



Multiperiod Portfolio Selection with Different Rates for Borrowing and Lending in Presence of Transaction Costs

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ABSTRACT

Portfolio management is one of the most important areas of research in financial engineering. This paper is concerned with multi period decision problem for financial asset allocation when the rate of borrowing is greater than the rate of lending. Transaction costs as a source of concern for portfolio managers is also considered in this paper. The proposed method of this paper is formulated in a form of dynamic linear programming which is capable of determining the amount of investment in different time periods. The method is also implemented using some numerical examples and the output results are discussed.

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1. Introduction

Portfolio management plays an important role for many financial institutions. Portfolio selection is defined as selecting a combination of assets among portfolios to reach the investment goal. In a typical portfolio management, one is responsible to allocate funding to different assets by buying and selling them. Modern portfolio theory (MPT) was introduced in 1952 by Harry Markowitz [1, 2, 3]. Markowitz's MPT has led to a new paradigm in portfolio selecting for investors in order to construct a portfolio with the highest expected return at a given level of risk (the lowest level of risk at a given expected return). Markowitz presents three nonlinear models and explained that the unique optimal solution for all three models is equal.

There are many researches have been performed by experts in order to solve and develop Markowitz's seminal model. Because of the limitations of a factual market, lots of these attempts have tried to make his model more useful and practical.

The portfolio selection strategy was extended for a planning horizon in stochastic form by Samuelson [4] and Merton [5].

In spite of comprehensive success of Markowitz's model, the single-period framework suffers from an important deficiency. It is impracticable and difficult to apply to long-term investors having goals at particular dates in the future, for which the investment decisions should be made with regard to temporal issues besides static risk-reward trade-offs. To satisfy this necessity, one may formulate from the beginning the allocation problem over a horizon composed of multiple periods ($T > 1$ periods), with the goal of minimizing the total risk over the investment path (or maximizing the return over the investment path), while satisfying constraints on the portfolio composition and on desired expected return at all the intermediate periods.

Merton [6, 7] presents a mathematical model for the optimum consumption and the portfolio rules in a continuous time horizon. Merton in his work shows how to construct and analyze optimal continuous-time allocation problems under uncertainty. Merton considers the model in which the prices of the risky assets are generated by correlated geometric Brownian motions, and assumes that the portfolio can be rebalanced instantly and free of cost. His objective is to maximize the net expected utility of consumption plus the expected utility of terminal wealth. Mossin [8] also presents a multi-period optimization technique.

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Chryssikou [9] uses approximate dynamic programming algorithms to provide a near-optimal dynamic trading strategy for special types of utility functions when a closed form solution to the discrete-time multi period problem with quadratic transaction costs is not attainable. Hakansson [10, 11] uses mean-variance and quadratic approximations in implementing dynamic investment strategies. Techniques from approximate dynamic programming have been successfully employed for efficient optimal policy computations: for example, Sadjadi et al. propose a dynamic programming approach to solve efficient frontier with the consideration of transaction cost in [12]. Their approach led to a closed form solution of the mean variance portfolio selection is presented by.

Li et al. [13, 14] consider a two-step method where a dynamic programming is employed to solve an auxiliary problem in the first phase and the solution to the auxiliary problem is then manipulated to obtain the optimal mean-variance portfolio policy and the corresponding efficient frontier. Note that the primary assumption with these models is that the rates of return of the assets during consecutive periods are uncorrelated. Leippold et al. [15] introduce a geometric approach to multi period mean variance optimization of assets and liabilities. Morey and Morey introduce the same idea in a multi-period or temporal setting in [16]. They propose two types of efficiency measures: The first efficiency measure attempts to contract all risk dimensions proportionally where the second one focuses on augmenting all return dimensions as much as possible in a proportional way.

Yan and Miao [17] present the multi-period semi-variance model where variance is substituted by semi-variance in Markowitz's portfolio selection model. They point out that for this class of portfolio model, that the hybrid GA with PSO is effective and feasible. Briec and Kerstens [18] develop multi-horizon mean-variance portfolio analysis in the [16] in several ways. First, instead of either proportionally contracting risk dimensions or proportionally expanding return dimensions, a more general efficiency measure simultaneously attempts to reduce the risk and to expand the return over all time periods.

