



## A Game-Theoretic Approach to Pricing in a Two-Level Supply Chain Considering Advertising and Servicing

Parinaz Esmaeili, Seyed Reza Hejazi & Morteza Rasti-Barzoki\*

*Parinaz Esmaeili, Department of Industrial and Systems Engineering, Isfahan University of Technology*

*Seyed Reza Hejazi, Department of Industrial and Systems Engineering, Isfahan University of Technology*

*Morteza Rasti-Barzoki, Department of Industrial and Systems Engineering, Isfahan University of Technology*

### KEYWORDS

Supply chain,  
Services,  
Advertising,  
Pricing,  
Game theory.

### ABSTRACT

*This paper considers advertising, pricing, and service decisions simultaneously to coordinate the supply chain with a manufacturer and a retailer. The amount of market demand is influenced by advertising, pricing, and service decisions. In this paper, three well-known approaches to the game theory, including the Nash, the Stackelberg-retailer, and the cooperative game, are exploited to study the effects of these policies on the supply chain. Using these approaches, we identify optimal strategies in each case for the manufacturer and retailer. Then, we will compare the outcomes of each developed strategy. The results show that, compared with the Nash game, the Stackelberg-retailer game yields higher profits for the retailer, the manufacturer, and the whole supply chain. The cooperative game yields the highest profits. Finally, the Nash bargaining model will be presented and explored to investigate the possibilities for profit sharing.*

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### 1. Introduction

“Supply chain (SC) management aims to increase the overall profit through improvement of various activities and components” [1]. Different aspects of the supply chain coordination have been studied in recent years. According to Bergen and John, vertical co-op advertising is an important strategy in the manufacturer-retailer channel, according to which the manufacturer agrees to pay part of the retailer’s local advertising costs in order to make more promotional initiatives aimed at increasing immediate sales [2]. The escalation of the total advertising expenditures in the United States from \$900 million in 1970 to more than

\$50 billion in 2012 is an indication of the importance of this strategy [3, 4]. Berger was the first one to investigate the vertical co-op advertising from a mathematical viewpoint aimed at developing optimal policies [5]. A common approach adopted for investigating the role of these models in the supply chain is the game theory which may be divided into two categories: static and dynamic. In the first category, co-op advertising is studied over a single period [2, 5-14].

In the second category, the customer’s goodwill function is considered for investigating the carryover effect of advertising [15-21]. Many researchers have also devoted their efforts to investigating methods of advertising and pricing [11, 12, 22, 23]. SeyedEsfahani et al. developed the relevant models by incorporating concave, convex, and linear price demand curves [24].

\* Corresponding author: Morteza Rasti-Barzoki

Email: [rasti@cc.iut.ac.ir](mailto:rasti@cc.iut.ac.ir)

Received 21 May 2017; revised 22 July 2017; accepted 13 August 2017

Aust and Buscher relaxed extended models of SeyedEsfahani et al. by including the restrictive assumption in equal margin profit [4]. Dietl et al. worked on the advertising and pricing model in media markets [25]. Helmes et al. studied the dynamic advertising and pricing based on constant demand elasticity, and they investigated advertising and pricing in the general new-product adoption models [26, 27]. Liu et al. investigated the joint decision-making process in pricing and advertising for competing retailers under emergency purchasing [28]. Giri and Sharma studied a two-level supply chain and manufacturer's pricing strategy with rival retailers and an advertising cost-dependent demand [29]. Nowadays, the role of service level in the increase of demand is as important as pricing. For example, in the PC market, the amount of market demand for PCs is dependent upon the service level. Companies such as IBM and DELL use this strategy to remain competitive [30]. Some scholars have studied the effect of both pricing and advertising strategies, while others have addressed that of simultaneous pricing and service policies. However, the effects of the combined advertising, pricing, and service level have not been studied yet. In the real-world situation, many companies use different policies, such as pricing, advertising, service, etc., for the purposes of competitiveness. It is the goal of the present study to investigate the three strategies together in an attempt to get closer to the real-world conditions and obtain more realistic results.

After-sales service forms a key strategy toward the durable product market and allows the manufacturer and retailer more profits [31]. Goffin presented seven elements involved in the after-sales service: installation, user training, documentation, maintenance and repair, online support, warranties, and upgrades [32]. The cost of attracting a new customer has been claimed to be five times greater than that of keeping a current customer satisfied [33]. Gaiardelli et al. defined after-sales service as those actions that take place after the purchase in the purpose of making the customer loyal [34].

The nature of service is different from that of advertising, in that service is an after-sales activity aimed at those who buy the product and whose aim is to make the customer loyal, while advertising is a before-sales activity that influences every potential customer and raises brand awareness and immediate sales. Hence,

service is a long-term strategy, while advertising is a short term one.

A brief survey of the studies within the field is as follows. Ishii reviewed the role of R&D in a supply chain based on a competition between pricing and service [35]. Tsay and Agrawal studied a distribution system in which a manufacturer supplies a common product to two independent retailers who, in turn, exploit both services and retail price to compete for end customers [36]. In a different study, the same authors surveyed the channel conflict and coordination where the manufacturer can sell his product through two channels: the retailer and e-commerce channels [37]. Charoensiriwath studied a case in which there are manufacturers producing competitive products in order to sell them through a common retailer. The manufacturers compete through providing services [30]. Bernstein studied a general equilibrium in a multi-retailer channel with price and service competition and the demand uncertainty [38]. Dumrong Siri et al. investigated pricing and services in a channel in which the manufacturer sells the products through both retailer and online channels and investigated the optimal policies for the members under demand uncertainty [39]. Their supply chain consists of one risk-neutral supplier and one risk-averse retailer. Dan et al. developed a dual-channel model to sell the manufacturer's product through both the retail and direct channels [40]. They examined optimal policies on retail service and pricing policies.

"Making a choice independently or integrating with some or all levels will be a critical decision, and therefore, affect the overall profit of the chain [41]." In this study, we investigate two non-cooperative games including: 1) the equal power as in the Nash game; 2) the powerful retailer such as the Wal-Mart in the Stackelberg-Retailer (SR) game and one cooperative game. For the first time, the demand function is considered that is influenced by the advertising, pricing, and service policies to investigate the simultaneous effects of these three important policies on customer's demand. The rest of this study is organized as follows: In Section 2, a description of the model is presented. In Section 3, the non-cooperative games and the cooperative one are presented. Illustrative results of the models are presented in Section 4. Section 5 deals with the use of the Nash bargaining problem for profit sharing. Conclusions, trends for future research, and the summary of results

are presented in Section 6. Finally, the proofs of the propositions are given in the Appendix. Table 1 presents a number of relevant key papers and the contributions made by the present study

to the field. The notations, which are used in the paper and this table, are described in Table 2.

