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Introducing a New Formulation for the Warehouse Inventory Management Systems: with Two Stochastic Demand Patterns

M. Mahootchi*, **T. Ahmadi & K. Ponnambalam**

Masoud Mahootchi, *Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, Tehran, Iran*
Taher Ahmadi, *Student Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, Tehran, Iran*
Kumaraswamy Ponnambalam, *Department of Systems Design Engineering, University of Waterloo, Waterloo, Canada*

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ABSTRACT

This paper presents a new formulation for warehouse inventory management in a stochastic situation. The primary source of this formulation is derived from FP model, which has been proposed by Fletcher and Ponnambalam for reservoir management. The new proposed mathematical model is based on the first and the second moments of storage as a stochastic variable. Using this model, the expected value of storage, the variance of storage, and the optimal ordering policies are determined. Moreover, the probability of within containment, surplus, and shortage are computable without adding any new variables. To validate the optimization model, a Monte Carlo simulation is used. Furthermore, to evaluate the performance of the optimal FP policy, It is compared to (s,S*) policy, as a very popular policy used in the literature, in terms of the expected total annual cost and the service level. It is also demonstrated that the FP policy has a superior performances than (s*,S*) policy*.

1. Introduction

Inventories are inactive stocks of goods stored for future need. For a manufacturing company, there must be some inventory of raw materials, work-in-process, and finished products stored to confront with stochastic demand, breakdown, lead time, and etc.

Nowadays, warehouse inventory management (WIM) is a significant aspect in the supply chain management. In a manufacturing company, inventory usually represents 20 to 60 percent of the total assets [1]. In other words, Inventory costs make up the second largest cost factor in many industries after production costs [2]. However, to determine the optimal cost of inventories and to stay competitive in today's fast changing business environment, manufacturing companies should adopt new and more efficient inventory control policies [1]. Inventory management problems have been studied for more than a half century. After developing the policy of economic order quantity (EOQ) proposed by Harris in 1913, many researchers and practitioners have been studied this issue under different operating parameters and modeling assumptions [3].

The two main streams of the research in the inventory management area have been continuous and periodic review control systems. In the continuous review system, the customer has the freedom to initiate an order anytime depending on the inventory level on hand. However, in the periodic review case, the orders can be only placed in a periodic framework [4].

So far, many inventory control systems have been proposed by combination of the s, r, R, Q, and S

* **Corresponding author: Masoud Mahootchi**
Email: *M.Mahootchi@aut.ac.ir*
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parameters in which s and r are reorder points, R is generally periodic review periods, Q is reorder quantities, and S is order-up-to levels. The control systems, such as (s, Q), (s, S), (R, S), (R, Q), (S-1, S) and (R, s, S), are called classic inventory control systems. However, finding the optimal values for the respective parameters of these control systems are usually challenging and problematic [5].

Karlin and Scarf [6] illustrated two periodic review inventory systems: the first system considered lost sales and the second one embedded the backordering into the model. In the second system, they introduced the optimality situations where a base-stock policy is used. However, they could not obtain an optimal policy for the first case.

Nahmias [7] considered a periodic review inventory system with consideration of lost sale, partial backordering, set-up costs, and random lead times. He developed two heuristics to find order-up-to level. Donselaar et al. [8] suggested a heuristic to find order-up-to level in systems, allowing shortage in form of lost sales. Johansen [9] introduced a modified base-stock policy.

In this paper, we developed a new control system for WIM. The primary source of the mathematical formulation of this control system arises from FP model, which was proposed by Fletcher and Ponnambalam (FP) for reservoir management in 1996 [10]. According to the similarities between reservoir and warehouse, the respective formulation is extended to WIM. To evaluate the model efficiency, a single-item inventory is considered in which warehouse can order from an external supplier to respond to a stochastic customers' demand which is assumed to be normally distributed.

