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Web Service Providers' Game on Price and Service Level

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Web Service Providers, Pricing, Game theory, Queuing theory, Competition, Complimentarity

ABSTRACT

Response time is one of the critical web service quality dimensions. It refers to how long it takes that a web service responds to request of a user. In order to manage the response time, pricing schemes can work as an efficient access control mechanism. In this paper, we study competition between two providers offering functionally same web services where there is a monopoly service provider. The monopoly offers a service that is complementary to their services. Each provider needs to decide a service level (L or H) and a corresponding price for the selected service level to meet the service level guarantee. We construct a Stackelberg game and benefit from queuing theory concept to propose a model that can examine strategic choices of the providers.

1. Introduction

There is an increasing acceptance of Service-Oriented Architectures (SOA) as a paradigm for integrating software applications within and across organizational boundaries. In this paradigm, independently developed and operated applications are exposed as (Web) services that communicate with each other using XML-based standards, most notably SOAP and associated specifications [1]. Web services permit different types of systems to share information without human intervention [2].

Web services provide an inexpensive and rapid solution for system development and integration. Typically, service providers create and publish components with specific functionalities.

A service consumer who needs certain functionality can resort to various web service discovery mechanisms to locate and invoke the service using standard protocols by paying a fee [3,4]. One characteristic of these services is that the integration needed by the user is no longer prohibitively expensive or time consuming. Therefore, Complementarity is an important characteristic of web services [5]. Independent services can be composed in processes to provide even greater value than the sum of component services [6].

There is a critical challenge on the performance of a web service caused by the network bandwidth and processing overhead associated with transferring the large and complex XML-based messages over the network. A service provider may not be able to handle the throughput, resulting in serious performance degradation. Therefore, launching a service is often associated with announcing a web service-level agreement (WSLA) along with the capability and interfaces of the service. WSLA defines agreed performance metrics and ways to evaluate and measure them [4,7,8,9]. You can see role of the WSLA in figure 1.

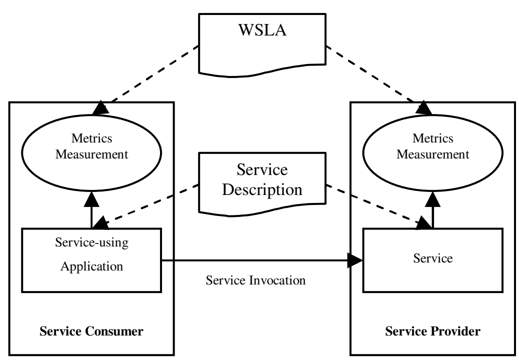

Fig. 1. Role of web service level agreement [4-9]

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A service-level agreement could include several quality metrics, such as response time or latency, availability, accessibility, reliability, and versioning [7,10]. Among the various quality dimensions, response time is often considered to be the most critical dimension and the most difficult one to manage [11]. Response time refers to how long it takes that a web service responds to request of a user and is typically measured as an average time over a specified time horizon.

In order to maintain a WSLA, especially with respect to the response time, designing and implementing a proper pricing scheme is necessary for web service providers. A pricing scheme can work as an efficient access control mechanism. Although there is a rich body of prior research on pricing products and services, these traditional pricing schemes do not usually work well for web services: First, the marginal cost of providing a service to an additional user is negligible, thereby reducing the traditional price to zero. Second, a very important aspect here is social cost of congestion; traditional pricing models do not capture this negative externality. On the other hand, non-pricing approaches to access control for reducing the congestion cost are either flawed or, more generally, have undesirable side effects [4,12]. Third, some web services are complementary to each other; hence, price of a web service can affect demand of its complementary service. Our work relates to the pricing of a service that is subject to queuing delays. As a classic paper, Mendelson (1985) [13] shows that the optimal price for a profit-maximizing firm is equal to the expected marginal delay cost.

A number of studies (such as [14,15,16]) have followed, all with an aim to improve the overall efficiency of a single network (or service facility). In the context of data network pricing, various pricing schemes (e.g., [16,18,19,20,21]) have been proposed. In recent years, these pricing schemes have also been applied to web services.

For example, Lin et al. (2005) [22] conduct a pilot study to demonstrate the use of dynamic pricing scheme to manage web service resources. Tang and Cheng (2005) [23] study the pricing and location strategy of a web service intermediary that provides time-sensitive integrated services from two complementary service providers.

Research on the duopoly competition in the presence of a delay cost dates back at least to Levhari and Luski (1978) [24], when they noted that there always exists a symmetric equilibrium if the market is covered. Similar studies [25-26] have been subsequently conducted under different assumptions. These models were also extended to other areas such as data networks or service industries (e.g., [27,28,29,30,31,32,33]). These studies demonstrated a common finding, i.e., the existence of symmetric equilibrium for identical firms in the presence of service guarantees.

This paper also relates to the vertical differentiation model. Some of the representative works include [34,35], which show that competing firms typically choose to locate at the extreme ends of quality spectrum to reduce price competition. Moorthy (1988) [36] extends the basic model by incorporating variable production costs and demonstrate that, in equilibrium, firms choose products that are differentiated. Many economists (e.g., [37,38,39,40,41,42]) have subsequently studied this game under various settings. Most of the vertical differentiation models however do not explicitly model capacity constraints (or service guarantees). A general finding from those models is that firms differentiate along the quality dimension at equilibrium. Recent research on price competition and service level guarantee in web services is more related to our study. Zhang et al. (2009) [4] develop a model to study a duopoly competition where service providers can provide either standard or premium service. They found that, in the long run, with reasonable market size and capacity costs, the principle of differentiation always holds. However, in the short run, service providers might choose to compete head to head by providing the same service level. When the traffic intensity is very high, providing the standard service level is beneficial. In this paper, we combine modeling constructs from game theory and queuing theory to propose a model that can provide useful insights to service providers. The paper considers duopoly competition between two providers offering web services with the same functionality. While there is a monopoly service provider who offers a web service that is complementary to their services. Facing a continuum of users who value the benefits from the service against its price (plus the delay cost), a provider needs to decide a service level (L or H) she would offer, and a corresponding price for the selected service level to meet the QoS guarantee (in terms of the average response time of the service). Actually, our model extends the model presented by Zhang et al. (2009) and investigates how the competitors' strategies affect by a monopoly complementary service strategy.

The rest of the paper is organized as follows. First, in section 2, we describe the basic modeling framework, and then analyze the equilibrium pricing strategy when two competitors choose service levels and prices simultaneously, while the monopoly as a leader had chosen service level H or L. section 4 investigates equilibrium strategies. Finally, section 5 concludes the paper.

2. The Model

We had presented our basic modeling framework in our previews paper [9], but the model is inevitably repeated in this section. Assume that there are three service providers including: 1) Two identical providers, indexed SP1 and SP2, offering web services with the same functionalities, S1, furthermore both of them have fixed processing capacity. 2) A monopoly service provider indexed SP3, with an unlimited processing capacity offering a given service, S2, that is complementary to S1. The target market is divided into three segments: customers that are interested in S1 only (Segment 1), customers who need both S1 and S2 (Segment 2) and customers that want S2 only (Segment 3). Figure 2 shows the market structure. The services can be delivered in one of two discrete QoS levels, *L* and *H*, each level is characterized by an expected total response time of the service. Let d_L and d_H be respectively the guaranteed response time for the service offered with level *L* and *H*, where $d_L > d_H$. We assume that d_L and d_H are exogenous and fixed. A provider *i (i=1,2,3)* can choose either *L* or *H* service level and charges a corresponding price, $p_i^{(j)}$ *(j=L or H)*, for it. Each provider guarantees that the actual expected response time for her service will be at most d_j. Since the marginal cost of providing a web service is negligible, we skip it and assume costs of developing web services are sunk.

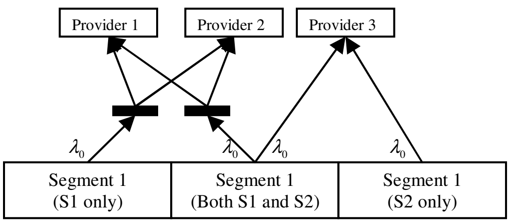

Fig. 2. Market segments and the total arrival of their users looking for web services

The users differ by two parameters: the value parameter, *v*, and the delay sensitivity parameter, *h*. *v* denotes the value that user assigns to the immediate provision of each single service (for simplicity we assume that the value of S1 and S2 for a given user are equal). *h* denotes the delay sensitivity of user. The raised disutility from delay is assumed to be linear in *h*; i.e. if the delay sensitivity is *h*, then the disutility is $h\gamma$ per time unit of delay, where γ is a constant. We assume that both *v* and *h* are uniformly distributed and are normalized between [0,1]. Assume that *v* and *h* are linearly related: $h = \varepsilon v + \eta$. This assumption is a reasonable since a high value of *v* usually means that the user is also more delay sensitive in obtaining the service. We should consider $\eta = 0$. This is because users with zero valuation of the service ($v = 0$) would not be sensitive to delay at all ($h = 0$). Given this, without any loss of generality, we can set $\varepsilon = 1$, i.e. $v = h$. Furthermore, assume that for each user of segment 2 who is interested to compose S1 and S2, there is a parameter v_2. v_2 denotes the value that a user assigns to the immediate provision of the composite web service ($v_2 \geq v$).

In order to simplify our analysis, we assume v_2 and v are linearly related and we have: $v_2 = \alpha v$, where $\alpha \geq 1$.