**Tab. 1. The relevant studies and the proposed model**

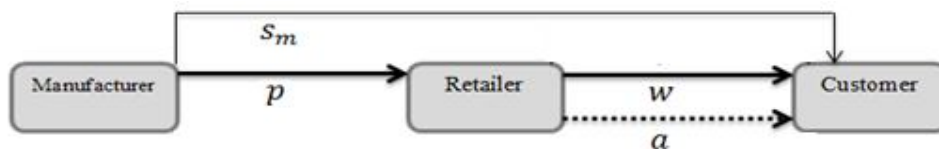
	Equality of margins	Price demand	Advertising demand	Services	Game structures
[24]	Assumed	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$k_r \sqrt{a} + k_m \sqrt{A}$	----	N, SR, SM and Co
[42]	Relaxed	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$k_r \sqrt{a} + k_m \sqrt{A}$	----	N, SR, SM and Co
Proposed Model	Assumed	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$k_r \sqrt{a}$	$b_m s_m$	N, SR and Co

N: Nash SR: Stackelberg-retailer SM: Stackelberg-manufacturer Co: Cooperativ

**2. The Model and Notations**

We have a single-manufacturer-single-retailer supply chain in which the manufacturer sells her/his products via the retail channel. The manufacturer specifies the wholesale price and

the manufacturer’s services budget. The retail price and local advertising budget are determined by the retailer. The structure of the considered supply chain is shown in Fig 1. Table 2 presents the decision variables and parameters used throughout this paper.



**Fig. 1 The structure of the considered supply chain**

**Tab. 2. Symbols used**

Variable	Parameters		
$w$	Wholesale price	$\alpha$	Price demand potential
$s_m$	Manufacturer’s services	$\beta$	Price sensitivity
$p$	Retail price	$\nu$	Shape parameter
$a$	Local advertising	$k_r$	Effectiveness of local advertising
$\Pi_m$	Manufacturer’s profit	$b_m$	Effectiveness of manufacturer’s services
$\Pi_r$	Retailer’s profit	$\eta_m$	Manufacturer’s services cost factor
$\Pi_{m+r}$	System’s profit	$c$	Manufacturer’s unit production cost
		$d$	Retailer’s unit handling cost

In this paper, we use the multiplicative form for the advertising and pricing relationship as it is well-known in the literature. The additive form between services and pricing is an example which is assumed based on the additive form between services and pricing in the model of Tsay and Agrawal [36], and the multiplicative form can be investigated for the future studies. Finally, we use the customer demand function as follows (one may consider other forms for future studies):

$$D = D_0(g(p)h(a) + I(s_m)) \tag{1}$$

$D_0$  is the base demand. The effects of the retail price, advertising, and services cost on the demand are shown by  $g(p)$ ,  $h(a)$ , and  $I(s_m)$ , respectively. The demand changes when the price changes within an inverse relationship. Based on SeyedEsfahani et al.,  $g(p)$  and  $h(a)$  are as follows [24]:

$$g(p) = (\alpha - \beta p)^{\frac{1}{\nu}} \tag{2}$$

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This general form can be convex, linear, or concave depending on whether  $\nu < 1$ ,  $\nu = 1$ , or  $\nu > 1$ , respectively.

$$h(a) = k_r \sqrt{a} \tag{3}$$

In most papers, the services' function form is based on that of Tsay and Agrawal [36] as follows:

$$I(s_m) = b_m s_m \tag{4}$$

In the proposed demand function, it is assumed that with an increase in the price, the customer's demand decreases and tends to a constant amount of  $D_0 I(s_m)$  rather zero. Due to the necessary nature of some goods, the customers are compelled to pay for it even though the price goes higher. According to Dan et al. (2012), the cost providing the sales effort level ( $s$ ) is represented by  $\frac{\eta s^2}{2}$ . "The quadratic form serves to bind system diminishing returns on the sales effort expenditures. Diminishing return is certainly natural if this notion of service has a significant store-level inventory component. Under the assumption of standard inventory models, moving from, say, 97% to 99% fill rate typically requires a great incremental investment compared to that from 95% to 97%. For other concepts of service, suppose that a rational manager will always target the "lowest-hanging fruit" so that subsequent improvements are progressively more difficult" [36].

Based on Eqs. (1)-(4), the demand function is written as follows:

$$D(p, a, s_m) = D_0 [(\alpha - \beta p)^{\frac{1}{\nu}} (k_r \sqrt{a}) + b_m s_m] \tag{5}$$

To avoid the negative effect of pricing and advertising on the demand when they are committed together, the following condition should be verified:

$$p < \frac{\alpha}{\beta} \tag{6}$$

The profit functions of the channel members and the system are as follows:

$$\Pi_m(w, s_m) = D_0 (w - c) \left[ (\alpha - \beta p)^{\frac{1}{\nu}} (k_r a) + b_m s_m \right] - \frac{\eta_m s_m^2}{2}; \tag{7}$$

$$\Pi_r(p, a) = D_0 (p - w - d) \left[ (\alpha - \beta p)^{\frac{1}{\nu}} (k_r a) + b_m s_m \right] - a; \tag{8}$$

$$\Pi_{m+r}(p, a, s_m) = D_0 (p - c - d) \left[ (\alpha - \beta p)^{\frac{1}{\nu}} (k_r a) + b_m s_m \right] - a - \frac{\eta_m s_m^2}{2}; \tag{9}$$

In this paper,  $m$ ,  $r$ , and  $m+r$  represent the manufacturer, the retailer, and the system,

respectively. Along the lines of SeyedEsfahani et al. (2011), Eqs. [7-9] should be verified in the following condition in order to avoid backwash effects [24]:

$$\Pi_m > 0 \rightarrow w > c;$$

$$\Pi_r > 0 \rightarrow p > w + d > w$$

And based on Eq. (6), it can be rewritten as  $\alpha - \beta(c + d) > 0$

$$\Pi_{m+r} > 0 \rightarrow p > c + d;$$

The variables are changed similar to the model of SeyedEsfahani et al. (2011) for ease of analysis.

$$\alpha' = \alpha - \beta(c + d) \quad b'_m = D_0 \frac{\alpha'}{\beta} b_m \quad w' = \frac{\beta}{\alpha'} (w - c) \quad p' = \frac{\beta}{\alpha'} (p + d)$$

$$p > w + d \rightarrow p - (c + d) > w$$

$$k'_r = \frac{\alpha'^{1+\frac{1}{\nu}}}{\beta} k_r \quad -c \rightarrow p' > w'$$