Storage of the warehouse is constrained to an upper and a lower bound. The excessive products (surplus) at the end of each period should be returned or sold off in the open market. The lead time for an order is assumed to be negligible. For this policy, the total expected value of cost consisting of ordering, holding, surplus, and shortage should be minimized in a steady-state for a warehouse.

The rest of this paper is organized as follows: Section 2 describes motivation and advantage of using this new control system for WIM. In Section 3, continues review (s,S) policy is introduced. Section 4 presents FP model formulation for stochastic WIM systems. Section 5 prsents a numerical example to compare optimal FP and (s,S) policy for a steady state situation based on annual cost evaluation and service level. Finally, Section 6 provides the conclusions and the future works.

2. Motivation and Advantages

By extending the FP model to the WIM, in addition to finding an optimal policy in a periodic framework, one can determine the reliability of meeting the specified demand in a warehouse with a limited capacity. The derived policy in FP model, called randomized policy in the literature [11], depends on the storage level. Moreover, this method can be used to find the capacity of a warehouse such that an acceptable level of reliability is guaranteed.

The work presented in this paper is also capable of obtaining an appropriate estimation for the two first moments of the system storage using the probability distribution of the demand. This mathematical formulation is also able to estimate the probabilities of within containment, surplus, and shortage for each period in the steady-state operation. The interesting point is that these probabilities are achieved without adding any new decision variables.

3. An Introduction of (s,S) Policy

One of the most popular inventory policies in a continues review system that has been studied extensively is the so-called (s,S) policy, reorder-point order-up-to policy, where s is the reorder point and S is the order-up-to level. With this policy, when the inventory is below or equal s, an order is placed to bring the inventory level up to S. When the inventory level is above s, there will be no order. It is introduced in the early inventory literature such as Arrow, Harris, and Marschak [12], where a single item periodic review inventory system is studied.

4. FP Model Formulation for WIM

In this section, the FP Model proposed by Fletcher and Ponnambalam in 1996 [10] is extended for WIM where the demand is stochastic and only one echelon is considered. In other words, the constraints (e.g., inventory balance equations) and the objective function should be expressed in terms of the first or the second moments. Moreover, the objective function is to minimize the expected value of the annual cost.

4-1. Notations

We will be using the following notations for the mathematical formulation of FP model extended for WIM:

4-1.1. Sets

$t = \{1, 2, \ldots, T\}$ **:** The number of time periods.

4-1-2. Parameters

d^t : The random demand variable in period t.

D^t : The natural mean of demand variable in period t.

η_{d^t} : The zero-mean random component of demand variable in period t.

$s^t_{\min}$: The minimum inventory level of warehouse in period t.

$s^t_{\max}$: The maximum inventory level of warehouse in period t.

4-1-3. Decision Variables

$1_{[\cdot]}$: The indicator functions as zero-one random variable.

u^t: Order quantity that is ordered from supplier in period t.

s^t: The end-of-period inventory level of warehouse in period t.

$E(s^t)$: The first moment of the storage state variable in period t.

$E\left\{(s^t)^2\right\}$: The second moment of the storage state variable in period t.

$\hat{s}^t$: The end-of-period available inventory warehouse in period t.

k^t: The fixed part of warehouse stochastic ordering policy in period t.

Remark 1: In this study ordering policy is randomized which means that depends on s^t as a random variable in our proposed method. This policy is expressed as $u^t = k^t - s^{t-1}$.

Remark 2: Random demand is broken down into two components, the constant part (the mean of demand) and the random part (white noise). This is written as $d^t = D^t - \eta_{d^t}$, where η_{d^t} is a zero-mean random normal variable ($N(0,\sigma^2_{d^t})$).