The multi period models have been developed in a variety of directions. Zenios et al [19] develop a fixed income portfolio model in a multi-stage form. The uncertainty which exists on the input and the output parameters of the multi-period portfolio optimization could be investigated in different forms. Leippold et al. [15] propose a method to minimize the variance between the assets and liabilities. Wei and Ye [20] introduce a multi period portfolio selection model constrained with bankruptcy control in a stochastic market. They use dynamic programming to solve developed model. Calafiore [21] proposed an asset allocation model which periodic optimal portfolio adjustments are determined with the objective of

minimizing a cumulative risk measure over the investment horizon. In developed model, portfolio diversity constraints at each period are satisfied. Celikyurt and Ozekici [22] consider a multi period portfolio model where the market consists of a riskless asset and several risky assets. They can describe the stochastic evaluation of market by a Markov chain. Oswaldo et al. [23] propose a generalized multi period mean-variance model with market parameters such as Markov switching parameters. They can obtain some closed formulas with necessary and sufficient conditions for obtaining an optimal control policy for this Markovian generalized multi period mean-variance problem.

Bertsimas and Pachamanova suggest robust optimization formulations of the multi period portfolio optimization problem that are linear and computationally efficient in [24]. Robust optimization models deal with future asset returns as uncertain coefficients in an optimization problem. Bertsimas and Pachamanova in [24] impose non negativity constraints on the investor's holdings at each time period which prevents any borrowing and short selling. Bertsimas and Pachamanova's model considers transaction cost as part of their model. However, the transaction cost does not play an important role in the optimization results since many brokerage houses are planning to remove transaction costs in order to create a motivation to absorb more investment. The multi-period portfolio optimization proposed by Bertsimas and Pachamanova could be developed to incorporate realistic features such as borrowing and lending rates. Shen and Zhang [25] also apply the concept of robust optimization to the portfolio selection problems. Their proposed model is formulated based on multi-stage scenario trees. They use SeDuMi to solve their robust portfolio selection problem. Quaranta and Zaffaroni [26] use robust optimization in portfolio selection problem for the minimization of the conditional value at risk of a portfolio of shares. They can obtain a linear robust copy of the bi-criteria minimization model. Chen and Tan [27] can successfully incorporate interval random chance-constrained programming to robust mean-variance portfolio selection under interval random uncertainty sets in the elements of mean vector and covariance matrix.

There are many approaches to deal with uncertainty in portfolio selection problem that we will use them in future researches.

In this paper a multi period portfolio selection with a new constraint that is borrowing and short selling constraint, i.e. for rendering the market incomplete, is suggested, this model is considered under different interest rates for borrowing and lending. In reality investors may charge a higher interest rate for borrowing money than the interest rate for saving money. Although many researchers have assumed the same riskless interest rate for borrowing and lending, the inconsistency between borrowing and lending rates

is determinant factor in the decisions of financial institutions. The proposed method of this paper considers the Transaction costs as a source of concern for portfolio managers and a part of market frictions.

We believe this features make our proposed method more realistic since most of the brokerage houses provide the opportunity to make an acquisition on different assets by borrowing the money from the brokerage. However, closed-form solutions of this kind can be derived only under strong assumptions on the investor's behavior and the structure of the asset price process. This paper is organized as follows. We first present the problem formulation in section 2. The numerical results are presented in section 3 and the conclusion remarks are given in section 4 to summarize the contribution of the paper.

2. The Proposed Model Formulation

In this section the model and its components are described. The following notations and parameters are used in the problem formulation,