Based on the above changes, Eqs. [7-9] can be rewritten as follows:

$$\Pi'_m(w', s_m) = w' \left[ (1 - p')^{\frac{1}{\nu}} (k'_r \sqrt{a}) + b'_m s_m \right] - \frac{\eta_m s_m^2}{2}; \tag{10}$$

$$\Pi'_r(p', a) = (p' - w') \left[ (1 - p')^{\frac{1}{\nu}} (k'_r \sqrt{a}) + b'_m s_m \right] - a; \tag{11}$$

$$\Pi'_{m+r}(p', a, s_m) = D_0 p' \left[ (1 - p')^{\frac{1}{\nu}} (k'_r \sqrt{a}) + b'_m s_m \right] - a - \frac{\eta_m s_m^2}{2}; \tag{12}$$

For simplicity, in the sequence of equations, superscript ( $'$ ) is removed.

### 3. Three Game Models

In this section, three games, consisting of two non-cooperative games (i.e., the Nash and Stackelberg-retailer (SR)) and one cooperative, are described. Another well-known non-cooperative game, the Stackelberg-manufacturer game, in which manufacturer is the leader of the retailer can be considered in the future studies.

#### 3-1. The nash game

The Nash game is particularly applicable to conditions where the members have equal power, and their policies are made simultaneously but independently. The solution for this game is called the 'Nash equilibrium' and achieved by solving the following two models:

$$\max \Pi_m(w, s_m) = w \left[ (1-p)^{\frac{1}{v}} (k_r \sqrt{a}) + b_m s_m \right] - \frac{\eta_m s_m^2}{2};$$

$$\text{st: } 0 \leq w \leq 1 \quad 0 \leq s_m \leq \mu_r \quad \mu_r \geq \mu_m \rightarrow p - w \geq w \rightarrow w \leq \frac{p}{2}$$

$$\max \Pi_r(p, a) = (p - w) \left[ (1-p)^{\frac{1}{v}} (k_r \sqrt{a}) + b_m s_m \right] - a;$$

$$\text{s.t: } w \leq p \leq 1$$

The optimal value of  $w$  is  $p$  because it has a positive coefficient, but  $p > w$ ; so, in order to make the profit accessible to both sides, we presume that the retailer will not sell the product if he does not get a minimum unit margin. This approach is similar to the models of Xie and Neyret and SeyedEsfahani et al. [11, 24]. They

**Case 1.**  $v = 1$

$$w^N = \frac{5}{12} - \frac{1}{12} \sqrt{1 - 48y}$$

$$p^N = \frac{5}{6} - \frac{1}{6} \sqrt{1 - 48y}$$

$$s_m^N = \frac{b_m}{\eta_m} \left( \frac{5}{12} - \frac{1}{12} \sqrt{1 - 48y} \right)$$

$$a^N = \frac{k_r^2}{16} \left( \frac{5}{6} - \frac{1}{6} \sqrt{1 - 48y} \right)^2 \left( \frac{1}{6} + \frac{1}{6} \sqrt{1 - 48y} \right)^2$$

**Case 2.**  $v = 2$

$$w^N = 0.4 + 0.8y$$

$$p^N = 0.8 + 1.6y$$

$$s_m^N = \frac{b_m}{\eta_m} (0.4 + 0.8y)$$

$$a^N = \frac{k_r^2}{16} (0.8 + 1.6y)^2 (0.2 - 1.6y)$$

**Case 3.**  $v = \frac{1}{2}$

$$z = (27y + 3\sqrt{81y^2 - 12288y^3})^{\frac{1}{3}} \quad x = \sqrt{3 + 8z + 348\frac{y}{z}}$$

$$w^N = \frac{7}{16} - \frac{\sqrt{3}}{48}x - \frac{1}{48} \sqrt{18 - 24z - 1152\frac{y}{z} + \frac{18}{x}\sqrt{3}}$$

$$s_m^N = \frac{b_m}{\eta_m} \left( \frac{7}{16} - \frac{\sqrt{3}}{48}x - \frac{1}{48} \sqrt{18 - 24z - 1152\frac{y}{z} + \frac{18}{x}\sqrt{3}} \right)$$

$$p^N = \frac{7}{8} - \frac{\sqrt{3}}{24}x - \frac{1}{24} \sqrt{18 - 24z - 1152\frac{y}{z} + \frac{18}{x}\sqrt{3}}$$

$$a^N = \frac{k_r^2}{16} \left( \frac{7}{8} - \frac{\sqrt{3}}{24}x - \frac{1}{24} \sqrt{18 - 24z - 1152\frac{y}{z} + \frac{18}{x}\sqrt{3}} \right)^2 \left( \frac{1}{8} + \frac{\sqrt{3}}{24}x + \frac{1}{24} \sqrt{18 - 24z - 1152\frac{y}{z} + \frac{18}{x}\sqrt{3}} \right)$$

$p^N$  shown above should be in the boundaries of  $[0,1]$  and leads to bigger profit to the retailer toward  $p = 1$ . Even if one of these situations is not verified,  $p = 1$  gets the optimal price; so, the equilibrium changes as follows:

$$p^N=1 \quad w^N=0.5 \quad a^N=0 \quad s_m^N = \frac{b_m}{2\eta_m}$$

took the manufacturer's minimum unit margin as the minimum level and replaced the wholesale price constraint with  $\mu_r > \mu_m \rightarrow p - w \geq w \rightarrow w \leq \frac{p}{2}$ .

The retailer and manufacturer's unit margins are shown as  $\mu_r = p - w$  and  $\mu_m = w$ . So, the highest possible value for  $w$  is  $\frac{p}{2}$ . As the model is rather complicated, three examples of  $v$  values, namely  $v = 1, v = \frac{1}{2}$ , and  $v = 2$ , representing examples of linear, convex, and concave price-demand curves, respectively, survived here to find the equilibrium. For simplification in all

mentioned cases, we assume that  $y = \frac{b_m}{k_r^2}$ .

**Proposition 1.** The Nash equilibrium is achieved as follows:

The proofs of all the propositions are presented in the Appendix.

