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An Integrated Production-Inventory Model with Backorder and Lot for Lot Policy

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ABSTRACT

In this paper, an inventory model for two-stage supply chain is investigated. A supply chain with single vendor and single buyer is considered. We assume that shortage as a backorder is allowed for the buyer and the vendor makes the production set up every time the buyer places an order and supplies on a lot for lot basis. With these assumptions, the joint economic lot size model is introduced and the minimum joint total relevant cost and optimal order quantity and optimal shortage quantity are obtained for both the buyer and the vendor at the same time. Numerical example is given and then Sensitivity analysis is performed to study the effects of changes in the parameters on optimum joint total relevant cost and optimal order quantity and optimal shortage quantity.

1. Introduction

When inventory decisions in supply chains are made independently at each stage, they are usually based on the local inventory status and local performance objectives (local policies). These policies are simple to be defined and implemented, but ignore the implications that decisions at one stage can have on the others, let alone the fact that local objectives are often conflicting among each other, which often leads to sub optimize the SC performance. In such cases, the economic lot size (ELS) of one stage may not result in an optimal policy for the other stages. To overcome this problem, researchers have come up with a joint economic lot size (JELS) model where the joint total relevant cost (JTRC) for all stages has been optimized. Goyal first introduced an integrated inventory policy for a single vendor and a single purchaser. He assumed that the demand for the item is uniform and there is no lead time for the supplier and the purchaser. Shortages were not permitted in this model. He showed that his proposed joint inventory approach could result in considerable savings for both the vendor and the purchaser [1]. Banerjee generated Goyal's joint economic lot-size model [1] by assuming that a vendor has a finite production rate and produces to order for a buyer on a lot-for-lot basis. He studied a case of a single buyer and single vendor. A deterministic mathematical model was developed to find the optimal lot size that minimizes the joint total relevant cost. He showed that the implementation of a jointly optimal ordering policy could be of economic benefit to both parties [2]. Later, Goyal [3] extended Banerjee's model [2] by relaxing the lot-for-lot policy and supposed that the vendor's economic production quantity should be an integer multiple of the buyer's purchase quantity that provided a lower joint total relevant cost. Goyal's model was derived based on the implied assumption that the vendor can supply to the purchaser only after completing the entire lot. He showed that his model provides a lower or equal total joint relevant cost compared to Banerjee's model [2]. Lu proposed an optimal solution to the single-vendor, single-buyer problem in which the delivering quantity of each shipment is identical. She assumed that the vendor can supply the purchaser even before completing the entire lot and shipments can occur during production. Further, the article considers the case of multiple

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buyers and a single vendor. She developed a heuristic approach for this integrated inventory problem [4]. In the same year, Goyal proposed an alternative shipment policy in which the quantity of products delivered to the purchaser is not identical in every shipment. At each delivery, the vendor supplies all available inventories to the purchaser [5]. Viswanathan proved that the Goyal model [5] gives a lower joint total annual relevant cost than the strategy proposed by Lu [4] only when the holding cost for the purchaser is not much higher than that for the vendor. In addition, this policy also results in an inconsistency in the delivery quantity and delivery period, which may cause operational planning and control problems [6]. Hill commented that neither of the two policies proposed by Lu [4] and Goyal [5] can obtain the optimal solution for all possible problem parameters. Hill's proposed optimal solution lies in $[n_1, n_2]$ where n_1 is the number of shipments per batch production for the equal-sized shipments policy and n_2 is that for the deliver what is produced policy. However, in his model, the sub-batch quantity delivered to the purchaser at every shipment may not necessarily be the same. This again can create operational planning problems [7]. Later, Hill determined the form of the globally-optimal production and shipping policy for a single vendor-single purchaser problem. He combined the policy proposed by Goyal [5] with an equal shipment size policy and he suggested that the successive shipment size of the first m shipments increases by a fixed factor and the remaining shipments would be equal sized. The objective is to minimize the mean total cost per unit time. Once again, no stock shortages are allowed to occur in the model [8].