4-2. Introduction of Methodology

Conceptually, the available inventory level in the end-of-period *t* can be written as:

$$\hat{s}^t = s^{t-1} + u^t - d^t. \tag{1}$$

Where by substitution of $u^t = k^t - s^{t-1}$ and $d^t = D^t - \eta_{d^t}$ in equation (1), equation (2) is obtained as bellow:

$$\hat{s}^t = k^t - D^t + \eta^t_{d^t}. \tag{2}$$

Now, the inventory balance equation for warehouse is given as:

$$s^t = \min\left\{s^t_{\max}, \max\left(s^t_{\min}, s^{t-1} + u^t - d^t\right)\right\}. \tag{3}$$

The above formulation can be rewritten according to indicator functions such as:

$$s^t = (k^t - D^t + \eta^t_{d^t}).1_{[s^t_{\min},s^t_{\max}]}(\hat{s}^t) + (s^t_{\min}).1_{(-\infty,s^t_{\min})}(\hat{s}^t) + (s^t_{\max}).1_{(s^t_{\max},+\infty)}(\hat{s}^t). \tag{4}$$

The indicator function of variable $\hat{s}^t$, takes zero or one in based on the following conditions:

$$1_{[s^t_{\min},s^t_{\max}]}(\hat{s}^t) = \begin{cases} 1 & s^t_{\min} \le \hat{s}^t \le s^t_{\max} \\ 0 & otherwise. \end{cases} \tag{5}$$

$$1_{(-\infty,s^t_{\min})}(\hat{s}^t) = \begin{cases} 1 & -\infty < \hat{s}^t < s^t_{\min} \\ 0 & otherwise. \end{cases} \tag{6}$$

$$1_{(s^t_{\max},+\infty)}(\hat{s}^t) = \begin{cases} 1 & s^t_{\max} < \hat{s}^t < +\infty \\ 0 & otherwise. \end{cases} \tag{7}$$

Note that, in equation (4), only one of the indicator functions of equations 5, 6, or 7 can have identity value in each arbitrary period. Furthermore, the expected value for indicator functions in expression (5), (6) and (7) gives the probabilities of within containment, shortage, and surplus in each period, respectively.

The objective function is constructed as expected value of annual cost consisting of ordering, holding, surplus and shortage costs such as:

$$\begin{aligned} &E\left\{f(k^t,s^t)\right\} = \\ &\sum_{t=1}^{T}(oc)^t.\left[k^t - E(s^t)\right] \\ &+(hc)^t.\left[\left\{E(s^t)+E(s^{t-1})\right\}/2\right] \\ &+(shc)^t.\left[\left(s^t_{\min}-(k^t-D^t)\right).E\left\{1_{(-\infty,s^t_{\min})}(\hat{s}^t)\right\} \right. \\ &\left. -E\left\{\eta_{d^t}1_{(-\infty,s^t_{\min})}(\hat{s}^t)\right\}\right] \\ &(suc)^t.\left[\left(k^t-D^t-s^t_{\max}\right).E\left\{1_{(s^t_{\max},+\infty)}(\hat{s}^t)\right\} \right. \\ &\left. +E\left\{\eta_{d^t}1_{(s^t_{\max},+\infty)}(\hat{s}^t)\right\}\right]. \end{aligned} \tag{8}$$

which must be minimized. Also $(oc)^t$ is ordering cost per item, $(stc)^t$ is storage or holding cost per item, $(suc)^t$ is surplus cost per item and $(shc)^t$ is shortage cost per item, all in period t. Finally, the first and the second moment of storage can be written as Equations (9) and (10), respectively. To figure out how to derive these equations, one can refer to Appendix.