### 3-2. The Stackelberg-retailer game

The players of this game consist of a leader and a follower. "In the Stackelberg game, first, a Stackelberg leader chooses his output, and then a Stackelberg follower, having information on the leader's choice at his disposal, makes decision on

his quantity [43].” In the Stackelberg-retailer game, the retailer has more power than the manufacturer. The solution to this game is called the ‘Stackelberg-retailer equilibrium’. In the SR game, the best response of the manufacturer is similar to the Nash game because it is the result of the first-order deviation of the manufacturer’s profit function:

**Case 1.**  $v = 1$

$$w^{SR} = \frac{3}{8} - \frac{1}{8}\sqrt{1 - 32y}$$

$$s_m^{SR} = \frac{b_m}{\eta_m} \left( \frac{3}{8} - \frac{1}{8}\sqrt{1 - 32y} \right)$$

**Case 2.**  $v = 2$

$$w^N = \frac{1}{3}(1 + 4y)$$

$$s_m^{SR} = \frac{b_m}{3\eta_m}(1 + 4y)$$

**Case 3.**  $v = \frac{1}{2}$

$$z = (y + \sqrt{y^2 - 64y^3})^{\frac{1}{3}}$$

$$w^N = \frac{5}{12} - \frac{x}{12} - \frac{1}{12}\sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}}$$

$$s_m^{SR} = \frac{b_m}{\eta_m} \left( \frac{5}{12} - \frac{x}{12} - \frac{1}{12}\sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}} \right)$$

$$p^{SR} = \frac{5}{6} - \frac{x}{6} - \frac{1}{6}\sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}}$$

$$a^{SR} = \frac{k_r^2}{16} \left( \frac{5}{6} - \frac{x}{6} - \frac{1}{6}\sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}} \right)^2 \left( \frac{1}{6} + \frac{x}{6} + \frac{1}{6}\sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}} \right)$$

$p^{SR}$  shown above should be in the boundaries of  $[0,1]$  and leads to bigger profit for the retailer toward  $p = 1$ . Even if one of these situations is not verified,  $p = 1$  gets the optimal price; so, the equilibrium changes as follows:

$$p^{SR} = 1 \quad w^{SR} = 0.5 \quad a^{SR} = 0 \quad s_m^{SR} = \frac{b_m}{2\eta_m}$$

### 3-3. The cooperative game

**Case 1.**  $v = 1$

$$p^{CO} = \frac{3}{4} - \frac{1}{4}\sqrt{1 - 16y}$$

$$s_m^{CO} = \frac{b_m}{\eta_m} \left( \frac{3}{4} - \frac{1}{4}\sqrt{1 - 16y} \right)$$

**Case 2.**  $v = 2$

$$p^{CO} = \frac{2}{3}(1 + 2y) \quad a^{CO} = \frac{k_r^2}{4} \left( \frac{2}{3}(1 + 2y) \right)^2 \left( \frac{1 - 4y}{3} \right) \quad s_m^{CO} = \frac{2b_m}{3\eta_m}(1 + 2y)$$

**Case 3.**  $v = \frac{1}{2}$

$$w^* = \frac{p}{2} \tag{13}$$

Now, to obtain the SR equilibrium, the retailer’s decision problem is solved using the optimal values of  $w$ .

**Proposition 2.** The equilibrium of the Stackelberg-retailer game is achieved as follows:

$$p^{SR} = \frac{3}{4} - \frac{1}{4}\sqrt{1 - 32y}$$

$$a^{SR} = \frac{k_r^2}{16} \left( \frac{3}{4} - \frac{1}{4}\sqrt{1 - 32y} \right)^2 \left( \frac{1}{4} + \frac{1}{4}\sqrt{1 - 32y} \right)^2$$

$$p^{SR} = \frac{2}{3}(1 + 4y)$$

$$a^{SR} = \frac{k_r^2}{16} \left( \frac{2}{3}(1 + 4y) \right)^2 \left( \frac{1 - 8y}{3} \right)$$

$$x = \sqrt{1 + 6z + 248\frac{y}{z}}$$

In this game, the manufacturer and the retailer first cooperate to maximize the profits of the whole system, and only then do they bargain to share the profit.

**Proposition 3.** The equilibrium of the cooperative game is achieved as follows:

$$a^{CO} = \frac{k_r^2}{4} \left( \left( \frac{3}{4} - \frac{1}{4}\sqrt{1 - 16y} \right) \left( \frac{1}{4} + \frac{1}{4}\sqrt{1 - 16y} \right) \right)^2$$

$$z = (27y + 3\sqrt{-12288y^3 + 81y^2})^{\frac{1}{3}} \quad x = \sqrt{3 + 8z + \frac{348y}{z}}$$

$$s_m^{co} = \frac{b_m}{\eta_m} \left( \frac{5}{6} - \frac{x}{6} - \frac{1}{6} \sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}} \right)$$

$$p^{co} = \frac{5}{6} - \frac{x}{6} - \frac{1}{6} \sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}}$$

$$a^{co} = \frac{k_r^2}{4} \left( \frac{5}{6} - \frac{x}{6} - \frac{1}{6} \sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}} \right)^2 \left( \frac{1}{6} + \frac{x}{6} + \frac{1}{6} \sqrt{2 - 6z - 24\frac{y}{z} + \frac{2}{z}} \right)^4$$

$p^{co}$  shown above should be in the boundaries of  $[0,1]$  and leads to bigger profit for the retailer toward  $p = 1$ . Even if one of these situations is not verified,  $p = 1$  gets the optimal price; so, the equilibrium changes as follows:

$$p^{co} = 1 \quad a^{co} = 0 \quad s_m^{co} = \frac{b_m}{2\eta_m}$$

#### 4. Discussion of The Results of the Above Examples

Discussion of the numerical results

In this section, the optimal solutions of one cooperative and two non-cooperative games will be compared. Because of the high complexity of the computations, all the comparisons are given based on the three values of  $v = 1, 2$ , and  $\frac{1}{2}$  for the linear, convex, and concave price-demand curves, respectively. Comparisons are made between price, advertising expenditures, and profits in above-mentioned games. The comparison results for retail price in the Nash, SR, and cooperative games are the same as those of wholesale price and manufacturer service, because these variables have a direct relationship with retail price.

Comparisons are made between price, advertising expenditures, and profits in above-mentioned games. The comparison results for retail price in the Nash, SR, and cooperative games are the

same as those of the comparison of wholesale price and manufacturer service, because these variables have a direct relationship with retail price.

In figure 2, the comparisons are made between the boundaries and extreme points of price; but in the other figures, the comparisons are down in the region in which the optimal price is the extreme points of  $p$ . The lines depend on the Nash, Stackelberg game, and  $p=1$  shown by dot, dash-dot, and dash-space, respectively. In all areas in the above-mentioned games, the price which is achieved by equilibrium solving leads to the higher retailer profit than the price at the beginning and end of the feasible interval, so it is proven that the achieved price by solving equations belongs to the games' equilibrium.

