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A Mathematical Model for Designing Optimal Urban Gas Networks, an Ant Colony Algorithm and A Case Study

Mojtaba Torkinejad, Iraj Mahdavi*, Nezam Mahdavi-Amiri & Mirmehdi Seyed Esfahani

Mojtaba Torkinejad, *Department of Industrial Engineering, Mazandaran University of Science and Technology*
Iraj Mahdavi, *Department of Industrial Engineering, Mazandaran University of Science and Technology*
Nezam Mahdavi-Amiri, *Faculty of Mathematical Sciences, Sharif University of Technology*
Mirmehdi Seyed Esfahani, *Department of Industrial Engineering & Management System, Amirkabir University of Technology*

KEYWORDS
Designing urban networks,
Optimization,
Tree structure,
Ant colony algorithm,
Pressure,
Velocity,
Metaheuristic algorithms.

ABSTRACT
Considering the high costs of the implementation and maintenance of gas distribution networks in urban areas, optimal design of such networks is vital. Today, urban gas networks are implemented within a tree structure. These networks receive gas from City Gate Stations (CGS) and deliver it to the consumers. This study presents a comprehensive model based on Mixed Integer Nonlinear Programming (MINLP) for the design of urban gas networks while taking into account topological limitations, gas pressure, and velocity and environmental limitations. An Ant Colony Optimization (ACO) algorithm is presented for solving the problem, and the results obtained by an implementation of ACO algorithm are compared with the ones obtained through an iterative method to demonstrate the efficiency of ACO algorithm. A case study of a real situation (gas distribution in Kelardasht, Iran) affirms the efficacy of the proposed approach.

1. Introduction

The expansion of gas industry as a source of clean fuel and the importance of this industry for the global and national economy have attracted much attention in the literature. Once extracted from the producing wells, gas is collected by a network of pipelines and delivered to a gas refinery plant to be purified, dehydrated and sweetened. This gas is then directed to the main gas transmission pipelines. To compensate for the pressure drop caused by gas consumption or friction in the pipelines, a compressor station is generally used per hundred kilometers to increase gas pressure up to about 1350 psig. Branches are split from these pipelines to supply gas to the consumers. As this gas gets closer to the consumption zones (cities, in general), its pressure is reduced to 250 psig through the City Gate Station (CGS). The network connecting gas

* Corresponding author: *Iraj Mahdavi*
Email: irajarash@rediffmail.com
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refinery plants to CGSs is called gas transmission network. The gas pressure ranges from 350 to 1350 psig in this network. Gas pressure decreases again once the gas is branched from the CGSs and transmitted to the Town Board Station (TBS). TBSs reduce gas pressure to 60 psig. The network transmitting gas from CGSs to TBSs is called feeding network. Gas pressure ranges from 150 to 250 psig in this network. Gas is delivered to urban consumption zones through branches split from the TBSs. The network transmitting gas from TBSs to urban consumption zones is called distribution network. The gas pressure ranges from 40 to 60 psig in this network. An urban gas network consists of a feeding network and a distribution network.

Transmission network projects are often strategic and costly. A major portion of gas studies is devoted to these networks. Some of these studies are collected and classified in [1] and [2]. However, out of various topics in this industry, the present study is focused on gas distribution in urban areas. The number of construction projects of these networks is high, having a great impact on the economy of the cities. This aspect of gas industry is less attractive to researchers than the transmission networks; therefore, available studies in this regard are scant.

Wu et al. [3] introduced a mathematical model for distribution networks and solved it for looped and tree networks. El-Mahdy et al. [4] introduced a method to determine the optimal pipe diameter in a given network using a Genetic Algorithm (GA). Djebedjian et al. [5] introduced a mathematical model to determine the optimal pipe diameter in a network; in addition to solving the model, they used a GA to solve the problem and worked through a case study to demonstrate the efficiency of GA. Mohajeri et al. [6] showed how to use Minimum Spanning Tree (MST) to design gas distribution networks. In another study, Mohajeri et al. [7] used an Ant Colony Optimization (ACO) algorithm to design distribution networks. Humpela et al. [8] proposed a MINLP model for topology of gas networks, and in order to speed up computations, they introduced a new class of inequalities to relax their proposed model. Shiono and Suzuki [9] introduced an algorithm for tree networks with a single supply source. They assumed that the pipe diameters were continuous. In their study, a heuristic algorithm was presented to convert optimal pipe diameters into approximate discrete pipe diameters. Mikolajkova et al. [10] presented a new model for the future extension of a given gas network in which fluid equations were considered. By providing a simple model for determining infeasibility, Humpola and Serrano [11] provided a method for pruning and increasing the computational speed in solving gas networks with MINLP models.

While reviewing the available studies in this area, researchers identified some major aspects of designing urban gas networks by considering the design departments in related companies (Table 1). Table 2 examines the relevant studies in two respects: first, subjects of Table 1 are covered; second, the specifications of the models and methods for solving them are given.

Herein, we present a comprehensive model for network design as provided in Table 2 and present new approaches to solving small and large gas urban networks. In the present study, feeding and distribution networks are modeled simultaneously for the first time. In addition, the model can determine the size of network components, such as diameters of pipes and capacities of TBSs, in the optimal network. Herein, to avoid solving a non-linear model, an innovative local search method is proposed and used in an ACO algorithm to compute the diameters of the network pipes.

Tab. 1. Coordinates of network elements for Example 2

CGS	TBS installation candidate sites	Consumption zones
CGS=[800 0]	TBS1=[0 800] TBS2=[100 200] TBS3=[500 100]	z1=[100 800] z2=[300 500] z3=[500 400] z4=[700 700] z5=[900 800]

Tab. 2. Values of parameters for Example 2

<table>
<tr><td>Parameter description</td><td>Parameter value</td></tr>
<tr><td>Number of consumption zones</td><td>$m = 5$</td></tr>
<tr><td>Distances in the distribution network</td><td>$dZ =$
<table>
<tr><td>j \ i</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>1</td><td>0</td><td>360.5</td><td>565.6</td><td>608.2</td><td>800</td><td>100</td><td>600</td><td>806.2</td></tr>
<tr><td>2</td><td>360.5</td><td>0</td><td>223.6</td><td>447.2</td><td>670.8</td><td>424.2</td><td>360.5</td><td>447.2</td></tr>
<tr><td>3</td><td>565.6</td><td>223.6</td><td>0</td><td>360.5</td><td>565.6</td><td>640.3</td><td>447.2</td><td>300</td></tr>
<tr><td>4</td><td>608.2</td><td>447.2</td><td>360.5</td><td>0</td><td>223.6</td><td>707.1</td><td>781.0</td><td>632.4</td></tr>
<tr><td>5</td><td>800</td><td>670.8</td><td>565.6</td><td>223.6</td><td>0</td><td>900</td><td>1000</td><td>806.2</td></tr>
<tr><td>6</td><td>100</td><td>424.2</td><td>640.3</td><td>707.1</td><td>900</td><td>0</td><td>608.2</td><td>860.2</td></tr>
<tr><td>7</td><td>600</td><td>360.5</td><td>447.2</td><td>781.0</td><td>1000</td><td>608.2</td><td>0</td><td>412.3</td></tr>
<tr><td>8</td><td>806.2</td><td>447.2</td><td>300</td><td>632.4</td><td>806.2</td><td>860.2</td><td>412.3</td><td>0</td></tr>
</table></td></tr>
<tr><td>Distances in the feeding network</td><td>$dT =$
<table>
<tr><td>j \ i</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>6</td><td>0</td><td>608.2</td><td>860.2</td></tr>
<tr><td>7</td><td>608.2</td><td>0</td><td>412.3</td></tr>
<tr><td>8</td><td>860.2</td><td>412.3</td><td>0</td></tr>
<tr><td>9</td><td>1131.3</td><td>728.0</td><td>316.2</td></tr>
</table></td></tr>
<tr><td>Volume of gas consumed in the consumption zones</td><td>D=[1000;4000;6000;8000;5000]</td></tr>
<tr><td>Number of candidate TBS installation sites</td><td>$n = 3$</td></tr>
<tr><td>Number of TBS varieties installable in the network</td><td>Ts=3</td></tr>
<tr><td>Volume of gas deliverable by each type of TBS</td><td>C=[5000 ; 10000 ;25000]</td></tr>
<tr><td>Cost of TBS types</td><td>CoT =[30000; 50000 ; 100000]</td></tr>
<tr><td>Cost of choosing each candidate TBS installation site</td><td>CoS=[10000;10000;10000]</td></tr>
<tr><td>Number of pipe varieties used in the distribution network</td><td>P=3</td></tr>
<tr><td>Diameter of the pipes used in the distribution network</td><td>SZ=[6;8;10]</td></tr>
<tr><td>Cost of the pipe varieties used in the distribution network</td><td>CLZ=[50;65;85]</td></tr>
<tr><td>Number of pipe varieties used in the feeding network</td><td>W=3</td></tr>
<tr><td>Diameter of the pipes used in the feeding network</td><td>ST =[10;12;14]</td></tr>
<tr><td>Cost of the pipe varieties used in the feeding network</td><td>CLT =[85;105;130]</td></tr>
<tr><td>Maximum gas velocity in the network (m/s)</td><td>$MaxV\_Znet = MaxV\_Tnet$ =20 m/s</td></tr>
<tr><td>Minimum pressure in the distribution network (psig)</td><td>$MinP\_Znet$ = 40, psig=54.7 psia</td></tr>
<tr><td>Minimum pressure in the feeding network (psig)</td><td>$MinP\_Tnet$=150 , psig=164.7 psia</td></tr>
</table>

