change in Non-conforming Items in High-Yield Processes.International Journal of Industrial Engineering & Production Management;Vol. 22, No. 4, 2012, pp. 319-330.

[4] Niaki, S. T. A., & Khedmati, M. Change point estimation of high-yield processes with a linear trend disturbance. International Journal of Advanced Manufacturing Technology; Vol. 69, 2013, pp. 491–497.

[5] Niaki, S. T. A., & Khedmati, M. Change point estimation of high-yield processes experiencing monotonic disturbances. Computers & Industrial Engineering; Vol. 67, 2014, pp. 82–92.

[6] Madsen, R. W. Generalised binomial distributions, *Communications in Statistics: Theory and Methods*, Vol. 22, 1993, pp. 3065 - 3086.

[7] Benneyan JC (1991): Statistical Control Charts Based on Geometric and Negative Binomial Populations, masters thesis, University of Massachusetts, Amherst.

[8] Kaminsky FC, Benneyan JC, Davis RB, and Burke RJ "Statistical Control Charts Based on a Geometric Distribution", *Journal of Quality Technology* Vol. 24, No. 2, 1992, pp. 63-69.

[9] Casella G, Berger RL(1990). *Statistical Inference*. Wadsworth and Brooks /Cole :Belmont, CA,.

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