Retailer's Replenishment and Pricing Decisions for Non-Instantaneous Deterioration and Price-Dependent Demand

Nita H. Shah* & Chetansinh R. Vaghela

Nita H. Shah, Department of Mathematics, Gujarat University, Ahmedabad-380009, Gujarat, India
Chetansinh R. Vaghela, Department of Mathematics, Marwadi University, Rajkot-360003, Gujarat, India

KEYWORDS	ABSTRACT
Inventory, Non-instantaneous Deterioration, Price-dependent demand, Variable holding cost, Dynamic pricing.	*In this research, an integrated inventory model for non-instantaneous deteriorating items is analyzed when demand is sensitive to changes in price. The price used in this research is a time-dependent function of the initial selling price and discount rate. To control the deterioration rate of items at the storage facility, investment in preservation technology is incorporated. To provide a general framework to the model, an arbitrary holding cost rate is used. Toward the end of the paper, a numerical case is given to approve the model and the impacts of the key parameters of the model are studied by sensitivity analysis to deduce managerial insights.*

1. Introduction

The deterioration of items is a very common phenomenon in daily life. Items in inventory are subject to several risks such as breakage, evaporation, and pilferage. Items, such as food grains, blood, radioactive chemical, and medicines, deteriorate very rapidly over time. The first attempt to incorporate deterioration was made by Ghare and Schrader (1963). They used exponentially decaying inventory system for their study. Covert and Philip (1973) extended the work of Ghare and Schrader (1963) by using Weibull distribution and Gamma distribution. Shah and Shah (2000), Goyal and Giri (2001), and Bakker *et al.* (2012) gave a fine literature review of inventory models consisting deteriorating items.

These researchers assume that the items in inventory start decaying instantly from the time of their arrival. However, in reality, there are some items that maintain their original condition for some time and start to perish afterwards. This phenomenon is called non-instantaneous deterioration. Wu *et al.* (2006) developed an optimal replenishment policy for a non-instantaneous deteriorating item with stock-dependent demand and partial backlogging. Numerous researchers, e.g., Ouyang et al. (2006, 2008), Wu et al. (2009), Jaggi and Verma (2010), Soni and Patel (2012), Shah et al. (2013), and Tsao (2016), have utilized non-instantaneous deterioration for their study.

Preservation technology investment plays a vital role in reducing deterioration rate. Therefore, it has received good attention from researchers in recent years. Hsu *et al.* (2010) formulated an inventory model with a constant demand rate. They incorporated preservation technology investment to reduce the deterioration rate of

items. On this basis, Lee and Dye (2012) developed an inventory model with stock-dependent selling rate and preservation technology investment. In their model, they allowed shortages and partial backlogging. Considering an inventory system for a non-instantaneous deteriorating item, Dye (2013) studied the effect of preservation technology investment on inventory decisions when inventory system contains the non-instantaneous deteriorating item. In his study, he demonstrated that a higher optimal service rate can be obtained by higher preservation technology investment. Dye and Hsieh (2013) connected particle swarm optimization to figure the optimal production and preservation technology investment for minimizing the aggregate cost. Zhang *et al.* (2014) developed an algorithm to address the problem of pricing, preservation technology investment, and inventory control for deteriorating items. Liu *et al.* (2015) studied the joint dynamic pricing and preservation technology investment strategy. Many researchers, like Dye and Hsieh (2012), Singh and Gupta (2014), and Yang *et al.* (2015), also used preservation technology investment for deteriorating items in their model.

Setting the optimal sale price and procurement quantity for deteriorating items was studied by Sana (2010), Ghosh *et al.* (2011), their cited references, and many more. Sarkar and Sarkar (2013) formulated an economic manufacturing quantity model with probabilistic deterioration during a production system. These citations considered demand rate constant and deterministic. However, in the market, it is observed to be dependent on selling price, time, stock-displayed, advertisement, etc. Burwell *et al.* (1997), Wee (1997), Mondal *et al.* (2003), You (2005) studied the effects of various parameters on the demand.

Price is one of the important variables that can be manipulated by enterprises in order to generate more demand. In most inventory problems, the price of the product is kept fixed throughout the product's market cycle. But, in practice, using appropriate pricing policies, enterprises can effectively control their losses caused by deterioration. Hence, dynamic pricing is an important strategy in today's global market. Kincaid and Darling (1963) were the first to use dynamic pricing in their study. They used dynamic programming in development of their model. Cai *et al.* (2013) proposed an inventory model with dynamic pricing which modeled price as a function of time. The optimal policy was obtained by considering the feedback of price on demand per time unit. Wang *et al.* (2015), Dye and Yang (2016), Tsao and Sheen (2008) also considered dynamic pricing in their study.