At the time of requesting a service, utility of a user, with *v* and *h*, who chooses to obtain S1 or S2 from provider *i* is: $v - p_i^{(j)} - h\gamma d_j$. Assume that in each segment the total arrival of users looking for web services follows a Poisson process with a base rate of λ_0 (as you can see in figure 2, in order to simplify our analysis we assume that the total arrival rate of users in all of the market segments are equal), i.e. λ_0 is the total arrival rate of users when the response time of the service is zero and the service is free. The effective arrival of users accessing service from provider *i* also follows a Poisson process, but with a rate λ_i^j proportional to λ_0 [43]. The service time of each request follows an arbitrary distribution with a mean service time of *b* second; in other words, the processing rate of each provider is $\frac{1}{b}$. The above system can be modeled as an M/G/1 processor-sharing queue, and the expected total response time W_i^j from provider *i* is given by [4,9,44]:

$$W_i^j = \frac{b}{1 - \rho\frac{\lambda_i^j}{\lambda_0}}$$

Where $\rho \equiv \lambda_0 b > 0$ represents the normalized total traffic intensity. W_i^j should not be greater than announced response time of the service, d_j (*j=L* and *H*). In our analysis we assume that service time and total response time of the composite web service follows the high watermark rule. That means they are equal to maximum of service times and total response times of S1 and S2. As mentioned above, each service provider can choose offering her service with level L or H. We assume that, SP3 first chooses her service level and sets a corresponding price. And then attention to choice of SP3, SP1 and SP2 choose their service levels and prices simultaneously and non-cooperatively. It can be modeled as a Stackelberg game in which one leader (SP3) moves first, and decides upon her service level and price. And then the other providers (followers), observing the choice of the leader, chooses her service level and the corresponding price. The leader can anticipates the reaction of the followers and using a backward mechanism, chooses a service level and the corresponding price to maximize her profit.

Therefore, there are eight cases for these three providers. Note that order of SP1 and SP2 is not important. So in cases which these two providers choose differentiated service levels, without loss of generality, we can assume that SP1 chooses H while SP2 chooses L. Hence we have six different cases, in three of them the monopoly (SP3) chooses level H and in others he chooses level L. Table1 shows these six cases.

Tab. 1. Thinkable cases

Case	Chosen Service Level			Abbreviation
	SP1	SP2	SP3	
1	H	H	H	HHH
2	L	L	H	LLH
3	H	L	H	HLH
4	H	H	L	HHL
5	L	L	L	LLL
6	H	L	L	HLL

In the next section, we have considered these cases and optimal prices for each provider are analyzed.

3. Optimal Pricing

3.1. The Leader Had Chosen Service Level H

In this section, we should consider the situation where SP3 had offered S2 with level H and price $P_3^{(H)}$. SP1 and SP2 choose their service levels and price decisions simultaneously and non-cooperatively. We had modeled, solved and analyzed the first three cases (where the leader had chosen service level H) in our previews paper [9] (see appendix A).

3.2. The Leader Had Chosen Service Level L

In this section, we consider the situation where SP3 had offered S2 with level L and price $P_3^{(L)}$. And then, SP1 and SP2 choose their service levels and price decisions simultaneously and non-cooperatively.

3.2.1. Case4: HHL

In this case, since $d_L > d_H$, total response time of composite web service for users of segment 2 is equal to d_L. Therefore, if only there is one service provider, her effective arrival rate is: $\lambda_H = \lambda_0(1-\frac{\overline{P_H}}{1-\gamma d_H}) + \lambda_0(1-\frac{\overline{P_H}+P_3^{(L)}}{\alpha(1-\gamma d_L)})$, and effective arrival rate of each competitor when they split the market equally and keep the response-time constraint binding is: $\lambda_i^{(H)} = \frac{1}{2}\lambda_0(2-\frac{P_H}{1-\gamma d_H}-\frac{P_H+P_3^{(L)}}{\alpha(1-\gamma d_L)})$, $(i=1,2)$. Hence, we have:

$$\frac{b}{1-\rho(2-\frac{P_H}{1-\gamma d_H}-\frac{P_H+P_3^{(L)}}{\alpha(1-\gamma d_L)})} = d_H$$

and

$$\frac{b}{1-\frac{1}{2}\rho(2-\frac{P_H}{1-\gamma d_H}-\frac{P_H+P_3^{(L)}}{\alpha(1-\gamma d_L)})} = d_H$$

Solving the above two equations, we get:

$\overline{P_H} = \max\{0 \ , \ x_8(2\alpha(1-\gamma d_L)(1-\frac{u_H}{2\rho})-P_3^{(L)})\}$ and

$\underline{P_H} = \max\{0 \ , \ x_8(2\alpha(1-\gamma d_L)(1-\frac{u_H}{\rho})-P_3^{(L)})\}$, Where $x_8 = \frac{1-\gamma d_H}{\alpha(1-\gamma d_L)+(1-\gamma d_H)}$.

Using results of Lemma a, In this case the effective demand to each provider is: $\lambda_H = \frac{1}{2}\lambda_0(2-\frac{P_H}{1-\gamma d_H}-\frac{P_H+P_3^{(L)}}{\alpha(1-\gamma d_L)})$. Hence, the optimization problem of each provider can be written as:

$$\begin{aligned} &\max_{P_H} \pi_i^{(L)} = \frac{1}{2}P_H\lambda_0(2 \quad \frac{P_H}{1 \quad \gamma d_H} \quad \frac{P_H+P_3^{(L)}}{\alpha(1 \quad \gamma d_L)}) \\ &s.t. \quad \frac{b}{1 \quad \frac{1}{2}\rho(2 \quad \frac{P_H}{1 \quad \gamma d_H} \quad \frac{P_H+P_3^{(L)}}{\alpha(1 \quad \gamma d_L)})} \leqslant d_H \\ &\qquad \underline{P_H} \leqslant P_H \leqslant \overline{P_H} \end{aligned} \tag{1}$$

Lemma 1: When the monopoly (leader) chooses level L and then the both competitors (followers) choose H, the Pareto optimal price for competitors is given by:

$$P_H = (1 \quad \gamma d_H)\times\begin{cases} 0 & if \quad \rho < r_9 \\ x_9(x_8+2\alpha)(1 \quad \frac{u_H}{2\rho}) \quad x_9 & if \quad r_9 \leqslant \rho < r_{10} \\ x_{10} \quad x_2(1 \quad \frac{x_{10}x_{11}}{2}) & if \quad r_{10} \leqslant \rho < r_{11} \\ x_9(x_8+2\alpha)(1 \quad \frac{u_H}{\rho}) \quad x_9 & Otherwise \end{cases} \tag{2}$$

Also, the optimal price for SP3 (leader) is:

$$P_3^{(L)} = \begin{cases} \frac{\alpha(1 \quad \gamma d_L)}{\alpha+1} & if \quad \rho < r_9 \\ x_9(\alpha(1 \quad \gamma d_L)+(1 \quad \gamma d_H)(\frac{u_H}{2\rho})) & if \quad r_9 \leqslant \rho < r_{10} \\ x_2 x_{11}(2\alpha(1 \quad \gamma d_L)+(1 \quad \gamma d_H)) & if \quad r_{10} \leqslant \rho < r_{11} \\ x_9(\alpha(1 \quad \gamma d_L)+(1 \quad \gamma d_H)(\frac{u_H}{\rho})) & Otherwise \end{cases} \tag{3}$$

Where:

$$x_9 = \frac{(1-\gamma d_L)}{(\alpha+1)(1-\gamma d_L)+(1-\gamma d_H)}, \quad x_{10} = \frac{\alpha(1-\gamma d_L)}{\alpha(1-\gamma d_L)+(1-\gamma d_H)},$$

$$x_{11} = \frac{1-\gamma d_H}{1-\gamma d_L}, \quad r_9 = u_H\left(\frac{2\alpha+x_8}{2(2\alpha+x_8-1)}\right),$$

$$r_{10} = u_H\left(\frac{2\alpha+x_8}{x_{11}(2\alpha+x_2-2x_{10})}\right) \text{ and } r_{11} = u_H\left(\frac{2(2\alpha+x_8)}{x_{11}(2\alpha+x_2-2x_{10})}\right).$$

For the proof of this lemma see appendix B.6.

3.2.2. Case5: LLL

In this case, like Case1, three providers choose same service levels. Therefore, if only there is one service provider, her effective arrival rate is: $\lambda_L = \lambda_0(1-\frac{\overline{P}_L}{1-\gamma d_L}) + \lambda_0(1-\frac{\overline{P}_L + P_3^{(L)}}{\alpha(1-\gamma d_L)})$, and effective arrival rate of each competitor when they split the market equally and keep the response-time constraint binding is: $\lambda_i^{(L)} = \frac{1}{2}\lambda_0(2-\frac{(\alpha+1)\underline{P_L}+P_3^{(L)}}{\alpha(1-\gamma d_L)})$, $(i=1,2)$.