Previous researches concerning the JELS model usually do not permit backorder. However, in a situation where backorder costs do exist and can be determined, an economic benefit may be realized by permitting stockouts to occur. By allowing stockouts, excess demand will be backordered and satisfied in the next shipment. Consequently, fewer products are held in the inventory as backup units and this strategy results in a lower inventory cost.

Now, we review the production-inventory models with shortage in supply chain and we focus on models with deterministic demands. Woo *et al.* considered an integrated inventory system where a single vendor purchases and processes raw materials in order to deliver finished items to multiple buyers. Shortages are not allowed for the vendor but are allowed for the buyers. The vendor and all buyers are willing to invest in reducing the ordering cost in order to decrease their joint total cost.

An analytical model is developed to derive the optimal investment amount and replenishment decisions for both vendor and buyers [9]. References [10]-[12], [13], [14], [15], [16], [17] considered single-setup-multiple-delivery policy for integrated inventory model. Yang and Wee, Wee and Chung, Chung determined the economic lot size without derivatives for the integrated single-vendor single-buyer inventory problem with backorder and multiple deliveries policy [10], [11], [16], [17]. Reference [12] extended the integrated vendor–buyer inventory problem by Yang and Wee [17] for three-stage supply Chain and optimized the economic lot size without derivatives.

Pourakbar *et al.* developed an integrated four-stage supply chain system, incorporating one supplier, multiple producers, multiple distributors multiple retailers. The aim of this model was to determine order quantity and shortage level of each stage such that the total cost of the supply chain to be minimized. They assumed that products from supplier to producer, from producer to distributor and from distributor to retailer deliver by multiple delivery policy. Then a heuristic approach based on genetic algorithm for solving this problem was presented [15]. Lo *et al.*, Law and Wee developed an integrated production-inventory model for single manufacturer and single retailer. They assumed a deteriorating product, partial backordering, inflation, and multiple deliveries.

The discounted cash flow and classical optimization technique were used to derive the optimal solution. Furthermore, Lo *et al.* considered imperfect production processes [13], [14]. Lin and Lin proposed a single supplier and a single buyer inventory model for deteriorating items and permitted the completed backorder in the problem. They solved the problem without the condition of equal replenishments periods during a specified planning horizon and presented a procedure to find the optimal solution [18].

As we see in literature, none of them considered production- inventory model with backorder and lot-for-lot policy for non-deteriorating items. Thus, in this paper, we have extended Banerjee's [2] JELS model with the assumption that the backorder for buyer is allowed. We assume there are one vendor and one buyer. First, in section2, we introduce the assumptions and notations of the model. In Section 3, a mathematical model of joint total relevant cost and optimum solutions of this model are determined. In section 4 and section 5 respectively a numerical example and sensitivity analysis is given. Section 6 provides the conclusions.

2.Assumptions and Notations

2-1. Assumptions

1. Single vendor and single buyer are considered.
2. There is a single product.
3. The demand rate and production rate are deterministic, constant and continuous.
4. The costs associated in the system, i.e., manufacturing set-up cost, ordering cost, unit inventory holding cost and backorder cost are known and constant.
5. Shortage is allowed for the buyer and fully backordered.

6. Production will first be used to satisfy all shortages and then later be used to satisfy current demand.
7. The vendor makes the production set up every time the buyer places an order and supplies on a lot for lot basis.
8. There is no lead time
9. Planning horizon is infinite.

2-2. Notations

D: Annual constant demand for the item
P: Vendor's annual constant rate of production for the item
C_V: The unit production cost for the item
C_P: The unit purchase cost paid by the buyer
A: The buyer's ordering cost per order
S: The vendor's setup cost per setup
r: The annual inventory carrying cost per dollar invested in stocks
π: The shortage cost per unit quantity per year
q: The order quantity (decision variable)
b: The shortage quantity(decision variable)

$TRC_B(q,b)$: Total relevant cost of buyer
$TRC_V(q)$: Total relevant cost of vendor
$JTRC(q,b)$: Joint total relevant cost of our model
$JTRC_{Banerjee}(q)$: Joint total relevant cost of Banerjee's model

3.Development of the Model

In the Banerjee's model, shortage isn't allowed. However, in a more realistic point of view, sometimes it is more cost efficient to allow shortages to occur if the estimated backordering cost penalty is lower than the corresponding buyer's inventory carrying cost. Therefore, we consider the integrated inventory model with shortage. In the proposed model, we assume that every time a shortage occurs, all the unsatisfied demands are backordered and we consider the lot for lot policy. In lot for lot, the optimal lot size is produced at one setup and delivered at one time.