In traditional inventory models, holding cost is known and constant. However, in practice, holding cost may not always be constant. Researchers, like Naddor (1966), Weiss (1982), Goh (1994), Giri *et al.* (1996), Roy (2008), Pando *et al.* (2012), Shah *et al.* (2013), and Rabbani *et al.* (2015), used various functions describing holding cost to provide generalized framework to their model.

The paper is organized as follows: In section 2, basic notation and assumptions of the problem are given. Section 3 demonstrates the proposed mathematical model. In section 4, a numerical example is illustrated to support the proposed model. The sensitivity analysis is carried out in section 5 followed by conclusion in section 6.

2. Notation and Assumptions

2-1. Notation

A	Ordering cost per order (in \$)
C	Purchase cost per unit (in \$)
$h(t)$	Holding cost per item per unit time (\$/unit)
θ	Constant rate of deterioration $(0 < \theta < 1)$
p_0	Initial selling price per unit (in \$) (decision variable)
$p(t)$	The dynamic price of product per unit at time t
$p'(t)$	Changes in price per unit time
$R(p(t))$	Demand rate $(t \geq 0)$ units
T	Length of production inventory cycle (Taken as one year)
t_d	Point of time at which deterioration of items starts $(t_d = \beta T,\ \beta > 0)$ (years)

$I_1(t)$	The Inventory level at time t, $0 \leq t \leq t_d$ (units)
$I_2(t)$	The Inventory level at time t, $t_d \leq t \leq T$ (units)
Q	The maximum inventory level
c_d	Cost due to deterioration per unit
u	Cost of preservation technology per unit time (decision variable)
$\pi_1(p(t), u)$	Seller's total profit when $t_d \leq T$
$\pi_2(p(t))$	Seller's total profit when $t_d \geq T$

2-2. Assumptions

1. The inventory system has a single item.
2. The deterioration rate is considered as constant and there is no repair or replacement of deteriorated items during cycle time.
3. The replenishment is instantaneous and time horizon is infinite.
4. Lead time is negligible or zero and shortages are not allowed.
5. Dynamic selling price is considered as (as in Herbon *et al.* (2014))

$$p(t) = \begin{cases} p_0 & 0 \leq t \leq t_d \\ p_0 \cdot e^{-r(t-t_d)} & t_d \leq t \leq T \end{cases}$$

Therefore, the change in the selling price is given as follows:

$$p'(t) = \begin{cases} 0 & 0 \leq t \leq t_d \\ -p_0 \cdot r \cdot e^{-r(t-t_d)} & t_d \leq t \leq T \end{cases}$$

6. Demand rate $R(p(t))$ is considered as

$$R(p(t)) = a - bp(t) + cp'(t) \qquad \text{where}$$

$a > 0$ is the scale demand, b and c are the parameters sensitive to price and change in price, respectively.

7. The holding cost considered in this study is the same as the one considered in Shah *et al.* (2013). The holding cost is considered as follows.

$$h(t) = \begin{cases} c_h & 0 \leq t \leq T \\ c_h + \delta(t - t_d) & t_d \leq t \leq T \end{cases} \quad \text{where}$$

$c_h, \delta > 0$

8. $\mu = 1 - f(u)$, where function $f(u) = 1 - \dfrac{1}{1+\lambda u}$ is continuous, increasing and concave function of u for $\lambda > 0$. Set $f(0) = 0$.

3. Mathematical Model

Based on the assumptions made in section 2, the inventory system evolves as follows. Q units of an item arrive in the inventory at the start of every cycle. Amid time interim $[0, t_d]$, the stock is exhausted just because of price-dependent demand rate. The inventory level reduces to zero due to demand and deterioration both during time interval $[t_d, T]$. The whole process is then repeated again. We analyze one cycle of length one year. Therefore, the differential equations that describe the inventory level in both cases are given by:

$$\frac{dI_1(t)}{dt} = -R(p(t)), \quad 0 \leq t \leq t_d \qquad (1)$$

$$\frac{dI_2(t)}{dt} = -R(p(t)) - \theta I_2(t)\mu, \quad t_d \leq t \leq T \qquad (2)$$

with boundary conditions $I_1(0) = Q$ and $I_2(T) = 0$. Solving equations (1) and (2) yields:

$$I_1(t) = (p_0 b - a)t + Q \qquad (3)$$

$$I_2(t) = \frac{a}{\mu\theta}\left(e^{\mu\theta(T-t)} - 1\right) - \frac{p_0(b-cr)}{(\mu\theta - r)}\left(e^{-r(T-t_d)}e^{\mu\theta(T-t)} - e^{-r(t-t_d)}\right) \qquad (4)$$