#### 1 Comparisons of the profits

In figure 2, it is obvious that the retailer's profit in the SR game is greater than the Nash game, because he is the game's leader. The retailer's profit and the difference between the profits in the SR and the Nash games increase with increasing values of  $v$ . Also, the retailer's profit increases with increasing  $k_r^2$ , or when the

proportion of  $\frac{b_m^2}{k_r^2}$  decreases.

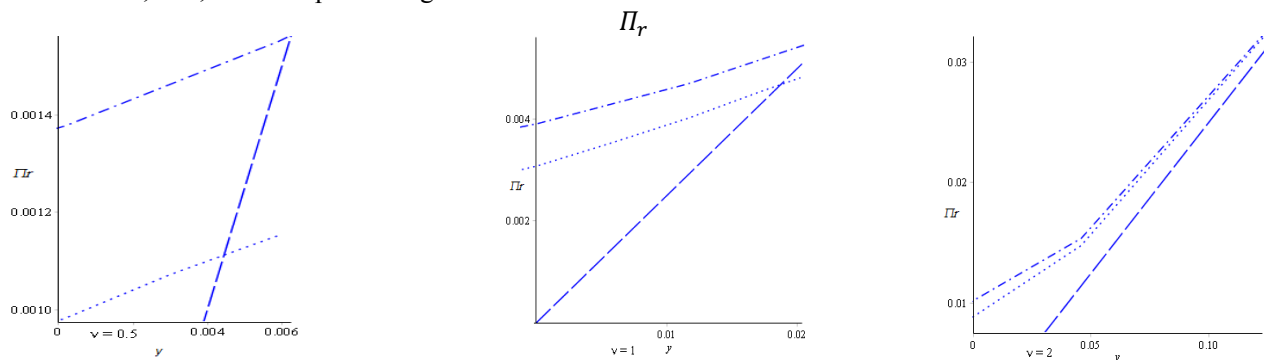


Fig. 2. Comparison of retailer profit.

The comparison results of the manufacturer's profit function among the three games are the same as those of the retailer's profit function, as shown in Fig. 3, because in each game, the strategies of manufacturer and retailer are the same.

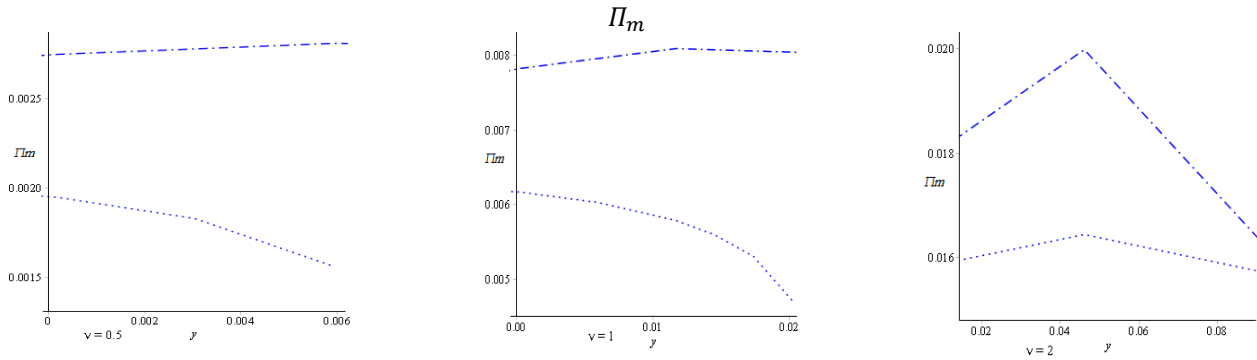


Fig. 3. Comparison of manufacturer profit.

4-2. Comparison of prices

In Fig. 4, the cooperative game is shown by a solid line and the results reveal that p in the Nash game is greater than those in the cooperative and SR games. The prices of the SR and cooperative games are very close to each other and could be considered to be almost equal. When v increases,

the price also increases. If  $k_r^2$  increases, i.e., if the proportion of  $\frac{b_m^2}{\eta_m k_r^2}$  decreases, the prices also decrease. The following is the outcome for the prices:

$$p^N > p^{SR} \approx p^{CO} \rightarrow s_m^N > s_m^{SR} \approx s_m^{CO}, w^N$$

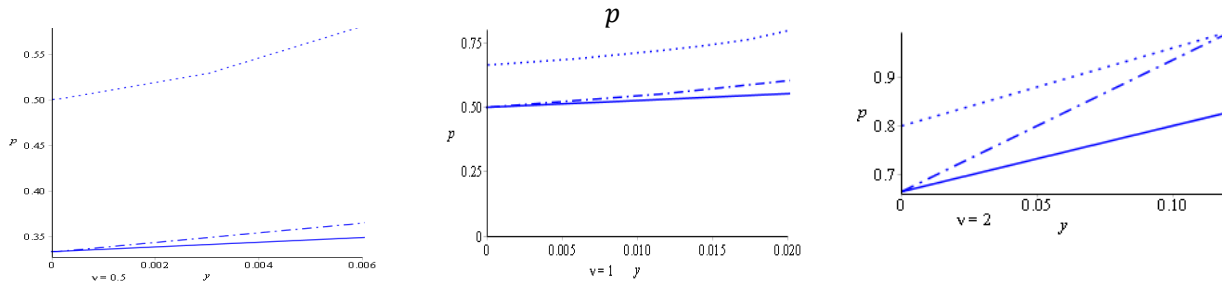


Fig.4. Comparison of the prices

Comparisons of advertising expenditures

Figure 5 is the comparison between the advertising expenditures. The advertising

expenditures in the cooperative game are biggest and at the lowest in the nash game.