M = the number of risky assets

N = the number of trading periods

X_t^m = the investor's dollar holdings in stock m at the beginning of period t , (which are fund with his capital); ($m=0, 1 \dots M$) & ($t=0, 1 \dots N$)

r_t^m = the return of stock m over time period ($t, t+1$]; ($m=1, 2 \dots M$)

r_t^b = the riskless borrowing rate over time period ($t, t+1$]; ($t=0, 1 \dots N$)

r_t^l = the riskless lending rate over time period ($t, t+1$]; ($t=0, 1 \dots N$)

u_t^m = the amount of stock m which is soled in period t ; ($m=1 \dots M$) & ($t=1 \dots N$)

v_t^m = the amount of stock m which is purchased in period t ; ($m=1 \dots M$) & ($t=1 \dots N$)

c_{sell} = proportional transaction cost for selling

c_{buy} = proportional transaction cost for buying

V = the maximum permitted amount of buying for each stock in each period

W_N = the investor's final wealth at period N

$U(X)$ = the investor utility function

In this model, we have M risky asset and one riskless asset (asset 0). The dynamics of the investor's holdings with a fix rate for riskless asset i.e. $r_t^l = r_t^b = r_t^0$, are then given by the equations:

$$X_t^m = (1+r_{t-1}^m)(X_{t-1}^m - u_{t-1}^m + v_{t-1}^m), \\ t = 1 \dots N, \quad m = 1 \dots M, \quad (1)$$

$$X_t^0 = (1+r_{t-1}^0) \left(X_{t-1}^0 + \sum_{m=1}^M (1-c_{sell}) u_{t-1}^m - \sum_{m=1}^M (1+c_{buy}) v_{t-1}^m \right), \\ t = 1 \dots N. \quad (2)$$

Note that there is not borrowing or lending rates used in (1) and (2). Many research works assume the same riskless interest rate for borrowing and lending, the inconsistency between borrowing and lending has determinant role in decisions of financial institutions. In reality, investors may be charge a higher interest rate for borrowing money than the interest rate for saving money. As mentioned before in this model r_t^b and r_t^l are the riskless borrowing rate and the riskless lending rate respectively and for aversion of arbitrage situations, it is assumed that $r_t^b \geq r_t^l$.

Now, one may invest using the existing cash or purchase more shares using the credit with the borrowing rate. Attending in (2), obviously, when the investor in a time period ventures to buy more than his affordability it will led to a negative value for X_t^0 in next time period. Negative value for investor holding in riskless asset (X_t^0) indicates that investor needs to use credit. Hence, if $X_t^0 \geq 0$, the investor short sell the portfolio of M risky assets and lend the proceeds in the riskless asset then $r_t^0 = r_t^l$ in (2). If $X_t^0 \leq 0$ the investor long sell the portfolio of M risky assets and borrow his holding shortage then $r_t^0 = r_t^b$ in (2). Therefore r_t^0 in (2) is defined by:

$$r_t^0 = \begin{cases} r_t^l, & \text{if } X_t^0 \geq 0 \\ r_t^b, & \text{if } X_t^0 \leq 0 \end{cases} \quad t = (1 \dots N)$$

Now, to formulate the model with different borrowing and lending rates, a binary variable y_t need to be considered as follow:

$$y_t = \begin{cases} 1, & \text{if } X_t^0 \geq 0 \\ 0, & \text{if } X_t^0 \leq 0 \end{cases} \quad t = (1 \dots N)$$

Therefore, we have:

$$(P) \quad \text{Max } U \left(\sum_{m=0}^M X_m^N \right) \\ \text{s.t.} \\ X_t^m = (1+r_{t-1}^m)(X_{t-1}^m - u_{t-1}^m + v_{t-1}^m), \\ t = (1 \dots N); \quad m = (1 \dots M), \quad (3)$$

$$X_t^0 = (1+y_{t-1} r_{t-1}^l - (1-y_{t-1} r_{t-1}^b)) \left(X_{t-1}^0 + \sum_{m=1}^M (1-c_{sell}) u_{t-1}^m - \sum_{m=1}^M (1+c_{buy}) v_{t-1}^m \right), \quad t = (1 \dots N), \quad (4)$$

$$\begin{aligned} v_t^m &\leq V \quad t = (1, \dots, N); \quad m = (1, \dots, M), \\ y_t &\in [0, 1] \end{aligned} \quad (5)$$

Note that presence of y_t in (4), ensure that when investor avoids from using credit in a time period and prefers investing in riskless assets, r_t^l is operate and when he decides to buy risky asset more than his affordability then he should borrow his need with rate of r_t^b that it is clearly shown in (4).