The buyer's total cost consists of an ordering cost, a holding cost, a backordering cost. So from figures1, the total relevant cost for the buyer is obtained as following:

$$TRC_B(q,b)=\frac{(rc_p+\pi)\cdot b^2}{2q}-rc_p\cdot b+\frac{rc_p\cdot q}{2}+\frac{D}{q}\cdot A \quad (1)$$

The vendor's cost function includes a set up cost and a holding cost. By considering figuare2, we have:

$$TRC_V(q)=(s+\frac{1}{2}.\frac{q}{p}\cdot q.rc_v).\frac{D}{q}=\frac{D}{q}\cdot s+\frac{Dq}{2p}\cdot rc_v \quad (2)$$

Therefore, the joint total relevant cost (JTRC) for the lot-for-lot case by considering backorder for the buyer is the sum of the buyer's total cost and the vendor's total cost. Thus, the joint total relevant cost (JTRC) is given by:

$$JTRC(q,b)\frac{D}{q}\cdot(s+A)+\frac{q}{2}\cdot r\cdot(\frac{D}{p}c_v+c_p)+\frac{(rc_p+\pi)\cdot b^2}{2q}-rc_p\cdot b \quad (3)$$

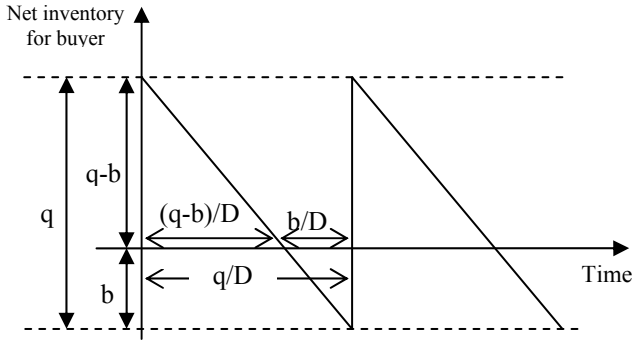

Fig. 1. Buyer's inventory level against time

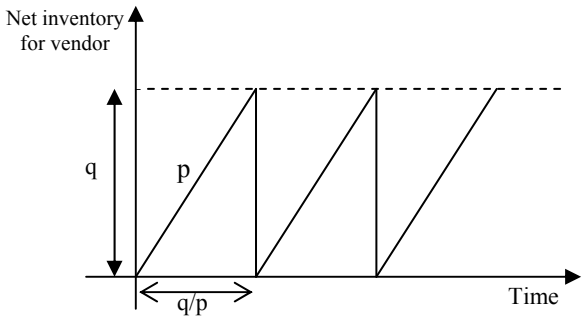

Fig. 2. Vendor's inventory level against time

As we showed in appendix, the JTRC is a convex function; thus for determining the optimal order quantity and the optimal backorder quantity, we use from the joint total relevant cost (JTRC) in Eq. (3). By taking the first derivatives of Eq. (3) with respect to b and q, setting them equal to zero, and solving for b and q simultaneously, we obtain the following formulas:

$$b^*=\frac{rc_p}{rc_p+\pi}\cdot q^* \quad (4)$$

$$q^*=\sqrt{\frac{2D\cdot(s+A)\cdot(rc_p+\pi)}{r\cdot(\frac{D}{p}c_v+c_p)\cdot(rc_p+\pi)-(rc_p)^2}} \quad (5)$$

As we know, under root have to be nonnegative. Now we show it:

$$\frac{2D\cdot(s+A)\cdot(rc_p+\pi)}{r\cdot(\frac{D}{p}c_v+c_p)\cdot(rc_p+\pi)-(rc_p)^2}\geq 0$$

The numerator is always a nonnegative number because it includes of summations and multiplications of nonnegative parameters. Therefore, it is sufficient to check the positivity of denominator.

$$\left[r\cdot(\frac{D}{p}c_v+c_p)\cdot(rc_p+\pi)\right]-(rc_p)^2\geq 0\Rightarrow$$

$$\left[(rc_p)^2+rc_p\pi+r^2\frac{D}{p}c_vc_p+r\frac{D}{p}c_v\pi\right]-(rc_p)^2\geq 0\Rightarrow$$

$$rc_p\pi+r^2\frac{D}{p}c_vc_p+r\frac{D}{p}c_v\pi\geq 0$$

With regarding this fact that all the parameters are nonnegative, we result the last inequality is always satisfied.