From the continuity of $I(t)$ at $t = t_d$, we have $I_1(t_d) = I_2(t_d)$. It follows equations (3) and (4), such that

$$Q = I(0) = (a - bp_0)t_d + \frac{a}{\mu\theta}\left(e^{\mu\theta(T-t_d)} - 1\right) - \frac{p_0(b-cr)}{(\mu\theta - r)}\left(e^{(\mu\theta - r)(T-t_d)} - 1\right) \qquad (5)$$

By substituting equation (5) into equation (3), equation (3) yields:

$$I_1(t) = (p_0 b - a)(t - t_d) + \frac{a}{\mu\theta}\left(e^{\mu\theta(T-t_d)} - 1\right) - \frac{p_0(b-cr)}{(\mu\theta - r)}\left(e^{(\mu\theta - r)(T - t_d)} - 1\right) \tag{6}$$

Based on the values of T and t_d, there may arise two cases viz. $t_d \leq T$ and $t_d \geq T$. The total profit of the inventory system is calculated in both cases.

Case-1: $t_d \leq T$

In this case, the inventory is consumed only due to demand rate over a time interval $[0, t_d]$ and the stock level is decreasing to zero inferable from demand and deterioration during time interval $[t_d, T]$. Therefore, the components of profit function of the inventory system are as follows:

$$SR = \text{Net sales revenue} = \frac{1}{T}\int_0^T p(t) R(p(t)) dt \tag{7}$$

$$PC = \text{Purchase cost} = \frac{CQ}{T} \tag{8}$$

$$OC = \text{Ordering cost} = \frac{A}{T} \tag{9}$$

$$HC = \text{Holding cost} = \frac{1}{T}\left[\int_0^{t_d} h(t) I_1(t) dt + \int_{t_d}^T h(t) I_2(t) dt\right] \tag{10}$$

$$DC = \text{Deterioration cost} = \frac{\theta \cdot c_d}{T}\int_{t_d}^T I_2(t) dt \tag{11}$$

$$PTC = \text{Preservation technology cost} = \frac{u \cdot T}{T} \tag{12}$$

Therefore, the total profit per unit time $\pi_1(p(t), u)$ is given by

$$\pi_1(p(t), u) = \frac{1}{T}(SR - OC - PC - HC - DC - PTC) \tag{13}$$

Case-2: $t_d \geq T$

For this situation, the model turns into the conventional inventory model. There is no deterioration of items in time interval $[0, T]$. The components of total profit, in this case, are as follows:

$$SR = \text{Net sales revenue} = \frac{1}{T}\int_0^T p(t) R(p(t)) dt \tag{14}$$

$$PC = \text{Purchase cost} = \frac{CQ}{T} \tag{15}$$

$$OC = \text{Ordering cost} = \frac{A}{T} \tag{16}$$

$$HC = \text{Holding cost} = \frac{1}{T}\left[\int_0^T h(t) I_1(t) dt\right] \tag{17}$$

Therefore, the total profit per unit time $\pi_2(p(t))$ is given by

$$\pi_2(p(t)) = \frac{1}{T}(SR - OC - PC - HC) \tag{18}$$

Using the classical optimization technique, we calculate maximum profit in the respective case using the following flow chart.
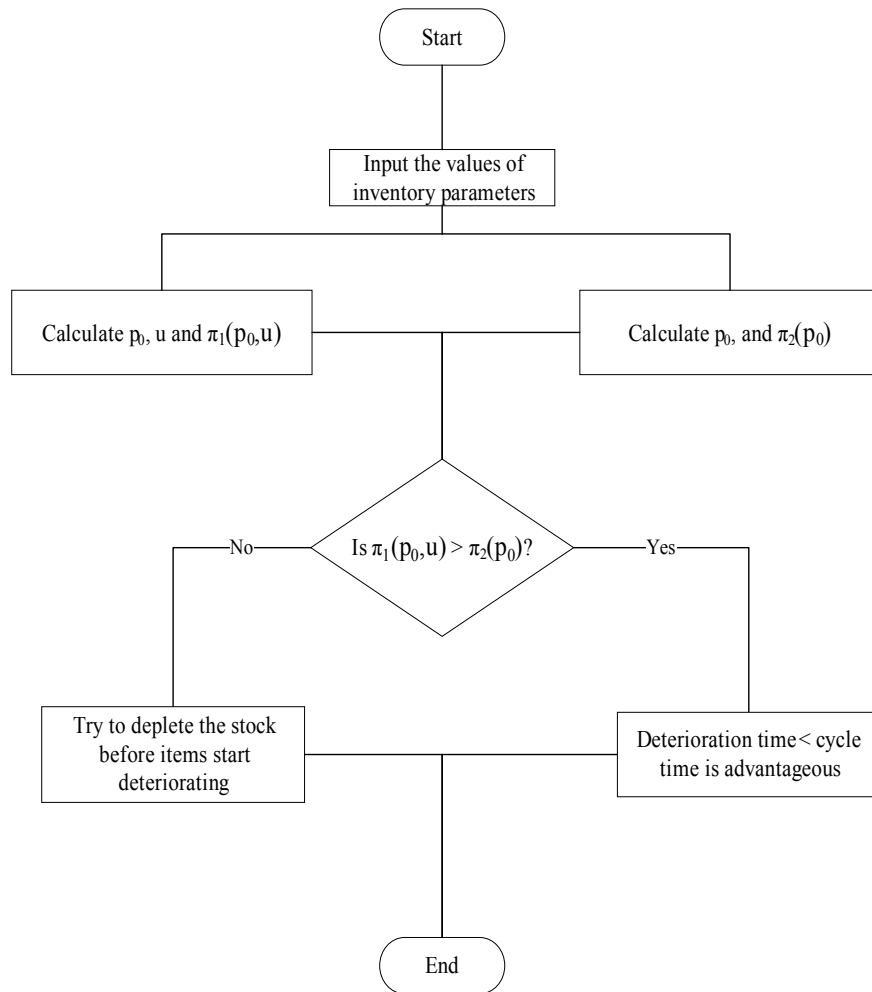
Fig. 1: Flow chart used for solution process