Future returns are not known at time 0, realistically. Practically, the investor has to treat the portfolio optimization problem as a rolling horizon problem, i.e., he has to act upon the information available at time t , and rebalance his portfolio at time $t+1$ after obtaining additional information over time period $(t, t+1]$. It is assumed that at each time period, the investor takes only the first step of the optimal allocation strategy computed with the information up to that time period. In the classical literature on portfolio optimization, the investor's utility function is assumed to be concave to reflect aversion to risk. We consider a linear objective instead:

$$U\left(\sum_{m=0}^M X_N^m\right) \approx \sum_{m=0}^M X_N^m$$

3. Numerical Results

Now an example is considered to illustrate the results of the proposed model, with $M=5$ (one risk free asset and four risky assets) and $N=4$. The borrowing and the lending rates and the return rates of risky assets are represented as follow:

$$r_b = [.08, .09, .08, .10]$$

$$r_l = [.08, .07, .09, .08]$$

$$r = \begin{bmatrix} .09 & .10 & .08 & .08 \\ .09 & .09 & .07 & .08 \\ .10 & .08 & .08 & .12 \\ .08 & .09 & .08 & .09 \end{bmatrix}$$

r_{ij} = return rate for risky asset i at time period j

The proposed method of this paper has been solved using this data. Moreover the initial values for investor's holding in the first period, transaction costs and the maximum permitted amount of buying for each stock in each period are need to be considered.

In this example it is supposed that $c_{sell} = 1\%$, and $c_{buy} = 0.5\%$. The maximum permitted amount of buying for each stock in each period, V is considered to be 1000, 2000 and 3000 in three run of model and results is discussed.

Table 1 demonstrates the details of the implementation of the proposed method with three maximum permitted

levels for buying of each stock in each period. As we can observe, as the V increases, value of investor utility in final period, U , will increase too i.e. when $V=1000$, objective of model, $U = 6681.543$, as it can be observed in table 1 when V increases to 2000, the U raises to 8629.844 and so on. This trend and correlation between V and U occurs because the value of V , indeed, controls the level of investor debts. Moreover in this numerical example it is supposed that $r \geq r_t^b \geq r_t^l$, so under this assumption, whatever the investor capital go up, he will be more interested to invest in risky assets, i.e. when his limit in buying in each period decreases (V increases) he would like to adopt credit and provide capital to buy risky asset with higher rate of return and finally this strategy will led to a higher U in model objective, as it can be seen in table 1.

Another observation in this example is that X_3^0 and X_4^0 values in table 1 are negative, it means that in spite of higher rate for borrowing than lending, in period 3 and 4, the investor encounters with dollar holdings shortage and he is required to use credit. Not that this observation shows that the investor charges higher rate for borrowing to gain higher benefit from buying risky assets.

Dynamic of portfolio selection can be seen easily in table 1, for example in first period it is assumed that the initial value for investor holding in asset 4, $X_1^4=1200$, when $V=3000$, this value decreases and $X_2^4=0$ in second period, as it can be seen in third period X_3^4 increases to 3270 and finally in forth period it increases too and $X_4^4 = 6771.6$. This going up and down in investors holdings depends on assets return rate in different periods and particularly represents the model's dynamic. Now we need to know when the investor doesn't need to borrowing, or when we can solve this portfolio selection model without any borrowing i.e. when we can have nonnegative values for all X_t^0 . To clarify this state, following example is considered and the results will be discussed. In new example, all parameters are similar to previous example, except matrix r that it is presented as follow:

$$r = \begin{bmatrix} .04 & .03 & .02 & .04 \\ .01 & .02 & .04 & .05 \\ .04 & .05 & .06 & .04 \\ .04 & .03 & .01 & .02 \end{bmatrix}$$

Results of implementation of the proposed model for second example and with three maximum permitted levels for buying of each stock in each period (V) is illustrated in Table 2.

As it can be observed there aren't any negative values for X_t^0 in Table 2, it means that the investor needn't to

borrowing in all time periods. As it was noted, second example differs from first one only in matrix r , so the new results also arise from new r .