$$a^{CO} > a^{SR} > a^N$$



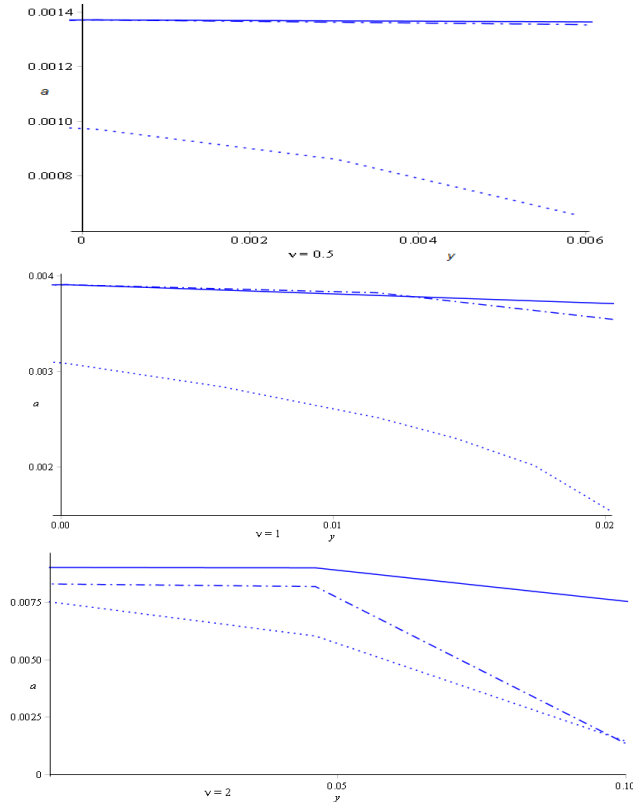


Fig. 5. Comparison of advertising expenditure

4-4. Feasibility of the cooperative game

The SR game yields more profits for the members than does the Nash game. Using the comparison of the results of profits, we verify the feasibility of the cooperative game. For the game to be feasible, the following conditions must hold:

$$\Pi_m^{co} = \Pi_m(p^{co}, w^{co}, a^{co}) \geq \max(\Pi_m^{SR}, \Pi_m^N) \quad (14)$$

$$= \Pi_m^{max} = \Pi_m^{SR}$$

$$\Pi_r^{co} = \Pi_r(p^{co}, w^{co}, a^{co}) \geq \max(\Pi_r^{SR}, \Pi_r^N) \quad (15)$$

$$= \Pi_r^{max} = \Pi_r^{SR}$$

We integrate Eqs. (14) and (15):

$$\Pi_{m+r}^{co} = \Pi_m^{co} + \Pi_r^{co} \geq \Pi_m^{max} + \Pi_r^{max} \quad (16)$$

$$= \Pi_m^{SR} + \Pi_r^{SR}$$

In the equation below,  $\Delta$  is the relative difference of the system's profits in the cooperative and non-cooperative games. Since the value of this parameter is positive (shown in figure 6), the condition in Eq. (16) is true and the feasible solution is sure to exist.

$$\Delta = \frac{\Pi_{m+r}^{co} - (\Pi_m^{SR} + \Pi_r^{SR})}{\Pi_{m+r}^{co}} \times 100 \quad (17)$$

$$= \frac{\Pi_m^{co} + \Pi_r^{co} - \Pi_m^{SR} - \Pi_r^{SR}}{\Pi_{m+r}^{co}} \times 100 > 0$$

$\Pi_{m+r}$

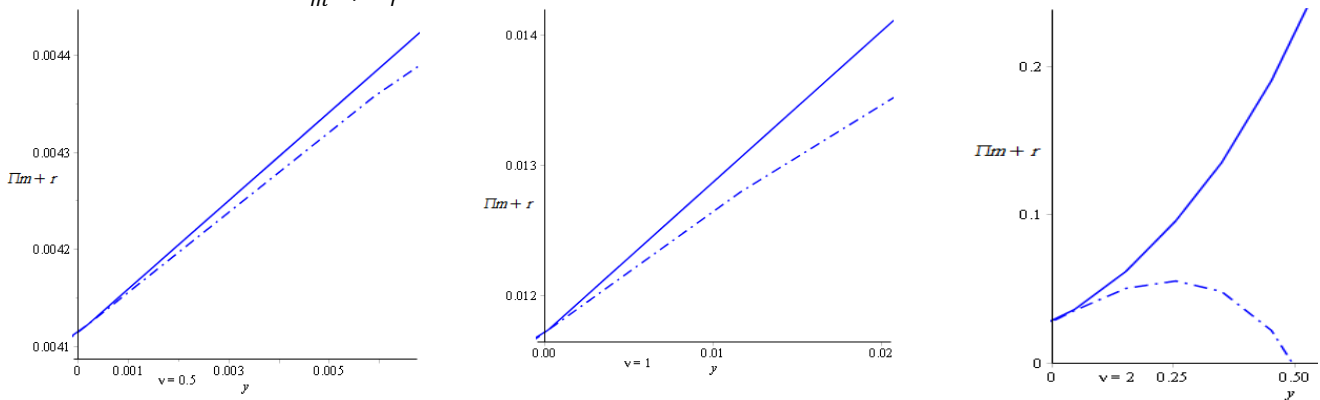


Fig. 6. Comparison of  $\Pi_{m+r}^{co}$  and  $\Pi_{m+r}^{SR}$

The feasibility of the cooperative game means that the manufacturer and retailer are willing to cooperate. In the next section, we will investigate the Nash bargaining model for sharing the extra profit gained.

### 5. Bargaining Problem

In this section, we use the Nash bargaining model in the same way that it was used by SeyedEsfahani et al. (2011) to fix how the profit can be shared between the members [24]. For this purpose, the feasible interval for variable  $w$  is first presented. The extra profit of the members is shown below:

$$\begin{aligned} \Delta \Pi_m &= \Pi_m^{co} - \Pi_m^{max} \\ &= w \left[ (1 - p^{co})^{\frac{1}{v}} (k_r \sqrt{a^{co}}) \right. \\ &\quad \left. + b_m s_m^{co} \right] - \frac{\eta_m s_m^{co2}}{2} - \Pi_m^{max} \end{aligned} \tag{18}$$

$$\begin{aligned} wB - C &> 0, \\ \Delta \Pi_m &= \Pi_r^{co} - \Pi_r^{max} \\ &= (p^{co} - w) \left[ (1 - p^{co})^{\frac{1}{v}} (k_r \sqrt{a^{co}}) \right. \\ &\quad \left. + b_m s_m^{co} \right] - a^{co} - \Pi_r^{max} \\ &= -wB + D > 0, \end{aligned} \tag{19}$$

$$B = (1 - p^{co})^{\frac{1}{v}} (k_r \sqrt{a^{co}}) + b_m s_m^{co} > 0,$$

$$C = -\frac{\eta_m s_m^{co2}}{2} - \Pi_m^{max} > 0,$$

$$D = p^{co} B - a^{co} - \Pi_r^{max} > 0,$$

The feasible interval for  $w$  is between inequalities (18) and (19). The manufacturer gains more from the extra profit if the solution gets nearer to  $\Pi_m = \Pi_m^{max} (\frac{D}{B})$  and the retailer's share will be less, or vice versa.