It should be noted that we are to obtain an optimal solution for designing urban gas networks using an algorithm in our mathematical model. An optimal solution consisting of a topology of the designed network shows the connection of nodes by pipes, called optimal design, chosen pipe diameters, and TBS capacities in the network, called optimal pipe diameters and optimal TBSs capacities, respectively.

The rest of our work is organized as follows. The statement of the problem, limitations, and hypotheses are described in Section 2. A mathematical model is introduced in Section 3 and validated in Section 4. An ACO algorithm is introduced in Section 5 for designing gas networks. Numerical results and discussions are presented in Section 6, and a case study is worked through in Section 7. We conclude the paper in Section 8.

2. Problem Statement

An overview of a gas network is shown in Fig. 1. In this figure, gas sub-networks, their locations, connections, equipment, and especially the components of the feeding and distribution networks, are clearly demonstrated.

An urban gas network can be regarded as a graph consisting of nodes, including CGSs, TBSs and consumers (zones) as well as edges representing the pipelines connecting the nodes. Designing a network for supplying gas to urban consumers presents four limitations including topological limitations, fluid limitations, material and equipment limitations, and physical-environmental limitations. An optimal gas network does not only supply the consumers' required gas, but also meets the limitations and is built with a minimal cost (including the costs of feeding and distribution networks). The cost of building the network involves the cost of equipment procurement (pipes and TBSs), construction cost, and the cost of land acquisition. The cost of land acquisition is comprised of the cost of buying land and satisfying other beneficiaries.

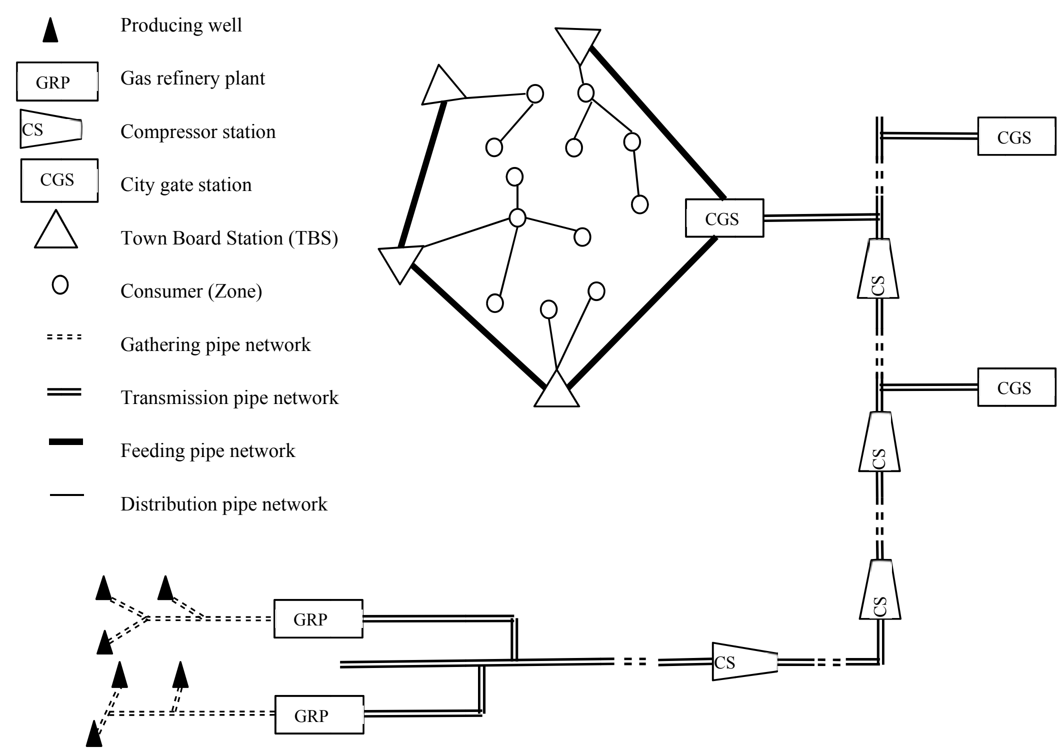

Fig. 1. An overview of a gas network

2-1. Topological limitations

- The network is developed within a tree structure.
- The network is directed, that is, gas can flow in only one of the modes as shown in Table 3.
- There is no connection from a node to itself.
- There can only be one connection from any node to another node.
- The pipe diameter is in a descending order, that is, the gas flows from pipes with larger diameters to pipes with the same or smaller diameters –not in the reverse order.

2-2. Fluid limitations

- Kirchhoff's circuit laws govern the network's nodes.
- The maximum gas velocity has an upper limit and the minimum gas pressure has a lower limit in the pipelines.

In gas distribution networks, the minimum acceptable pressure is 40 psig for the consumers and 150 psig for the efficient performance of the TBS, which are equivalent to 54.7 and 164.7 psia. The maximum gas velocity is usually less than 20 m/s. There are various methods for calculating the pressure between two points in a pipeline [14]. A simplified IGT equation is used here:

$$P_1^2\text{-}P_2^2=\left[\frac{L}{1076\times d^{4.8}}\right]Q^{1.8}, \quad (1)$$

where L is pipe length in meters, Q is volume flow rate in m^3/h and d is pipe's inside diameter in inches, and P_1 and P_2 are respectively the upstream and downstream pressures in psia. The gas velocity is calculated using the following equations:

where

V is the gas velocity in m/s,

Q is the volume flow rate in m^3/h,

T is the temperature in Rankine (T = 520 ^{oR}),

P_{ave} is the average pressure in psia,

P_1 is the upstream pressure in psia,

P_2 is the downstream pressure in psia, and

d is the pipe inside diameter in inch.

2-3. Material and equipment limitations

Pipes available in the market have specific diameters. TBSs are also produced with specific capacities. They offer limited choices for application.

2-4. Physical-environmental limitations

Depending on the location where the urban gas network is to be built, land dues, rivers, historical sites, environmentally protected areas, etc., present limitations to the choice of a place for the installation of TBSs and the transition of pipelines, which may not freely be installed between any two selected nodes.

2-5. Assumptions

The following assumptions generally hold in the design of urban gas networks.

(1) Pipe’s material is the same in the entire network.

(2) Pipes are installed horizontally on the same level, that is, the upstream and downstream elevations are the same for all the pipes.

$$V=0.0155\frac{Q\times T}{P_{\text{ave}}\times d^2}\ , \qquad (2)$$

$$P_{\text{ave}}=\frac{2}{3}[(P_1+P_2)-\frac{P_1\times P_2}{P_1+P_2}]\ , \qquad (3)$$

(3) There are no active elements such as compressors in the network.

(4) Flow in the network is assumed to be steady, that is, pressure and velocity do not vary with time.

3. Urban Network Modeling

Herein, a mathematical model is presented containing binary and continuous variables. Binary variables in the model are used to show the presence or absence of pipes between two nodes and also to show the installation of a TBS on a TBS installation candidate site. A number of sites are first selected as TBS installation candidate sites before starting the design of the urban network, and designers are to select the most appropriate one among them for installation of the TBS. Continuous variables are used to show the amount of flow and the average gas velocity between any two nodes and to also determine the pressure in each node. The parameters, variables and limitations of the model are presented in the following section.