$$\begin{aligned} &E(s^t) = \\ &\left(k^t-D^t\right).E\left\{1_{[s^t_{\min},s^t_{\max}]}(\hat{s}^t)\right\} + E\left\{\eta_{d^t}.1_{[s^t_{\min},s^t_{\max}]}(\hat{s}^t)\right\} \\ &+\left(s^t_{\min}\right).E\left\{1_{(-\infty,s^t_{\min})}(\hat{s}^t)\right\} + \left(s^t_{\max}\right).E\left\{1_{(s^t_{\max},+\infty)}(\hat{s}^t)\right\}. \end{aligned} \tag{9}$$

$$\begin{aligned} &E\left\{(s^t)^2\right\} = \\ &\left(k^t-D^t\right)^2.E\left\{1_{[s^t_{\min},s^t_{\max}]}(\hat{s}^t)\right\} + 2\left(k^t-D^t\right).E\left\{\eta_{d^t}.1_{[s^t_{\min},s^t_{\max}]}(\hat{s}^t)\right\} \\ &+E\left\{(\eta_{d^t})^2.1_{[s^t_{\min},s^t_{\max}]}(\hat{s}^t)\right\} + \left(s^t_{\min}\right)^2.E\left\{1_{(-\infty,s^t_{\min})}(\hat{s}^t)\right\} \\ &+\left(s^t_{\max}\right)^2.E\left\{1_{(s^t_{\max},+\infty)}(\hat{s}^t)\right\}. \end{aligned} \tag{10}$$

4-3. Nonlinear Programming

For implementation, using the rules of probabilities and Taylor series approximation, equations (8), (9) and (10) are obtained, in terms of error function as expressions (11), (12) and (13). For more details about mathematical operations, see [10] and [11]. The mathematical model is presented as:

$$\begin{aligned}
&Min \quad E\left\{f(k^t,s^t)\right\}=\sum_{t=1}^{T}(oc)^t.\left[k^t-E(s^t)\right]+(hc)^t.\left[E(s^t)+E(s^{t-1})/2\right]\\
&+(suc)^t.\left[\left(k^t-D^t-s^t_{\max}\right).\left\{\frac{1}{2}\left[1-erf\left(\frac{s^t_{\max}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)\right]\right\}+\left\{\sqrt{\frac{Var(\eta_{d^t})}{2\pi}}\right\}.\left(e^{-\frac{\left(s^t_{\max}-k^t+D^t\right)^2}{2Var(\eta_{d^t})}}\right)\right]\\
&+(shc)^t.\left[\left(s^t_{\min}-(k^t-D^t)\right).\left\{\frac{1}{2}\left[1+erf\left(\frac{s^t_{\min}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)\right]\right\}+\left\{\sqrt{\frac{Var(\eta_{d^t})}{2\pi}}\right\}.\left(e^{-\frac{\left(s^t_{\min}-k^t+D^t\right)^2}{2Var(\eta_{d^t})}}\right)\right].
\end{aligned} \tag{11}$$

subject to:

$$\begin{aligned}
&E(s^t)=\left(k^t-D^t\right).\left\{\frac{1}{2}\left[erf\left(\frac{s^t_{\max}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)-erf\left(\frac{s^t_{\min}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)\right]\right\}\\
&+\left(\sqrt{\frac{Var(\eta_{d^t})}{2\pi}}\right).\left\{e^{-\left(\frac{(s^t_{\min}-k^t+D^t)^2}{2Var(\eta_{d^t})}\right)}-e^{-\left(\frac{(s^t_{\max}-k^t+D^t)^2}{2Var(\eta_{d^t})}\right)}\right\}+\left(s^t_{\min}\right).\left\{\frac{1}{2}\left[1+erf\left(\frac{s^t_{\min}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)\right]\right\}\\
&+\left(s^t_{\max}\right).\left\{\frac{1}{2}\left[1-erf\left(\frac{s^t_{\max}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)\right]\right\}.
\end{aligned} \tag{12}$$