Clearly when the return rates of risky assets are lower than riskless lending rate (as we can see in second example), the investor will invest his holdings in riskless asset with lending rate, and he will never confront with lack of cash (negative value for X_t^0) for

buying risky asset, so he wouldn't have any tendency to borrow. Therefore, as it can be observed in Table 2, investor's holdings in risky assets are equal to 0.

Note that the value of U for different V s is equal in Table 2. It indicates that the second example hasn't any sensitivity to values of V , because there aren't any transaction of buying and selling of risky assets in second example.

Tab. 1. Solutions of first example for different value of V

| Variable | | $V=1000$ | $V=2000$ | $V=3000$ |
|-----------------|---------|----------------------------|----------------------------|----------------------------|
| U | | 6681.543 | 8629.844 | 10577.09 |
| t = 1 | X_1^0 | 1000 | 1000 | 1000 |
| | X_1^1 | 500 | 500 | 500 |
| | X_1^2 | 600 | 600 | 600 |
| | X_1^3 | 400 | 400 | 400 |
| | X_1^4 | 1200 | 1200 | 1200 |
| | X_2^0 | 0 | 0 | 0 |
| t = 2 | X_2^1 | 1635 | 2725 | 3562.294 |
| | X_2^2 | 648.4949 | 0 | 0 |
| | X_2^3 | 440 | 440 | 440 |
| | X_2^4 | 1296 | 842.1818 | 0 |
| | X_3^0 | -3658.20 | -7316.40 | -10974.60 |
| t = 3 | X_3^1 | 2898.50 | 5197.50 | 7218.523 |
| | X_3^2 | 1796.859 | 2180 | 3270 |
| | X_3^3 | 1555.20 | 2635.20 | 3715.20 |
| | X_3^4 | 2502.640 | 3097.978 | 3270 |
| t = 4 | X_4^0 | -7063.944 | -14127.89 | -21191.83 |
| | X_4^1 | 4210.380 | 7773.30 | 11036 |
| | X_4^2 | 2992.640 | 4472.60 | 6708.90 |
| | X_4^3 | 2759.616 | 5006.016 | 7252.416 |
| | X_4^4 | 3782.851 | 5505.816 | 6771.60 |

4. Conclusions

In this paper a multi period portfolio selection model has been presented where the rates of borrowing are greater than the lending and transaction costs as a source of concern for portfolio managers is also considered.

We have discussed that one easy way to treat the difference in the lending and the borrowing rates is to use binary variables. The proposed model is considered under different interest rates for borrowing and lending in a practical example. The model is capable of imposing a balance between the lending and the cash

purchases which makes it easy to adjust the portfolio when lender changes its regulation. In numerical result, two different examples are implemented with proposed model and the sensitivity of model for V and matrix r is discussed.

Finally for future researches three areas are proposed: first adding other constraints of real market like margin account limitations to ensure investor from margin call, second adding strategies like fuzzy, interval or robust programming to overcome uncertainty in estimating rate of returns and at last verifying multi period model to time continuous dynamic model.

Tab. 2. Solutions of second example for different value of V

| Variable | V= 1000 | V=2000 | V= 3000 |
|-------------|----------|----------|----------|
| U | 4626.525 | 4626.525 | 4626.525 |
| t =1 | X_1^0 | 1000 | 1000 |
| | X_1^1 | 500 | 500 |
| | X_1^2 | 600 | 600 |
| | X_1^3 | 400 | 400 |
| | X_1^4 | 1200 | 1200 |
| t =2 | X_2^0 | 3966.84 | 3966.84 |
| | X_2^1 | 0 | 0 |
| | X_2^2 | 0 | 0 |
| | X_2^3 | 0 | 0 |
| | X_2^4 | 0 | 0 |
| t =3 | X_3^0 | 4244.519 | 4244.519 |
| | X_3^1 | 0 | 0 |
| | X_3^2 | 0 | 0 |
| | X_3^3 | 0 | 0 |
| | X_3^4 | 0 | 0 |
| t =4 | X_4^0 | 4626.525 | 4626.525 |
| | X_4^1 | 0 | 0 |
| | X_4^2 | 0 | 0 |
| | X_4^3 | 0 | 0 |
| | X_4^4 | 0 | 0 |

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