Now, in order to compute the $JTRC(q_*,b_*)$, we replace equations (4) and (5) in equation (3). Therefore, the optimum joint total relevant cost obtains as follow

$$JTRC(q_*,b_*)=\sqrt{\frac{2D\cdot(s+A)\cdot\left[r\cdot(\frac{D}{p}c_v+c_p)\cdot(rc_p+\pi)-(rc_p)^2\right]}{(rc_p+\pi)}} \quad (6)$$

4.Numerical Example

Let, we consider the values of parameters as mentioned in Banerjee's article:

D =1000, P =3200, A =100, S =400, ,C_P=250, C_V=20, r=0/20

Then the value of q_* and $JTRC(q_*)$ in Banerjee's model are 400, 2500. In our model, furthermore of parameters in Banerjee's model, we have parameter π .By considering π ١٠ = and from equations (4), (5) and (6), we have the optimal order quantity $q_*=467$, the optimal shortage quantity $b_*=1557$ and the minimal total cost $JTRC(q_*,b_*)=2140.9$. The $JTRC(q_*,b_*)$ thus obtained is about -14.4% less than \$2500 as obtained by Banerjee. This is a result from permitting a shortage to occur in every delivery cycle.

5.Sensitivity Analysis

To study the effects of changes in the system parameters D, P, A, S, C_P, C_V, r and π on the optimal order quantity, optimal shortage quantity and optimal cost, a sensitivity analysis is performed. The sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 5%, 10%, 15%, 20%, 25%, 30% taking one at a time, keeping the remaining parameters at their original values and we calculate the following deviations for different quantity of these parameter:

$$r_1=\frac{JTRC(q_*,b_*)-2140.9}{2140.9}*100$$

$$r_2=\frac{JTRC(q_*,b_*)-JTRC_{Banerjee}(q_*)}{JTRC_{Banerjee}(q_*)}*100$$

Tab. 1. Sensitivity analysis

Variation parameter	Change in parameter (%)												
	-30%	-25%	-20%	-15%	-10%	5%	0%	5%	10%	15%	20%	25%	30%
(i) Changing the parameter *D*													
D	700	750	800	850	900	950	1000	1050	1100	1150	1200	1250	1300
q_*	407.8	419.1	429. 7	439.7	449.3	458.4	467.1	475.4	483.4	491	498.3	505.3	512
b_*	135.9	139.7	143.2	146.6	149.8	152.8	155.7	158.5	161.1	163.7	166.1	168.4	170.7
$JTRC(q_*,b_*)$	1716.3	1789.7	1861.9	1933.0	2003.1	2072.4	2140.9	2208.6	2275.8	2342.3	2408.3	2473.8	2538.9
r_1	-19.83	-16.40	-13.03	-9.71	-6.43	-3.20	0.00	3.17	6.30	9.41	12.49	15.55	18.59
$JTRC_{Banerjee}(q_*)$	2027.9	2110.2	2190.9	2270	2347.9	2424.5	2500	2574.5	2648.1	2720.9	2792.8	2864.1	2934.7
r_2	-15.4	-15.2	-15	-14.8	-14.7	-14.5	-14.4	-14.2	-14.1	-13.9	-13.8	-13.6	-13.5
(ii) Changing the parameter *P*													
P	2240	2400	2560	2720	2880	3040	3200	3360	3520	3680	3840	4000	4160
q_*	442.0	447.2	451.9	456.2	460.2	463.8	467.1	470.2	473.0	475.6	478.1	480.4	482.5
b_*	147.3	149.1	150.6	152.1	153.4	154.6	155.7	156.7	157.7	158.5	159.4	160.1	160.8