According to Nash, the optimal values of  $w$  are found by maximization of the product of the members' utility function [44]. In our case, the utility function is assumed to be the same as the one used in SeyedEsfahani et al. [24]:

$$U_m(w) = \Delta \Pi_m(w)^{\lambda_m}$$

$$U_r(w) = \Delta \Pi_r(w)^{\lambda_r}$$

Parameter  $\lambda$  shows the member's attitude to the risk. If  $\lambda = 1$ , the player is indifferent to the risk; if  $\lambda > 1$ , the player is a risk seeker; and if  $\lambda < 1$ , he will be a risk averser. The members gain the profit according to their risk attitude. A higher risk-seeking attitude leads to a higher profit. The Nash bargaining model is solved as follows:

$$Max U_m(w) U_r(w) = \Delta \Pi_m(w)^{\lambda_m} \Delta \Pi_r(w)^{\lambda_r}$$

The profit is divided with respect to  $\lambda$ .

$$Max U_m(w) U_r(w) = \Delta \Pi_m(w)^{\lambda_m} \Delta \Pi_r(w)^{\lambda_r}$$

$$\begin{aligned} \Delta \Pi_m(w^*) &= \frac{\lambda_m}{\lambda_m + \lambda_r} \Delta \Pi = \frac{\lambda_m}{\lambda_m + \lambda_r} (D - C) \\ \Delta \Pi_r(w^*) &= \frac{\lambda_r}{\lambda_m + \lambda_r} \Delta \Pi = \frac{\lambda_r}{\lambda_m + \lambda_r} (D - C) \\ \Rightarrow w^* B &= \frac{C \lambda_m + D \lambda_r}{\lambda_m + \lambda_r}. \end{aligned} \tag{20}$$

The optimal value for  $w$  can be achieved only if the other variable can be determined.

### 6. Conclusion

In this paper, a single-retailer-single-manufacturer supply chain was studied in which pricing, advertising, and service policies are used simultaneously to affect the customer's demand function. Optimal policies derived in the Nash, SR, and Cooperative game show that the retail price, wholesale price, and manufacturer's services are always greater when retailer is the leader or when the members cooperate. These policies' variables in SR and games are close. National and local advertising expenditures have the highest value in the Cooperative game and the lowest value in the Nash game. The retailer, the manufacturer, and the whole system gain extra profits in the cooperative game, and the Nash game provides the least profit for the members and the whole system; therefore, it would be better for the members to cooperate with each other. However, if they are not willing to cooperate, then the manufacturer prefers to be the retailer's follower rather than having an equal power and participating in the Nash game. When  $v$  increases the price level, the advertising cost, and the services cost, the profits of the members and the whole system as well as the difference between these levels among the three games will increase.

This problem in the multi-member or multi-channel supply chain can be an interesting area for future studies. Another area of interest is the application of other games or bargaining methods to study the same problem. By considering feathers such as risks, uncertainties, and so on, this study will be more realistic. Finally, different demand functions may be employed to solve the problem.

### Appendix

#### Proof of Proposition 1

The first partial derivative of  $\Pi_m$  and  $\Pi_r$  w.r.t. its variables should be initially fixed. Once accomplished, all the equations should be solved simultaneously.

The result is negative; so, the maximum value accrues at the beginning of the interval.

$$\frac{\partial \Pi_m}{\partial w} = (1-p)^{\frac{1}{v}}(k_r \sqrt{a}) + b_m s_m > 0 \rightarrow w^* = \frac{p}{2} \tag{A. 1}$$

The result is positive; so, the maximum value accrues at the end of the interval.

$$\frac{\partial \Pi_m}{\partial s_m} = w b_m - \eta_m s_m \rightarrow \frac{\partial \Pi_m}{\partial s_m} = 0 \rightarrow s_m^* = \frac{w b_m}{\eta_m} \tag{A. 2}$$

$$\frac{\partial \Pi_r}{\partial a} = \frac{1}{2}(p-w)(1-p)^{\frac{1}{v}} k_r a^{-\frac{1}{2}} \rightarrow \frac{\partial \Pi_r}{\partial a} = 0 \rightarrow a^* = (k_r(p-w)(1-p)^{\frac{1}{v}})^2 \tag{A. 3}$$

$$\frac{\partial \Pi_r}{\partial p} = \frac{1}{v}(1-p)^{\frac{1}{v}-1}(k_r \sqrt{a} + k_m \sqrt{A})(v-p(v+1)+w) + (b_r s_r + b_m s_m) \tag{A. 4}$$

In order to find the optimal value of p, the above equation should be set equal to zero. Because it is so difficult to solve such an equation, three

values of v (i.e., 1, 2, and 0.5) representing the linear, convex, and concave shapes of the price-demand function, respectively, survived.

**1. v = 1**

$$a^* = \left(\frac{k_r(p-w)(1-p)}{2}\right)^2 \quad p^* = \frac{1}{2}(1+w + \frac{b_m s_m}{k_r \sqrt{a}}) \quad w^* = \frac{p}{2} \quad s_m^* = \frac{w b_m}{\eta_m}$$

All the computations are accomplished using the Maple software. The above equations are solved simultaneously and three values are achieved for p.

$$y = \frac{\frac{b_m^2}{\eta_m}}{k_m^2} \quad (y \leq \frac{1}{48}) \quad p_1 = 0 \quad p_2 = \frac{5}{6} - \frac{1}{6}\sqrt{1-48y} \quad p_3 = \frac{5}{6} + \frac{1}{6}\sqrt{1-48y}$$

It is difficult to determine whether the retailer's hessian matrix is positive or negative.

$$H(\Pi_r) = \begin{bmatrix} \frac{\partial^2 \Pi_r}{\partial a^2} & \frac{\partial^2 \Pi_r}{\partial a \partial p} \\ \frac{\partial^2 \Pi_r}{\partial p \partial a} & \frac{\partial^2 \Pi_r}{\partial p^2} \end{bmatrix} \tag{A. 5}$$

$$= \begin{bmatrix} -\frac{k_r}{4} a^{\frac{-3}{2}}(p-w)(1-p)^{\frac{1}{v}} & \frac{k_r}{2} a^{\frac{-1}{2}}((1-p)^{\frac{1}{v}} - \frac{1}{v}(1-p)^{\frac{1-v}{v}}(p-w)) \\ \frac{k_r}{2v} a^{\frac{-1}{2}}(v-p(v+1)+w)(1-p)^{\frac{1-v}{v}} & \frac{(1-p)^{\frac{1}{v}-1} k_r \sqrt{a}}{v} ((1-\frac{1}{v})(1-p)^{-1}(v-p(v+1)+w) - (v+1)) \end{bmatrix}$$

In some region of the feasible interval,  $\Pi_r$  is concave, and in another, it is convex. The second partial derivative of  $\Pi_m$  w.r.t.  $s_m$  is negative; so, the extreme points show the maximum point.