$$\begin{aligned}
&E\left\{(s^t)^2\right\}=\left\{\left(k^t-D^t\right)^2+Var(\eta_{d^t})\right\}.\left\{\frac{1}{2}\left[erf\left(\frac{s^t_{\max}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)-erf\left(\frac{s^t_{\min}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)\right]\right\}\\
&+2\left(k^t-D^t\right).\left(\sqrt{\frac{Var(\eta_{d^t})}{2\pi}}\right).\left\{e^{-\left(\frac{(s^t_{\min}-k^t+D^t)^2}{2Var(\eta_{d^t})}\right)}-e^{-\left(\frac{(s^t_{\max}-k^t+D^t)^2}{2Var(\eta_{d^t})}\right)}\right\}\\
&+\left(\sqrt{\frac{Var(\eta_{d^t})}{2\pi}}\right).\left\{\left(s^t_{\min}-k^t+D^t\right).e^{-\left(\frac{(s^t_{\min}-k^t+D^t)^2}{2Var(\eta_{d^t})}\right)}-\left(s^t_{\max}-k^t+D^t\right).e^{-\left(\frac{(s^t_{\max}-k^t+D^t)^2}{2Var(\eta_{d^t})}\right)}\right\}\\
&+\left(s^t_{\min}\right)^2.\left\{\frac{1}{2}\left[1+erf\left(\frac{s^t_{\min}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)\right]\right\}+\left(s^t_{\max}\right)^2.\left\{\frac{1}{2}\left[1-erf\left(\frac{s^t_{\max}-k^t+D^t}{\sqrt{2Var(\eta_{d^t})}}\right)\right]\right\}.
\end{aligned} \tag{13}$$

Note that all variables except the constant part of order policy (k^t) which is a free variable are positives variables.

5. Numerical Example

In this section, we consider two cases. The first one considers the stationary demand in which the pattern of demands is analogous for all periods. In the second case, called non-stationary demand, the pattern of demand is unique for each period. For both cases, the nonlinear programming (FP model) is formulated and solved by MATLAB Solver. The optimal policies obtained using FP models are evaluated by a Monte Carlo simulation.

The total objective function and the service level are then compared with the optimal objective achieved and the service level achieved through the (s,S) policy. The problem parameters for two cases are demonstrated in Tables 1 and 2.

5-1. Results of FP Model

By solving the nonlinear programming model, we can obtain the optimal ordering policy, expected value of storage, variance, and the probabilities of within containment, surplus, and shortage in each period. The results of the optimization and simulation for two cases are presented in Tables 3 and 4.

Tab. 1. Related parameters of inventory system;case1

	Capacity		Cost parameters				Demand	
Periods	$s^t_{\min}$	$s^t_{\max}$	oc	stc	suc	shc	D^t	$Var(d^t)$
{1,2,… 12}	0	200	10	5	2	20	100	100

Tab. 2. Related parameters of inventory system; case2

Period	Capacity		Cost parameters				Demand	
	$s^t_{\min}$	$s^t_{\max}$	*oc*	*stc*	*suc*	*shc*	D^t	$Var(d^t)$
1	0	200	10	2	30	100	100	25
2	0	200	5	3	50	50	300	100
3	0	200	5	4	30	50	100	40
4	0	200	2	1	30	80	250	50
5	0	200	15	6	10	120	160	30
6	0	200	7	7	70	110	170	25
7	0	200	10	10	10	80	300	100
8	0	200	5	3	50	150	350	120
9	0	200	5	2	50	150	100	20
10	0	200	5	5	50	100	250	50
11	0	200	2	2	20	95	200	80
12	0	200	1	1	10	170	100	25