Hence, we have:

$$\frac{b}{1-\rho(2-\frac{(\alpha+1)\overline{P}_L+P_3^{(L)}}{\alpha(1-\gamma d_L)})} = d_L \quad \text{and}$$

$$\frac{b}{1-\frac{1}{2}\rho(2-\frac{(\alpha+1)\underline{P_L}+P_3^{(L)}}{\alpha(1-\gamma d_L)})} = d_L$$

Solving the above two equations, we get:

$$\overline{P_L} = \max\{0 \;\;,\;\; \frac{\alpha(1-\gamma d_L)}{\alpha+1}(2-\frac{u_L}{\rho}) - \frac{P_3^{(L)}}{\alpha+1}\}$$

and

$$\underline{P_L} = \max\{0 \;\;,\;\; \frac{2\alpha(1-\gamma d_L)}{\alpha+1}(1-\frac{u_L}{\rho}) - \frac{P_3^{(L)}}{\alpha+1}\}$$

Using results of Lemma a, In this case the effective demand to each provider is: $\lambda_L = \frac{1}{2}\lambda_0(2-\frac{(\alpha+1)P_L+P_3^{(L)}}{\alpha(1-\gamma d_L)})$. Hence, the optimization problem of each provider can be written as:

$$\begin{aligned} &\max_{P_L} \pi_i^{(L)} = \frac{1}{2}\lambda_0 P_L \left(2-\frac{(\alpha+1)P_L+P_3^{(L)}}{\alpha(1-\gamma d_L)}\right) \\ &s.t. \quad \frac{b}{1-\frac{1}{2}\rho\left(2-\frac{(\alpha+1)P_L+P_3^{(L)}}{\alpha(1-\gamma d_L)}\right)} \leqslant d_L \\ &\qquad \underline{P_L} \leqslant P_L \leqslant \overline{P_L} \end{aligned} \qquad (4)$$

Lemma 2: When the monopoly (leader) chooses level L and then the both competitors (followers) choose L too, the Pareto optimal price for competitors is given by:

$$P_L = \frac{(1-\gamma d_L)}{\alpha+1} \times \begin{cases} 0 & if \quad \rho < r_{12} \\ \frac{\alpha(2\alpha+3)}{\alpha+2} - (\frac{2\alpha^2+4\alpha+1}{\alpha+2})(\frac{u_L}{2\rho}) & if \quad r_{12} \leqslant \rho < r_{13} \\ \alpha(1-\frac{2\alpha+1}{2(2\alpha^2+4\alpha+1)}) & if \quad r_{13} \leqslant \rho < r_{14} \\ \frac{\alpha(2\alpha+1)}{\alpha+2} - (\frac{2\alpha^2+4\alpha+1}{\alpha+2})(\frac{u_L}{\rho}) & Otherwise \end{cases} \qquad (5)$$

Also, the optimal price for SP3 (leader) is:

$$P_3^{(L)} = (1-\gamma d_L) \times \begin{cases} \frac{\alpha}{\alpha+1} & if \quad \rho < r_{12} \\ \frac{1}{\alpha+2}(\alpha+\frac{u_L}{2\rho}) & if \quad r_{12} \leqslant \rho < r_{13} \\ \frac{\alpha(2\alpha+1)}{2\alpha^2+4\alpha+1} & if \quad r_{13} \leqslant \rho < r_{14} \\ \frac{1}{\alpha+2}(\alpha+\frac{u_L}{\rho}) & Otherwise \end{cases} \qquad (6)$$

Where: $r_{12} = u_L(\frac{2\alpha^2+4\alpha+1}{2\alpha(2\alpha+3)})$,

$$r_{13} = u_L\left(\frac{(2\alpha^2+4\alpha+1)^2}{2\alpha(\alpha+1)(2\alpha^2+4\alpha+1)+\alpha(2\alpha+1)(\alpha+2)}\right) \quad \text{and}$$

$$r_{14} = u_L\left(\frac{2(2\alpha^2+4\alpha+1)^2}{2\alpha(\alpha+1)(2\alpha^2+4\alpha+1)+\alpha(2\alpha+1)(\alpha+2)}\right).$$

For the proof of this lemma see appendix B.7.

3.2.3. Case6: HLH

In this case we assume that SP1 and SP2 choose differentiated service levels. This, of course, means that SP1 must charge a price $P_1^{(H)}$ higher than $P_2^{(L)}$ charged by SP2.

In order to find the equilibrium prices, we need to estimate the expected demand for each provider. For users of segment 1, please see description of case3, the effective arrival rates from users of segment 1 for the two providers are:

$$\lambda_{1,1}^{(H)} = \lambda_0(1-\max\{V, V_{H,1}\}) \quad and \quad \lambda_{2,1}^{(L)} = \lambda_0(\max\{V, V_{L,1}\} - V_{L,1})$$

Also for users of segment 2, let V_2 be the v-value of the marginal user who is indifferent between two providers. Hence, we have:

$$\alpha V_2 - P_1^{(H)} - P_3^{(L)} - \alpha V_2 \gamma d_L = \alpha V_2 - P_2^{(L)} - P_3^{(L)} - \alpha V_2 \gamma d_L \quad or \quad P_1^{(H)} = P_2^{(L)}$$

Therefore, in this case there is no demand from users of segment 2 for service provider who offers S1 with level H. Using results of Lemma d we can obtain the total effective arrival rates for the two providers as:

$$\lambda_1^{(H)} = \lambda_{1,1}^{(H)} + \lambda_{1,2}^{(H)} = \lambda_0(1 - \frac{P_1^{(H)} - P_2^{(L)}}{\gamma(d_L - d_H)} + 0)$$

$$\lambda_2^{(L)} = \lambda_{2,1}^{(L)} + \lambda_{2,2}^{(L)} = \lambda_0(1 + \frac{P_1^{(H)} - P_2^{(L)}}{\gamma(d_L - d_H)} - \frac{(\alpha+1)P_2^{(L)} + P_3^{(L)}}{\alpha(1-\gamma d_L)})$$

Lemma 3: When the monopoly (leader) chooses level L and then the two competitors (followers) make a differentiated market, the optimal prices are given by:

$$P_1^{(H)} = \begin{cases} x_{12}(3\alpha + 4\gamma((\alpha+1)d_L \;\; d_H) \;\; \frac{\alpha}{2}(1 \;\; \gamma d_L)x_{13}) & if \quad \rho < r_{14} \\ x_{14}(3\alpha + 2\gamma(d_L \;\; d_H)) + x_{15}(1 \;\; x_{14}) \;\; x_{14}(2\alpha + \frac{x_{15}}{2}\gamma(d_L \;\; d_H))(\frac{u_H}{\rho}) & if \quad r_{14} \leqslant \rho < r_{15} \\ \frac{1 \;\; \gamma d_L}{\alpha+1}((\frac{\alpha}{\alpha+2})(2 \;\; \frac{u_L + u_H}{2\rho})) + \gamma(d_L \;\; d_H)(1 \;\; \frac{u_H}{\rho}) & Otherwise \end{cases} \tag{7}$$

$$P_2^{(L)} = \begin{cases} x_{12}\alpha(1 \;\; \gamma d_L)(3 \;\; x_{13}) & if \quad \rho < r_{14} \\ x_{14}(2\alpha + x_{15}\gamma(d_L \;\; d_H))(1 \;\; \frac{u_H}{2\rho}) \;\; x_{15}\gamma(d_L \;\; d_H) & if \quad r_{14} \leqslant \rho < r_{15} \\ \frac{1 \;\; \gamma d_L}{\alpha+1}(2\alpha \;\; \frac{\alpha}{\alpha+2} \;\; (2\alpha + \frac{1}{\alpha+2})(\frac{u_H + u_L}{2\rho})) & Otherwise \end{cases} \tag{8}$$

$$P_3^{(H)} = \begin{cases} x_{13}\frac{\alpha(1 \;\; \gamma d_L)}{2} & if \quad \rho < r_{14} \\ x_{15}(\alpha(1 \;\; \gamma d_H) + \gamma(d_L \;\; d_H)(\alpha + 1 + \frac{u_H}{2\rho})) & if \quad r_{14} \leqslant \rho < r_{15} \\ \frac{(1 \;\; \gamma d_L)}{\alpha+2}(\alpha + \frac{u_L + u_H}{2\rho}) & Otherwise \end{cases} \tag{9}$$

Where:

$$x_{12} = \frac{\gamma(d_L - d_H)}{3\alpha - \gamma(8d_H - (\alpha+4)d_L)},$$

$$x_{13} = \frac{2\alpha(3 - \gamma d_L) - \gamma(5d_L - 13d_H)}{(\alpha+1)(3\alpha - \gamma(8d_H - (\alpha+4)d_L) - 2\gamma(d_L - d_H))},$$

$$x_{14} = \frac{\gamma(d_L - d_H)}{\alpha(1-\gamma d_H) + (\alpha+2)\gamma(d_L - d_H)},$$

$$x_{15} = \frac{\alpha(1 - \gamma d_L)}{\alpha(\alpha+1)(1-\gamma d_H) + (\alpha^2 + 3\alpha + 1)\gamma(d_L - d_H)},$$

$$r_{14} = \frac{u_H(x_{14}x_{15}\gamma(d_L - d_H))}{2(2\alpha x_{14} + x_{15}(x_{14} - 1)\gamma(d_L - d_H) - x_{12}(3 - x_{13}))} \text{ and}$$

$$r_{15} = \frac{u_H(x_{15}(1 - \gamma d_H) - 2\alpha) - u_L(2\alpha(1 - \gamma d_L))}{2((\alpha+2)(1-\gamma d_L)(x_{14} - 1) - (\alpha+1)(x_{14}(2\alpha + x_{15}\gamma(d_L - d_H) - 1))}.$$

For the proof of this lemma see appendix B.8.