4. Numerical Example

In order to illustrate the above solution procedure, we consider the following numerical examples. We have calculated maximum profit in each case using Maple 18 software for the inventory parameters.

Example 1: We consider an inventory system with the following parameters in appropriate units:

$a = 500$, $b = 41$, $c = 90$, $\theta = 10\%$, $\lambda = 0.9$, $C = \$8$, $A = \$50$, $c_h = \$0.1$, $\delta = 50\%$, $r = 80\%$, $c_d = \$1$,

$T = 1\,year$, and $\beta = 0.6$. In this example, we assume that deterioration starts before the inventory is depleted to zero.

In this case, the optimal solution is $\left(p_0^*, u^* \right) = (18.83, 7.239)$. That means by setting initial selling price as \$18.83 and investing \$7.239 in preservation technology to reduce deterioration rate, the decision-maker attains maximum profit of

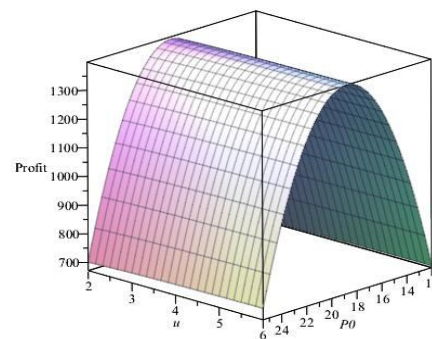$\pi_1\left(p_0^*, u^* \right) = \$1398.70$. The concavity of the profit function is shown in figure 2.

Fig. 2. Concavity of profit function in case 1

Example 2: Again, the data are same as in Example 1 except that $\beta = 1.2$. Also, θ, λ, and c_d are not used here as there is no deterioration during interval $[0, T]$.

In this case, the profit function is concave with respect to p_0 and the optimal value of initial selling price is $p_0^* = 10.22$. The maximum profit obtained in this case is $\pi_2\left(p_0^*\right) = \$149.53$. Figure 3 projects the concavity of the profit function in case-2.
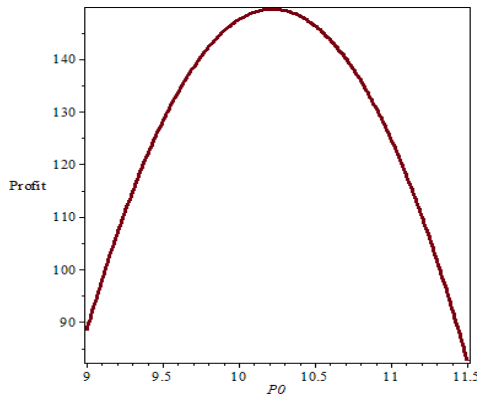

Fig. 3. Concavity of profit function in case 2

5. Sensitivity Analysis and Managerial Insights

In this section, we extend some managerial insights based on the sensitivity analysis of the inventory parameters. The sensitivity analysis is performed by changing each value of the parameter by -20%, -10%, 10%, and 20% while keeping remaining parameters unchanged. The computational results are shown in Figures 4 and 5.