Tab. 3. The data gained through FP model for case 1

Period (Month)	Optimal policy (k^t)	Exp. value of storage	Variance of storage	Probability of storage	Probability of surplus	Probability of shortage	Ordering cost	Storage cost	Surplus cost	Shortage cost
1	10	6.5	52.	0.6	0.0	0.3	542.	141.	0.	44.
2	10	6.5	52.	0.6	0.0	0.3	977.	32.5	0.	44.
3	10	6.5	52.	0.6	0.0	0.3	978.	32.5	0.	44.
4	10	6.5	52.	0.6	0.0	0.3	978.	32.5	0.	44.
5	10	6.5	52.	0.6	0.0	0.3	978.	32.6	0.	44.
6	10	6.5	52.	0.6	0.0	0.3	978.	32.5	0.	44.
7	10	6.5	52.	0.6	0.0	0.3	978.	32.5	0.	44.
8	10	6.5	52.	0.6	0.0	0.3	978.	32.5	0.	44.
9	10	6.5	52.	0.6	0.0	0.3	978.	32.5	0.	44.
10	10	6.5	52.	0.6	0.0	0.3	978.	32.5	0.	44.
11	10	6.5	52.	0.6	0.0	0.3	978.	32.5	0.	44.
12	99	3.3	28.	0.4	0.0	0.5	921.	24.6	0.	94.

Tab. 4. The data gained through FP model for case 2

Period (Month)	Optimal policy (k^t)	Exp. value of storage	Variance of storage	Probability of storage	Probability of surplus	Probability of shortage	Ordering cost	Storage cost	Surplus cost	Shortage cost
1	10	6.6	20.	0.9	0.	0.1	563.8	56.	0.	24.
2	31	14.	87.	0.9	0.	0.0	1537.	31.	0.	17.
3	10	8.4	33.	0.9	0.	0.1	467.7	45.	0.	15.
4	41	169	50.	1.0	0.	0.0	820.9	88.	0.	0.0
5	16	8.9	27.	0.9	0.	0.0	0.00	533	0.	15.
6	17	7.5	22.	0.9	0.	0.0	1179.	57.	0.	17.
7	31	12.	79.	0.8	0.	0.1	3038.	97.	0.	50.
8	37	23.	116	0.9	0.	0.0	1806.	53.	0.	9.6
9	10	8.6	19.	0.9	0.	0.0	425.4	32.	0.	7.2
10	26	11.	45.	0.9	0.	0.0	1265.	51.	0.	14.
11	22	22.	79.	0.9	0.	0.0	420.6	33.	0.	1.9
12	13	30.	25.	1.0	0.	0.0	108.1	26.	0.	0.0

5-2. Validation of the Optimal FP Policy

The result of the optimization model and the simulation in terms of the expected value and the variance of the storage using the optimal policies are compared to each other in Figures 1- 4. As it can be seen, there is a little gap between these results. This verifies that the proposed optimization model can be a suitable representative of the real system.

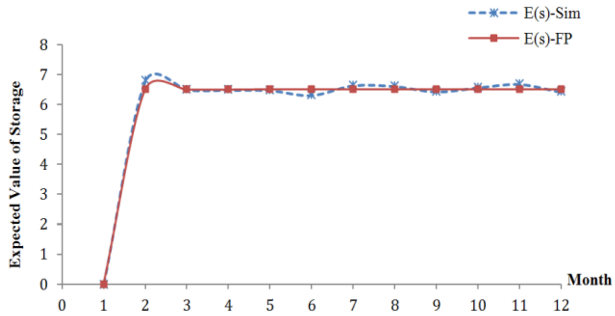

Fig. 1. The evaluation of optimal FP policy by simulation for case 1; Expected Value of Storage

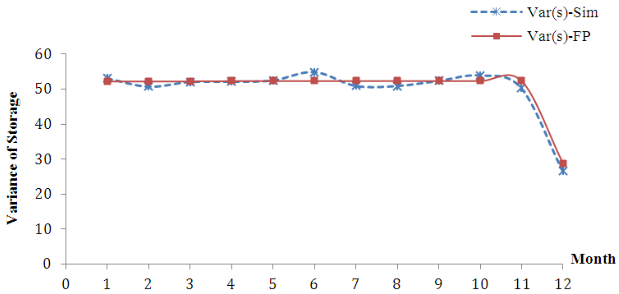

Fig. 2. The evaluation of optimal FP policy by simulation for case 1; Variance of Storage.