4. Equilibrium Strategies

Before we can decide the equilibrium strategies, for simplifying the model and making it traceable, we apply two reasonable assumptions to the model:

1. The value of composite web service for a given user is equal to the sum of the values of single services, i.e. $\alpha = 2$.

2. We assume that $u_L \frac{1-\gamma d_L}{1-\gamma d_H} > 1.2u_H$. This assumption implies that:

$$\gamma d_H d_L^2 + d_L(0.2d_H - 1.2b - 1.2\gamma d_H^2 + 0.2\gamma b d_H) + bd_H < 0$$

This is a quadratic equation in d_L and can be solved to obtain d_L that is bounded in both directions. The lower bound ensures that d_L and d_H are well separated, while the upper bound ensures that the service capacity is limited.

As it mentioned above, the payoff to a provider depends not only on her own choice, but also on the choice by her competitor and the choice of monopoly provider offering a service that is complementary to their services. We can obtain payoffs of each provider using following simple equation:

$$\pi_i^{(j)} = P_i^{(j)} \times \lambda_i^{(j)} \tag{10}$$

This implies that, in each case, payoffs of each provider can be obtained by multiplying her optimal price to her total effective arrival rate. Since the functional forms of these profits vary with overall traffic intensity, ρ, different Equilibrium strategies could exist.

Proposition 1: When the monopoly had chosen service level H, there exists ρ_1, such that, when $\rho < \rho_1$ two competitors choose differentiated service; and else, both of them offer their services with level H.

Proposition 2: When the monopoly had chosen service level L, there exists ρ_1 and ρ_2, such that, when $\rho < \rho_1$ two competitors choose differentiated service; when $\rho_1 < \rho < \rho_2$, both of them offer their services with level H; and when $\rho > \rho_2$, both providers choose service level L. For the proofs of these propositions see appendix B.9 and B.10.

These propositions have important managerial implications for service providers who need to establish service-level agreements with their customers.

In order to describe the propositions, we have presented a numerical example, assume that $b = 0.25s$, $a = 2$, $d_H = 0.5s$, $d_L = 0.8$ and $\gamma = 1$.

Assume that the monopoly service provider (SP3) chooses H (Cases 1,2, and 3). For this situation, optimal profits of SP1 and SP2 against ρ respectively are plotted in figures 3, 4. In the figures, filled markers show the equilibrium strategies.

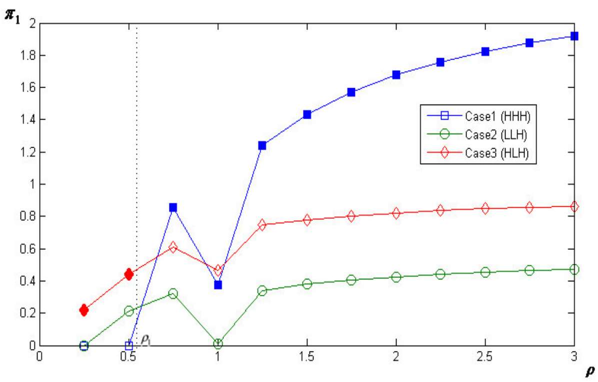

Fig. 3. Optimal profits of provider 1

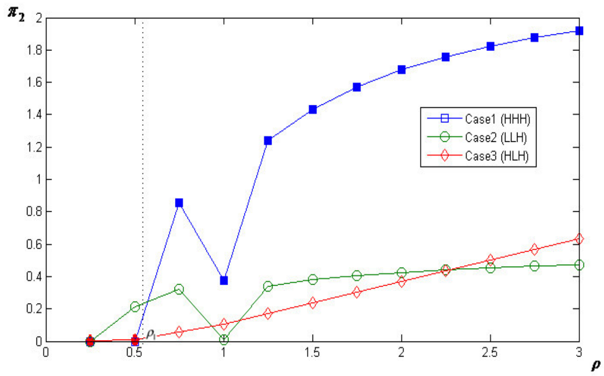

Fig. 4. Optimal profits of provider 2

According to proposition 1 and as you can see in figure 3 and 4, if the monopoly had chosen level H, When the traffic intensity is low ($\rho < \rho_1$) the capacity constraint is not relevant, and the service-level agreement does not have an impact on the providers' decision.

Analogous to the traditional setting (e.g., [36]), both the providers benefit by locating far away from each other in the service dimension, since the competition in that position is the weakest. Consequently the providers charge different prices for differentiated services.

The figures also show that, as the traffic intensity increases above ρ_1, SP2 becomes interested in the more beneficial market of service level H. In response, SP1 chooses not to move.

Hence, both providers end up in the same market, charging the same price and splitting the demand equally (see figures 5 to 8). Even though price competition intensifies in this situation, SP2 still enjoys a higher profit by switching because of the higher price she can charge. The symmetric equilibrium may appear a bit surprising at first, but because of fixed processing capacity, service guarantee and the choice of SP3 (service level H), it is not surprising.

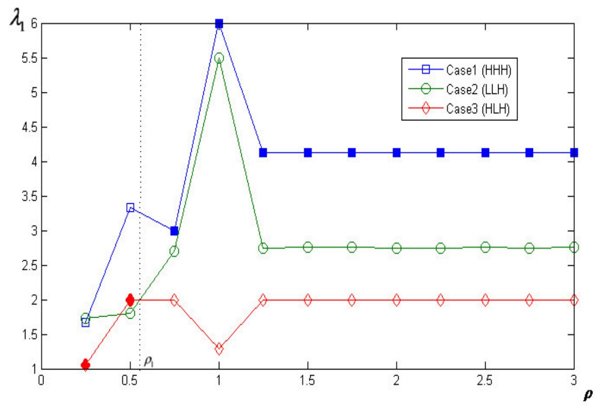

Fig. 5. Optimal demands of provider 1

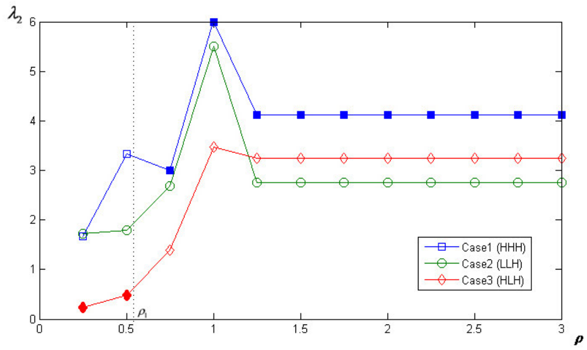

Fig. 6. Optimal demands of provider 2

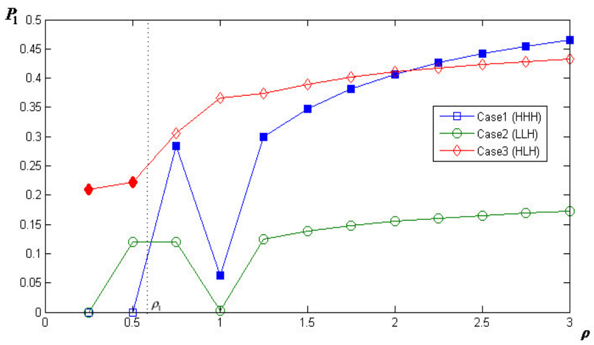

Fig. 7. Optimal prices of provider 1

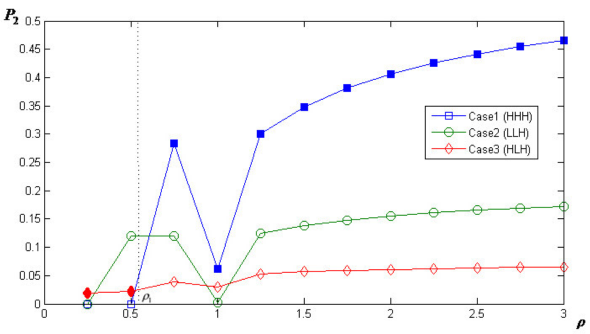

Fig. 8. Optimal prices of provider 2

Now, assume that the monopoly offers her service with level L (Cases 4,5, and 6). Figures 9 and 10 shows optimal profits of SP1 and SP2 against ρ. As you can see in these figures, When the traffic intensity is low ($\rho < \rho_1$), both the providers benefit by locating far away from each other in the service dimension, therefore the competitors charge different prices for differentiated services.