3-1. Parameters

M		Number of consumption zones
N		Number of candidate TBS installation sites
P		Number of pipe varieties used in the distribution network
W		Number of pipe varieties used in the feeding network
SZ_p ,	$p=1,\dots,P$	Diameter of pipe type p in the distribution network
ST_w ,	$w=1,\dots,W$	Diameter of pipe type w in the feeding network
Ts		Number of TBS types installable in the network
dZ_{ij} ,	$i,j=1,\dots,m+n$, $i\neq j$	Distance between the zones and the TBSs in the distribution network
dT_{ij} ,	$i=m+1,\dots,m+n+1$, $j=m+1,\dots,m+n$, $i\neq j$	Distance between the CGS and the TBSs in the feeding network
S ,	$S= S_1+ S_2$, $S_1= m(m-1)+mn$, $S_2= n^2$	Number of connections in the network
L		Number of physical-environmental limitations
R		Coefficient matrix of the physical-environmental limitations

b		Matrix on the right-hand side related to the physical-environmental limitations
D_i ,	i=1,…,m	Volume of gas consumed by each consumer per unit of time
C_t ,	t= 1,…,Ts	Volume of gas delivered by TBS type t per unit of time
CLZ_p ,	p=1,…,P	Cost of installing pipe type p per unit of length in the distribution network
CLT_w ,	w=1,…,W	Cost of installing pipe type w per unit of length in the feeding network
CoS_i ,	i=m+1,…,m+n	Cost of choosing location i for TBS installation
CoT_t ,	t= 1,…,Ts	Cost of TBS type t
CaT_i ,	i= m+1,…, m+n	Maximum TBS capacity installable in location i
T		Temperature in rankine (T = 520 ^{oR})
ZD_{ij}	i,j=1,…,m+n , $i \neq j$	Diameter of the pipe connecting node i to node j in the distribution network
TD_{ij}	i=m+1,…,m+n+1 , j=m+1,…,m+n , $i \neq j$	Diameter of the pipe connecting node i to node j in the feeding network
ZP_{ij}^{ave}	i,j=1,…,m+n , $i \neq j$	Average pressure in the pipe connecting node i to node j in the distribution network
TP_{ij}^{ave}	i=m+1,…,m+n+1 , j=m+1,…,m+n , $i \neq j$	Average pressure in the pipe connecting node i to node j in the feeding network
MaxV_Znet		Maximum gas velocity in the distribution networks in m/s
MaxV_TNet		Maximum gas velocity in the feeding network in m/s
MinP_Znet		Minimum gas pressure in the distribution network in psia
MinP_Tnet		Minimum gas pressure in the feeding network in psia.

3-2. Variables

X_{ijp} ,	i,j=1,…,m+n , $i \neq j$ p=1,…,P	The arc between two nodes in the distribution network If node i is connected to node j through a type p pipe, this variable is one; otherwise, it is zero
X_{ij} ,	i,j=1,…,m+n , $i \neq j$	Connections in the distribution network If node i is connected to node j, this variable is one; otherwise, it is zero
Y_{ijw} ,	i =m+1,…,m+n+1 j=m+1,…,m+n, $i \neq j$ w=1,…,W	The arc between two nodes in the feeding network If node i is connected to node j through a type w pipe, this variable is one; otherwise, it is zero
Y_{ij} ,	i =m+1,…,m+n+1 j=m+1,…,m+n, $i \neq j$	Connections in the feeding network If node i is connected to node j, this variable is one; otherwise, it is zero
T_{it} ,	i= m+1,…,m+n t=1,…,Ts	If TBS type t is installed in location i, this variable is one; otherwise, it is zero
fZ_{ij} ,	i,j=1,…,m+n, $i \neq j$	Volume of gas passing through each edge in the distribution network
fT_{ij} ,	i =m+1,…,m+n+1 j=m+1,…,m+n, $i \neq j$	Volume of gas passing through each edge in the feeding network

ZP_i	$i = 1,\dots,m$	Gas pressure at node i in the distribution network in psia
TP_i	$i = m+1,\dots,m+n$	Gas pressure at node i in the feeding network in psia
ZV_{ij}	$i,j=1,\dots,m+n$, $i\neq j$	Velocity of gas passing through each edge in the distribution network in m/s
TV_{ij}	$i=m+1,\dots,m+n+1$ $j=m+1,\dots,m+n$, $i\neq j$	Velocity of gas passing through each edge in the feeding network in m/s.

3-3. Mathematical model

Our proposed model is a mathematical programming model with linear and non-linear constraints with the aim of minimizing the total cost of network's installation including installment cost of distribution pipelines (Z_1), cost of installing feeding pipelines (Z_2), cost of procurement and installation of TBSs (Z_3), and cost of choosing candidate TBS sites (Z_4):

$$\min \quad Z=Z_1+Z_2+Z_3+Z_4 \tag{4}$$

$$Z_1=\sum_{i=1}^{m+n}\sum_{j=1,\ j\neq i}^{m}\sum_{p=1}^{P} X_{ijp}\,.dZ_{ij}\ \ .CLZ_p \tag{5}$$

$$Z_2=\sum_{i=m+1}^{m+n+1}\sum_{j=m+1,\ j\neq i}^{m+n}\sum_{w=1}^{W} Y_{ijw}\,.dT_{ij}\ \ .CLT_w \tag{6}$$

$$Z_3=\sum_{i=m+1}^{m+n}\sum_{t=1}^{T_s} CoS_i.T_{it} \tag{7}$$

$$Z_4=\sum_{i=m+1}^{m+n}\sum_{t=1}^{T_s} CoT_t.T_{it} \tag{8}$$

s.t.

$$\sum_{p=1}^{P}(X_{ijp}+X_{jip})\leq 1\ , \qquad i,j=1,\dots,m\ ,\ i<j \tag{9}$$

$$\sum_{w=1}^{W}(Y_{ijw}+Y_{jiw})\leq 1\ , \qquad i,j=m+1,\dots,m+n\ ,\ i<j \tag{10}$$

$$\sum_{p=1}^{P}(X_{jkp}\ .\ SZ_p)\leq\sum_{p=1}^{P}(X_{ijp}\ .\ SZ_p)\ , \qquad i=1,\dots,m+n;\ j,k=1,\dots,m,\ i\neq j\neq k \tag{11}$$

$$\sum_{w=1}^{W}(Y_{jkw}\ .\ ST_w)\leq\sum_{w=1}^{W}(Y_{ijw}\ .\ ST_w)\ , \qquad i=m+1,\dots,m+n+1;\ j,k=m+1,\dots,m+n,\ i\neq j\neq k \tag{12}$$

$$\sum_{p=1}^{P}X_{ijp}\ \leq 1\ , \qquad i=m+1,\dots,m+n,\ j=1,\dots,m \tag{13}$$

$$\sum_{w=1}^{W}Y_{ijw}\ \leq 1\ , \qquad i=m+n+1,\ \ j=m+1,\dots,m+n \tag{14}$$

$$\sum_{i=1}^{m+n}\sum_{p=1}^{P}X_{ijp}\ =1\ , \qquad j=1,\dots,m,\ j\neq i \tag{15}$$

$$\Big(\sum_{j=m+1}^{m+n}\sum_{w=1}^{W}Y_{ijw}+\sum_{k=1}^{m}\sum_{p=1}^{P}X_{ikp}\Big)\Big(1-\sum_{j=m+1}^{m+n+1}\ \sum_{w=1}^{W}Y_{jiw}\Big)=0\ , \qquad i=m+1,\dots,m+n,\ \ i\neq j\neq k \tag{16}$$

$$\sum_{i=1}^{m+n}\sum_{j=1,j\neq i}^{m}\ \sum_{p=1}^{P}X_{ijp}\ =m \tag{17}$$

$$\sum_{i=m+1}^{m+n+1}\sum_{j=m+1,j\neq i}^{m+n}\sum_{w=1}^{W}Y_{ijw}=\sum_{i=m+1}^{m+n}\sum_{t=1}^{T_s}T_{it} \qquad (18)$$