From figure 4, we can observe that

- Selling price is highly sensitive to parameters β and markup of selling price b. It decreases drastically when parameters β and b increase. Parameters r, c and scale demand a moderately increase selling price. Other parameters have minor effect on selling price.
- Preservation technology cost u reduces heavily when parameter β is increased. Preservation technology cost u observes significant decrease when parameter λ and markup of selling price b increase. Preservation technology cost u is very less sensitive to parameter deterioration cost c_d. All other parameters bring a significant increase in preservation technology cost.

Scale demand a has a large impact on profit function. It is obvious that when holding cost c_h, purchase cost C and ordering cost A increase, then total profit decreases. Profit function is highly sensitive to parameter β and markup of selling price b. Profit increases significantly when parameters r and c increase. Profit decreases when replenishment time T is increased. Other parameters have minor effect on profit.
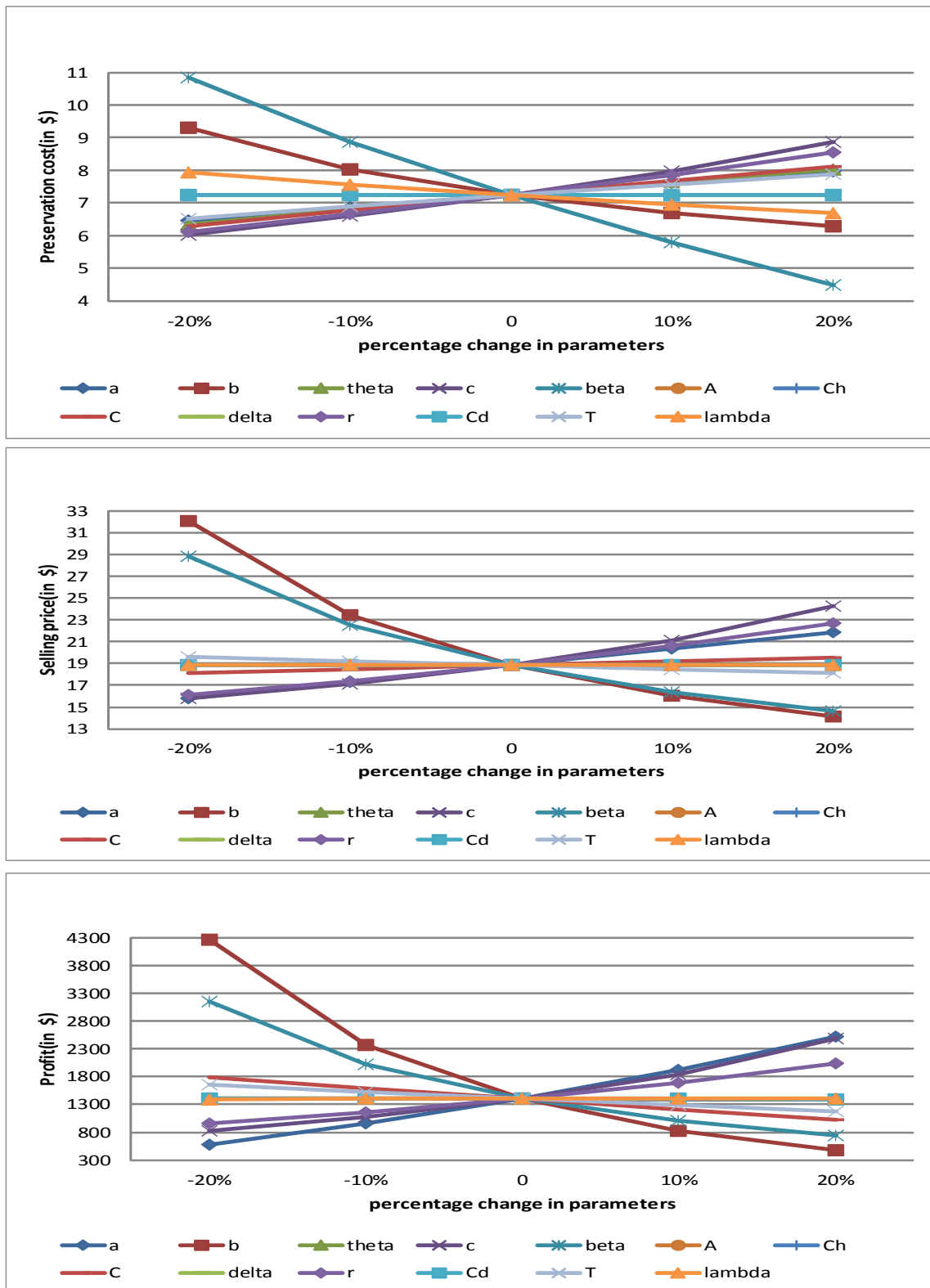
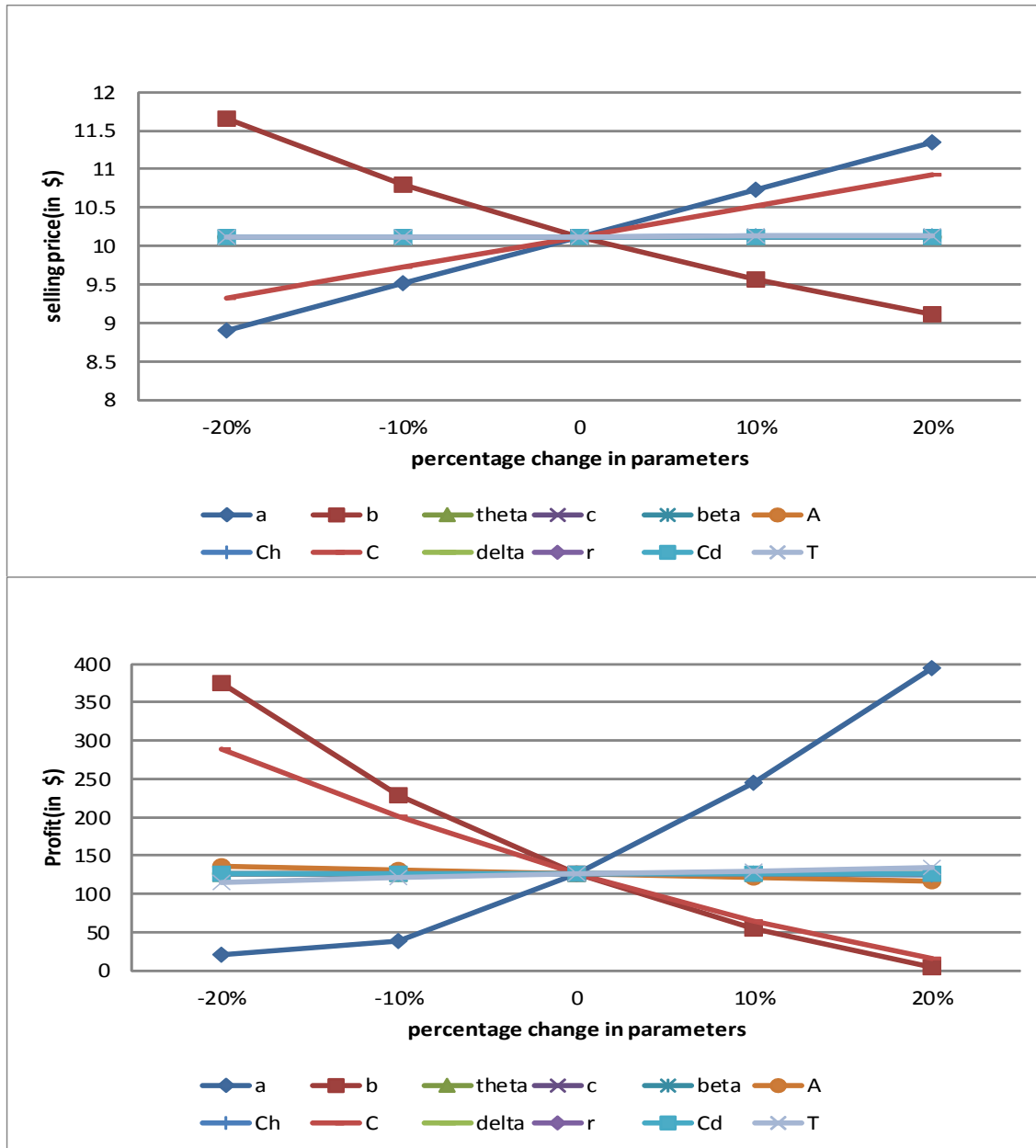**Fig. 4. Sensitivity analysis for case-1**