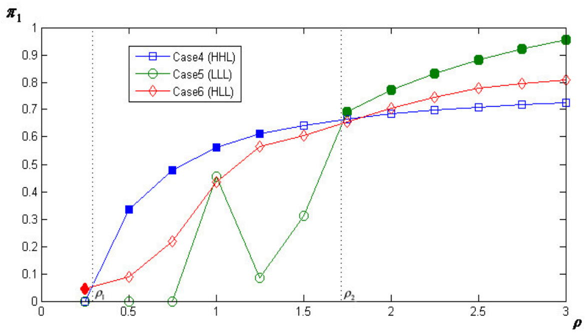

Fig. 9. Optimal profits of provider 1

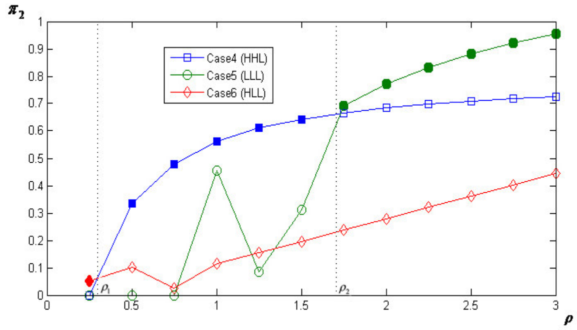

Fig. 10. Optimal profits of provider 2

As the traffic intensity increases above ρ_1, SP2 becomes interested in the more beneficial market of service level H. Hence, both providers end up in charging same price and splitting the demand equally. When ρ is beyond ρ_2, the competitors benefit by changing their service level strategies as well as their prices. In this situation, SP1 and SP2, in order to obtain more benefit, offer their services with level L and a low price (see figures 11 to 14). According to [4] in a duopoly when the traffic intensity is very high, providing the service with level L is beneficial. This is because, under a heavy traffic, the high negative externality imposed by the service level H requires a provider to drastically increase prices (thus resulting in significantly low demand and profit). But in our model because of presence of the monopoly provider, who offers a complementarity service to SP1 and SP2's services, the result is different. Our analysis shows, when the traffic intensity is very high, the monopoly's power increases and she can force the competitors to accompany with her. Therefore, under high traffic intensity, SP1 and SP2 choose their service level same as SP3.

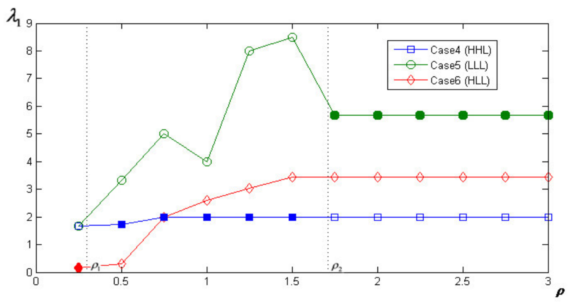

Fig. 11. Optimal demands of provider 1

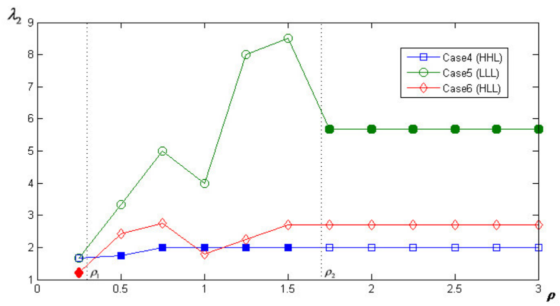

Fig. 12. Optimal demands of provider 2

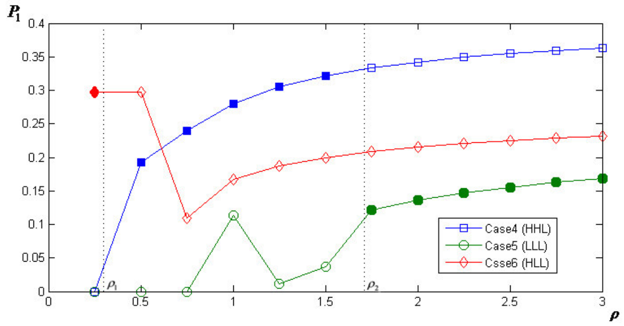

Fig. 13. Optimal prices of provider 1

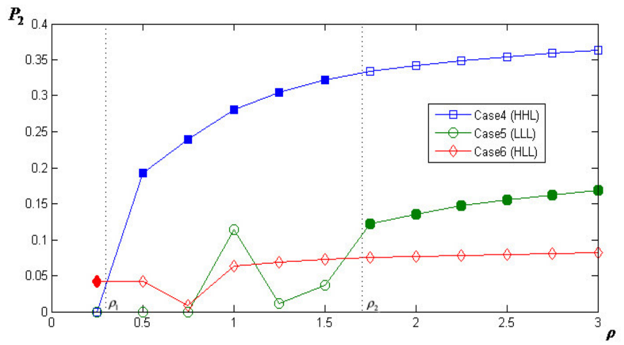

Fig. 14. Optimal prices of provider 2

5. Conclusion

Pricing has been used as an incentive mechanism to control traffic in many areas. As web services become popular, consumers are asking for service level agreements that guarantee the QoS they pay for. In this context, it is particularly important that service providers design an efficient pricing mechanism to enforce service-level agreements. The paper, combining game theory and queuing theory, has developed a model to study the duopoly competition where their services are complementary to a service that is offered by a monopoly service provider. The main contribution of this paper is the investigation of the role of the monopoly complementarity service in the modeled market. Service providers can provide either H or L service level. We found that, in low traffic intensity both the providers benefit by locating far away from each other in the service dimension and they choose differentiated strategy. While, in high traffic intensity, the competitors might choose to compete head to head by providing the same service level. Furthermore, our analysis attains to proof that when the traffic intensity is very high, the monopoly's power increases and the competitors make their service levels alike the monopoly's service level.

Appendix A

A.1. Case1: HHH

When SP1 and SP2 offer their services with same service level, Consumers of segment 1 and 2 buy from the provider who charges the lowest price. If they both charge the same price, each provider would face a demand equal to the half of the market demand at that price. Because of the capacity constraint and the service-level agreement, however, a provider may not be able to reduce her price to the marginal cost. We now investigate the range of prices a service provider is allowed to charge.

Let $\overline{P_H}$ be the price charged by only one provider offering S1 (when the other competitor is out of the market) such that the response-time constraint due to the service level agreement is binding. Since v is uniformly distributed in [0, 1], in segment 1 only a portion of users with $v-\overline{P_H}-v\gamma d_H>0$ and in segment 2 a portion of users with $\alpha v-\overline{P_H}-P_3^{(H)}-\alpha v\gamma d_H>0$ will choose service from the provider. In other words the effective provider's arrival rate is: $\lambda_H=\lambda_0\left(1-\frac{\overline{P_H}}{1-\gamma d_H}\right)+\lambda_0\left(1-\frac{\overline{P_H}+P_3^{(H)}}{\alpha(1-\gamma d_H)}\right)$.

Hence, $\overline{P_H}$ solves the following equation:

$$\frac{b}{1-\rho\left(2-\frac{(\alpha+1)\overline{P_H}+P_3^{(H)}}{\alpha(1-\gamma d_H)}\right)}=d_H$$

Further, let $\underline{P_H}$ be the common price charged by each competitor when they split the market equally and keep the response-time constraint binding. In this situation, the effective arrival rate for each provider can be obtained as: $\lambda_i^{(H)}=\frac{1}{2}\lambda_0\left(2-\frac{(\alpha+1)\underline{P_H}+P_3^{(H)}}{\alpha(1-\gamma d_H)}\right)$, ($i$=*1,2*). Hence, $\underline{P_H}$ solves the following equation:

$$\frac{b}{1-\frac{1}{2}\rho\left(2-\frac{(\alpha+1)\underline{P_H}+P_3^{(H)}}{\alpha(1-\gamma d_H)}\right)}=d_H$$

Solving the above two equations, we get:

$$\overline{P_H}=\frac{\alpha(1-\gamma d_H)}{\alpha+1}\left(2-\frac{u_H}{\rho}\right)-\frac{P_3^{(H)}}{\alpha+1}$$ and

$$\underline{P_H}=\frac{2\alpha(1-\gamma d_H)}{\alpha+1}\left(1-\frac{u_H}{\rho}\right)-\frac{P_3^{(H)}}{\alpha+1}$$

Where $u_H=1-\frac{b}{d_H}$. It can be easily verified that both $\overline{P_H}$ and $\underline{P_H}$, as defined above, can be negative if the total traffic intensity, ρ, be very low. This is because, when ρ is small, the delay constraint could become slack, artificially forcing the delay constraint to be binding would lead to negative prices. Of course, prices cannot be negative in reality, so we re-define:

$$\overline{P_H}=\max\{0\ ,\ \frac{\alpha(1-\gamma d_H)}{\alpha+1}\left(2-\frac{u_H}{\rho}\right)-\frac{P_3^{(H)}}{\alpha+1}\}$$

and $\underline{P_H}=\max\{0\ ,\ \frac{2\alpha(1-\gamma d_H)}{\alpha+1}\left(1-\frac{u_H}{\rho}\right)-\frac{P_3^{(H)}}{\alpha+1}\}$

Lemma a: When two competitors choose same service level, *(i)* the equilibrium price $P_i^{(j)}$ must satisfy $\underline{P_j} \le P_i^{(j)} \le \overline{P_j}$, and *(ii)* every symmetric price choice, $P_1^{(j)} = P_2^{(j)} = P_j \in [\overline{P_j}, \underline{P_j}]$, is an equilibrium.

For the proof of this lemma see appendix B.1.

From all of the possible symmetric Nash equilibria, we consider only the Pareto-dominant one. Let P_H be the common price charged by both the providers if they both choose service level H.