Variation parameter	Change in parameter (%)												
	-30%	-25%	-20%	-15%	-10%	5%	0%	5%	10%	15%	20%	25%	30%
$JTRC(q_*, b_*)$	2262.5	2236.1	2212.7	2191.8	2173.1	2156.2	2140.9	2126.9	2114.2	2102.4	2091.7	2081.7	2072.4
r_1	5.68	4.45	3.35	2.38	1.50	0.72	0.00	-0.65	-1.25	-1.79	-2.30	-2.77	-3.20
$JTRC_{Banerjee}(q_*)$	2604.9	2582	2561.7	2543.7	2527.6	2513.1	2500	2488.1	2477.2	2467.2	2458	2449.5	2441.6
r_2	-13.1	-13.4	-13.6	-13.8	-14	-14.2	-14.4	-14.5	-14.6	-14.8	-14.9	-15	-15.1
(iii) Changing the parameter *A*													
A	70	75	80	85	90	95	100	105	110	115	120	125	130
q_*	452.9	455.3	457.7	460.0	462.4	464.8	467.1	469.4	471.7	474.1	476.3	478.6	480.9
b_*	151.0	151.8	152.6	153.3	154.1	154.9	155.7	156.5	157.2	158.0	158.8	159.5	160.3
$JTRC(q_*, b_*)$	2075.7	2086.7	2097.6	2108.5	2119.4	2130.1	2140.9	2151.5	2162.2	2172.7	2183.3	2193.7	2204.2
r_1	-3.05	-2.53	-2.02	-1.51	-1.01	-0.50	0.00	0.50	1.00	1.49	1.98	2.47	2.96
$JTRC_{Banerjee}(q_*)$	2423.8	2436.7	2449.5	2462.2	2474.9	2487.5	2500	2512.5	2524.9	2537.2	2549.5	2561.7	2573.9
r_2	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4
(iv) Changing the parameter *S*													
S	280	300	320	340	360	380	400	420	440	460	480	500	520
q_*	407.2	417.8	428.1	438.2	448.0	457.7	467.1	476.3	485.4	494.3	503.1	511.7	520.1
b_*	135.7	139.3	142.7	146.1	149.3	152.6	155.7	158.8	161.8	164.8	167.7	170.6	173.4
$JTRC(q_*, b_*)$	1866.4	1914.9	1962.1	2008.3	2053.5	2097.6	2140.9	2183.3	2224.9	2265.7	2305.8	2345.2	2384.0
r_1	-12.82	-10.56	-8.35	-6.19	-4.08	-2.02	0.00	1.98	3.92	5.83	7.70	9.54	11.36
$JTRC_{Banerjee}(q_*)$	2179.4	2236.1	2291.3	2345.2	2397.9	2449.5	2500	2549.5	2598.1	2645.8	2692.6	2738.6	2783.9
r_2	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4	-14.4
(v) Changing the parameter C_P													
C_P	17.5	18.75	20	21.25	22.5	23.75	25	26.25	27.5	28.75	30	31.25	32.5
q_*	510.1	501.4	493.4	486.1	479.3	473.0	467.1	461.6	456.5	451.7	447.2	443.0	439.0
b_*	132.3	136.8	141.0	145.0	148.7	152.3	155.7	158.9	162.0	164.9	167.7	170.4	172.9
$JTRC(q_*, b_*)$	1960.3	1994.3	2026.6	2057.3	2086.5	2114.3	2140.9	2166.2	2190.5	2213.8	2236.1	2257.5	2278.0
r_1	-8.44	-6.85	-5.34	-3.90	-2.54	-1.24	0.00	1.19	2.32	3.41	4.45	5.45	6.41
$JTRC_{Banerjee}(q_*)$	2179.4	2236.1	2291.3	2345.2	2397.9	2449.5	2500	2549.5	2598.1	2645.8	2692.6	2738.6	2783.9
r_2	-10.1	-10.8	-11.5	-12.3	-13	-13.7	-14.4	-15	-15.7	-16.3	-16.9	-17.6	-18.2
(vi) Changing the parameter C_V													
C_V	14	15	16	17	18	19	20	21	22	23	24	25	26