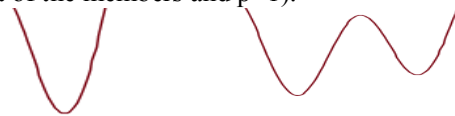
derivatives of  $\Pi_m$  and  $\Pi_r$  w.r.t its variables which maximize the profit of the members in the function of  $\Pi_r$ . The one p value should now be found, which maximizes  $\Pi_r$  among the three values of p achieved by  $\frac{\partial \Pi_r}{\partial p} = 0$ . After replacing the variables,  $\Pi_r$  is a quartic function of p and the coefficient of the biggest p is positive.

But,  $\frac{\partial^2 \Pi_r}{\partial p^2}$  is sometimes negative and sometimes positive. In order to fix which extreme point of p maximizes  $\Pi_r$ , we replace the optimal values of the variables achieved from the first partial

$$\begin{aligned} \Pi_r(p, a) &= (p-w)[(1-p)k_r \sqrt{a} + b_m s_m] - a = p^2(1-p)^2 k_r^2 + p^2 \frac{b_m^2}{4\eta_m} \\ &= p^4 k_r^2 - 2p^3 k_r^2 + p^2 (\frac{b_m^2}{4\eta_m} + k_r^2) \end{aligned} \tag{A. 6}$$

The shape of the quartic functions with positive coefficient of the biggest power will be as follows. Since we have three extreme points, the function shape is the right one in Fig. 7. After sorting the extreme points of p, the second root will be the relative maximum and the equilibrium can be found by comparing the relative maximum with the first and last points of the feasible

interval of p (p=0 which leads to the minimum profit of the members and p=1).



**Fig. 7. The shapes of the quartic functions with positive coefficient of the biggest power.**

The proofs below follow the same logic as defined above.

In this situation, two roots are achieved as the extreme points for  $p$  are achieved.

**2.  $v = 2$**

$$y = \frac{\frac{b_m^2}{k_r^2}}{\frac{\eta_m}{k_r^2}} \quad (y \leq \frac{3}{8})$$

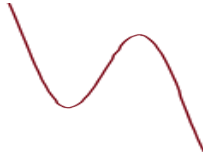
$$p_1 = 0$$

$$p_2 = 0.8 + 1.6y$$

$$\Pi_r(p, a) = (p - w) \left[ (1 - p)^{\frac{1}{2}} k_r \sqrt{a} + b_m s_m \right] - a = p^2 (1 - p) \frac{k_r^2}{16} + p^2 \frac{b_m^2}{4\eta_m} = -p^3 \frac{k_r^2}{16} + p^2 \left( \frac{b_m^2}{4\eta_m} + \frac{k_r^2}{16} \right) \quad (A. 7)$$

The cubic polynomial equation with the negative coefficient of the third degree and two extreme points will be of the form described in Fig. 8. So, once the roots are sorted, the second one will be

the relative maximum and the equilibrium will then be found by comparing it with the terminal point of the feasible interval of  $p$  ( $p=1$ ).



**Fig. 8. The cubic polynomials function with the negative coefficient of the third degree and two extreme points.**

**3.  $v = 0.5$**

After solving the equations simultaneously, we have five extreme points for  $p$  as follows:

$$y = \frac{\frac{b_m^2}{k_r^2}}{\frac{\eta_m}{k_r^2}} \quad (y \leq 0.006) \quad z = (108y + 12\sqrt{81y^2 - 6144y^3})^{\frac{1}{3}} \quad X = \sqrt{3 + 4z + 384\frac{y}{z}}$$

$$p_1 = 0$$

$$p_2 = \frac{7}{8} - \frac{\sqrt{3}}{24}x - \frac{1}{24} \sqrt{18 - 12z - 1152\frac{y}{z} + \frac{18}{x}\sqrt{3}}$$

$$p_3 = \frac{7}{8} + \frac{\sqrt{3}}{24}x - \frac{1}{24} \sqrt{18 - 12z - 1152\frac{y}{z} - \frac{18}{x}\sqrt{3}}$$

$$p_4 = \frac{7}{8} - \frac{\sqrt{3}}{24}x + \frac{1}{24} \sqrt{18 - 12z - 1152\frac{y}{z} + \frac{18}{x}\sqrt{3}}$$

$$p_5 = \frac{7}{8} + \frac{\sqrt{3}}{24}x + \frac{1}{24} \sqrt{18 - 12z - 1152\frac{y}{z} - \frac{18}{x}\sqrt{3}}$$

These roots are highly complex; we prefer to sort them by numerating  $y$ . Two of the roots are irrational and there remain three real roots.

**Tab. A .1. The price value in the Nash game achieved by numerating  $y$  for  $v=0.5$**

$y$	0.001	0.002	0.003	0.004	0.005	0.006
$p_1$	0	0	0	0	0	0
$p_2$	-	-	-	-	-	-
$p_3$	-	-	-	-	-	-
$p_4$	0.859	0.815	0.779	0.747	0.714	0.675
$p_5$	0.508	0.518	0.529	0.541	0.558	0.582

It could be understood that  $p_5$  is the relative maximum of the feasible interval.

$$\Pi_r(p, a) = \frac{p}{2} \left[ (1-p)^2 k r^2 \left( \frac{p}{4} (1-p)^2 \right) + \frac{p b_m^2}{2 \eta_m} \right] - \frac{p(1-p)^2}{4} k r^2 = 0 \rightarrow p^6 \frac{k_r^2}{8} - p^5 \frac{k_r^2}{2} + \frac{3}{4} p^4 k_r^2 - p^3 \frac{k_r^2}{2} + p^2 \left( \frac{b_m^2}{4 \eta_m} + \frac{5 k_r^2}{8} \right) - p \frac{k_r^2}{4} \quad (A. 8)$$

The curve of the above polynomial is the one shown in right side of Fig. 7, because we have three extreme points for  $p$ . So,  $p_5$  is the relative maximum and the equilibrium should be determined by comparing it with the first point of the feasible interval of  $p$  ( $p=1$ ).

The proof of propositions 2 and 3 is similar to proposition 1.

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