Then, the effective demand to each provider is $\lambda_H = \frac{1}{2}\lambda_0(2 - \frac{(\alpha+1)P_H + P_3^{(H)}}{\alpha(1-\gamma d_H)})$. Hence, the optimization problem of each provider can be written as:

$$\max_{P_H} \pi_i^{(H)} = \frac{1}{2}\lambda_0 P_H (2 - \frac{(\alpha+1)P_H + P_3^{(H)}}{\alpha(1-\gamma d_H)}) \qquad (a)$$
$$s.t. \quad \frac{b}{1-\frac{1}{2}\rho(2-\frac{(\alpha+1)P_H + P_3^{(H)}}{\alpha(1-\gamma d_H)})} \le d_H$$
$$\underline{P_H} \le P_H \le \overline{P_H}$$

Lemma b: When the monopoly (leader) chooses level H and then the both competitors (followers) choose H too, the Pareto optimal price for competitors is given by:

$$P_H = \frac{(1-\gamma d_H)}{\alpha+1} \times \begin{cases} 0 & if \quad \rho < r_1 \\ \frac{\alpha(2\alpha+3)}{\alpha+2} - (\frac{2\alpha^2+4\alpha+1}{\alpha+2})(\frac{u_H}{2\rho}) & if \quad r_1 \le \rho < r_2 \\ \alpha(1-\frac{2\alpha+1}{2(2\alpha^2+4\alpha+1)}) & if \quad r_2 \le \rho < r_3 \\ \frac{\alpha(2\alpha+1)}{\alpha+2} - (\frac{2\alpha^2+4\alpha+1}{\alpha+2})(\frac{u_H}{\rho}) & Otherwise \end{cases} \qquad (b)$$

Also, the optimal price for SP3 (leader) is:

$$P_3^{(H)} = (1 \quad \gamma d_H) \times \begin{cases} \frac{\alpha}{\alpha+1} & if \quad \rho < r_1 \\ \frac{1}{\alpha+2}(\alpha + \frac{u_H}{2\rho}) & if \quad r_1 \leqslant \rho < r_2 \\ \frac{\alpha(2\alpha+1)}{2\alpha^2+4\alpha+1} & if \quad r_2 \leqslant \rho < r_3 \\ \frac{1}{\alpha+2}(\alpha + \frac{u_H}{\rho}) & Otherwise \end{cases} \qquad (c)$$

Where: $r_1 = u_H(\frac{2\alpha^2+4\alpha+1}{2\alpha(2\alpha+3)})$,

$r_2 = u_H(\frac{(2\alpha^2+4\alpha+1)^2}{2\alpha(\alpha+1)(2\alpha^2+4\alpha+1)+\alpha(2\alpha+1)(\alpha+2)})$ and

$r_3 = u_H(\frac{2(2\alpha^2+4\alpha+1)^2}{2\alpha(\alpha+1)(2\alpha^2+4\alpha+1)+\alpha(2\alpha+1)(\alpha+2)})$.

For the proof of this lemma see appendix B.2.

A.2. Case2: LLH

In this case, since $d_L > d_H$, total response time of composite web service for users of segment 2 is equal to d_L.

Therefore, if only there is one service provider, her effective arrival rate is:

$\lambda_L = \lambda_0(1 - \frac{\overline{P}_L}{1-\gamma d_L}) + \lambda_0(1 - \frac{\overline{P}_L + P_3^{(H)}}{\alpha(1-\gamma d_L)})$, and effective arrival rate of each competitor when they split the market equally and keep the response-time constraint binding is: $\lambda_i^{(L)} = \frac{1}{2}\lambda_0(2 - \frac{(\alpha+1)P_L + P_3^{(H)}}{\alpha(1-\gamma d_L)})$, (*i=1,2*). Hence, we have:

$$\frac{b}{1-\rho(2-\frac{(\alpha+1)\overline{P}_L + P_3^{(H)}}{\alpha(1-\gamma d_L)})} = d_L$$

and

$$\frac{b}{1-\frac{1}{2}\rho(2-\frac{(\alpha+1)\underline{P_L} + P_3^{(H)}}{\alpha(1-\gamma d_L)})} = d_L$$

Solving the above two equations, we get:

$$\overline{P_L} = \max\{0 \ , \ \frac{\alpha(1-\gamma d_L)}{\alpha+1}(2 - \frac{u_L}{\rho}) - \frac{P_3^{(H)}}{\alpha+1}\} \quad and \quad \underline{P_L} = \max\{0 \ , \ \frac{2\alpha(1-\gamma d_L)}{\alpha+1}(1 - \frac{u_L}{\rho}) - \frac{P_3^{(H)}}{\alpha+1}\}$$

Using results of Lemma a, In this case the effective demand to each provider is: $\lambda_L = \frac{1}{2}\lambda_0(2 - \frac{(\alpha+1)P_L + P_3^{(H)}}{\alpha(1-\gamma d_L)})$.

Hence, the optimization problem of each provider can be written as:

$$\max_{P_L} \pi_i^{(L)} = \frac{1}{2}\lambda_0 P_L (2 - \frac{(\alpha+1)P_L + P_3^{(H)}}{\alpha(1-\gamma d_L)}) \qquad (d)$$
$$s.t. \quad \frac{b}{1-\frac{1}{2}\rho(2-\frac{(\alpha+1)P_H + P_3^{(H)}}{\alpha(1-\gamma d_L)})} \le d_L$$
$$\underline{P_L} \le P_L \le \overline{P_L}$$

Lemma c: When the monopoly (leader) chooses level H and then the both competitors (followers) choose L, the Pareto optimal price for competitors is given by:

$$P_L = \frac{(1 \quad \gamma d_L)}{\alpha+1} \times \begin{cases} 0 & if \quad \rho < r_4 \\ \alpha(2 \quad x_1) \quad (2\alpha + x_1)(\frac{u_L}{2\rho}) & if \quad r_4 \leqslant \rho < r_5 \\ \alpha(1 \quad \frac{x_2}{2}) & if \quad r_5 \leqslant \rho < r_6 \\ \alpha(2 \quad x_1) \quad (2\alpha + x_1)(\frac{u_L}{\rho}) & Otherwise \end{cases} \qquad (e)$$

Also, the optimal price for SP3 (leader) is:

$$P_3^{(H)} = (1 \quad \gamma d_L) \times \begin{cases} \frac{1 \quad \gamma d_H}{\alpha(1 \quad \gamma d_L) + (1 \quad \gamma d_H)} & if \quad \rho < r_4 \\ x_1(\alpha + \frac{u_L}{2\rho}) & if \quad r_4 \leqslant \rho < r_5 \\ \alpha x_2 & if \quad r_5 \leqslant \rho < r_6 \\ x_1(\alpha + \frac{u_L}{\rho}) & Otherwise \end{cases} \qquad (f)$$

Where:

$x_1 = \dfrac{(1-\gamma d_H)}{(\alpha+1)(1-\gamma d_L) + (1-\gamma d_H)}$,

$x_2 = \dfrac{\alpha(1-\gamma d_H)}{2\alpha(\alpha+1)(1-\gamma d_L) + (2\alpha+1)(1-\gamma d_H)}$,

$r_4 = u_L(\dfrac{2\alpha + x_1}{2\alpha(2-x_1)})$, $\quad r_5 = u_L(\dfrac{2\alpha + x_1}{\alpha(2+x_2-2x_1)})$ and

$r_6 = u_L(\dfrac{2(2\alpha + x_1)}{\alpha(2+x_2-2x_1)})$.

For the proof of this lemma see appendix B.3.

A.3. Case3: HLH

In this case we assume that SP1 and SP2 choose differentiated service levels. This, of course, means that SP1 must charge a price $P_1^{(H)}$ higher than $P_2^{(L)}$ charged by SP2.

In order to find the equilibrium prices, we need to estimate the expected demand for each provider. For users of segment 1, let V be the v-value of the marginal user who is indifferent between two providers. This implies that $V - P_1^{(H)} - V\gamma d_H = V - P_2^{(L)} - V\gamma d_L$ or $V = \dfrac{P_1^{(H)} - P_2^{(L)}}{\gamma(d_L - d_H)}$.

A user with v should prefer service level H if $v \in [V,1]$, or level L if $v \in [0,V]$; this is the incentive compatibility constraint (ICC).

Even though one level of service may dominate the other, it would be chosen only if the user obtains a non-negative net utility from it. This implies that $v - P_i^{(j)} - v\gamma d_j \ge 0$ or $v \ge \dfrac{P_i^{(j)}}{1-\gamma d_j}$, this is the individual rationality constraint (IRC). Let $V_{L,1} = \dfrac{P_2^{(L)}}{1-\gamma d_L}$ then combining ICC and IRC, we can express the effective arrival rates from users of segment 1 for the two providers as:

$$\lambda_{1,1}^{(H)} = \lambda_0(1 - \max\{V, V_{H,1}\}) \quad and \quad \lambda_{2,1}^{(L)} = \lambda_0(\max\{V, V_{L,1}\} - V_{L,1})$$

Also for users of segment 2, let V_2 be the v-value of the marginal user who is indifferent between two providers. Hence, we have: $\alpha V_2 - P_1^{(H)} - \alpha V_2 \gamma d_H = \alpha V_2 - P_2^{(L)} - \alpha V_2 \gamma d_L$ or $V_2 = \dfrac{P_1^{(H)} - P_2^{(L)}}{\alpha\gamma(d_L - d_H)}$.