Variation parameter	-30%	-25%	-20%	-15%	-10%	5%	0%	5%	10%	15%	20%	25%	30%
						Change in parameter (%)							
q_*	487.5	483.9	480.4	477.0	473.6	470.3	467.1	463.9	460.9	457.8	454.9	451.9	449.1
b_*	162.5	161.3	160.1	159.0	157.9	156.8	155.7	154.6	153.6	152.6	151.6	150.6	149.7
$JTRC(q_*, b_*)$	2051.4	2066.6	2081.7	2096.6	2111.5	2126.2	2140.9	2155.4	2169.9	2184.2	2198.5	2212.7	2226.7
r_1	-4.18	-3.47	-2.77	-2.07	-1.37	-0.68	0.00	0.68	1.35	2.02	2.69	3.35	4.01
$JTRC_{Banerjee}(q_*)$	2423.8	2436.7	2449.5	2462.2	2474.9	2487.5	2500	2512.5	2524.9	2537.2	2549.5	2561.7	2573.9
r_2	-15.4	-15.2	-15	-14.8	-14.7	-14.5	-14.4	-14.2	-14.1	-13.9	-13.8	-13.6	-13.5
(vii) Changing the parameter *r*													
r	0.14	0.15	0.16	0.17	0.18	0.19	0.2	0.21	0.22	0.23	0.24	0.25	0.26
q_*	537.0	522.4	509.2	497.2	486.3	476.3	467.1	458.6	450.7	443.3	436.4	430.0	423.9
b_*	139.2	142.5	145.5	148.3	150.9	153.4	155.7	157.9	159.9	161.8	163.7	165.4	167.0
$JTRC(q_*, b_*)$	1862.1	1914.4	1964.0	2011.2	2056.3	2099.5	2140.9	2180.6	2218.9	2255.7	2291.3	2325.7	2358.9
r_1	-13.02	-10.58	-8.26	-6.06	-3.95	-1.93	0.00	1.86	3.64	5.36	7.03	8.63	10.18
$JTRC_{Banerjee}(q_*)$	2091.7	2165.1	2236.1	2304.9	2371.7	2436.7	2500	2561.7	2622	2681	2738.6	2795.1	2850.4
r_2	-11	-11.6	-12.2	-12.7	-13.3	-13.8	-14.4	-14.9	-15.4	-16.3	-16.9	-17.6	-18.2
(viii) Changing the parameter π													
π	7	7.5	8	8.5	9	9.5	10	10.5	11	11.5	12	12.5	13
q_*	489.9	485.1	480.7	476.8	473.3	470.1	467.1	464.4	461.9	459.6	457.4	455.4	453.6
b_*	204.1	194.0	184.9	176.6	169.0	162.1	155.7	149.8	144.3	139.3	134.5	130.1	126.0
$JTRC(q_*, b_*)$	2041.2	2061.6	2080.1	2097.2	2112.9	2127.4	2140.9	2153.4	2165.1	2176.0	2186.2	2195.8	2204.8
r_1	-4.65	-3.70	-2.84	-2.04	-1.31	-0.63	0.00	0.58	1.13	1.64	2.12	2.56	2.99
$JTRC_{Banerjee}(q_*)$	2500	2500	2500	2500	2500	2500	2500	2500	2500	2500	2500	2500	2500
r_2	-18.3	-17.5	-16.8	-16.1	-15.5	-14.9	-14.4	-13.9	-13.4	-13	-12.5	-12.2	-11.8

The following inferences can be made from the sensitivity analysis based on Tables 1.

1. When the parameter *D* increases (decreases) and other parameters remain unchanged, the optimal order quantity, optimal shortage quantity and optimal joint total cost increase (decrease).
2. When the parameter *P* increases, the optimal order quantity and optimal shortage quantity increase and optimal joint total cost decreases. In the other hand, as we see in table1, when P increases 30%, the optimal joint total cost decreases 3.2% and when P decreases 30%, the optimal joint total cost increases 5.68%. Therefore, the decrease of parameter P has more effect on optimal joint total cost relative to increase of parameter P.
3. When the parameter *A, S* increases, the optimal order quantity, optimal shortage quantity and optimal joint total cost increase and vice versa.
4. When the parameter C_P increases, the optimal order quantity decreases and optimal shortage quantity and optimal joint total cost increase. Increasing of C_P means more holding cost, so the system prefers to hold less quantity of products in warehouse.
5. When the parameter C_V increases, the optimal order quantity and optimal shortage quantity decrease and

optimal joint total cost increases. Like increasing of C_P, we will have more holding cost and so less order quantity.