Fig. 5. Sensitivity analysis for case .2

From figure 5, we can observe that

• Selling price is highly sensitive to scale-demand a, markup of selling price b, and purchase cost C. It increases drastically when scale demand a, as well as purchase cost C increase. A heavy decrease is observed in selling price when the markup of selling price b is decreased. Other parameters have minor effect on selling price.

• Similar to selling price, total profit is also very sensitive to scale demand a, markup of selling price b, and purchase cost C. Profit shows a heavy rise when scale-demand a, increases which is an obvious observation. Profit reduces drastically when markup of selling price b and purchase cost C increases. Other parameters have minor effect on profit.

In the second case, if the scale demand is increased by 80%, profit is increased by nearly 15 times. Inventory depletes faster before deterioration starts. In addition, if markup of selling price b decreases by 50%, profit is higher when a player depletes stock before deterioration starts. It suggests to the player that increase in

scale demand a , mark up of selling price b and c are beneficial if items are getting sold off before they start deteriorating.

6. Conclusion

In this research, we have developed an inventory model for a non-instantaneous deteriorating item. To enhance the dynamic feature of the problem, we have considered demand function dependent on dynamic price and changes in price. We have also incorporated preservation technology investment to reduce deterioration and maximize seller's total profit. Apart from the above features, we have also considered time-dependent holding cost which generalizes the developed model. A solution procedure is developed for the proposed model to attain maximum profit per unit item. The study concludes with a couple of numerical examples and sensitivity analysis which helps provide some important managerial insights.

This research can be extended to permissible delay in payment, allowed shortages, variable deterioration, etc.

References

[1] Bakker, M., Riezebos, J., and Teunter, R. H. Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, Vol. 221, No. 2, (2012). pp. 275-284.

[2] Burwell, T. H., Dave, D. S., Fitzpatrick, K. E., and Roy, M. R. Economic lot size model for price-dependent demand under quantity and freight discounts. *International Journal of Production Economics*, Vol. 48, No. 2, (1997), pp. 141-155.

[3] Cai, X., Feng, Y., Li, Y., and Shi, D. Optimal pricing policy for a deteriorating product by dynamic tracking control. *International Journal of Production Research*, Vol. 51, No. 8, (2013), pp. 2491-2504.

[4] Covert, R. P., and Philip, G. C. An EOQ model for items with Weibull distribution deterioration. *AIIE transactions*, Vol. 5, No. 4, (1973), pp. 323-326.

[5] Dye, C. Y. The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. *Omega*, Vol. 41, No. 5, (2013), pp. 872-880.

[6] Dye, C. Y., and Hsieh, T. P. An optimal replenishment policy for deteriorating items with effective investment in preservation technology. *European Journal of Operational Research*, Vol. 218, No. 1, (2012), pp. 106-112.

[7] Dye, C. Y., and Hsieh, T. P. A particle swarm optimization for solving lot-sizing problem with fluctuating demand and preservation technology cost under trade credit. *Journal of Global Optimization*, Vol. 55, No. 3, (2013), pp. 655-679.

[8] Dye, C. Y., and Yang, C. T. Optimal dynamic pricing and preservation technology investment for deteriorating products with reference price effects. *Omega*, Vol. 62, (2016), pp. 52-67.

[9] Ghare, P. M., Schrader, G. F. A model for exponentially decaying inventory. *Journal of Industrial and Engineering Chemistry*, Vol. 14, No. 2, (1963), pp. 238-243.

[10] Ghosh, S. K., Khanra, S., and Chaudhuri, K. S. Optimal price and lot size determination for a perishable product under conditions of finite production, partial backordering and lost sale. *Applied Mathematics and Computation*, Vol. 217, No. 13, (2011), pp. 6047-6053.

[11] Giri, B. C., Goswami, A., and Chaudhuri, K. S. An EOQ model for deteriorating items with time varying demand and costs. *Journal of the Operational Research Society*, Vol. 47, No. 11, (1996), pp. 1398-1405.

[12] Goh, M. EOQ models with general demand and holding cost functions. *European Journal of Operational Research*, Vol. 73, No. 1, (1994), pp. 50-54.

[13] Goyal, S. K., and Giri, B. C. Recent trends in modeling of deteriorating inventory. *European Journal of operational research*, Vol. 134, No. 1, (2001), pp. 1-16.

[14] Herbon, A., Levner, E., and Cheng, T. C. E. Perishable inventory management with dynamic pricing using time–temperature indicators linked to automatic detecting

devices. *International Journal of Production Economics*, Vol. 147, (2014), pp. 605-613.

[15] Hsu, P. H., Wee, H. M., and Teng, H. M. Preservation technology investment for deteriorating inventory. *International Journal of Production Economics*, Vol. 124, No. 2, (2010), pp. 388-394.

[16] Jaggi, C. K., and Verma, P. An optimal replenishment policy for non-instantaneous deteriorating items with two storage facilities. *International Journal of Services Operations and Informatics*, Vol. 5, No. 3, (2010). 209-230.

[17] Kincaid, W. M., and Darling, D. A. An inventory pricing problem. *Journal of Mathematical Analysis and Applications*, Vol.7, No. 2, (1963), pp. 183-208.

[18] Lee, Y. P., and Dye, C. Y. An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. *Computers and Industrial Engineering*, Vol. 63, No. 2, (2012), pp. 474-482.

[19] Liu, G., Zhang, J., and Tang, W. Joint dynamic pricing and investment strategy for perishable foods with price-quality dependent demand. *Annals of Operations Research*, Vol. 226, No. 1, (2015), pp. 397-416.

[20] Mondal, B., Bhunia, A. K., and Maiti, M. An inventory system of ameliorating items for price dependent demand rate. *Computers and industrial engineering*, Vol. 45, No. 3, (2003), pp. 443-456.