Further, we have $\alpha v - P_i^{(j)} - P_3^{(H)} - \alpha v\gamma d_j \ge 0$ or $v = \dfrac{P_i^{(j)} + P_3^{(H)}}{\alpha(1-\gamma d_j)}$. Now let $V_{L,2} = \dfrac{P_2^{(L)} + P_3^{(H)}}{1-\gamma d_L}$. So, we can express the effective arrival rates from users of segment 2 for the two providers as:

$$\lambda_{1,2}^{(H)} = \lambda_0(1 - \max\{V_2, V_{H,2}\}) \quad and \quad \lambda_{2,2}^{(L)} = \lambda_0(\max\{V_2, V_{L,2}\} - V_{L,2})$$

Lemma d: At equilibrium $V_{H,1} < V$, $V_{L,1} < V$, $V_{H,2} < V_2$ and $V_{L,2} < V_2$. Therefore, the total effective arrival rates for the two providers are:

$$\lambda_1^{(H)} = \lambda_{1,1}^{(H)} + \lambda_{1,2}^{(H)} = \lambda_0(2 - \frac{(\alpha+1)(P_1^{(H)} - P_2^{(L)})}{\alpha\gamma(d_L - d_H)})$$

$$\lambda_2^{(L)} = \lambda_{2,1}^{(L)} + \lambda_{2,2}^{(L)} = \lambda_0(\frac{(\alpha+1)(P_1^{(H)} - P_2^{(L)})}{\alpha\gamma(d_L - d_H)} - \frac{(\alpha+1)P_2^{(L)} + P_3^{(H)}}{\alpha(1-\gamma d_L)})$$

For the proof of this lemma see appendix B.4.

Lemma e: When the monopoly (leader) chooses level H and then the two competitors (followers) make a differentiated market, the optimal prices are given by:

$$P_1^{(H)} = \frac{1}{\alpha+1} \times \begin{cases} x_3(4\alpha(1 \quad \gamma d_H) + x_5(x_3 \quad \alpha \quad 1)) & if \quad \rho < r_7 \\ x_4((4\alpha(1 \quad \gamma d_H) + x_4 x_6)(1 \quad \frac{u_H}{2\rho}) \quad x_6(\alpha+1)) & if \quad r_7 \leqslant \rho < r_8 \\ 2\alpha(1 \quad \gamma d_H) \quad \alpha x_7 \quad (x_7 + 2\alpha(1 \quad \gamma d_H))(\frac{u_H}{2\rho}) \quad (x_7 + 2\alpha(1 \quad \gamma d_L))(\frac{u_L}{2\rho}) & Otherwise \end{cases} \qquad (g)$$

$$P_2^{(L)} = \frac{1}{\alpha+1} \times \begin{cases} x_3(2\alpha(1 \quad \gamma d_L) + 2x_5(x_3 \quad \alpha \quad 1)) & if \quad \rho < r_7 \\ x_4((2\alpha(1 \quad \gamma d_L) + x_4 x_6)(1 \quad \frac{u_H}{2\rho}) \quad x_6(\alpha+1)) & if \quad r_7 \leqslant \rho < r_8 \\ (2(1 \quad \gamma d_L) \quad x_7)(\alpha \quad \frac{u_H + u_L}{2\rho}) & Otherwise \end{cases} \qquad (h)$$

$$P_3^{(H)} = \begin{cases} x_5(\alpha+1-x_3) & if \quad \rho < r_7 \\ x_6(\alpha+1) - x_4 x_6(1-\frac{u_H}{2\rho}) & if \quad r_7 \le \rho < r_8 \\ x_7(\alpha+1) - x_7(1-\frac{u_H+u_L}{2\rho}) & Otherwise \end{cases} \qquad (i)$$

Where: $x_3 = \frac{\gamma(d_L - d_H)}{3-\gamma(4d_H - d_L)}$, $x_4 = \frac{\gamma(d_L - d_H)}{1-\gamma(2d_H - d_L)}$,

$$x_5 = \frac{\alpha(1-\gamma d_L)(1-\gamma d_H)}{\alpha(\alpha+1)(1-\gamma d_L)+(1-\gamma d_H)(\alpha+1-2x_3)}$$

$$x_6 = \frac{\alpha(1-\gamma d_L)(1-\gamma d_H)}{\alpha(\alpha+1)(1-\gamma d_L)+(1-\gamma d_H)(\alpha+1-x_4)},$$

$$x_7 = \frac{(1-\gamma d_L)(1-\gamma d_H)}{(\alpha+1)(1-\gamma d_L)+(1-\gamma d_H)},$$

$$r_7 = \frac{u_H(2\alpha(1-\gamma d_L)+x_4x_6)}{2(2\alpha(1-\gamma d_L)(x_4-x_3)-(\alpha+1)(x_4x_6+2x_3x_5)+x_4^2x_6-2x_3^2x_5)},$$

and $r_8 = \frac{u_H(2\alpha(1-x_4)(1-\gamma d_L)+x_7+x_4^2x_6)+u_L(2\alpha(1-\gamma d_L)+x_7)}{2(2\alpha(1-\gamma d_L)(1-x_4)-(\alpha+1)(x_7+x_4x_6)+x_7-2x_4^2x_6)}$.

For the proof of this lemma see appendix B.5.

Appendix B

B.1. Proof of Lemma a

(i) If provider 1 charges a price $P_1^{(j)} > \underline{P_j}$, provider 2 can simply set $P_2^{(j)} = \overline{P_j}$ and get the entire market while meeting the service-level agreement. Hence provider 1would be better off by reducing her price to $\overline{P_j}$. On the other hand, provider 1 would not charge a price $P_1^{(j)} < \overline{P_j}$. This is because if she does, provider 2 would charge a price $P_2^{(j)} > P_1^{(j)}$ in order to commit to her service level guarantee. All the consumers would then choose service from provider 1, resulting in violation of her service level guarantee. ***(ii)*** Assume that provider 1 offers a price $P_1^{(j)} \in [\overline{P_j}, \underline{P_j}]$. If provider 2 chooses $P_2^{(j)} < P_1^{(j)}$, all consumers would prefer the service from 2, leading to the violation of provider 2's service level guarantee. Provider 2 also would not choose $P_2^{(j)} > P_1^{(j)}$ either, because in that case all consumers would choose the service from 1, resulting in zero profit for 2. Hence, the only choice for provider 2 is to charge $P_1^{(j)} = P_2^{(j)}$. Therefore, any price choice $P_1^{(j)} = P_2^{(j)} = P_j \in [\overline{P_j}, \underline{P_j}]$ is a Nash equilibrium.

B.2. Proof of Lemma b

We can solve (a) to obtain the equilibrium price. The formulation is a simple quadratic optimization problem with one decision variable P_H. So, we have:

$$P_H = \frac{(1-\gamma d_H)}{\alpha+1} \times \begin{cases} 0 & if \quad \rho < r_1 \\ 2\alpha(1-\frac{u_H}{2\rho}) - P_3^{(H)} & if \quad r_1 \le \rho < r_2 \\ \alpha - \frac{P_3^{(H)}}{2} & if \quad r_2 \le \rho < r_3 \\ 2\alpha(1-\frac{u_H}{\rho}) - P_3^{(H)} & Otherwise \end{cases} \qquad (j)$$

Further, in this case, the effective arrival rate for provider 3 from users of segment 3 and segment 2 can be obtained as $\lambda_3^{(H)} = \lambda_0(1-\frac{P_3^{(H)}}{1-\gamma d_H}) + \lambda_0(1-\frac{P_3^{(H)}+P_H}{\alpha(1-\gamma d_H)})$. Since there is no capacity limitation for provider 3, the optimization problem can be written as:

$$\max_{P_3^{(H)}} \pi_3^{(H)} = \lambda_0 P_3^{(H)}(2-\frac{(\alpha+1)P_3^{(H)}+P_H}{\alpha(1-\gamma d_H)}) \qquad (k)$$

Now, Substituting (*j*) into (*k*) using first order condition of $\pi_3^{(H)}$ we can obtain $P_3^{(H)}$ and then substituting it into (*j*), P_H will be obtained. In order to complete the proof, the thresholds of ρ can be found by comparing the optimal prices.

B.3. Proof of Lemma c

Please see proof of Lemma b. Note that in this case we have:

$$\lambda_3^{(H)}=\lambda_0(1-\frac{P_3^{(H)}}{1-\gamma d_H})+\lambda_0(1-\frac{P_3^{(H)}+P_L}{\alpha(1-\gamma d_L)})=\lambda_0(2-\frac{P_3^{(H)}}{1-\gamma d_H}-\frac{P_3^{(H)}+P_L}{\alpha(1-\gamma d_L)})$$

B.4. Proof of Lemma d

First, note that $V_{L,1}<V$, otherwise, we have $\lambda_{2,1}=0$, and hence payoff of provider 2 in segment 1 will be equal to zero. So, provider 2 can increase her profit by decreasing $P_2^{(L)}$ till $V_{L,1}$ drops to a value just below *V*. because, in that case, $\lambda_{2,1}>0$ and provider 2 enjoys a positive profit from users of segment 1 while satisfying the service level agreement. Now, we can write:

$$\frac{P_2^{(L)}}{1\quad\gamma d_L}<\frac{P_1^{(H)}\quad P_2^{(L)}}{\gamma(d_L\quad d_H)}\Rightarrow\frac{P_1^{(H)}\quad P_2^{(L)}}{P_2^{(L)}}>$$

$$\frac{\gamma(d_L\quad d_H)}{1\quad\gamma d_L}\Rightarrow\frac{P_1^{(H)}}{P_2^{(L)}}>\frac{\gamma(d_L\quad d_H)}{1\quad\gamma d_L}+1=\frac{1\quad\gamma d_H}{1\quad\gamma d_L}$$

This means

$$\frac{P_2^{(L)}}{P_1^{(H)}}<\frac{1-\gamma d_L}{1-\gamma d_H}\Rightarrow 1-\frac{P_2^{(L)}}{P_1^{(H)}}>1-\frac{1-\gamma d_L}{1-\gamma d_H}\Rightarrow\frac{P_1^{(H)}-P_2^{(L)}}{P_1^{(H)}}>\frac{\gamma(d_L-d_H)}{1-\gamma d_H}$$

Therefore

$$V_{H,1}=\frac{P_1^{(H)}}{1-\gamma d_H}<\frac{P_1^{(H)}-P_2^{(L)}}{\gamma(d_L-d_H)}=V$$

This completes proof of $V_{H,1}<V$ and $V_{L,1}<V$. Using same way for users of segment 2 we can proof that $V_{H,2}<V_2$ and $V_{L,2}<V_2$.