$$\sum_{t=1}^{T_s}T_{it}\leq 1\,, \qquad i=m+1,\ldots,m+n \qquad (19)$$

$$\sum_{j=m+1,j\neq i}^{m+n+1}\sum_{w=1}^{W}Y_{jiw}=\sum_{t=1}^{T_s}T_{it}\,, \qquad i=m+1,\ldots,m+n \qquad (20)$$

$$\sum_{s=1}^{S}r_{ls}.x_s=b_l\,, \qquad l=1,\ldots,L \qquad (21)$$

$$\sum_{i=m+1}^{m+n}\sum_{t=1}^{T_s}T_{it}\geq 1 \qquad (22)$$

$$\sum_{j=m+1}^{m+n}\sum_{w=1}^{W}Y_{ijw}\geq 1\,, \qquad i=m+n+1 \qquad (23)$$

$$D_i+\sum_{j=1,\ j\neq i}^{m}fZ_{ij}=\sum_{j=1,\ j\neq i}^{m+n}fZ_{ji}\,, \qquad i=1,\ldots,m \qquad (24)$$

$$\sum_{j=1,\ j\neq i}^{m}fZ_{ij}+\sum_{j=m+1,\ j\neq i}^{m+n}fT_{ij}=\sum_{j=m+1,\ j\neq i}^{m+n+1}fT_{ji}\,, \qquad i=m+1,\ldots,m+n \qquad (25)$$

$$\sum_{j=1}^{m}fZ_{ij}\leq CTM_i\,, \qquad i=m+1,\ldots,m+n \qquad (26)$$

$$fZ_{ij}(1-\sum_{p=1}^{P}X_{ijp})=0\,, \qquad i=1,\ldots,m+n;\ j=1,\ldots,m,\ i\neq j \qquad (27)$$

$$fT_{ij}(1-\sum_{w=1}^{W}Y_{ijw})=0\,, \qquad i=m+1,\ldots,m+n+1,\ j=m+1,\ldots,m+n,\ j\neq i \qquad (28)$$

$$\sum_{i=m+1}^{m+n}\sum_{t=1}^{T_s}C_t.T_{it}\geq\sum_{i=1}^{m}D_i \qquad (29)$$

$$(\sum_{j=1}^{m}fZ_{ij})\ (1-\sum_{t=1}^{T_s}T_{it})=0\,, \qquad i=m+1,\ldots,m+n \qquad (30)$$

$$\sum_{j=1}^{m}fZ_{ij}\leq\sum_{t=1}^{T_s}CaT_i.T_{it}\,, \qquad i=m+1,\ldots,m+n \qquad (31)$$

$$(ZP_i^2-ZP_j^2-\left[\frac{d_{ij}}{1076\times ZD_{ij}^{4.8}}\times fZ_{ij}^{\ 1.8}\right])\times X_{ijp}=0\,, \qquad i=1,\ldots,m+n;\ j=1,\ldots,m,\ i\neq j \qquad (32)$$

$$(TP_i^2-TP_j^2-\left[\frac{d_{ij}}{1076\times TD_{ij}^{4.8}}\times fT_{ij}^{\ 1.8}\right])\times Y_{ijw}=0\,, \qquad i=m+1,\ldots,m+n+1,\ j=m+1,\ldots,m+n,\ i\neq j \qquad (33)$$

$$ZV_{ij}=0.0155\frac{fZ_{ij}\times T}{ZP_{ij}^{ave}\times ZD_{ij}^2}\,, \qquad i=1,\ldots,m+n;\ j=1,\ldots,m,\ i\neq j \qquad (34)$$

$$TV_{ij}=0.0155\frac{fT_{ij}\times T}{TP_{ij}^{ave}\times TD_{ij}^2}\,, \qquad i=m+1,\ldots,m+n+1,\ j=m+1,\ldots,m+n,\ i\neq j \qquad (35)$$

$ZV_{ij}\times(1-\sum_{p=1}^{P} X_{ijp})=0$,	$i=1,\ldots,m+n$ $j=1,\ldots,m,\ i\neq j$	(36)
$TV_{ij}\times(1-\sum_{w=1}^{W} Y_{ijw})=0$,	$i=m+1,\ldots,m+n+1,$ $j=m+1,\ldots,m+n,\ i\neq j$	(37)
$ZP_i\geq MinP\_Znet$,	$i=1,\ldots,m$	(38)
$TP_i\geq MinP\_Tnet$,	$i=m+1,\ldots,m+n$	(39)
$ZV_{ij}\leq MaxV\_Znet$,	$i=1,\ldots,m+n,\ i\neq j$ $j=1,\ldots,m+n$	(40)
$TV_{ij}\leq MaxV\_Tnet$,	$i=m+1,\ldots,m+n+1,\ i\neq j$ $j=m+1,\ldots,m+n$	(41)
$ZP_i=74.7$,	$i=m+1,\ldots,m+n$	(42)
$TP_i=264.7$,	$i=m+n+1$	(43)
$X_{ijp}\in\{0,1\}$,	$i,j=1,\ldots,m+n,\ i\neq j$ $p=1,\ldots,P$	(44)
$Y_{ijw},T_{it}\in\{0,1\}$,	$i,j=m+1,\ldots,m+n,\ i\neq j,\ j\neq m+n+1$ $w=1,\ldots,W,\ t=1,\ldots,T_s$	(45)
$fZ_{ij}\geq 0$,	$i,j=1,\ldots,m+n,\ i\neq j$	(46)
$fT_{ij}\geq 0$,	$i,j=m+1,\ldots,m+n, i\neq j,\ j\neq m+n+1.$	(47)

Equation (4), the objective function of the problem, involves four terms, as shown by equations (5)-(8), to show the total cost comprising cost of distribution network, feeding network, choosing candidate TBS sites, and cost of TBS installation. Constraints (9)-(21) are concerned with the network topology, constraints (22)-(43) pertain to gas flow in the network, and constraints (44)-(47) show the model's variable types. Constraints (9) reveal that each of two zones in the distribution network is interconnected with no more than one type of pipeline, and constraints (10) show the same in the feeding network. Constraints (11) consider the pipe diameters in the distribution network in a descending order. Constraints (12) do the same for the feeding network. Constraints (13) show that each TBS node can be connected to a zone with only one type of pipe in the distribution network. Constraints (14) show the same for the feeding network, i.e., each TBS node can be connected to a CGS node with only one type of pipe. Constraints (15) indicate that each zone should have only one input that can branch from a TBS or another zone. Constraints (16) show that if a pipe is connected from a TBS to a zone or another TBS, that TBS should have an input that can have branched from another TBS or a CGS. The number of edges or the pipes between the nodes is exactly equal to that of zones in the distribution network, as shown in constraints (17). Constraint (18) shows the same for the feeding network, i.e., the number of edges equals that of active TBSs. Constraints (19) imply that no more than one TBS can be determined for a candidate site in the final solution. Constraints (20) show that a TBS considered for a candidate site should have at least one pipeline as its input. Constraints (21) show the physical-environmental limitations presented in the design of networks. Binary variables X_{ij} and Y_{ij}, signifying the connection of node i to node j in the distribution network and the feeding network, are used to define these limitations. These variables are calculated using the following equations:

$$X_{ij}=\sum_{p=1}^{P} X_{ijp}\ , \tag{48}$$

$$Y_{ij}=\sum_{w=1}^{W} Y_{ijw}\ . \tag{49}$$

Assuming m consumers and n candidate TBS installation sites, the number of connections in model S equals S_1+S_2, in which S_1 is the number of connections in the distribution network and equals $m(m\text{-}1)+mn$, and S_2 is the number of connections in the feeding network which is equal to n^2. Matrix R and vectors X and b are used to indicate the constraints. Vector X is a row vector consisting of all the network connections (with S elements). Each element of this vector is shown by x_s in the following general order:

$X=[X_{1,2}\ \ X_{1,3}\ \ldots\ X_{1,n}\ \ldots\ X_{n\text{-}1,1}\ \ X_{n\text{-}1,2}\ \ldots\ X_{n\text{-}1,n}$

$Y_{n+1,n+2}$ $Y_{n+1,n+3}$ … $Y_{n+1,n+m}$
… $Y_{n+m,n+1}$ $Y_{n+m,n+2}$ … $Y_{n+m,n+m-1}$
$Y_{n+m+1,n+1}$ $Y_{n+m+1,n+2}$ … $Y_{n+m+1,n+m}$].