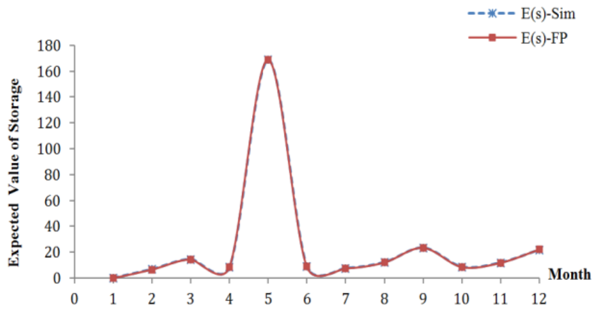

Fig. 3. The evaluation of optimal FP policy by simulation for case 2; Expected Value of Storage.

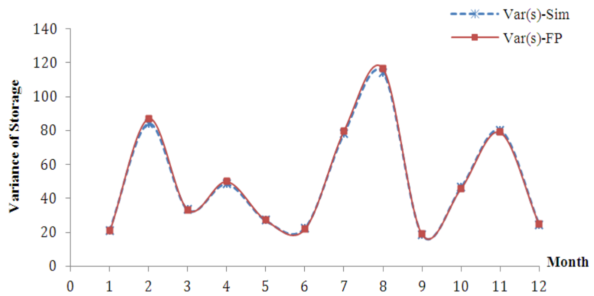

Fig. 4. The evaluation of optimal FP policy by simulation for case 2; Variance of Storage.

5-3. Evaluation of FP Policy

In the both cases, FP optimal policies are compared with optimal (s,S) policy in terms of the expected value of the total cost and the service level in each period. The optimal parameters of (s,S) policy for the first and second cases are (s*=53, S*=104) and (s*=121, S*=200), respectively. In this section, optimal FP policy is compared with the policy (s*,S*) for both cases and the results are graphically presented in Figures 5-8.

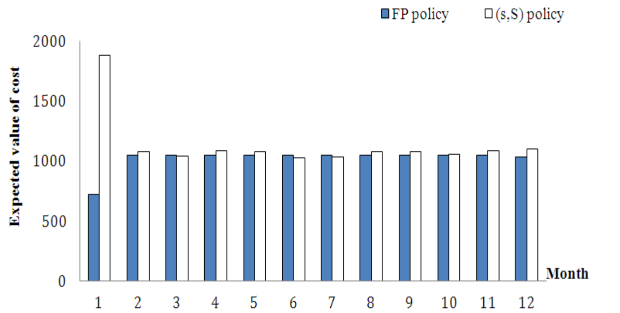

Fig. 5. Comparison FP and (s,S) policy for case 1; Expected Value of monthly total cost.

Fig. 6. Comparison FP and (s,S) policy for case 1; Expected Value of monthly service level

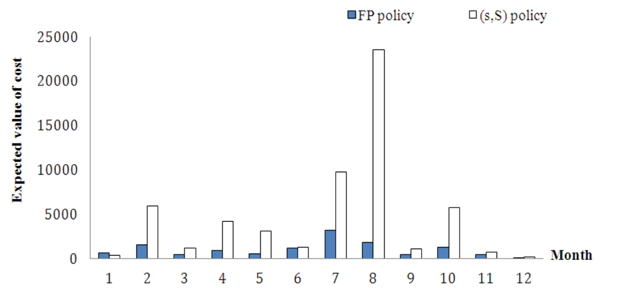

Fig. 7. Comparison FP and (s,S) policy for case 2: Expected Value of monthly total cost.