B.5. Proof of Lemma e

When one of the competitors chooses service level H and the other L, the providers' optimization problems can be written as:

$$\max_{P_1^{(H)}}\pi_1^{(H)}=P_1^{(H)}\lambda_0(2-V-V_2)$$

$$s.t.\quad\frac{b}{1-\rho(2-V-V_2)}\le d_H$$

and

$$\max_{P_2^{(L)}}\pi_2^{(L)}=P_2^{(L)}\lambda_0(V+V_2-V_L-V_{L,2})$$

$$s.t.\quad\frac{b}{1-\rho(V+V_2-V_L-V_{L,2})}\le d_L$$

Each one is a non-linear constrained optimization problem with one decision variable and one constraint (due to service-level agreement). If both of the constraints are slack, we can obtain the best response functions from the first order conditions as:

$$P_1^{(H)}=\frac{\alpha\gamma(d_L-d_H)}{\alpha+1}+\frac{P_2^{(L)}}{2}\ \text{and}$$

$$P_2^{(L)}=\frac{(1-\gamma d_L)P_1^{(H)}}{2(1-\gamma d_H)}-\frac{\gamma(d_L-d_H)P_3^{(H)}}{2(\alpha+1)(1-\gamma d_H)}$$

Which then yield:

$$P_2^{(L)}=\frac{2\alpha(1-\gamma d_L)x_3}{(\alpha+1)}-\frac{2x_3P_3^{(H)}}{(\alpha+1)}\ \text{and}$$

$$P_1^{(H)}=\frac{4\alpha(1-\gamma d_H)x_3}{(\alpha+1)}-\frac{x_3P_3^{(H)}}{(\alpha+1)}$$

Because of our assumption that the total response time, d_L and d_H guarantees are well separated, if only one of the constraints is binding, it must be the one for the service with level H; that for the service with level L must be slack. In that case, the best response functions are given by:

$$P_1^{(H)}=\frac{\alpha\gamma(d_L\text{ - }d_H)}{(\alpha+1)}(2-\frac{u_H}{\rho})+P_2^{(L)}\ \text{and}$$

$$P_2^{(L)}=\frac{(1-\gamma d_L)P_1^{(H)}}{2(1-\gamma d_H)}-\frac{\gamma(d_L-d_H)P_3^{(H)}}{2(\alpha+1)(1-\gamma d_H)}$$

Solving the two equations simultaneously, we get:

$$P_1^{(H)}=\frac{2\alpha x_4(1-\gamma d_H)}{(\alpha+1)}(2-\frac{u_H}{\rho})-\frac{x_4P_3^{(H)}}{(\alpha+1)}\ \text{and}$$

$$P_2^{(L)}=\frac{\alpha x_4(1-\gamma d_L)}{(\alpha+1)}(2-\frac{u_H}{\rho})-\frac{x_4P_3^{(H)}}{(\alpha+1)}$$

Finally, if both the constraints are binding, the optimal prices can be obtained directly from the constraints as:

$$P_1^{(H)}=\frac{\alpha(1-\gamma d_H)}{(\alpha+1)}(2-\frac{u_H}{\rho})-\frac{\alpha(1-\gamma d_L)}{(\alpha+1)}(\frac{u_L}{\rho})-\frac{P_3^{(H)}}{(\alpha+1)}$$

$$P_2^{(L)}=\frac{\alpha(1-\gamma d_L)}{(\alpha+1)}(2-\frac{u_H+u_L}{\rho})-\frac{P_3^{(H)}}{(\alpha+1)}$$

Further, in this case, the effective arrival rate for provider 3 from users of segment 3 and segment 2 can be obtained as $\lambda_3^{(H)}=\lambda_0(2-\frac{P_3^{(H)}}{1-\gamma d_H}-\frac{P_3^{(H)}+P_2^{(L)}}{\alpha(1-\gamma d_L)})$ since there is no capacity limitation for provider 3, the optimization problem can be written as:

$$\max_{P_3^{(H)}}\pi_3^{(H)}=\lambda_0P_3^{(H)}(2-\frac{P_3^{(H)}}{1-\gamma d_H}-\frac{P_3^{(H)}+P_2^{(L)}}{\alpha(1-\gamma d_L)})\qquad(l)$$

Now, Substituting $P_2^{(L)}$ from above equations into (*l*) using first order condition of $\pi_3^{(H)}$ we can obtain $P_3^{(H)}$ in each situation and then $P_2^{(L)}$ and $P_1^{(H)}$ will be obtained. In order to complete the proof, the thresholds of ρ can be found by comparing the optimal prices.

B.6. Proof of Lemma 1

Please see proof of Lemma b. Note that in this case we have:

$$\lambda_3^{(L)}=\lambda_0(1-\frac{P_3^{(L)}}{1-\gamma d_L})+\lambda_0(1-\frac{P_3^{(L)}+P_H}{\alpha(1-\gamma d_L)})=\lambda_0(2-\frac{(\alpha+1)P_3^{(L)}+P_H}{\alpha(1-\gamma d_L)})$$

B.7. Proof of Lemma 2

Please see proof of Lemma b. Note that in this case we have:

$$\lambda_3^{(L)} = \lambda_0(1-\frac{P_3^{(L)}}{1-\gamma d_L}) + \lambda_0(1-\frac{P_3^{(L)}+P_L}{\alpha(1-\gamma d_L)})$$

B.8. Proof of Lemma 3

Please see proof of Lemma e. Note that in this case we have:

$$\lambda_3^{(L)} = \lambda_0(1-\frac{P_3^{(L)}}{1-\gamma d_L}) + \lambda_0(1-\frac{P_3^{(L)}+P_2^{(L)}}{\alpha(1-\gamma d_L)}) = \lambda_0(2-\frac{(\alpha+1)P_3^{(L)}+P_2^{(L)}}{\alpha(1-\gamma d_L)})$$

B.9. Proof of Proposition 1

Let π_H and π_L denote the profit functions of SP1 and SP2, respectively in case1 and case2. Further, in case3, let $\pi_1^{(H)}$ and $\pi_2^{(L)}$ denote respectively the profit functions of SP1 and SP2. To prove the existence of ρ_1 and the various equilibrium strategies, we need to find out where the profit functions $\pi_1^{(H)}$ and π_L, $\pi_2^{(L)}$ and π_H intersect. First, using first order condition of profit functions it can be easily verified that they are non-decreasing with ρ in their bounds. For simplicity, assume $r_3 < r_8$, Also note that when $u_L\frac{1-\gamma d_L}{1-\gamma d_H} > 1.2u_H$, we will have: $r_1 < r_7 < r_2 < r_4 < r_8 < r_5$. Let us now look at where $\pi_2^{(L)}$ and π_H intersect. It is clear that $\pi_2^{(L)}\big|_{\rho=r_1} > \pi_H\big|_{\rho=r_1} = 0$. One can also show that $\pi_2^{(L)}\big|_{\rho=r_7} < \pi_H\big|_{\rho=r_7}$. Hence, $\pi_2^{(L)}$ and π_H intersect at $\rho_1 \in [r_1, r_7]$. Also we can show that $\pi_2^{(L)}\big|_{\rho=r_2} < \pi_H\big|_{\rho=r_2}$, $\pi_2^{(L)}\big|_{\rho=r_3} < \pi_H\big|_{\rho=r_3}$, $\pi_2^{(L)}\big|_{\rho=r_8} < \pi_H\big|_{\rho=r_8}$, and $\pi_2^{(L)}\big|_{\rho=\infty} < \pi_H\big|_{\rho=\infty}$. Hence, there is no more intercept through these two functions.

In order to find where $\pi_1^{(H)}$ and π_L intersect, we can use above mechanism again. One can show that in all of $[r_7, r_4]$, $[r_4, r_8]$, $[r_8, r_5]$, $[r_5, r_6]$, and $[r_6, \infty)$, and therefore always, $\pi_1^{(H)} > \pi_L$. Hence there is no intercept through these two functions. Finally, the equilibrium strategies can be obtained by comparing the profits for the two firms under different regions. Specifically, if $\rho < \rho_1$, then $\pi_1^{(H)} > \pi_L$ and $\pi_2^{(L)} > \pi_H$. So, providers would choose differentiated service levels. But if $\rho > \rho_1$, then $\pi_2^{(L)} < \pi_H$ while $\pi_1^{(H)} > \pi_L$. So, both providers choose level H.

B.10. Proof of Proposition 2

Please see proof of Proposition 1. Note that in this proof we assume $r_{11} < r_{16}$. And also, note that $u_L\frac{1-\gamma d_L}{1-\gamma d_H} > 1.2u_H$ implies that $r_9 < r_{15} < r_{10} < r_{12} < r_{16} < r_{13}$.

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