Matrix R consists of limitation coefficients, and each row of it contains the coefficients of the variables related to one limitation. Each element in this matrix is shown by r_{ls}, in which s=1,…,S and l=1,…,L (L is the number of limitations). Vector b is a column vector and includes values of the right-hand side of the physical-environmental limitations. Example 1 elaborates on this point further.

Example 1:

Assume that gas needs to be supplied to four consumers (m=4), shown by nodes 1 to 4, from one TBS (n=1), shown by node 5. Examining the project location leads to the identification of two limitations (L=2):

- Node 3 cannot be connected to node 2.
- Node 2 should be connected to node 4, and node 4 should be connected to node 3.

Assuming that three pipe diameters (P=3) are available for the project, these limitations are defined as follows:

$$\sum_{p=1}^{P} X_{3,2,p}=0 \ ,$$

$$\sum_{p=1}^{P} X_{2,4,p}+\sum_{p=1}^{P} X_{4,3,p}=2 \ .$$

The matrix representation of these limitations is as follows:

X=[$X_{1,2}$ $X_{1,3}$ $X_{1,4}$ $X_{2,1}$ $X_{2,3}$ $X_{2,4}$ $X_{3,1}$ $X_{3,2}$ $X_{3,4}$ $X_{4,1}$ $X_{4,2}$ $X_{4,3}$ $X_{5,1}$ $X_{5,2}$ $X_{5,3}$ $X_{5,4}$ $X_{6,5}$]

$$R=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$b=\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \ RX^{T}=b \ .$$

The limitations of design in this example are thus shown by Constraints (22) and (23) applying two similar limitations, that is, at least one TBS should be selected in the distribution network and the CGS should be selected in the feeding network. Constraints (24) show Kirchhoff's law in the consumption nodes, that is, the amount of input gas in each zone equals the zone's consumption and its total output gas. Constraints (25) show the same for TBSs, that is, the amount of each TBS' output gas transmitted to the distribution network plus its output gas delivered to other TBSs equals the input gas of that TBS. This input gas is supplied by one of the TBSs or the CGS. Constraints (26) ensure that the total gas transmitted from a TBS candidate site to the consumptions sites should not exceed the maximum capacity of the TBS installable in that site. Constraints (27) ensure that when gas flows between two nodes in the distribution network, the two nodes are interconnected. Constraints (28) imply the same for feeding networks. Constraint (29) clarifies that the capacity of the TBSs installed in the candidate sites should exceed the total consumption in all the consumption zones. Constraints (30) ensure that if gas flows from one TBS candidate site toward a consumer, one TBS type should have been selected in that candidate site. Constraints (31) show that the total output gas of a TBS candidate site should be less than or equal to the capacity of the TBS installed in that site. Constraints (32) show that if there is a pipe between two nodes in the distribution network, Equation (1) should hold between the two nodes. In constraints (32), the average pressure between two nodes is obtained through $ZP_{ij}^{ave}=\frac{2}{3}[(P_i+P_j)-\frac{P_i \times P_j}{P_i+P_j}]$, and the diameter of the pipe connecting these two nodes is obtained through $ZD_{ij}=\sum_{p=1}^{P} SZ_p \times X_{ijp}$. Constraints (33) show the same for the feeding network. Constraints (34) and (35) show the gas velocity in the distribution network and the feeding network based on Equation (2). Constraints (36) and (37) indicate that gas velocity is a number other than zero if there is a connection between the two nodes in the distribution and feeding networks. The minimum acceptable gas pressure in the distribution and feeding networks is ensured with constraints (38) and (39). Constraints (40) and (41) impose limitations on the maximum gas velocity in the distribution and feeding networks. Constraints (42) and (43) show the output pressure of the TBSs and the CGS. Constraints (44) and (45)

$$\sum_{s=1}^{17} r_{ls} .x_s=b_l \ , \qquad l=1,2 \quad , \ b=\begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

show the binary variables, and constraints (46) and (47) clarify that flow cannot take a negative value in a pipe.

4. Model Validation

Pipesim software is used to verify the model and its proper performance in the exact calculation of gas pressure and velocity. This software is an instrument for simulating oil and gas networks and can be used to analyze an existing network (Pipesim cannot be used to design a network). Example 2 gives the optimal solution for a gas supply network. The topology of an optimal network is determined and the gas pressure and velocity in the network are also obtained once the model is solved. To ensure the exact calculation of gas pressure and velocity, the topology obtained from the model is solved again using Pipesim, and gas pressure and velocity are also calculated in the network, and the solutions obtained through the two methods are compared.

Example 2:

Five municipal districts are supposed to receive gas from three TBS installation candidate sites. The coordinates of the sites and the values of the parameters for the example in accordance with what is formerly introduced in Section 3-1 are provided in the Appendix.

Table 4 presents the data obtained by optimally solving the example including the selected optimal pipe diameters and nodes, the upstream and downstream pressures, gas velocities in the pipes and cost of the network, and Fig. 2 presents its corresponding gas network.

The problem was solved by Pipesim through entering the topology obtained by solving the model in the same software. Table 4 presents the results obtained from this software in percentages along with comparison with the results obtained from the previous method using the following equations:

$$\text{Pressure deviation} = \left|\frac{P_{Optimal}-P_{Pipesim}}{P_{Optimal}}\right| \times 100 \qquad (50)$$

$$\text{Velocity deviation} = \left|\frac{V_{Optimal}-V_{Pipesim}}{V_{Optimal}}\right| \times 100. \qquad (51)$$

Fig. 3 shows the distribution network of Example 2 in Pipesim. As shown, the calculations performed by both methods for obtaining gas pressure and velocity are acceptably close to each other, and the maximum deviation in either of them does not exceed 2.4%. The proposed mathematical model gives a value as the average gas velocity in each pipe, while Pipesim calculates the upstream and downstream gas velocities. Table 4 shows the difference between the two methods in the calculation of gas velocity based on the comparison of gas velocity obtained from the model with the average upstream and downstream velocities obtained from Pipesim.

5. Algorithm for Designing Gas Networks

The shortest route somehow matters when designing gas distribution and feeding networks, that is, the consumers' required gas should be transmitted through the shortest route, and the limitations set forth in Section 2 should be considered when determining this route. The goal here is to find the shortest route from a specific node to any other node in a network. Several methods have been proposed to date for solving this problem, and Cherkassky et al. [15] examined the weaknesses and strengths of some of those methods. Also, several metaheuristic algorithms, including ACO, have been proposed for solving this problem [16].

Given that the problem of distribution and its relevant issues are NP-complete, a metaheuristic algorithm is proposed to solve this problem. Algorithm 1, an algorithm for designing networks (Design_Net function), uses the ACO_ACS method to provide a list of solutions that meet all the constraints noted in Section 2, sorted in an ascending order in terms of costs. The basic steps of Algorithm 1 are shown as a function in MATLAB.

As indicated, the Design_Net function receives the coordinates of the zones, TBS installation candidate sites, the coordinates of the CGS, the correction matrix and the matrices involving *SZ*, *ST*, *CLZ*, *CLT*, *CoS*, *CoT*, and *CaT* variables (defined in Section 3-1) as input. The physical-environmental limitations stated in Section 2-4 are applied to the network design using the correction matrix. The entries of the correction matrix are binary. The element *(i,j)* refers to a connection if it is one, and the lack of a connection between *i* and *j* if it is zero, as in the following cases:

- When the connection between two nodes is physically impossible (e.g., passing through a protected area).
- A node of the zone type cannot be connected to the TBS or CGS.
- The CGS node cannot be connected to a zone.

At first, the function Cal_Distance receives the location of the zones, TBSs, and CGS and calculates the distances between the locations and stores them in a matrix. The distances are corrected using the correction matrix and stored in Corrected_Mat. Then, the values of the parameters in the Design_Net function are determined with Set_Parameters(). The function ACO-ACS(), described in detail in Section 5-1, provides a list of topologies that meet all the limitations noted in Section 2 along with their lengths. The function Find_Sizes(), described in detail in Section 5-2, determines the optimal sizes of the TBS and pipes for each topology in a way that the relevant network cost is minimized. If it is possible to determine the optimal size of the TBS or pipes and meet the minimum pressure and maximum velocity limitations, this function will return the variable Net_Ok with a value of one, and with a value of zero, otherwise. The cost of topology is calculated using Cal_Cost(). The function Save_Sol() stores the network topology; the sizes of the pipes, the TBSs, and the cost of the network are used in Sol_List.