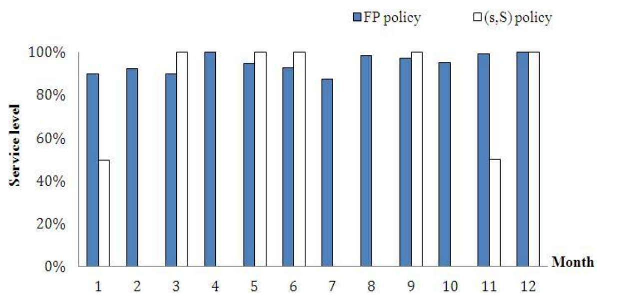

Fig. 8. Comparison FP and (s,S) policy for case 2; Expected Value of monthly service level

According to results in Table I and the details in Figures 5 and 6, this can be figured out that FP policy provides higher service level and less annual cost compared to other policy in both cases, stationary and non-stationary situations. Furthermore, the difference between the corresponding measures in two cases is mostly remarkable especially in the second case.

Tab. 5. Final result to comparison optimal FP and (s,S) policy

Measure	Case1		Case 2	
	FP policy	*(s,S) policy*	*FP policy*	*(s,S) policy*
Serv	68 %	60 %	95 %	50 %
Ann	12588	13667	13149	57463
Bett	FP policy		FP policy	

6. Conclusion and Future Work

A new modeling for WIM with stochastic demand was proposed which is originated from Fletcher and Ponnambalam model developed for reservoir management. To evaluate the performance of the optimal policy obtained from this model, two demand patterns were considered in form of stationary and non-stationary as two different cases. The FP policy is compared with the continues review (s,S) policy for these two cases. According to the demonstrated results, the FP policy gives superior service level and annual cost than other policy for two cases especially in the second case.

Future work could include reliability analysis and risk concepts of warehouse inventory system in FP model, which is provided as the failure probability of system in each period. Also, the inputs and outputs of FP model such as the lower and upper bounds, expected value of storage could be used as essential information to design optimal capacity or variable optimal capacity of warehouse in each period.

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Appendix

Consider the following term in Equation (9):

$$E\left\{1_{[s_{\min}^t, s_{\max}^t]}(\hat{s}^t)\right\}.$$

This term can be written as:

$$E\left\{1_{[s_{\min}^t, s_{\max}^t]}(\hat{s}^t)\right\} = \Pr\left\{s_{\min}^t \le \hat{s}^t \le s_{\max}^t\right\}.$$

Recall that the available storage based on the balance equation can be calculated as
$\hat{s}^t = k^t - D^t + \eta_{d^t}^t$.

The respective probability in the above equation can be expanded as follows:

$$\Pr\left\{s_{\min}^t \le \hat{s}^t \le s_{\max}^t\right\} = \Pr\left\{s_{\min}^t \le k^t - D^t + \eta_{d^t}^t \le s_{\max}^t\right\},$$

or, equivalently,

$$\Pr\left\{s_{\min}^t - (k^t - D^t) \le \eta_{d^t}^t \le s_{\max}^t - (k^t - D^t)\right\} = \int_{s_{\min}^t - (k^t - D^t)}^{s_{\max}^t - (k^t - D^t)} f_{\eta_{d^t}^t}(\eta_{d^t}^t).d\eta_{d^t}^t \qquad (A1),$$

where $f_{\eta_{d^t}^t}(\eta_{d^t}^t)$ is the density function for a zero-mean random variable $\eta_{d^t}^t$.

As $\eta_{d^t}^t$ is assumed to be normally distributed with zero mean (i.e., N(0, $Var(\eta_{d^t}^t)$)), the error function can be used for the finding the numerical integration at (A1). Therefore, Equation (A1) can be written as:

$$\Pr\left\{s_{\min}^t - (k^t - D^t) \le \eta_{d^t}^t \le s_{\max}^t - (k^t - D^t)\right\} = \frac{1}{2}\left\{erf\left[\frac{s_{\max}^t - (k^t - D^t)}{\sqrt{2Var(\eta_{d^t}^t)}}\right] - erf\left[\frac{s_{\min}^t - (k^t - D^t)}{\sqrt{2Var(\eta_{d^t}^t)}}\right]\right\}.$$