6. When the parameter r increases, the optimal order quantity decreases and optimal shortage quantity and optimal joint total cost increase. As illustrated in table1, an increase in r results in an increase in the cost difference between the two models. The proposed model is preferred in all cases. A high r value indicates that perhaps it is no longer economical to hold a large inventory. Rather, an economic benefit can be realized by allowing some units to be backordered.
7. When the parameter π increases and other parameters remain unchanged, the optimal order quantity and optimal shortage quantity decrease and optimal joint total cost increase. After comparing the value of r_2 for different values of π, we find that Since Banerjee[2] did not permit demand shortage in their model, backordering cost has no effect in the joint total relevant cost. Therefore, it remains unchanged as π varies. However, the effect of backordering cost can be clearly seen in the proposed model as presented in table 1. From this, it is obvious that the proposed model is more advantageous for the lower values of π.
8. As we can see in table 1, the effect of increase of parameters is not equal to the effect of decrease of parameters. For example, consider the parameter π, when it changes +30%, JTRC changes +2.99% but when it changes -30%, JTRC changes -4.65%.
9. The optimal joint total cost is sensitive to parameters D, S, r more than other parameters. Tables 1 show the computed results.
10. From table 1, when each of the parameters D and C_V becomes smaller, r_2 becomes larger. From this, it is obvious that the proposed model is more advantageous for the lower values of D or C_V ; and also, when each of the parameters C_p and P becomes smaller, r_2 becomes smaller.

5. Conclusion

This paper extended the Banerjee's [2] JELS model with the assumption that the backorder for buyer is allowable and then obtained the minimum joint total relevant cost and optimal order quantity and shortage quantity for both buyer and vendor at the same time. Then, the numerical example is given to explain the solution. Sensitivity analysis is performed to study the effect of changes in the system parameters D, P, A, S, C_P, C_V, r and π on the optimal order quantity, optimal shortage quantity and optimal cost. We found that the optimum joint total cost of model with backorder is smaller than model without backorder.

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Appendix

$$\nabla JTRC(q,b) = \begin{bmatrix} \frac{\partial JTRC(q,b)}{\partial q} \\ \frac{\partial JTRC(q,b)}{\partial b} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-D(A+S)}{q^2} + \frac{r(\frac{D}{p}c_v + c_p)}{2} - \frac{(rc_p+\pi)b^2}{2q^2} \\ \frac{(rc_p+\pi)b}{q} - rc_p \end{bmatrix} \Rightarrow$$

$$\nabla^2 JTRC(q,b) = \begin{bmatrix} q & b \end{bmatrix} \begin{bmatrix} \frac{\partial^2 JTRC(q,b)}{\partial q^2} & \frac{\partial^2 JTRC(q,b)}{\partial q \partial b} \\ \frac{\partial^2 JTRC(q,b)}{\partial b \partial q} & \frac{\partial^2 JTRC(q,b)}{\partial b^2} \end{bmatrix} \begin{bmatrix} q \\ b \end{bmatrix}$$

$$= \begin{bmatrix} q & b \end{bmatrix} \begin{bmatrix} \frac{2D(A+S)}{q^3} + \frac{(rc_p+\pi)b^2}{q^3} & \frac{-(rc_p+\pi)b}{q^2} \\ \frac{-(rc_p+\pi)b}{q^2} & \frac{(rc_p+\pi)b}{q} \end{bmatrix} \begin{bmatrix} q \\ b \end{bmatrix}$$

$$= \frac{2D(A+S)}{q}$$

We know that parameters D, A, S, q are nonnegative; consequently, the Heissian matrix is always equal or greater than zero and so, JTRC is a convex function.