Algorithm 1- The network design algorithm (Design_Net function).

```
function [Sol_List]=Design_Net (Zone_Mat, TBS_Mat, CGS, Correction, SZ_Mat, ST_Mat, CLZ_Mat,
CLT_Mat, CoS_Mat, CoT_Mat, CaT_Mat)
    Dis_Mat=Cal_Distance(Zone_Mat, TBS_Mat, CGS);
    Corrected_Mat=Dis_Mat.*Correction;
    Set_Parameters();      % tau0, No_of_Ants, I_Max, T_Max, T_Max, R_Max,  beta, rho, q0
    k=1;
    Net_Ok=0;
    for i=1:D_Max
       [Top_Bank, No_of_Top]=ACO_ACS(Zone_Mat, TBS_Mat, CGS, Corrected_Mat, tau0, No_of_Ants,
I_Max,                              T_Max, S_Max, beta, rho, q0);
       for j=1: No_of_Top
           [Net_sizes, Net_Ok]= Find_Sizes(Top_Bank, j, R_Max, SZ_Mat, ST_Mat, CaT_Mat);
           if Net_Ok
             Cost=Cal_Cost(Top_Bank, j, Net_sizes, Dis_Mat, CLZ_Mat, CLT_Mat, CoS_Mat, CoT_Mat);
             Sol_List=Save_Sol(Top_Bank, j, Sol_List, Net_sizes,Cost,Pres_Vel_Mat,k);
             Net_Ok=0;
             k=k+1;
           end
       end
    end
end
```

5-1. ACO-ACS Algorithm for determining network topology

Of the numerous methods introduced for the implementation of ACO algorithm [17], the Ant Colony System (ACS) was chosen for use in our study. Bunabio et al. [18] provided the details of using this method. We present a pseudo-code of the algorithm developed for ACS as Algorithm 2 or the ACO-ACS algorithm. Implementing this algorithm yields a series of network topologies along with their individual lengths as stored in Topology_bank. The symbols used in this algorithm are:

No_of_Ants	Number of all the colony ants
I_Max	Number of times a zone is randomly chosen
Coe	A coefficient for determining I_Max
T_Max	Maximum number of iterations allowed for the algorithm
S_Max	Number of times the topology is obtained from the arranged pheromone matrix
τ_0	Amount of initial pheromone (determined in Algorithm 1)
τ_{ij}	Amount of pheromone placed on arc (*i,j*)
η_{ij}	Inversion distance between nodes *i* and *j*
J_i^k	A set of nodes adjacent to node *i* to which the k^{th} ant can be transported (in the first

	step)
N_i	A set of nodes adjacent to node i to which it can be transported without forming loops (in the second step)
β	Parameter determining the importance of pheromone against distance
q	A random quantity with a uniform distribution in the range [0, 1]
q_0	A problem parameter for determining the method of choosing the next node ($0 \le q_0 \le 1$)
$R^k(t)$	Route that transports the k^{th} ant from a given zone to the CGS in the t^{th} iteration
$L^k(t)$	Length of the route that transports the k^{th} ant from a given zone to the CGS in the t^{th} iteration
T^s	Topology obtained in the s^{th} iteration
ρ	Coefficient of evaporation of pheromone from the edges
R_Max	Maximum number of searches for obtaining the optimal TBS and pipe sizes for a topology.

Algorithm 2 has two main steps. In the first step, pheromone in the constant amount of τ_0 is initially placed on all the possible routes based on Corrected_Mat. This parameter is experimentally obtained from $\tau_0 = m \times L^{-1}$, where m is the number of zones and L is the sum of all elements in Corrected_Mat. The shortest route from a zone to the CGS is chosen for each zone determined in the For loop. Once the route is selected, some routes will have more pheromones, and once the next zone is selected, the colony ants select the shortest route to the CGS depending on the current pheromone placed on the routes. In other words, each time a route is selected from a zone to the CGS, the decision is based on the experience accumulated so far. The For loop is iterated for I_Max times. The amount of I_Max should be big enough so that all the zones are selected at least once by the end of this loop. Therefore, this parameter is chosen as $\text{I\_max} = \text{Coe} \times m$, where m is the number of zones. The shortest route selected by the colony ants from a zone to the CGS is stored in R+ matrix and the length of the route is stored in L+ variable. The algorithm shows how the next node is chosen and the pheromones updated. In the second step of the algorithm, effort is made to obtain a network topology from the pheromone placed on the routes for S_Max times. Example 3 elaborates on this in further detail.

Example 3:

Assume a network with three consumers (zones) and two TBS installation candidate sites (Coordinates of elements in the network are provided in the Appendix.). The data in Table 5 provide the amount of pheromone in each edge of the network as the output of the first step of the ACO-ACS algorithm.

In the first step of Algorithm 2, the ants' movement is assumed to be from the zone to the CGS, which is opposite of the real direction of gas flow. Table 5 is therefore arranged into Table 6 (the array S) after correction of the direction. Table 6 does not copy all the entries of Table 5; for instance, given a choice between entries (2,3) and (3,2), the second entry, which has more pheromone, is inserted into the table with the order of its row and column also changed. Assume one of the iterations in the second loop of the second step (e.g., s=1). The following probabilities are first calculated based on Equation (57) in order to choose a TBS:

$$p_{6,4} = \frac{16.725}{16.725+4.0787} = 0.8 \,,$$

$$p_{6,5} = \frac{4.0787}{16.725+4.0787} = 0.2 \,.$$

Algorithm 2- ACO-ACS algorithm for determining the network topology.

Start

/*first step*/
/*Assume that τ_0 is the initial number of pheromones on each arc.*/
Set Z to be 1.
Repeat
Randomly select *the current node i* among the consumers.
Put all the No_of_Ants in *the current node i*.
/*Assume that R^+ is the shortest route between the selected consumer and the CGS and its length is L^+.*/
Set t to be 1.
Repeat
Set k to be 1.
Repeat
Put all the zones and TBSs in the Candidate List.
Repeat
Select next node j from the members of the Candidate List based on the following equation:

$$j=\begin{cases}\arg\max_{u\in J_i^k}\left\{[\tau_{iu}(t)]\cdot\left[\eta_{ij}\right]^{\beta}\right\}, & \text{if } q\leq q_0\\ J\,, & \text{if } q>q_0\,,\end{cases} \quad (52)$$

where $J\in J_i^k$ is obtained based on the following probability:

$$p_{iJ}^{k}(t)=\frac{[\tau_{iJ}(t)]\cdot\left[\eta_{iJ}\right]^{\beta}}{\sum_{l\in J_i^k}[\tau_{il}(t)]\cdot\left[\eta_{il}\right]^{\beta}}\,. \quad (53)$$

Omit node j from the Candidate List.
Apply the local update rule for the pheromone in each ant transmission based on the following equation:

$$\tau_{ij}(t)\leftarrow(1\text{-}\rho)\cdot\tau_{ij}(t)+\rho\cdot\tau_0\,. \quad (54)$$

Until reaching the CGS node,
Set k to be k+1.
Until k > No_of_Ants
Set k to be 1.
Repeat
Put the route chosen by ant k and its length respectively in $R^k(t)$ and $L^k(t)$.
Set k to be k+1.
Until k > No_of_Ants
If a route shorter than L^+ is found, **Then** update R^+ and L^+.
Update the pheromones in the arcs of the shortest route ($(i,j)\in R^+$) using the equation below:

$$\tau_{ij}(t)\leftarrow\left(1\text{-}\rho\right)\cdot\tau_{ij}(t)+\rho\cdot\Delta\tau_{ij}(t),\ \Delta\tau_{ij}(t)=\frac{1}{L^+}\,. \quad (55)$$

Set the pheromone of each arc in the t+1[th] iteration as the pheromone obtained in the t^{th} iteration:

$$\tau_{ij}(t+1)=\tau_{ij}(t)\,. \quad (56)$$

Set t to be t+1.
Until t > T_Max
Set z to be z+1.
Until z > T_Max
/*Second step*/
Develop the S matrix with the triple (j, i, τ_{ij}) and arrange it in an ascending order based on τ_{ij} values.
Of the two arcs (i,j) and (j,i) in the S matrix, omit the arc with fewer pheromones.
Set s to be 1.
Repeat
Repeat
Of the TBSs connected to the CGS in the S matrix, find the next node i using the equation following and store it in the T^s matrix:

$$p_{\text{CGS},i}=\frac{\tau_{\text{CGS},i}}{\sum_{l\in\text{TBS's}}\tau_{\text{CGS},l}}\,. \quad (57)$$