[21] Naddor, E. Inventory systems. Wiley, New York (1966).

[22] Ouyang, L. Y., Wu, K. S., and Yang, C. T. A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. *Computers and Industrial Engineering*, Vol. 51, No. 4, (2006), pp. 637-651.

[23] Ouyang, L. Y., Wu, K. S., and Yang, C. T. Retailer's ordering policy for non-instantaneous deteriorating items with quantity discount, stock-dependent demand

and stochastic backorder rate. *Journal of the Chinese Institute of Industrial Engineers*, Vol. 25, No. 1, (2008), pp. 62-72.

[24] Pando, V., Garcı, J., San-José, L. A., and Sicilia, J. Maximizing profits in an inventory model with both demand rate and holding cost per unit time dependent on the stock level. *Computers and Industrial Engineering*, Vol. 62, No. 2, (2012), pp. 599-608.

[25] Rabbani, M., Zia, N. P., and Rafiei, H. Coordinated replenishment and marketing policies for non-instantaneous stock deterioration problem. *Computers and Industrial Engineering*, Vol. 88, (2015), pp. 49-62.

[26] Roy, A. An inventory model for deteriorating items with price dependent demand and time varying holding cost. *Advanced modeling and optimization*, Vol. 10, No. 1, (2008), pp. 25-37.

[27] Sana, S. S. Optimal selling price and lot size with time varying deterioration and partial backlogging. *Applied Mathematics and Computation*, Vol. 217, No. 1, (2010), pp. 185-194.

[28] Sarkar, M., and Sarkar, B. An economic manufacturing quantity model with probabilistic deterioration in a production system. *Economic Modelling*, Vol. 31, (2013), pp. 245-252.

[29] Shah, N. H., and Shah, Y. K. Literature survey on inventory models for deteriorating items. *Economic Annals*, Vol. 44, No. 1, (2000), pp. 221-237.

[30] Shah, N. H., Soni, H. N., and Patel, K. A. Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. *Omega*, Vol. 41, No. 2, (2013), pp. 421-430.

[31] Singh, S., and Gupta, V. Supply chain production model with preservation technology under fuzzy environment. *International Journal of Industrial Engineering Computations*, Vol. 5, No. 3, (2014), pp. 459-474.

[32] Soni, H., and Patel, K. Optimal pricing and inventory policies for non-instantaneous deteriorating items with permissible delay in payment: Fuzzy expected value model. *International journal of industrial engineering computations*, Vol. 3, No. 3, (2012), pp. 281-300.

[33] Tsao, Y. C. Joint location, inventory, and preservation decisions for non-instantaneous deterioration items under delay in payments. *International Journal of Systems Science*, Vol. 47, No. 3, (2016), pp. 572-585.

[34] Tsao, Y. C., and Sheen, G. J. Dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payments. *Computers and Operations Research*, Vol. 35, No. 11, (2008), pp. 3562-3580.

[35] Wang, Y., Zhang, J., and Tang, W. Dynamic pricing for non-instantaneous deteriorating items. *Journal of Intelligent Manufacturing*, Vol. 26, No. 4, (2015), pp. 629-640.

[36] Weiss, H. J. Economic order quantity models with nonlinear holding costs. *European Journal of Operational Research*, Vol. 9, No. 1, (1982), pp. 56-60.

[37] Wee, H. M. A replenishment policy for items with a price-dependent demand and a varying rate of deterioration. *Production Planning and Control*, Vol. 8, No. 5, (1997), pp. 494-499.

[38] Wu, K. S., Ouyang, L. Y., and Yang, C. T. An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. *International Journal of Production Economics*, Vol. 101, No. 2, (2006), pp. 369-384.

[39] Wu, K. S., Ouyang, L. Y., and Yang, C. T. Coordinating replenishment and pricing policies for non-instantaneous deteriorating items with price-sensitive demand. *International Journal of Systems Science*, Vol. 40, No. 12, (2009), pp. 1273-1281.

[40] Yang, C. T., Dye, C. Y., and Ding, J. F. Optimal dynamic trade credit and preservation technology allocation for a deteriorating inventory model. *Computers and Industrial Engineering*, Vol. 87, (2015), pp. 356-369.

[41] You, S.P. Inventory Policy for Products with Price and Time-Dependent Demands. *The Journal of the Operational Research Society,* Vol. 56, No. 7, (2005), pp. 870-873.

[42] *Zhang, J. X., Bai, Z. Y., and Tang, W. S. Optimal pricing policy for deteriorating items with preservation technology investment. Journal of Industrial and Management Optimization, Vol. 10, No. 4, (2014), pp. 1261-1277.*