Repeat
find the next node j using the following equation and store it in the T^s matrix:

$$p_{i,j}=\frac{\tau_{i,j}}{\sum_{l\in N_i}\tau_{i,l}}. \tag{58}$$

Until no other node is available
If the T^s matrix involves all the consumers (zones) and the topological limitations are met **Then**
Calculate the length of the network in the T^s matrix.
Store the T^s matrix and its corresponding length of network in Topology_Bank.
Until all the consumers are selected or the T^s matrix is found
Set s to be s+1.
Until s > I_Max
Print the minimum length of network obtained from the T^s matrix and its corresponding network as the output.
End

According to these calculations, the probability of choosing the TBS with node 4 is 0.8 and the probability of choosing TBS with node 5 is 0.2. Assume that the TBS with node 4 is chosen in this step. The next node is chosen from nodes 1, 2, and 3 with probabilities 0.72, 0.16, and 0.12, respectively. Assume that node 1 is selected as the next node. It is impossible to enter another node from node 1. These steps are therefore iterated by returning to array S and choosing another TBS. Assume that the TBS with node 4 is chosen again, and node 2 is chosen next and node 3 afterwards. When moving from one node to the next, the selected route is stored in Table 7, which contains array T^1. Given that all the zones have been chosen and since none of these choices violates the topological limitations presented in Section 2-1, the total length of the network is calculated and stored in Topology_Bank together with array T^1. Fig. 4 shows the topology of this network.

5-2. Parameter Determination

The parameters of Algorithm 2 have a significant effect on the obtained results; choosing their correct values is very essential. To obtain the best value for these parameters, first, a set of values for each parameter was chosen according to the second column of Table 8 and then the network cost for each of the 4374 possible combinations was calculated by running the algorithm. The deviations of the results from the optimal costs obtained by Algorithm 1 were then assessed by Equation (59). The value that creates the least deviation was selected from among the different values of each parameter. For instance, from the three values (0.5, 2 and 4) allocated to parameter, β=2 produced the least deviation from the result obtained by Algorithm 1. The best value of this parameter is therefore 2. The selected values for the parameters are shown in Table 8.

$$Gap=\frac{\left|Cost_{Optimal}\text{-}Cost_{ACO\_ACS}\right|}{Cost_{Optimal}} \tag{59}$$

5-3. Algorithm to determine network items

This algorithm, with the pseudo-code as given by Algorithm 3, is used to determine the capacity of TBSs and pipe diameters in the problem. Algorithm 3 receives the solution obtained from Algorithm 2 and uses it to develop a mathematical model. Once the model is solved, the minimum diameter of each network pipe with minimum piping cost is obtained. The amount of gas flowing in each pipe is then determined, and the optimal capacities for the TBSs are determined based on that amount. In this step, the data required for calculating gas pressure and velocity in each pipe are obtained according to equations (1) and (2). If the obtained values meet the minimum pressure and maximum velocity limitations of gas, the solution is optimal; otherwise, the solution is discarded and the search continues for finding the optimal solution or reaching the upper bound of the number of iterations for the algorithm (R-Max). Example 4 presents how each of these steps is performed.

Algorithm 3- Method for determining items of the network.

Develop a mathematical model based on the topology of the network received.
Set i to be 1.
Repeat
Solve the mathematical model to determine the diameter of each pipe in the network.
Calculate the amount of gas flowing in each pipe.

Determine the minimum capacity needed for each TBS.
Calculate the gas pressure and velocity in each pipe.
If the calculated values do not meet the minimum pressure and maximum velocity limitations of gas, **Then**
Discard the obtained solution from the mathematical model by applying the relevant constraint.
Set i to be i+1.
Until i > R_Max or the calculated values meet the minimum pressure and maximum velocity limitations of gas.

Example 4:

Herein, the execution of Algorithm 3 is examined for Example 3. The values of parameters for the example are provided in the Appendix.

Based on the input received from Algorithm 2, i.e., Table 7, which can also be shown as $Y_{6,4}=X_{4,1}=X_{4,2}=X_{2,3}=1$, part of the model in Section 3-3 is selected to develop the new model. The parameters and variables of the new model are the same as those provided in Sections 3-1 and 3-2 and the objective function, i.e., equation (60) below, including equations (5) and (6). The rule that the pipe diameters must be in a descending order, stated by constraint (11), is applied according to constraint (61), as shown below. Moreover, constraint (12), which is associated with this part of the network, cannot be expanded because the feeding network in this example has only one member. Constraints (62)-(65) below ensure that the two selected nodes are connected to each other with exactly the same type of a pipe. Constraints (66) and (67) show that the variables in the model are binary. Given these explanations, the model is stated as follows:

$$\min \ Z=\sum_{p=1}^{P} X_{4,1,p}\cdot dZ_{4,1}\cdot CLZ_p+\sum_{p=1}^{P} X_{4,2,p}\cdot dZ_{4,2}\cdot CLZ_p+\sum_{p=1}^{P} X_{2,3,p}\cdot dZ_{2,3}\cdot CLZ_p+\sum_{w=1}^{W} Y_{6,4,w}\cdot dT_{6,4}\cdot CLT_w \quad (60)$$

s.t.

$$\sum_{p=1}^{P} (X_{2,3,p}\cdot SZ_p)\leq \sum_{p=1}^{P} X_{4,2,p}\cdot SZ_p \quad (61)$$

$$\sum_{p=1}^{P} X_{4,1,p}=1 \quad (62)$$

$$\sum_{p=1}^{P} X_{4,2,p}=1 \quad (63)$$

$$\sum_{p=1}^{P} \mathrm{X}_{2,3,p}=1 \quad (64)$$

$$\sum_{w=1}^{W} Y_{6,4,w}=1 \quad (65)$$

$$X_{4,1,p}\ ,X_{4,2,p}\ ,X_{2,3,p}\in\{0,1\}\ , \qquad p=1,\ldots,P \quad (66)$$

$$X_{6,4,w}\in\{0,1\}\ , \qquad w=1,\ldots,W\ . \quad (67)$$

The solution obtained from solving the model is $Y_{6,4,1}=X_{4,1,1}=X_{4,2,1}=X_{2,3,1}=1$, based on which the flow rate in pipes is obtained simply in accordance with constraints (24) and (25) as follows:

$fZ_{4,1}=D_1=2000\ {m^3}/{h}$,

$fZ_{2,3}=D_3=4000\ {m^3}/{h}$,

$fZ_{4,2}=D_2+D_3=3000+4000=7000\ {m^3}/{h}$,

$fY_{6,4}=fZ_{4,1}+fZ_{4,2}=7000+2000=9000\ {m^3}/{h}$.

Since node 6, which is a candidate TBS installation site, requires 9000 m3/h of gas, the optimal choice is to choose TBS type 2 with a capacity of 10000 m^3/h. Calculating gas pressure and velocity begins from the CGS in the feeding network and from the TBS in the distribution network, since their pressures are known (the pressure of the TBS in the distribution network is 74.7 psia, and the pressure of the CGS in the feeding network is 264.7 psia). For instance, the pressure at node 4, which is a TBS, is P_1 = 74.7 psia, and since $X_{4,2,1}=1$, nodes 2 and 4 are connected to each other through pipeline type 1 with a diameter of 6 inches. This pipe is 78.1 m long, and its gas flow, i.e., $fZ_{4,2}=D_2+D_3$, is 7000 m^3/h. The pressure at node 2 is thus calculated using (1) and gas velocity using (2) and (3) (P_2 = 73.15 psia and V = 21.19 m/s). With the pressure at node 2 being known, the diameter, length, and gas flow of the pipe connecting this node to node 3 are used to calculate the pressure and average velocity at this node (P_2 = 71.93 psia and V = 12.25 m/s). Given that gas velocity in the pipe connecting node 4 to node 2 (V = 21.19 m/s) is above the limit, this solution is not considered to be an optimal solution and should be omitted. The solution is discarded by adding constraint (68) to the model (this constraint ensures that the option will not be chosen again):

$$X_{4,1,1}+X_{4,2,1}+X_{2,3,1}+Y_{6,4,1}<4 . \qquad (68)$$

To obtain a general constraint in relation to constraint (68), assume that each edge of the obtained solution can be shown by an ordered triple of the source node, the destination node and the type of pipe used between the two nodes, and M and N series contain these triples for the feeding network and the distribution network, respectively. These series have K_1 ($K_1 = m$) and K_2 ($K_2 \leq n$) members. Adding the following constraint to the mathematical model prevents the reselection of this solution:

$$\sum_{(r,s,p)\in M} X_{rsp} + \sum_{(r,s,w)\in N} Y_{rsw} < k_1+k_2 . \qquad (69)$$

Further execution of Algorithm 3 in the next iterations leads to the solution provided in Table 9.

6. Numerical Results and Discussions

To compare the result obtained by Algorithm 1 with the optimal solution obtained by solving the mathematical model in Section 3-3, six locations were first assumed for the zones and six locations for the TBSs. Based on the data provided in the Appendix, 36 problems were designed and then solved in a computer with 8G RAM, 2 GHz CPU, Intel core i7 and 64-bit Windows 8, and the optimal solution for each problem was obtained. Algorithm 1 was then implemented in the 64-bit version of MATLAB and validated in Pipesim-2011.1. The results are shown in Table 10.

As shown in Table 10, the cost obtained by both methods is the same in cases where there are three zones or less. Also, in the two cases pertaining to rows 28 and 34 of the table, we see the favorable performance of Algorithm 1 for networks with small number of nodes. The cost predicted by Algorithm 1 for networks with more nodes is higher than the cost predicted in the optimal solution. Fig. 5 compares the costs obtained by the two different methods for six problems using the following equation:

$$\frac{Cost_{Algorithm1}-Cost_{Optimal}}{Cost_{Optimal}} . \qquad (70)$$

The number of zones in each of the six problems is six and the number of TBS is one in the first problem, two in the second, three in the third, four in the fourth, five in the fifth, and six in the sixth problem.

Based on this figure, although the solution of Algorithm 1 is further distanced from the optimal solution when the number of nodes increases, the total difference is still less than 6.8%. Furthermore, based on Table 10, it can be easily demonstrated that the execution time for obtaining the optimal solution and the execution time for Algorithm 1 depend more on the number of zones than the number of TBSs, as shown in Fig. 6. According to Fig. 6, Algorithm 1 needs a considerable execution time in the beginning, but as the number of zones and TBSs increases, the time required to obtain solution decreases at a lower rate than the one required for reaching the optimal solution; in cases with more than five consumers (zones), the time required for executing Algorithm 1 is less than the one required for reaching the optimal solution. This trend is fully justified considering that the number of constraints in the mathematical model follows the following equation:

Number of constraints in the model =

$$m^3+n^3+nm^2+\frac{1}{2}\left(m^2+n^2\right)+2m\,n+\frac{1}{2}(3m+17 \qquad (71)$$

where m is the number of zones and n is the number of TBSs, and the number of constraints grows further once the number of zones increases at a faster rate than the number of TBSs, given the nm^2 term.

In Algorithm 1, the ants begin moving from each zone and any increase in the number of zones therefore directly affects the computing time.

To validate the gas pressure and velocity calculations, all the problems were solved in Pipesim according to Table 10, and the average ratios of the deviations for the two methods were obtained using the following equations:

Average pressure deviation=

$$\frac{\sum_{i=1}^{n+m}\left|\frac{P^i_{Algorithm\ 1}-P^i_{PIPESIM}}{P^i_{Algorithm\ 1}}\right|}{n+m}\times 100 \quad (73)$$

Average velocity deviation =

$$\frac{\sum_{i=1}^{n+m}\left|\frac{V^i_{Algorithm\ 1}-V^i_{PIPESIM}}{V^i_{Algorithm\ 1}}\right|}{n+m}\times 100\ , \quad (74)$$

where $P^i_{Algorithm\ 1}$ and $P^i_{PIPESIM}$ respectively indicate pressure at node i obtained by solving the model using Algorithm 1 and solving the topology obtained from Pipesim; similarly, $V^i_{Algorithm\ 1}$ and $V^i_{PIPESIM}$ respectively indicate the average gas velocity in the pipe transmitting gas to node i obtained by the two noted methods.

As shown in Table 10, the maximum difference between the two methods is 1.597% and pertains to the calculation of gas velocity in the problem in row 32 of the table. Such a difference is well accepted in designing gas networks.

Solutions provided for small and large network problems can only serve as an assisting tool for network designers, who should also use their ingenuity in designing networks. According to Fig. 6, as dimensions of the problem increase, the designer is compelled to use the ACO method due to the time needed to solve mathematical models. According to Fig. 5, deviation of the solution obtained by this method from the optimal solution increases with increasing dimension of the problem. In such cases, the designer can reduce the number of zones by combining the consumption of two or several zones and develop a new problem to be solved. In this case, the obtained solution is not necessarily an optimal solution to the initial problem. It should be noted that according to equation (71), the number of zones has a greater effect on the time required to solve the mathematical model as compared to the number of TBSs. In addition, the designer can assign more pheromones on the network obtained from optimal solution to the new problem and enter these values in the ACO method as τ_0 to solve the initial problem in order to increase the likelihood of producing a closer solution to the optimal one. These approaches are used in the case study.

7. Case Study

The performance of Algorithm 1 was examined on a gas supply project in Kelardasht, Mazandaran, Iran, and the results were compared with those obtained by the project's value engineering team [19]. In this project, gas is supplied to 256 zones, and Table 11 shows any pertinent data based on the documentations and maps received from the Gas Company of Mazandaran Province. To narrow down the problem and solve it within a reasonable time, the number of consumption zones was reduced to 31 large zones in this study and the demand for gas in each zone was obtained by adding the demands from the small zones; pertinent data are presented in the Appendix. Six candidate TBS installation sites were determined in this project. All the data required for solving the problem resemble the data for the previous examples, as shown in the Appendix.

According to the study performed in [19], the distribution network presented by the project's consulting design team included TBSs 2, 4 and 5 with cost 1,398,726,382, and the network presented by the project's value engineering team included TBSs 1, 4 and 5 with cost 1,372,374,925. Table 12 presents the results obtained by executing Algorithm 1. Based on this table, the cost predicted by this algorithm was 1,252,739,300, and this algorithm also selected TBSs 1, 4 and 5 for the network, but gave different consumption zones and connections for the gas supply. Fig. 7 shows the proposed network. In addition to the better results obtained from the algorithm, the discussed study [19] has not determined the types of pipes for the proposed network and has only given estimates of the different pipe diameters that can be used in the network, and the gas pressure and velocity have not been determined in the study either.

8. Conclusion and Future Research

We proposed a comprehensive mathematical model for solving urban gas network problems, such as problems involving distribution and feeding networks. This model does not consider optimization of a network with a predefined design; rather, it provides an optimal design for gas supply to consumption zones considering the different limitations in designing gas networks. The model was applied to small networks and was validated through an example, and the results were compared with the ones obtained by Pipesim. To solve large problems, an Ant Colony System (ACS) method, as one of the most efficient methods for the implementation of Ant Colony Optimization (ACO) algorithms, was developed and its performance was demonstrated with an example. A local search method was also used in combination with the ACS algorithm, as a mathematical program determining the minimum pipe diameter so that the pressure and velocity limitations of gas networks were met. The time required for the execution and the costs of the network designed by the two algorithms were compared after solving 36 sample problems, and the execution times for the optimal solution and the ACS algorithm were found to further grow, as the number of consumption nodes increased at a faster rate than the number of candidate TBS installation sites. Furthermore, the time required for the execution of the optimal method grew at a faster rate than the time required for executing the ACS algorithm. Both algorithms were found to yield similar solutions to small problems, but, as the scale of the problem grew, the costs predicted by the ACS algorithm were more than the ones predicted by the optimal algorithm. The proposed model was also applied to a case study and was shown to yield a more appropriate solution.

The following cases are identified as subjects for future studies:

- Use of the Steiner tree (Steiner points) in the design of urban networks, taking into account underground gas pipelines passing through alleys and streets.
- In the present study, the assumption is that candidate sites for TBS installations have already been determined. However, choosing these sites can be a subject for urban network design studies.
- Repair, maintenance, and risk issues can also be investigated in modeling urban gas networks.
- Applying other metaheuristic methods may prove to be useful.
- The presented approach can also be used in other networks such as power, water, computer, etc.

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